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On the Performance of NOMA and Coded Multicasting in Cache-Aided Wireless Networks

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Abstract—Coded multicasting has been widely studied as an effective content delivery solution for wireless caching at user equipment. Recently, non-orthogonal multiple access (NOMA) has been studied as an alternative delivery approach in cache-aided network. Both schemes allow multiple users to access the same frequency/time/code resource and thus offer enhanced spectral efficiency. By exploiting the information available in the cache, the users can successfully decode the requested message through cache-enabled interference cancellation (CIC) in NOMA or XOR decoding in coded multicasting. These techniques require user pairing/clustering in order to address the complexity issue arising from handling large number of users. Nevertheless, it is not known which delivery technique performs better under different user pairing scenarios. Based on two-user pairing, we study the performance of NOMA and coded multicasting in terms of probability of sum rate comparison and outage probability under the situation in which the users are randomly located in a circular cell. Both analytical and simulation results show that NOMA is better than coded multicasting when the channel gains of the paired users are highly distinctive while coded multicasting is preferable when the users’ channel gains are similar.

I. INTRODUCTION

Wireless caching is one of the promising technologies which address the growing capacity demands for rich multimedia traffic in the next generation cellular networks [1], [2]. The main idea of wireless caching is to bring contents closer to the users via the caches distributed across the network (e.g. routers, base stations (BSs), user equipment (UEs) etc.). The process of populating content into the caches, which is commonly referred to as placement phase, is executed during off-peak traffic periods. During delivery phase, users are allowed to retrieve the cached content and thus alleviating the network traffic.

Caching at the UEs is deemed to be the most preferable technique as it potentially reduces both backhaul and over-the-air traffic. However, UEs are associated with small cache memory sizes which pose greater challenges to the design of content placement and delivery. A substantial amount of studies have analyzed coded multicasting as a promising solution for content delivery for caching at the UEs [2]–[4]. In coded multicasting, each content file is divided into a number of portions. Only several portions of each selected content file are stored in users’ cache during the content placement phase. During the delivery phase, the missing portions of the contents requested by multiple users are combined through XOR coding to form a single coded stream. The stream is then delivered over a shared resource to these users via multicasting and hence offers increased spectral efficiency. Nevertheless, the transmission rate of coded multicasting is restricted by the least channel gain user in order to ensure successful detection of coded stream by all users. In addition, coded multicasting exhibits increasing computational complexity with larger number of users and portions of content files [2].

Another next generation cellular technology which offers enhanced spectral efficiency over conventional schemes is non-orthogonal multiple access (NOMA) [5], [6]. NOMA can be directly applied as a content delivery approach for caching at the UEs. Moreover, the concept of cache-enabled interference cancellation (CIC) is explored in [7] to cancel the interference in each user’s received signal by exploiting the contents cached in the user. In CIC-based NOMA, a user need to first cancel the interfering signals by utilizing the contents available in the cache. Prior to the detection of its own signal, the user may then cancels the remaining interfering signals via successive interference cancellation (SIC) depending on the SIC detection order. By applying CIC, the achievable downlink rate region for all the users improves significantly as studied in [7]. Other works on the application of NOMA in cache-enabled network focus on caching at the BSs or dedicated content server. For instance, the works in [8] and [9] exploits the benefits of NOMA in wireless caching to allow the BS to frequently update the content in cache-equipped local servers. The content placement can either be performed during off-peak hours or jointly with content delivery during on-peak hours via NOMA transmission. These studies show that NOMA can efficiently enhance the performance of cached-enabled network in terms of hit probability and delivery outage probability. In addition, the coverage performance studied in [10] demonstrates the performance gain attained by NOMA over OMA system. In [11], multicasting and content pushing are jointly performed via NOMA transmission in order to efficiently utilize the spectral resource.

