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Optimal Capital Structure, Ambiguity Aversion, and Leverage Puzzles

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Abstract

We introduce ambiguity about the asset value dynamics of a firm into a trade-off framework of capital structure. We characterize investors’ preferences by recursive multiple priors utility. We show that equity holders’ ambiguity aversion significantly reduces the optimal leverage and debt capacity. In particular, equity holders with sufficiently strong ambiguity aversion perceive their asset value dynamics to be “too valuable to lose” upon bankruptcy and therefore optimally choose zero leverage and forgo the tax benefits of debt to avoid possible default. We also show that the distortion effect of ambiguity on the leverage choice is stronger (weaker) when a firm’s equity and debt markets are segmented (integrated).

Key Words — Ambiguity aversion, capital structure, debt conservatism, trade-off theory, market segmentation.

JEL code — G32.

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1 Introduction

The zero-leverage (ZL) phenomenon, which has been observed widely in the U.S. (Strebulaev and Yang, 2013) and internationally El Ghoul et al. (2018), suggests that up to 20% of public firms completely forgo the net tax benefits of debt by choosing zero leverage. While various models have been developed to explain the low-leverage puzzle on the grounds of dynamic trade-off (e.g., Fischer et al., 1989; Goldstein et al., 2001; Strebulaev, 2007) or agency frictions (Leland, 1998; Titman and Tsyplakov, 2007; DeAngelo et al., 2011), our understanding of ZL as an equilibrium outcome of the above-mentioned trade-off models is still limited. For instance, Strebulaev and Yang (2013) highlight that “What we show is that to explain the low-leverage puzzle one needs to explain why some firms tend not to have debt at all instead of why firms on average have lower outstanding debt than expected, and most of extant models fail on this dimension.” The primary objective of this paper is to demonstrate that ZL can be an optimal policy in a static trade-off framework with ambiguity-averse investors.

In our model, equity and debt investors price contingent claims on a firm’s asset value dynamics that is perceived to be ambiguous. We characterize their preferences with the recursive multiple priors utility (RMPU) function proposed by Chen and Epstein (2002). In the multiple priors framework, investors’ beliefs are represented by a set of priors over the stochastic process of the asset value. In line with the existing studies on applications of multiple priors utility in economics and finance, the adoption of RMPU implies that investors value their holdings under their respective worst-case bankruptcy scenarios.

We maintain the fundamental assumptions of the trade-off framework of Leland (1994) in that the default boundary is endogenous and that immediate costly liquidation occurs upon default. We further assume that investors face ambiguity about the “true” dynamic process of

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1In the multiple priors framework such as Gilboa and Schmeidler (1989) and Chen and Epstein (2002), the set of priors characterizes both “ambiguity” and “ambiguity aversion”, and thus the two concepts are not distinguished. We follow this convention and use “ambiguity” and “ambiguity aversion” interchangeably throughout the paper.

2In the economics and finance literature, many studies have investigated portfolio choice, asset prices or real options under RMPU. For example, see Epstein and Schneider (2007), Epstein and Schneider (2008), Miao and Wang (2011), Liu (2011), and Jeong et al. (2015). Epstein and Schneider (2010) and Guidolin and Rinaldi (2013) provide excellent literature reviews.
the asset value and consider a set of candidate models with different mean growth rates, in light of ample empirical evidence that it is often challenging to estimate mean returns of financial assets (Merton, 1980). Consequently, ambiguity aversion, characterized by RMPU, dictates that equity holders make optimal capital structure choice based on the worst-case probability measure. Throughout the paper, we refer to “default ambiguity” as the scenario in which equity holders of the firm are uncertain about the asset value dynamics that they will lose in bankruptcy.

We first highlight that default risk increases equity holders’ default option value while default ambiguity does the opposite. Under limited liability, equity investors of a levered firm hold a perpetual American put on the assets with a strike equals to the after-tax coupon stream’s value. In the absence of ambiguity, uncertainty only originates from the Brownian component and is characterized by asset growth volatility. The value of the default option increases in default risk, following standard arguments that volatility increases option values. In the presence of ambiguity, uncertainty also stems from the lack of knowledge about the true dynamics of the asset value. Importantly, we show that the worst-case scenario within the set of priors arises when the default option attains the lowest value under the highest possible growth of the asset value dynamics. Intuitively, this worst case implies that equity holders worry about losing the most valuable asset to debt holders upon default. Thus, the default option value decreases in default ambiguity.

We then show that equity holders with sufficiently high ambiguity aversion will optimally choose not to borrow because they view their assets as “too valuable to lose”. The economic intuition is as follows. If the firm were to be levered, the worst-case scenario for equity holders in default would be to lose the asset value dynamics with the highest growth constrained by the set of priors. If equity holders perceive the asset growth to be high, they have the incentive to keep the default option alive by postponing default indefinitely. However, to postpone default to any horizon in this case, equity holders must continue to inject sufficiently high capital into the firm, which would then result in negative payout at all times hence negative equity value as perceived by equity holders. This unrealistic scenario contradicts limited liability, and as such,
equity holders are unable to postpone default in any case when they hold a default option with strictly negative values. Consequently, equity holders fear that they would lose their “valuable” assets if the firm were to be levered and thus optimally choose not to borrow.

Next, we show that for equity holders who are less ambiguity averse, the firm issues debt. Moreover, debt investors have a distinct worst-case measure than the equity holders, even when both equity and debt holders have the same belief on the set of priors over the asset value dynamics. For debt holders, their worst case arises when the asset value dynamics has the lowest possible growth constrained by the set of priors, leading to the correspondingly high likelihood of default. This finding indicates that both default risk and default ambiguity command positive premia to debt investors. Our analysis suggests that ambiguity aversion can make equity investors and debt investors inherently heterogeneous in terms of pricing ambiguity in the asset value dynamics and therefore precludes the assumption of a representative investor in our model.

We further extend our analysis by examining the effect of ambiguity on optimal leverage conditional on whether or not a firm’s equity and debt markets are partially segmented in the spirit of Rubinstein (1973). When the markets are partially segmented, the set of equity investors and that of debt investors are disjoint. Hence, equity (debt) is priced under the worst-case measure for equity (debt) investors.\(^3\)

The opposite case is when a firm’s equity and debt markets are perfectly integrated. We find that all securities are priced with debt holders’ measure. To investigate this case, we adopt the general framework of Rubinstein (1974) and Jouini and Napp (2006), among others, on aggregating heterogeneous pricing measures in competitive markets. Since perfectly integrated equity and debt markets are competitive, the First Welfare Theorem holds. We recover the aggregated measure from the first-best outcome. In our setting, we follow Broadie et al. (2007) to define the social planner’s objective as the total firm value instead of equity value. In bankruptcy, the total firm value is penalized with the deadweight bankruptcy cost. Hence, the

\(^3\)Numerous stylized facts support the assumption of segmented markets. See Mankiw and Zeldes (1991), Huang and Huang (2012), Kapadia and Pu (2012), and Choi and Kim (2018).
worst-case scenario occurs when default happens with the highest probability implied by the constrained set of priors, under debt holders’ pricing measure.