Both NOMA and coded multicasting offer enhanced spectral efficiency since multiple users are allowed to access the same time/frequency/code resource. Nevertheless, the performance of NOMA may be affected due to the sharing of power among the users while, in coded multicasting, the performance is limited by the user with the poorest channel quality. In fact, NOMA achieves significant performance gain over OMA by scheduling the users whose channel qualities are highly...
analytical expressions and Section V concludes the paper. Therefore, limited number of users can only be served in an orthogonal resource. In NOMA, this problem can be solved by user pairing/clustering, in which the users are divided into groups and each group is allocated with different orthogonal resources. This technique can also be applied to coded multicasting. Therefore, it is also possible for NOMA or coded multicasting to be simultaneously applied in different orthogonal resources. However, the key question is which users are grouped or paired together to perform either NOMA or coded multicasting.

In this paper, we compare the performance of NOMA and coded multicasting under different user pairing scenarios. We particularly focus on the delivery phase in which multiple cache-equipped users are requesting for missing portions of contents from a BS. The users are sorted in ascending order of channel gains which takes into account both path loss and Rayleigh fading. We consider that the BS is only allowed to serve two users in a single resource at a time. In particular, the \( m \)-th user and the \( n \)-th user from the users’ sorted order are selected, where \( m < n \). The main contributions of this paper are as follows:

- The probability that the sum rate of NOMA achieves a specific performance gain over coded multicasting is studied. An asymptotic expression of the probability is derived by considering high SNR approximation. The analytical result provides an insight on the performance achieved by NOMA over coded multicasting depending on which users from the users’ channel order are selected.
- In order to examine the performance of individual users, the outage probability is also studied, in which an approximate analytical expression is obtained. In NOMA, the individual transmission rate may be affected due to the sharing of power and hence influence the outage performance, particularly for the weaker user \( m \) who may possess poor channel quality. It is crucial to examine the quantitative individual performance using analytical results in order to obtain better insight on the overall performance.
- We demonstrate the accuracy on the asymptotic performance of the analytical expressions through simulation. The simulation results show the impact of SNR on the performance gain achieved by NOMA over coded multicasting.

The remainder of this paper is organized as follows. Section II introduces the system model of NOMA and coded multicasting in cache-enabled network. Section III examines the analytical performance of NOMA and coded multicasting. Numerical results are presented in Section IV to verify the developed analytical expressions and Section V concludes the paper.

II. System Model

We consider a downlink system that consists of a BS located at the centre of a circular cell of radius \( R_C \) within which \( K \) cache-equipped users are uniformly distributed. The channel gain between the \( k \)-th user and the BS is expressed as \( |h_k|^2 = |g_k|^2 \frac{\sin^2 (\theta_k)}{4} \), where \( g_k \) is the Rayleigh fading channel gain, \( \alpha \) is the path loss exponent, and \( d_k \) is the distance from the \( k \)-th user to the BS. Let the set of users in which the index are sorted according to increasing channel gains be \( K \subseteq \{1, 2, \ldots, K\} \). The random variables \( |h_k|^2 \) are arranged in a specific order. It is crucial to determine the CDF of the ordered channel gain using order statistics \([13]\) for the asymptotic study of the sum rates and the evaluation of outage probability. This is derived from the CDF of an unordered channel gain \(|\tilde{h}|^2\). However, for the scenario where the users are randomly positioned in a circular cell and affected by Rayleigh fading, it is difficult to obtain an insightful expression for this CDF, particularly when \( \alpha > 2 \). Therefore, the CDF of the unordered channel gain \(|\tilde{h}|^2\) is approximated using Gaussian Chebyshev quadrature \([14]\) as

\[
F_{|\tilde{h}|^2}(y) \approx \frac{1}{RC} \sum_{n=1}^{N} \frac{\pi}{N} g(\theta_n) \tag{1}
\]

where \( g(\theta_n) = \sqrt{1 - \theta_n^2} (1 - e^{-\alpha y}) \left( \frac{R_C}{2} \theta_n + \frac{R_C}{2} \right) \), \( N \) is a parameter that influence the accuracy-complexity tradeoff, \( \alpha_n = 1 + \left( \frac{R_C}{2} \theta_n + \frac{R_C}{2} \right) \), and \( \theta_n = \cos \left( \frac{2n-1}{N} \pi \right) \).

From (1), the approximate PDF of the unordered channel gain \(|\tilde{h}|^2\) can be derived as

\[
f_{|\tilde{h}|^2}(y) \approx \frac{1}{RC} \sum_{n=1}^{N} \beta_n e^{-\alpha_n y} \tag{2}
\]

where \( \beta_n = \frac{\pi}{N} \sqrt{1 - \theta_n^2} \left( \frac{R_C}{2} \theta_n + \frac{R_C}{2} \right) \).

The system will be too complex and hence impractical if all the users’ signals are multiplexed using either NOMA or coded multicasting. Therefore, in this paper, we consider only two users are paired together, that is, the weaker user \( m \) is paired with the stronger user \( n \) for \( m < n \) (i.e. \(|h_m|^2 \leq |h_n|^2\)). Assume that all the files in the content server have equal sizes and these files are partitioned into two portions of equal sizes (each portion is referred to as a subfile). During the content placement phase (i.e. prior to the time of request), we assume that each paired user \( i \in \{m, n \} \) stores the first portion of the file which will be requested by this user and this subfile is denoted as \( x_{i,1} \). Therefore, each user only requires the second portion of the requested file \( x_{i,2} \) during the delivery phase. We further assume that each user also stores the second subfile of the unwanted file \( x_{j,2} \) for \( j \neq i \) in order to make CIC-based NOMA and coded multicasting feasible for the paired users. In coded multicasting, a user can decode the requested subfile from a coded stream (which contains a mixture of requested and unwanted subfiles) if and only if the user possesses all the unwanted subfiles in the cache. Similarly, in NOMA, the cached information can also be used to cancel the interference via CIC and then the requested subfiles can be
directly decoded. The content placement and delivery strategy is shown in Fig. 1.

Having the information of the unwanted subfile \(x_{n,2} (x_{m,2})\) in the cache of user \(m (n)\), the requested subfile \(x_{m,2} (x_{n,2})\) can be successfully retrieved through XOR decoding. Thus, in coded multicasting, the individual achievable rate is represented as

\[
R_i^{CM} = \min_{i \in \{m,n\}} \left\{ \log \left( 1 + \rho |h_i|^2 \right) \right\} = \log \left( 1 + \rho |h_m|^2 \right)
\]

(9)

since \(|h_m|^2 \leq |h_n|^2\). Therefore, the sum rates of NOMA and coded multicasting are expressed respectively as

\[
R_{\text{sum}}^{NOMA} = \log \left( 1 + \rho a_{m,2} |h_m|^2 \right) + \log \left( 1 + \rho a_{n,2} |h_n|^2 \right)
\]

(10)

\[
R_{\text{sum}}^{CM} = 2 \log \left( 1 + \rho |h_m|^2 \right).
\]

(11)

Note that, from (11), the whole power is utilized to transmit the message in coded multicasting and the sum rate grows with increase in total power. However, the sum rate is constrained by the channel gain of the weaker user \(m\). Meanwhile, in NOMA, the two users share the total power according to the specified power allocation coefficients. Higher sum rates can be achieved by allocating more power to the stronger user \(n\). Nevertheless, fairness must be also taken into account and hence higher power should be allocated to the weaker user \(m\) in order to achieve the minimum individual rate comparable to coded multicasting.

III. ANALYTICAL PERFORMANCE STUDIES OF NOMA AND CODED MULTICASTING

A. Asymptotic Sum Rate Comparison

We compare the performance of NOMA and coded multicasting in terms of sum rates which are given in (10) and (11). This is performed by examining the probability that the sum rate of NOMA achieves a specific performance gain over that of coded multicasting in different user pairing scenarios, i.e.

\[
P \left( R_{\text{sum}}^{NOMA} - R_{\text{sum}}^{CM} > R \right) = 1 - P \left( R_{\text{sum}}^{NOMA} - R_{\text{sum}}^{CM} < R \right)
\]