Under these two market settings, we find that ambiguity aversion lowers optimal leverage and debt capacity. The impact of ambiguity aversion is more pronounced when a firm’s equity and debt markets are partially segmented. Specifically, under market segmentation and given that equity and debt investors have the same belief on the set of priors over the asset value dynamics, equity investors’ worst-case scenario has the highest possible growth, and thus the optimal default is associated with a relatively high default boundary. In contrast, under market integration, the integrated worst-case scenario has the lowest possible growth constrained by the set of priors. In this case, equity investors view default more likely, and thus the optimal default is associated with a relatively low default boundary. Although debt investors’ worst-case scenario coincide under these two market settings, they view the default probability lower under market integration. This lowered probability follows from taking into account equity holders’ optimal default boundary under market integration. Consequently, market integration increases equity holders’ willingness to borrow and debt holders’ willingness to lend, resulting in higher optimal leverage and debt capacity relative to the case of market segmentation.

Our theoretical results are connected to several studies on capital structure. The result that ambiguity lowers debt capacity is consistent with the findings of Devos et al. (2012), Bessler et al. (2013) and Dang (2013) that debt capacity constraint, instead of corporate governance, is the main determinant of ZL. Additionally, we show that when the equity and debt markets are integrated, the distortion effects of ambiguity on leverage and debt capacity weaken. This result echoes the empirical studies on dual holders who invest in the same firm’s equity and debt. Jiang et al. (2010) find that syndicated loans with dual holder participation have loan yield spreads that are 18-32 bps lower than those in the case without dual holder participation. If dual ownership better align the interests of equity holders to those of debt holders, our theoretical analysis suggests that the enhanced uncertainty sharing through market integration fulfills the goal. That is, the enhanced uncertainty sharing can induce equity holders to deviate from operating under their worst-case scenario and thereby improve social welfare.
Our paper is closely related to several papers on optimal capital structure. Lee (2016, 2017) and Izhakian et al. (2021) also study optimal capital structure under ambiguity. However, none of them has ZL as an equilibrium outcome. Both Lee (2017) and Izhakian et al. (2021) utilize a one-period trade-off framework with alternative frameworks for modelling ambiguity. Because the utility preferences that they adopt are inherently smooth, the optimal leverage in their models is always positive. Like our model, Lee (2016) integrates RMPU into a static trade-off model of capital structure. The model of Lee (2016) can be viewed equivalently as modeling earnings before interest and taxes (EBIT) with a geometric Brownian motion. However, the model still cannot obtain ZL as an optimal outcome because the total payout to debt and equity holders is implicitly restricted to be positive in the model. In our model, the total payout rate is the difference between the risk-free rate and the asset growth under equity holders’ worst-case measure, which can be negative when the worst-case asset growth exceeds the risk-free rate. We show that ZL will arise as an equilibrium outcome in such a case. Moreover, Lee (2016) does not investigate the role of the market setting (market segmentation versus integration) as we do in this paper.

Yang (2013) considers a model in which insiders and potential outside investors (in both equity and debt) have heterogeneous beliefs and the firm will refinance. Yang (2013) shows that the model can generate ZL. Instead, heterogeneous beliefs of equity holders and debt holders emerge endogenously in our model from the assumption that both equity holders and debt holders have the same set of priors. Additionally, in contrast to Yang (2013), our model does not require the firm to refinance in order to generate low and realistic optimal leverage ratios. Lotfaliei (2018) explains the ZL puzzle based on the value in waiting to have debt and shows that ZL is transient, which is, however, inconsistent with the evidence of Strebulaev and Yang (2013) and Graham (2000) that both ZL and debt conservatism are persistent phenomena.

The rest of our paper is organized as follows. Section 2 presents the model of capital
structure incorporating ambiguity. Section 3 shows that ZL can be an optimal outcome in equilibrium under certain conditions. Section 4 provides theoretical analysis on the impacts of ambiguity on optimal leverage and debt capacity in different market settings. Section 5 discusses numerical results. Section 6 concludes. Additional discussion and proofs of main results are included in the Appendix.

2 The Framework

Ambiguity refers to the situation in which investors have imperfect knowledge about the probability distribution of a random variable, which is the value of an asset, $A_t$, in our model. To model ambiguity, we adopt the recursive multiple priors utility framework of Chen and Epstein (2002), which extends the static multiple priors utility model of Gilboa and Schmeidler (1989) to the continuous-time setting. We first present the benchmark model, based on Leland (1994), and then introduce ambiguity into the model.

2.1 The benchmark model under risk aversion

Let $(\Omega, \mathcal{F}, Q)$ be a complete probability space, $(W_t)_{0 \leq t \leq T}$ be a standard Brownian motion with respect to the risk-neutral measure $Q$, and $\{F_t\}_{0 \leq t \leq T}$ is the filtration for $(W_t)_{0 \leq t \leq T}$. Under the risk-neutral measure $Q$, a firm has the asset value dynamics $A_t$ that is governed by the following stochastic differential equation:

$$dA_t = \mu A_t dt + \sigma A_t dW_t,$$

where $\mu$ is mean growth of the asset, and $\sigma$ is the volatility of asset growth. Given the risk-free rate $r$ satisfying $r > \mu$, the difference $\eta = r - \mu$ is the total payout rate to both equity holders and debt holders. We assume that $r$, $\eta$ and $\sigma$ are constants (Leland, 1994, 1998).

It is well known that there is a trade-off between the tax benefits of issuing debt and the associated bankruptcy costs. At time $0^-$, equity holders issue a consol that pays a constant
continuous coupon $C$, which is tax-deductible at a corporate tax rate $q$, to maximize total firm value, the sum of equity and debt values immediately after the issuance at $0^+$. Both equity and debt values at $0^+$ and afterwards depend on equity holders’ default policy, a function of $C$. Specifically, default occurs at a random time $\tau$, when the asset value is sufficiently low such that the total payout does not cover debt obligations. Upon default, the firm is bankrupt and hence liquidated, and the absolute priority rule is enforced. Under liquidation, equity holders stop the coupon payment and lose the asset value plus the tax benefits of debt. In contrast, creditors lose the coupon stream and receive all the asset value net of bankruptcy costs. The cost of bankruptcy is a proportion ($\alpha \in [0, 1]$) of the asset value upon default, and hence debt holders recover $(1 - \alpha)A_{\tau}$. Additionally, equity holders are assumed to inject equity capital under financial distress and use the proceeds to meet net debt service requirements. That is, they choose the optimal timing to default to maximize equity value. This case corresponds to the optimal default or endogenous default specification, where $A_{\tau}$ is determined from solving an optimal stopping problem for equity holders (Leland, 1994).

In this setting, under a general default policy, equity value at any time $t < \tau$ is given by:

$$E_t = \mathbb{E}^Q \left[ A_t - \frac{(1 - q)C}{r} \left( 1 - e^{-r(\tau-t)} \right) - e^{-r(\tau-t)} A_{\tau} | \mathcal{F}_t \right] ,$$

(2.2)

where $\mathcal{F}_t$ is the time-$t$ filtration of $W_t$, and $\tau$ is an $\mathcal{F}_t$-stopping time. Equation (2.2) states that equity value equals the current asset value less the value of the annuity of tax-deducted coupons until default, less the discounted asset value lost in default. Furthermore, optimal default is given by the following optimal stopping problem

$$E_t^* = \sup_{\tau} E_t ,$$

(2.3)

Namely, equity holders in the future time $\tau^*$ give up the assets to debt holders in exchange for the stopped coupon obligations.
Equation (2.2) also admits another expression

\[ E_t = \mathbb{E}^Q \left[ \int_{\tau}^{\infty} e^{-r(s-t)}((r - \mu)A_s - (1 - q)C)ds | \mathcal{F}_t \right], \tag{2.4} \]

which follows from the Dynkin’s formula.\(^5\) Equation (2.4) depicts equity value as the sum of present value of dividends, and the instantaneous dividend rate is \((r - \mu)A_s - (1 - q)C\). When the firm is liquid instantaneously at time \(s\), \((r - \mu)A_s - (1 - q)C \geq 0\), equity holders collect nonnegative dividends. Otherwise, they realize negative dividends by injecting capital to meet coupon payments and therefore keep the firm alive for future upside prospects. In Leland’s framework, under optimal default, equity holders will stop injecting capital until equity value is zero.

Under full information, debt holders are endowed with the same belief about the asset value dynamics, and they know equity holders’ decision rule \(A_r\). Hence, the value of debt prior to default is given by

\[ D_t = \mathbb{E}^Q \left[ \frac{C}{r}(1 - e^{-r(\tau-t)}) + e^{-r(\tau-t)}(1 - \alpha)A_r | \mathcal{F}_t \right], \tag{2.5} \]

where the first term inside the conditional expectation operator is the value of the annuity of coupon until default, and the second term is the discounted net asset value recovered in default.

At time \(0^-\), equity holders choose the coupon to maximize the sum of the debt and equity values:

\[ V_0^* = \max_C E_0 + D_0. \tag{2.6} \]

The optimal coupon then determines the optimal capital structure of the firm.

### 2.2 Incorporating ambiguity aversion

We assume that investors have RMPU with a set of priors. The set of priors represents the alternative models that investors are willing to consider. These priors are equivalent probability

\(^5\)The details are in the proof of Proposition 2; see the Appendix.
measures that are absolutely continuous and linked by density generators. We use \( \kappa \)-ignorance as in Chen and Epstein (2002) to model ambiguity, where \( \kappa \) determines the size of the set of priors and also the extent of ambiguity aversion. The set of priors is constructed from \( \{F_t\}_{t=0}^T \)-adapted density generators defined by \( \theta \equiv \{\theta_t\}_{t=0}^T \in \Theta \) satisfying \( \sup |\theta_t| \leq \kappa \) with \( \kappa \geq 0 \). Unlike risk, which is characterized by the Brownian motion, ambiguity is characterized by multiplicity of the set of priors. A higher \( \kappa \) indicates that more events will be considered ambiguous, leading to a higher degree of ambiguity aversion. In the special case of \( \kappa = 0 \), investors are ambiguity neutral and risk averse only.

Following Chen and Epstein (2002), the construction of the set of priors is as follows. Each density generator \( \theta_t \) delivers a martingale \( z_t^\theta \) under \( Q \):

\[
dz_t^\theta = -\theta_t z_t^\theta dW_t, \quad \text{for } z_0^\theta = 1. \tag{2.7}
\]

We define a probability measure \( Q^\theta \) (an alternative model)

\[
Q^\theta(A) = \int_A z_T^\theta(\omega) dQ(\omega), \quad \text{for } A \in \mathcal{F}_T, \tag{2.8}
\]

where \( Q^\theta \) is absolutely continuous with respect to \( Q \). The set of priors, \( Q^\Theta \), is defined in terms of the Radon-Nikodym derivative of \( Q^\theta \) with respect to \( Q \):

\[
Q^\Theta \equiv \left\{ Q^\theta : \theta \in \Theta, \frac{dQ^\theta}{dQ} = z_T^\theta \right\}. \tag{2.9}
\]

Define \( dW_t^\theta = dW_t + \theta dt \). Then \( W_t^\theta \) is a Brownian motion under \( Q^\theta \), which is also referred to as the distorted measure. Equation (2.1) can be rewritten as

\[
dA_t = (\mu - \theta_t \sigma)A_t dt + \sigma A_t dW_t^\theta. \tag{2.10}
\]

The total payout rate under \( Q^\theta \), \( \eta_{\theta} = r - \mu + \theta_t \sigma \), is higher (lower) than that under \( Q \), \( \eta = r - \mu \), for \( \theta_t > 0 \) \((< 0)\). Under the distorted measure \( Q^\theta \), \( \theta_t \sigma > 0 \) \((< 0)\) represents the required reward
We now present the valuation of the asset under multiple priors. Consider an investment strategy with a payoff \( f \) that depends on \( A_T \), the asset value at a future time \( T \). According to the multiple priors model of Chen and Epstein (2002), the value of this investment to an ambiguity averse investor at time 0 is

\[
V_0 = \inf_{Q^\theta \in \mathcal{Q}} E^{Q^\theta}[e^{-rT} f(A_T)|\mathcal{F}_0].
\]

The interpretation is that the value of an investment with an ambiguous payoff is determined by the worst-case scenario.

### 3 Optimal Zero Leverage

In this section, we show that when ambiguity aversion \( \kappa \) exceeds a certain level, they optimally choose ZL. Under optimal default, equity value at time \( t \) prior to default time \( \tau \) is given by

\[
E^*_t = \sup_{\tau} \inf_{Q^\theta \in \mathcal{Q}} E^{Q^\theta}[A_t - (1 - q)C + e^{-r(\tau-t)} \left((1 - q)C - A_\tau\right) |\mathcal{F}_t].
\]

Equation (3.1) suggests that the total uncertainty associated with default in equity holders’ view originates from two sources. The first one is default risk, the probability that the future asset value will be sufficiently low, for a given model \( Q^\theta \). The second one is default ambiguity, the possibility of exchanging the asset value dynamics with ambiguous growth potential for stopped deterministic coupons in bankruptcy.

We solve the problem (3.1) in two steps. First, we find the worst-case measure for all stopping times. Second, we find the optimal stopping time under the worst-case measure.\(^6\)

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\(^6\)Cheng and Riedel (2013) provide a formal approach for solving optimal stopping problems under ambiguity. They show that the worst-case measure and the optimal stopping time exist for a class of problems under general conditions. Furthermore, they show that the worst-case measure and the optimal stopping time satisfy the minimax relation. The optimal stopping problems under ambiguity that we consider belong to the class of problems examined by Cheng and Riedel (2013).
Proposition 1. For all stopping times, the worst-case probability measure for equity holders is \( Q^{-\kappa} \), under which the asset value dynamics has the drift \( \mu + \kappa \sigma \).

To interpret this result, we focus on the value of equity holders’ default option at time \( t \), the term \( \mathbb{E}^{Q^\theta}[e^{-r(\tau-t)}((1-q)\frac{C}{r} - A_\tau)|\mathcal{F}_t] \) in Equation (3.1). In the absence of ambiguity, standard arguments suggest that this option value increases with default risk, characterized by asset growth volatility. Under ambiguity, the worst-case scenario for equity holders is when this option has the lowest value implied by the set of priors. Since equity holders lose the asset to creditors in exchange for the stopped coupon stream upon default, the option value attains its minimum when the ambiguity-averse equity holders perceive the asset to be most valuable, i.e., when the asset has the highest growth constrained by the set of priors.

Our analysis above suggests that ambiguity can reduce equity holders’ incentive to borrow by lowering their perceived equity value. Hence, when equity holders’s ambiguity aversion is sufficiently high, limited liability implies that they will optimally choose not to borrow. This result is illustrated in the following proposition.

Proposition 2. If equity holders’ \( \kappa > \frac{r-\mu}{\sigma} \), then \( C^* = 0 \), i.e., it is optimal to have zero leverage.