(12)

where \(R\) is the target performance gain. Note that, the probability that NOMA achieves better performance than coded multicasting can be determined by setting \(R = 0\). Based on high SNR approximation (\(\rho \to \infty\)), the asymptotic expression of the sum rate gap can be simplified as

\[
R_{\text{sum}}^{NOMA} - R_{\text{sum}}^{CM} = \log \left( 1 + \rho a_{m,2} |h_m|^2 \right) + \log \left( 1 + \rho a_{n,2} |h_n|^2 \right) - 2 \log (\rho |h_m|^2)
\]

\[
\approx \log \left( \frac{a_{m,2} a_{n,2} |h_m|^2}{|h_n|^2} \right) + \log \left( \frac{a_{m,2} a_{n,2} |h_n|^2}{|h_m|^2} \right) - 2 \log (\rho |h_m|^2)
\]

(13)

Thus, the probability can be asymptotically expressed as

\[
P \left( R_{\text{sum}}^{NOMA} - R_{\text{sum}}^{CM} < R \right) = P \left( \log \left( \frac{a_{m,2} a_{n,2} |h_m|^2}{|h_n|^2} \right) < R \right)
\]

\[
= P \left( \frac{|h_n|^2}{|h_m|^2} < \frac{2R}{a_{m,2} a_{n,2}} \right).
\]

(14)

The PDF for the ratio of the channel gains of two users for the case of Rayleigh distributed channel gains can be found in literature, specifically in [12], but not for the condition where
users are uniformly located in circular cell and are affected by Rayleigh fading. First, the joint PDF of $|h_m|^2$ and $|h_n|^2$ is obtained from (1) and (2) according to [13]. Then, by applying the Mellin transforms [15] to this joint PDF, the PDF for the ratio of two order statistics is expressed as (15) in the following page where \( \omega = (m-1)!/(n-m-1)!(K-n)! \), \( \tau_1 = \sum_{n=1}^N c_n p_n + \sum_{n=1}^N c_n r_n \), and \( \tau_2 = \sum_{n=1}^N c_n q_n + \sum_{n=1}^N c_n s_n \). By applying (15), the probability that NOMA achieves a performance gain over coded multicasting is given by (16).

Since (16) is affected by the index of both user \( m \) and \( n \) in users’ set \( K \), this expression gives a rough idea on the sum rate performance attained by NOMA over coded multicasting based on different pairing scenarios. Therefore, the analytical result, which will be shown in Section IV, will give an understanding on which delivery technique performs better under different pairing situations. It is also worth mentioning that (16) is not a function of SNR \( (\rho) \), and hence only provide asymptotic result. Since the derivation involves high SNR approximation, it is anticipated that (16) is accurate at larger SNR. The accuracy of (16) will be verified through the simulation results presented in Section IV.

B. Outage Performance

For NOMA, the outage probability for each user \( i \in \{m, n\} \) can be obtained by analyzing order statistics and thus is expressed as

\[
P_i^{NOMA} = P\left(R_i^{NOMA} < \tilde{R}_i\right) = P\left(|h_i|^2 < \frac{2\tilde{R}_i - 1}{\rho a_{i,2}}\right)
\]

\[
= \frac{2\tilde{R}_i - 1}{\rho a_{i,2}} \int_0^{\tilde{R}_i} K[(K-i)!/((K-1)!)] f_{|h|^2}(x) \, dx
\]

where \( \tilde{R}_i \) is the minimum rate requirement for each user \( i \in \{m, n\} \). It is difficult to solve the integral in (17). In order to solve this, the CDF and PDF of the unordered channel gains can be further simplified using Taylor’s approximation as \( F_{|h|^2}(y) \approx \frac{1}{\mathcal{R}_c} \sum_{n=1}^N \beta_n y \) and \( f_{|h|^2}(y) \approx \frac{1}{\mathcal{R}_c} \sum_{n=1}^N \beta_n (1 - \alpha_n y) \) respectively. By following the steps in [6], the outage probability is estimated using high SNR as