Proposition 2 indicates that equity holders with sufficiently high ambiguity aversion will optimally choose not to borrow because they view their assets as “too valuable to lose”.\(^7\) The formal proof of Proposition 2 is given in the Appendix. We discuss the intuition of our proof here. Suppose otherwise equity holders choose to borrow at time \( 0^- \) with an optimal coupon \( C^* > 0 \). We show that in such a case, equity value would become strictly negative for any default time \( \tau > 0 \), thereby contradicting limited liability. Specifically, since the asset growth under the worst-case scenario for equity holders is \( \mu + \kappa \sigma \), the total payout ratio as perceived by equity holders is \( r - \mu - \kappa \sigma \), implying that equity holders require a negative default ambiguity premium given by \( -\kappa \sigma \); see the discussion after Equation (2.10). When

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\(^7\)In the Appendix, we further show that this result holds under an alternative default settlement, the debt-equity swap considered in Fan and Sundaresan (2000).
\( \kappa > \frac{r - \mu}{\sigma} \), equity holders perceive both the total payout rate \( r - \mu - \kappa \sigma \) and the dividend \((r - \mu - \kappa \sigma)A_t - (1 - q)C \) to be strictly negative at any time \( t > 0 \). That is, by postponing default to any horizon \( \tau > 0 \), equity holders must inject sufficient capital into the firm and would never collect positive dividends. As such, the implied negative equity value contradicts limited liability. This is in stark contrast with the case for \( r - \mu - \kappa \sigma > 0 \), when equity holders can collect positive dividends for high \( A_t \) values (i.e., \((r - \mu - \kappa \sigma)A_t > (1 - q)C \)) and inject capital for low \( A_t \) values (i.e., \((r - \mu - \kappa \sigma)A_t < (1 - q)C \)) to postpone default until equity value approaches zero.\(^8\)

We can further define a ZL domain, \( \mathcal{D}^{ZL} = \{\kappa | \kappa > \eta/\sigma \} \). This domain categories all firms into those that will adopt zero leverage and those that will borrow. The domain shrinks with payout \( \eta \) under the reference measure but expands with asset growth volatility \( \sigma \). This observation suggests that both total payout and asset growth volatility can predict ZL choices. Keeping \( \kappa \) constant, firms with lower total payout and higher asset growth volatility are more likely to choose ZL. This implication is consistent with certain stylized facts documented by Devos et al. (2012), Bessler et al. (2013), and Dang (2013) among others, who find that many ZL firms do not pay dividends.

4 Optimal Low Leverage under Ambiguity Aversion

Now we consider the other set of firms whose equity holders have ambiguity aversion \( \kappa \) outside the ZL domain \( \mathcal{D}^{ZL} = \{\kappa | \kappa > \eta/\sigma \} \). The equity holders of these firms have the incentive to borrow and face default uncertainty. Using the results from Propositions 1 and 2, we can derive the equity value under \( Q^{-\kappa} \) as

**Proposition 3.** If \( \kappa \leq \frac{r - \mu}{\sigma} \), for \( C > 0 \), equity value at time \( t \) under optimal default \( (E_t^*)^* \)

\(^8\)For the case \( r - \mu - \kappa \sigma = 0 \), or more precisely, when the total payout rate is zero, we follow the treatment of Leland (1994), who states that equity holders will issue equity to cover the coupon payments.
given by Equation (3.1)) is

\[ E_t^{-\kappa} = A_t - \frac{(1-q)C}{r} + \left( \frac{(1-q)C}{r} - A_{D}^{*} \right) \left( \frac{A_t}{A_{D}^{*}} \right)^{\gamma_2^{-\kappa}}, \]  

(4.1)

where \( \gamma_2^{-\kappa} \) is

\[ \gamma_2^{-\kappa} = \sigma^{-2} \left[ \frac{1}{2} \sigma^2 - (\mu + \sigma \kappa) - \sqrt{(\mu + \sigma \kappa - \frac{1}{2} \sigma^2)^2 + 2r\sigma^2} \right], \]  

(4.2)

and \( A_{D}^{*} \) is

\[ A_{D}^{*} = \frac{(1-q)C}{r} \frac{\gamma_2^{-\kappa}}{\gamma_2^{-\kappa} - 1}. \]  

(4.3)

4.1 The valuation of debt under ambiguity aversion

Provided that equity holders decide to issue debt, we analyze the demand for this security. In the analysis below, we maintain two assumptions. First, we assume that debt holders have the same set of priors as equity holders do. We do not consider that one group of investors has superior information or abilities that help resolve ambiguity. Second, the same set of priors is common knowledge for both debt holders and equity holders, on which contingent claims are priced. In particular, debt holders know the optimal default boundary, \( A_{D}^{*} \), chosen by equity holders.

Given \( C > 0 \) and \( A_{D}^{*} \) chosen by equity holders, ambiguity-averse debt holders value debt at time \( t \) prior to default as

\[ D_t = \inf_{Q^{\theta} \in \mathcal{Q}} \mathbb{E}^{Q^{\theta}} \left[ \frac{C}{r} + \left( 1 - \alpha \right) A_{\tau} - C_{\tau} \right] e^{-r(\tau-t)} |\mathcal{F}_t], \]  

(4.4)

To evaluate debt value, we begin with the following Lemma.

Lemma 1. Let \( \tau_D = \inf_{s>t} \{ A_s \leq A_D \} \) for \( A_D \in (0, A_t) \). Then, \( \tau_D \) is stochastically smallest under \( Q^{\theta} \) for all \( Q^{\theta} \in \mathcal{Q} \).

Proposition 4. When \( \kappa \leq \frac{r - \mu}{\sigma} \), anticipating equity holders’ default policy \( A_{D}^{*} \), debt holders’
valuation of debt at time $t$ prior to default is

$$D^*_t = \frac{C}{r} + \left(1 - \alpha \right) A_D^* - \frac{C}{r} \right) \left( \frac{A_t}{A_D^*} \right)^{\gamma_2^*}, \quad (4.5)$$

where $\gamma_2^* < 0$ is given by

$$\gamma_2^* = \sigma^{-2} \left[ \frac{1}{2} \sigma^2 - (\mu - \sigma \kappa) - \sqrt{(\mu - \kappa \sigma - \frac{1}{2} \sigma^2)^2 + 2r \sigma^2} \right]. \quad (4.6)$$

**Corollary 1.** For an arbitrary $C > 0$, and for $\kappa \leq \frac{r - \mu}{\sigma}$, debt holders’ valuation of debt $D^*_t$ is less than equity holder’s valuation of debt $D^{-\kappa}_t$.

This Corollary is an immediate outcome from the result that $Q^\kappa$ yields the lowest debt value. Proposition 4 suggests that ambiguity-averse debt holders perceive default to occur more likely. The intuition is that because debt holders take over the asset in bankruptcy, their worst-case scenario is when the asset has the lowest growth implied by the set of priors. Proposition 4 further implies that ambiguity-averse debt holders require compensation for bearing the default ambiguity.

4.2 Partially segmented debt and equity markets

Corollary 1 indicates that equity holders and debt holders value debt differently under their respective worst-case measures. Under the rationale that the sets containing investors and available securities in different markets are disjoint (Rubinstein, 1973), we analyze the pricing of debt and optimal leverage in equilibrium with partially segmented equity and debt markets.

The assumption of partially segmented equity and debt markets is supported by numerous stylized facts that fall broadly into two categories. The first one relates to the clientele phenomenon initially documented by Mankiw and Zeldes (1991). The study shows that only one-fourth of U.S. families own stocks. Among those consumers holding other liquid assets over 0.1 million U.S. dollars, only 47.6% hold equities. Manolache (2018) recently documents that between 2000 and 2018, households account for around 38% of direct ownership in the
equity market but only 13% of direct ownership in the corporate bond market. In contrast, insurance companies and mutual funds are dominant investors in the corporate bond market. He further finds that insurance companies own a relatively small share of the equity market. More importantly, mutual funds that operate in equity and corporate bond markets appear to be mutually exclusive.

The second category of facts relates directly to the discrepancies in risk premia in these two markets. For example, Huang and Huang (2012), Kapadia and Pu (2012), and Choi and Kim (2018) document that the premia for equivalent risk factor exposures differ in these two markets. In particular, Titman (2002) interprets the finding of Huang and Huang (2012) as evidence of a larger risk premium in the corporate bond market than in the equity market. Furthermore, Collin-Dufresne et al. (2001) and Gabaix et al. (2007) show that there are specific risk factors priced in the bond market.

Collectively, these two strands of literature provide foundations for us to examine the equilibrium market price of debt and optimal leverage under market segmentation. Since equity holders and debt holders disagree on the value of debt, one possibility is that equity holders refuse to issue debt for a lower price than their valuation. Consequently, equity holders will not borrow, implying that the firm value does not increase above its all-equity level. Besides, debt holders will not lend, indicating an opportunity loss.

Alternatively, equity holders can undersell this bond and still capitalize on tax benefits to improve the firm value. In this case, the coupon $C$ and the market price of debt $D$ are determined in the following equilibrium.

**Definition 1.** The equilibrium is characterized by $(C^*, D^*_0)$. Equity holders of a firm issue a consol with coupon $C^*$ for an amount of $D^*_0$ such that the firm value at time-$0$ $V_0 = E^*_0 + D^*_0$ is maximized. Debt holders buy this consol for $D^*_0$ only if $D^*_0$ is not higher than their valuation $D^*_0$.

It is clear that the market price of debt $D^*_0$ equals debt holders’ valuation $D^*_0$. If $D^*_0 < D^*_0$, equity holders can raise more cash by increasing the selling price of debt, so long as the asking
price is less than debt holders’ valuation. Thus, the firm value is

\[ V_0 = E_0^* + D_0^*, \]  

(4.7)

and equity holders choose \( C^* \) to maximize \( V_0 \). From the first-order condition, we have \( C^* \) as

\[
\frac{q}{r} C^{\gamma_2^{-\kappa} + \gamma_2} + (1 - \gamma_2^\kappa)(1 - \alpha)X - \frac{1}{r} \left( \frac{A_0}{X} \right)^{\gamma_2^{\kappa}} C^{\gamma_2^{-\kappa}} + (1 - \gamma_2^{-\kappa})(\frac{1 - q}{r} - X) \left( \frac{A_0}{X} \right)^{\gamma_2^{\kappa}} C^{\gamma_2^{-\kappa}} = 0,
\]

(4.8)

where

\[ X = \frac{1 - q}{r} \frac{\gamma_2^{-\kappa}}{\gamma_2^{\kappa} - 1}. \]

Equation (4.8) is not analytically tractable when \( \kappa > 0 \). We resort to numerical analysis to examine the properties of \( C^* \) and optimal leverage. Note that the equilibrium is well-defined if \( C^* > 0 \).

4.3 Perfectly integrated equity markets and debt markets

We further explore the possibility that equity and debt markets are perfectly integrated. Under market segmentation, investors holding either equity or debt have their respective worst-case pricing measures. We adopt the general framework of Rubinstein (1974) and Jouini and Napp (2006) on aggregating heterogeneous pricing measures in competitive markets. Since perfectly integrated equity and debt markets are competitive, the First Welfare Theorem holds. The aggregated measure is recovered from the first-best outcome.

We follow Broadie et al. (2007) and formulate the social planner’s problem as choosing the optimal stopping time that maximizes total firm value subject to the limited liability constraint. Hart (2000) and Broadie et al. (2007) state that the bankruptcy court determines liquidation based on socially optimal considerations. That is, a firm will not be liquidated so long as there is positive firm value remaining. Hence, under ambiguity aversion, the social planner maximizes
welfare as
\[
\sup_{\tau} \inf_{Q^\theta \in \mathbb{Q}} \{E_0^\theta + D_0^\theta\},
\tag{4.9}
\]
where \( \tau = \inf_{s > t} \{A_s \leq A_D, E_s^\theta \geq 0\} \) denotes the constrained stopping time. It follows that social welfare in this case dominates that in the case of segmented markets. That is,
\[
\inf_{Q^\theta \in \mathbb{Q}} \{E_0^\theta + D_0^\theta\} \geq \inf_{Q^\theta \in \mathbb{Q}} \{E_0^\theta\} + \inf_{Q^\theta \in \mathbb{Q}} \{D_0^\theta\}
\tag{4.10}
\]
for any coupon and stopping time.\(^9\) It is worth discussing that the limited liability constraint admits bankruptcy scenarios. Without this constraint, the optimal stopping time should be infinite. If bankruptcy never occurs, the social welfare will not be penalized by the deadweight bankruptcy cost. However, this implies that equity holders’ wealth is also liable.

With the objective defined above, we find that the first-best outcome materializes when both equity holders and debt holders accept debt holders’ worst-case measure \( Q^\kappa \). For any \( C \) and admissible \( \tau \) we have
\[
V_0^S = \inf_{Q^\theta \in \mathbb{Q}} \{E_0^\theta + D_0^\theta\}
= \inf_{Q^\theta \in \mathbb{Q}} E_{Q^\theta}^\theta \left[ A_0 + \frac{qC}{r} - e^{-r\tau} \left( \alpha A_r + \frac{qC}{r} \right) |F_0 \right]
= A_0 + \frac{qC}{r} + \inf_{Q^\theta \in \mathbb{Q}} E_{Q^\theta}^\theta \left[ e^{-r\tau} \left( -\alpha A_r - \frac{qC}{r} \right) |F_0 \right].
\tag{4.11}
\]
Since \(-\alpha A_r - \frac{qC}{r}\) in Equation 4.11 is strictly negative and for any default boundary \( A_D \), and Lemma 1 shows that \( \tau_D \) under \( Q^\kappa \) is stochastically smallest, the minimum in the above is reached under \( Q^\kappa \) by the first-order stochastic dominance.

Furthermore, we show that the optimal default boundary in this first-best outcome is lower than the one derived under segmented markets. By choosing \( A_D \) that maximizes \( V_0^S \) subject to \( E_0 \geq 0 \) in Equation (4.11), we find the expression for the optimal boundary, which is given
\[^9\text{This follows from the general inequality } \text{ess.inf}\{f + g\} \geq \text{ess.inf}\{f\} + \text{ess.inf}\{g\}, \text{ when all three are well-defined.}\]
by\textsuperscript{10} \[ A^*_D|_\kappa = (1 - q)C \frac{\gamma_2^\kappa}{r} \gamma_2^{\kappa - 1}. \] (4.12)

Obviously, the relation $A^*_D|_\kappa < A^*_D|_{-\kappa}$ holds, in which $A^*_D|_{-\kappa}$ is the optimal boundary under $Q^{-\kappa}$ obtained when equity holders are only concerned with their own interests. Hence, in this first-best benchmark, equity holders will postpone default by defaulting at a lower threshold. Correspondingly, we can find the optimal coupon that maximizes $V_0^S$ in (4.11) as

\[ C^* = \left( 1 - \left( 1 + \frac{1 - q}{q} \right) \gamma_2^\kappa \right) \frac{\sqrt{\gamma_2^\kappa - 1}}{(1 - q)\gamma_2^\kappa} A_0. \] (4.13)

The result that equity holders will postpone default, relative to the case of segmented markets, is in line with recent empirical findings on the effects of dual equity and debt ownership. For instance, Jiang et al. (2010) find that syndicated loans with dual holder participation have loan yield spreads that are 18-32 bps lower than those in the case without dual holder participation. Chu et al. (2018) show that simultaneous holdings are positively associated with the likelihood of out-of-court restructuring versus bankruptcy filing. This effect is more substantial when the expected bankruptcy costs are higher.