\[
P_i^{NOMA} \approx \frac{\mu_i}{\mathcal{R}_c} \int_0^{\frac{2\tilde{R}_i - 1}{\rho a_{i,2}}} \eta y^{K-i-1} (1 - \eta y)^{K-i} \frac{1}{\mathcal{R}_c} \sum_{n=1}^N \beta_n (1 - \alpha_n y) \, dy
\]

\[
\approx \frac{\mu_i}{\mathcal{R}_c} \eta^{K-i} \left(\frac{2\tilde{R}_i - 1}{\rho a_{i,2}}\right)^i
\]

where \( \eta = \frac{1}{\mathcal{R}_c} \sum_{n=1}^N \beta_n \) and \( \mu_i = \frac{K!}{(i-1)!(K-i)!} \). From (18), it is obvious that the outage probability decreases with increasing SNR. In addition, the channel ranks of the paired users also influence the outage performance. Since \( m < n \), the stronger user \( n \) tends to outperform weaker user \( m \). Nevertheless, this is not always the case as the power allocation coefficient \( a_{i,2} \) also affects the outage probability. The outage probability of the weaker user \( m \) can be improved by allocating higher power allocation coefficient and thus enhancing the user fairness.

Using similar steps used in NOMA, the outage probability for both users in coded multicasting is represented as

\[
P_i^{CM} \approx \frac{\mu_m}{\mathcal{R}_c} \left(\frac{2\tilde{R}_m - 1}{\rho} \right)^m
\]

Note that, from (18) and (19), the outage performance of the users in coded multicasting is always better than the weaker user \( m \) in NOMA since the power allocation factor \( a_{m,2} \leq 1 \). Therefore, in order to maintain comparable outage performance for user \( m \), \( a_{m,2} \) should be closer to 1. It is worth pointing out that significant performance gain can be achieved by the stronger user \( n \) in NOMA over the users in coded multicasting particularly when index of the user \( n \in K \) is much greater than \( m \) in \( K \) i.e. \( n \gg m \). This indicates that NOMA potentially offers significant performance gain when the channel gains between the paired users are highly distinctive as shown in Section IV.

IV. Numerical Results

In this section, simulations are used to assess the performance of NOMA and coded multicasting, and to verify the accuracy of the derived analytical expressions. The total number of users is set as \( K = 5 \), the radius of the cell is \( R_c = 100 \, \text{m} \) and the path loss exponent is \( \alpha = 3 \). For NOMA, the power allocation coefficients for the paired users are \( a_{m,2} = 0.8 \) and \( a_{n,2} = 0.2 \). The parameter for Chebyshev-Gauss quadrature \( N \) is set as 10. We consider two extreme user pairing cases. The first case is a scenario in which the user with the best channel condition \( (n = K = 5) \) is paired with the one having the worst channel \( (m = 1) \). This means the channel gain difference between the two users is always large. The second case is a situation in which a user \( (n = 5) \) is paired with the nearest ordered user \( (m = 4) \). Here, the channel gains for these users are often very close to each other.

Fig. 2. Probability that the sum rate difference between NOMA and coded multicasting is greater than \( R \) when \( n = 5 \) is paired with \( m = 1 \), and \( m = 4 \).
pairing the users with distinctive channel quality to achieve the performance. When the channel gains of the paired users are similar, the performance of NOMA improves significantly because the total power is shared between the users, allowing for higher rates. However, when the channel gains are significantly different, the performance improvement is limited.

For NOMA, the total power is shared between the users, and the individual transmission rate of the stronger user increases with higher SNR. However, the rate of the weaker user is comparable to that of the strong user, which increases substantially with higher SNR.

The extent of performance gain over coded multicasting at higher SNR is significant. The probability that NOMA performs better than coded multicasting at any SNR since $a_{m,2} = 0.8$.

Fig. 3 shows the probability that NOMA performs better than coded multicasting against the power allocation coefficient of the weaker user $m$. Firstly, the results demonstrate the accuracy of the analytical expression in (16) particularly at high SNR. Secondly, it can be observed that the performance of NOMA improves as more power is allocated to the stronger user $n$. This indicates that NOMA can exploit the individual rate of the stronger user to enhance the sum rate. However, this will result in lower achievable transmission rates for the weaker user $m$ and hence disregard the user fairness requirement.