5 Numerical Analysis

We perform numerical analyses to examine the effect of ambiguity on the firm’s leverage choice. The ZL domain under ambiguity is $D^{ZL} = \{ \kappa|\kappa > \eta/\sigma \}$ while the positive leverage domain is $D^{PL} = \{ \kappa|\kappa \in [0, \eta/\sigma] \}$. To study the impacts of risk and ambiguity on the firm’s optimal leverage choice, we first create a grid of realistic values on $\sigma$ according to $\sigma_i = i \times 0.05$, $i = 1, \ldots, 7$, and then for each $\sigma_i$, we construct an equally spaced grid $\{ \kappa_{i,j} \}^N_{j=1}$ on the positive leverage domain $[0, \eta/\sigma_i]$. This grid partition corresponds to a cross-section of firms with different levels of asset growth volatility and ambiguity. We numerically compute optimal leverage choices for these firms. As shown in our theoretical analysis, the distortion effect of ambiguity on the

\textsuperscript{10}Broadie et al. (2007) show that this constrained problem is equivalent to equity value maximization.
optimal leverage depends on parameters such as $\sigma$ and $\eta$, as well as market segmentation versus integration. We set other parameter values in line with previous studies (e.g., Leland, 1998; Goldstein et al., 2001; Strebulaev, 2007, among others). Our parameter choices are shown in Table 1.

5.1 Results for segmented markets

In segmented markets, equity holders and debt holders optimally choose diverged beliefs specified by the set of priors, as studied in Section 4.2. Table 2 shows that without ambiguity ($\kappa_{i,1} = 0$), the predicted optimal leverage decreases from 0.84 to 0.53 for $\sigma$ increasing from 0.05 to 0.35, which is comparable with previous results in Leland (1994). Moreover, Table 2 shows that the optimal leverage monotonically decreases in the degree of ambiguity aversion for each $\sigma$ value when $\kappa$ is within the positive leverage domain. For a small $\sigma$ value of 0.05, optimal leverage decreases from 0.84 to 0.45, for $\kappa$ increasing from 0 to the ZL limit $\eta/\sigma = 0.03/0.05 = 0.6$. For a large $\sigma$ value of 0.35, optimal leverage decreases from 0.53 to 0.11, for $\kappa$ increasing from 0 to the ZL limit $\eta/\sigma = 0.03/0.35 \approx 0.09$.

This result addresses a common critique of the static trade-off model. It helps explain the low leverage puzzle introduced by Graham (2000) and further analyzed by Korteweg (2010), among others. Previous studies (e.g., Goldstein et al., 2001) find that the optimal leverage in the static trade-off model is much higher than the empirically observed market leverage. Indeed, the mean market leverage for nonfinancial public US firms is 0.28 with a standard deviation of 0.26 (Frank and Goyal, 2009). In contrast, it is in the range of $[0.53, 0.84]$ in the benchmark model with risk aversion only. Moreover, Graham (2000) and Korteweg (2010) show that firms on average do not borrow to the optimal level predicted by their models with risk aversion only. Hence, they rely on a substantial amount of tax benefits to explain their empirical finding. Our results suggest that ambiguity leads to conservative leverage choices that are more consistent with empirical evidence.

The notable difference in optimal leverage between the ambiguity-neutral and ambiguity-
averse cases can be interpreted by the difference in the optimal coupon in the two cases. In the absence of ambiguity \((\kappa_{i,1} = 0)\), the optimal coupon exhibits a U-shaped relation with \(\sigma\), consistent with the benchmark result (e.g., Leland, 1994). The interpretation is standard and based on the two offsetting mechanisms of \(\sigma\), the default acceleration mechanism, and the upside prospect mechanism. Higher \(\sigma\) values can increase the risk of default, which implies a lower optimal coupon. However, at the same time, higher \(\sigma\) values can increase the upside prospect, implying high potential tax benefits. For smaller \(\sigma\) values, the default effect dominates, leading to a negative relation between \(\sigma\) and the optimal coupon. For higher \(\sigma\) values, the upside prospect effect dominates, leading to a positive relation between \(\sigma\) and the optimal coupon.

Under ambiguity, for each \(\sigma\), the optimal coupon decreases monotonically in the positive leverage domain of \(\kappa\). For a small \(\sigma\) value of 0.05, the optimal coupon decreases from 4.06 to 2.06, for \(\kappa\) increasing from 0 to the ZL limit \(\eta/\sigma = 0.03/0.05 = 0.6\). For a large \(\sigma\) value of 0.35, the optimal coupon decreases from 3.77 to 0.56, for \(\kappa\) increasing from 0 to the ZL limit \(\eta/\sigma = 0.03/0.35 \approx 0.09\). The interpretation is that ambiguity affects optimal coupon through a debt demand reduction mechanism. As equity holders’ ambiguity aversion approaches the ZL threshold, the demand for debt falls toward zero. This demand reduction mechanism driven by changing \(\kappa\) dominates the two mechanisms induced by changing \(\sigma\).

Another important quantity to examine is debt capacity, the maximum amount that a firm can borrow against its assets in place. In our model, debt capacity \(D_{0}^{\text{max}}\) is given by

\[
D_{0}^{\text{max}} = \max_{C} D_{0}^{\kappa},
\]

where \(D_{0}^{\kappa}\) is given by Equation (4.5). From the first-order condition, we can derive the expression for \(C^{\text{max}}\) as

\[
C^{\text{max}} = \left[ (1 - \gamma_{2}^{\kappa}) + (1 - \alpha)(1 - q)\frac{\gamma_{2}^{-\kappa}(\gamma_{2}^{\kappa} - 1)}{\gamma_{2}^{-\kappa} - 1} \right]^{1/\gamma_{2}^{\kappa}} \frac{r(\gamma_{2}^{-\kappa} - 1)}{(1 - q)\gamma_{2}^{-\kappa}} A_{0}.
\]

Table 3 shows that ambiguity leads to reduced debt capacity. For a small \(\sigma\) value of
0.05, debt capacity decreases from 106.55 to 76.80, for $\kappa$ increasing from 0 to the ZL limit $\eta/\sigma = 0.03/0.05 = 0.6$. For a large $\sigma$ value of 0.35, debt capacity decreases from 86.76 to 76.85, for $\kappa$ increasing from 0 to the ZL limit $\eta/\sigma = 0.03/0.35 \approx 0.09$. In the absence of ambiguity ($\kappa_{i,1} = 0$), debt capacity decreases monotonically from 106.55 to 86.76, for $\sigma$ increasing from 0.05 to 0.35. Adding ambiguity to the model does not change this pattern.\footnote{It is important to note that as for Table 3, we cannot directly compare results across different values of $\sigma$ columnwise for a $\kappa$ value in a given row because of the grid partition adopted. In unreported results, we find that for any $\kappa$ in the common positive leverage domains specified over a range of $\sigma$ values, debt capacity monotonically decreases with $\sigma$.}

It is interesting to note that for a given $\sigma$, debt capacity coupon exhibits a U-shaped pattern when $\kappa$ increases from 0 to the ZL limit. To interpret this result, we note that there are two interacting mechanisms for $\kappa$ to exert impact on the value of debt, the higher default boundary mechanism, and the default acceleration mechanism. On the one hand, ambiguity increases equity holders’ optimal default boundary, leading to a higher value of the recovered assets and hence debt. On the other hand, ambiguity also increases debt holders’ valuation of the Arrow-Debreu security that pays in default. This default acceleration effect results in lower debt values. Thus, for small $\kappa$ values, the higher default boundary mechanism dominates the default acceleration mechanism, explaining the initially decreased coupon. For larger $\kappa$ values, the default acceleration mechanism dominates, leading to subsequently increased coupon.