Moreover, the results indicate that NOMA performs better than coded multicasting for the first case (i.e. user $n = 5$ paired user $m = 1$). In NOMA, the individual transmission rate of the strong user ($n = 5$) is significantly high due to the best channel condition while the weak user ($m = 1$) maintains a considerable rate since most of the power is allocated to this user i.e. $a_{m,2} = 0.8$. This contributes to higher sum rate for NOMA. Whereas, the individual rates of both users in coded multicasting are limited by the channel gain of the weakest user ($m = 1$) and thus resulting in lower sum rate. However, coded multicasting performs better than NOMA for the second case (i.e. user $n = 5$ paired user $m = 4$). With total transmission power fully allocated to transmit the single coded message, the individual rates of both users in coded multicasting improves significantly when the channel gain of the weakest user in the pair is comparable to that of the stronger user. For NOMA, the total power is shared among the users and therefore greater performance gain will not be achieved when the channel gains of the paired users are relatively similar. The results show that NOMA prefers pairing the users with distinctive channel quality to achieve significant performance over coded multicasting. On the other hand, the probability performance obviously deteriorates when considering higher $R$ as shown in Fig. 2 since it becomes more difficult to achieve higher performance target. The extent of the performance gain achieved by NOMA against SNR cannot be observed when $R = 0$. However, for $R = 0.25 BPCU$ and $R = 0.5 BPCU$, the probability drops with decreasing SNR which shows that NOMA can only achieve significant performance gain over coded multicasting at higher SNR.

The performance gain of NOMA is dominated by the rate of the strong user which increase substantially with higher SNR while the rate of the weak user is comparable to that of the users in coded multicasting at any SNR since $a_{m,2} = 0.8$. 

The impact of the power allocation coefficient on the performance of NOMA over coded multicasting.
the index of the user \( n \) is much greater than \( m \) i.e. \( n \gg m \), which implies that the channel gains of the paired users are becoming more distinctive.

Finally, the outage probability of the users in NOMA and coded multicasting systems is presented in Fig. 4 as a function of transmit SNR, where the minimum individual target rate \( \tilde{R}_i \) for \( i \in \{ m, n \} \) is 0.25 BPCU. The results for the first and second extreme cases are separately shown in Fig. 4(a) and Fig. 4(b) respectively. Fig. 4 shows that the analytical expressions in (18) and (19) are accurate at high SNR due to high SNR estimation. As observed in Fig. 4(a), the stronger user (\( n = 5 \)) in NOMA achieves significant outage performance while the performance of the weaker user (\( n = 1 \)) is comparable to that of the users in coded multicasting. Nevertheless, coded multicasting outperforms NOMA for the case of a user paired with nearest ranked user (i.e. user \( n = 5 \) paired with \( m = 4 \)) as demonstrated in Fig. 4(b). Therefore, coded multicasting is suitable to be implemented in a case where the users have relatively similar channel conditions. Note that, in Fig. 4(b), the weaker user outperforms the stronger user for NOMA. This is because of the higher power allocated to weaker user while having almost similar channel condition to the stronger user. It is worth mentioning that the user fairness and QoS requirements can be controlled by dynamically adjusting the power allocation coefficients [16], which is our future work.

V. Conclusions

In this paper, we have investigated the performance of NOMA and coded multicasting in cache-aided networks. Both analytical and simulation results demonstrate that NOMA offer significant performance gain over coded multicasting when selecting users whose channel gain difference are very large. However, coded multicasting is favoured when the channel gains of the paired users are relatively similar. Depending on the channel gains of the scheduled users, both NOMA and coded multicasting can be applied to enhance the performance of a cache-aided network. The results developed in this paper provides an insight for the design of pairing schemes in cache-aided NOMA, and also a hybrid NOMA and coded multicasting scheme.

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