The result for debt capacity is in line with several empirical studies regarding ZL and debt conservatism. Devos et al. (2012) and Bessler et al. (2013) find that the disciplinary role of debt is insufficient to explain ZL, because they do not find weaker corporate governance in ZL firms. Instead, they show that ZL firms have relatively small debt capacity. Our model predicts that low debt capacity arises due to ambiguity.

### 5.2 Results for perfectly integrated markets

We show in Section 4.3 that market integration can mitigate inefficiencies due to market segmentation in which equity and debt investors hold diverged beliefs. We now numerically assess the impact of market integration on improving welfare. Specifically, we examine whether market...
Market integration can alleviate the distortion effects of ambiguity on optimal leverage and debt capacity.

Table 4 shows that market integration unambiguously leads to higher firm value. For a small \( \sigma (\sigma = 0.05) \), the range of firm value is \([106.87, 118.50]\) under market segmentation and \([111.18, 118.50]\) under market integration, when \( \kappa \) decreases from the ZL limit \( \eta/\sigma = 0.03/0.05 = 0.6 \) to 0. For a large \( \sigma (\sigma = 0.35) \), the range of firm value is \([100.63, 106.21]\) under market segmentation and \([105.72, 106.21]\) under market integration, when \( \kappa \) decreases from the ZL limit \( \eta/\sigma = 0.03/0.35 \approx 0.09 \) to 0. Collectively, these findings highlight the welfare improving property of market integration.

Table 5 also reports that market integration leads to efficiency gains by attenuating the distortion effect of ambiguity on optimal leverage. Indeed, the range of optimal leverage under ambiguity is much reduced. For \( \sigma = 0.05 \), the optimal leverage decreases from 0.84 to 0.66 when \( \kappa \) increases from 0 to the ZL limit \( \eta/\sigma = 0.03/0.05 = 0.6 \). For \( \sigma = 0.35 \), the optimal leverage decreases from 0.53 to 0.51 as \( \kappa \) increases from 0 to the ZL limit \( \eta/\sigma = 0.03/0.35 \approx 0.09 \). Together with results in Table 4, we observe that higher optimal leverages are associated with higher total firm values.

Next, we observe that the distortion effect of ambiguity is stronger for firms with low \( \sigma \) in terms of proportional reduction in optimal leverage. This asymmetry is more prominent here than the pattern obtained under market segmentation, as shown in Table 2. The interpretation of this asymmetry lies in that the positive leverage domain for low-\( \sigma \) firms contains that for high-\( \sigma \) firms. To further understand this result, we proceed to analyze the optimal coupon choice.

The optimal coupons provide additional insights regarding the underlying mechanism through which market integration achieves efficient outcomes. First, the optimal coupons under market integration are higher than those shown in Table 2 under market segmentation. For instance, for a small \( \sigma (\sigma = 0.05) \), the optimal coupon ranges from 3.35 to 4.06, higher than the corresponding values in Table 2. Similar results are obtained for other values of \( \sigma \). Interestingly,
while the optimal coupon decreases with $\kappa$ under market segmentation, the relation between the optimal coupon and $\kappa$ depends on the level of $\sigma$ here. We see that the optimal coupon decreases with $\kappa$ for low $\sigma$ values but increases with $\kappa$ when $\sigma$ is high. These results can be interpreted by an interacting mechanism between $\kappa$ and $\sigma$. Market integration implies a unique worst-case measure induced by ambiguity aversion for both types of investors, and the resulting distorted drift of $\mu - \kappa \sigma$ for the asset value dynamics. With low $\sigma$ values, the impact of the drift dominates that of the diffusion. Thus, both claimants implement a conservative strategy by decreasing the coupon as $\kappa$ increases. On the other hand, for high $\sigma$ values, the diffusion effect dominates. Thus, both equity and debt holders are willing to engage in high borrowing.

Panel A of Table 6 presents results on debt capacity under ambiguity and market integration. Higher ambiguity leads to lower debt capacity. For a small $\sigma$ value of 0.05, debt capacity decreases from 106.55 to 93.17, for $\kappa$ increasing from 0 to the ZL limit $\eta/\sigma = 0.03/0.05 = 0.6$. Results are similar for higher values of $\sigma$ (e.g., $\sigma = 0.35$). However, the impact of ambiguity on debt capacity is mitigated due to market integration.

Panel B of Table 6 reports results on the debt capacity coupon pattern. We note that under market integration, debt capacity coupon increases monotonically with ambiguity for each volatility value. Recall that in market segmentation, we observe a U-shaped pattern for the relation between debt capacity coupon and ambiguity because debt capacity coupon is determined by an interaction between a higher optimal default boundary mechanism and a default acceleration mechanism due to enhanced ambiguity. In the case of market integration, the higher optimal default boundary mechanism disappears, because according to Equation (4.12), the optimal default boundary decreases with $\kappa$. Hence, ambiguity affects equity and debt values only through the default acceleration mechanism, which leads to increased debt capacity coupons for higher $\kappa$ values.
6 Conclusion

We introduce ambiguity about the asset value dynamics of a firm into a trade-off model of capital structure with infinite debt maturity and immediate liquidation upon default in the spirit of Leland (1994). Investors in our model have recursive multiple priors utility function of Chen and Epstein (2002). Both equity investors and debt investors are ambiguous about the asset value dynamics and have a set of priors over the dynamics. We show that equity holders’ ambiguity aversion significantly decreases the optimal leverage and debt capacity. In particular, equity holders with sufficiently high ambiguity aversion perceive their asset value dynamics to be “too valuable to lose” upon bankruptcy and therefore optimally choose zero leverage and forgo the tax benefits of debt to avoid possible default. We also analyze the impacts of ambiguity on the leverage choice and debt capacity, respectively, when the debt market and the equity market of a firm are segmented or integrated. Our model predictions are consistent with the empirical findings of zero-leverage or low leverage for many firms that are widely documented in the literature (e.g. Strebulaev and Yang, 2013; El Ghoul et al., 2018).

Future research could extend our model by considering the role of learning in dynamic capital structure models. For instance, it would be interesting to study the dynamics of the zero-leverage domain and optimal leverage by taking into account learning about ambiguous parameters or states. One could use the multiple priors framework, or alternatively, the smooth ambiguity framework developed by Klibanoff et al. (2009) to model the interaction of learning and ambiguity. Such studies would contribute to the ongoing discussion on leverage dynamics as in Admati et al. (2018) and DeMarzo and He (2021). Future research could also estimate some augmented version of our model to examine the empirical distribution of ambiguity across firms.
Table 1: **Parameter values.** This table shows the values of the parameters used in our numerical analyses. We create a grid (7 points) for $\sigma$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default cost: $\alpha$</td>
<td>$0.25$</td>
</tr>
<tr>
<td>Payout rate: $\eta$</td>
<td>$0.03$</td>
</tr>
<tr>
<td>Tax rate: $q$</td>
<td>$0.2$</td>
</tr>
<tr>
<td>Risk-free interest rate: $r$</td>
<td>$0.04$</td>
</tr>
<tr>
<td>Initial asset value: $A_0$</td>
<td>$100$</td>
</tr>
<tr>
<td>Asset growth volatility: $\sigma_i$</td>
<td>$i \times 0.05$, for $i = 1$ to 7</td>
</tr>
</tbody>
</table>
Table 2: The range for feasible optimal leverage (coupon) under ambiguity aversion and market segmentation. This table shows the range for feasible optimal leverage (coupon) based on a grid of $\kappa$ (the row index) and $\sigma$ (the column index). Optimal leverage is the ratio of debt over debt plus equity, where the coupon argument in debt value and equity value takes the optimal value given by Equation (4.8). An optimal leverage (coupon) is feasible under a pair $(\kappa, \sigma)$, when $\kappa$ is in the positive leverage domain: $[0, \eta/\sigma]$. To find the range for feasible optimal leverage (coupon) for each $\sigma_i$, we construct an equally spaced grid $\{\kappa_{i,j}\}_{j=1}^N$ on $[0, \eta/\sigma_i]$. Other parameter values are from Table 1.

<table>
<thead>
<tr>
<th>$(\kappa, \sigma_i)$</th>
<th>0.05</th>
<th>0.1</th>
<th>0.15</th>
<th>0.2</th>
<th>0.25</th>
<th>0.3</th>
<th>0.35</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_{i,1} = 0$</td>
<td>0.84</td>
<td>0.72</td>
<td>0.65</td>
<td>0.61</td>
<td>0.57</td>
<td>0.55</td>
<td>0.53</td>
</tr>
<tr>
<td>$\kappa_{i,2}$</td>
<td>0.78</td>
<td>0.65</td>
<td>0.58</td>
<td>0.53</td>
<td>0.50</td>
<td>0.48</td>
<td>0.47</td>
</tr>
<tr>
<td>$\kappa_{i,3}$</td>
<td>0.72</td>
<td>0.58</td>
<td>0.50</td>
<td>0.45</td>
<td>0.42</td>
<td>0.41</td>
<td>0.40</td>
</tr>
<tr>
<td>$\kappa_{i,4}$</td>
<td>0.66</td>
<td>0.53</td>
<td>0.44</td>
<td>0.38</td>
<td>0.35</td>
<td>0.33</td>
<td>0.32</td>
</tr>
<tr>
<td>$\kappa_{i,5}$</td>
<td>0.60</td>
<td>0.48</td>
<td>0.38</td>
<td>0.32</td>
<td>0.28</td>
<td>0.26</td>
<td>0.25</td>
</tr>
<tr>
<td>$\kappa_{i,6}$</td>
<td>0.55</td>
<td>0.43</td>
<td>0.34</td>
<td>0.27</td>
<td>0.23</td>
<td>0.20</td>
<td>0.19</td>
</tr>
<tr>
<td>$\kappa_{i,7}$</td>
<td>0.50</td>
<td>0.40</td>
<td>0.31</td>
<td>0.24</td>
<td>0.19</td>
<td>0.16</td>
<td>0.14</td>
</tr>
<tr>
<td>$\kappa_{i,8}$</td>
<td>0.45</td>
<td>0.36</td>
<td>0.28</td>
<td>0.21</td>
<td>0.16</td>
<td>0.13</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Panel B: Feasible optimal coupon ($\$)$

<table>
<thead>
<tr>
<th>$\kappa_{i,1} = 0$</th>
<th>4.06</th>
<th>3.53</th>
<th>3.33</th>
<th>3.31</th>
<th>3.39</th>
<th>3.54</th>
<th>3.77</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_{i,2}$</td>
<td>3.73</td>
<td>3.09</td>
<td>2.83</td>
<td>2.77</td>
<td>2.82</td>
<td>2.96</td>
<td>3.16</td>
</tr>
<tr>
<td>$\kappa_{i,3}$</td>
<td>3.42</td>
<td>2.73</td>
<td>2.40</td>
<td>2.27</td>
<td>2.28</td>
<td>2.37</td>
<td>2.55</td>
</tr>
<tr>
<td>$\kappa_{i,4}$</td>
<td>3.11</td>
<td>2.43</td>
<td>2.05</td>
<td>1.86</td>
<td>1.80</td>
<td>1.84</td>
<td>1.96</td>
</tr>
<tr>
<td>$\kappa_{i,5}$</td>
<td>2.81</td>
<td>2.19</td>
<td>1.78</td>
<td>1.53</td>
<td>1.41</td>
<td>1.39</td>
<td>1.46</td>
</tr>
<tr>
<td>$\kappa_{i,6}$</td>
<td>2.53</td>
<td>1.99</td>
<td>1.57</td>
<td>1.29</td>
<td>1.12</td>
<td>1.05</td>
<td>1.06</td>
</tr>
<tr>
<td>$\kappa_{i,7}$</td>
<td>2.28</td>
<td>1.82</td>
<td>1.40</td>
<td>1.10</td>
<td>0.91</td>
<td>0.80</td>
<td>0.76</td>
</tr>
<tr>
<td>$\kappa_{i,8}$</td>
<td>2.06</td>
<td>1.66</td>
<td>1.26</td>
<td>0.96</td>
<td>0.75</td>
<td>0.62</td>
<td>0.56</td>
</tr>
</tbody>
</table>
Table 3: The range for feasible debt capacity (coupon) under ambiguity aversion and market segmentation. This table shows the range for debt capacity (coupon) based on a grid of $\kappa$ (the row index) and $\sigma$ (the column index). Debt capacity is the the maximum amount a firm can borrow against its assets in place and is given by Equation (5.1). Debt capacity coupon is the argument such that the maximum in Equation (5.1) is obtained. A debt capacity (coupon) is feasible under a pair $(\kappa, \sigma)$, when $\kappa$ is in the positive leverage domain: $[0, \eta/\sigma]$. To find the range for feasible debt capacity (coupon) for each $\sigma_i$, we construct an equally spaced grid $\{\kappa_{i,j}\}_{j=1}^N$ on $[0, \eta/\sigma_i]$. Other parameter values are from Table 1.

<table>
<thead>
<tr>
<th>$(\kappa, \sigma_i)$</th>
<th>0.05</th>
<th>0.1</th>
<th>0.15</th>
<th>0.2</th>
<th>0.25</th>
<th>0.3</th>
<th>0.35</th>
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</thead>
<tbody>
<tr>
<td>$\kappa_{i,1} = 0$</td>
<td>106.55</td>
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<td>79.75</td>
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<td>77.92</td>
<td>77.83</td>
<td>78.05</td>
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<tr>
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<td>76.55</td>
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<td>76.87</td>
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<td>77.74</td>
</tr>
<tr>
<td>$\kappa_{i,8}$</td>
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<td>75.86</td>
<td>75.62</td>
<td>75.71</td>
<td>75.99</td>
<td>76.38</td>
<td>76.85</td>
</tr>
</tbody>
</table>

Panel A: Feasible debt capacity ($$)

| $\kappa_{i,1} = 0$ | 4.68 | 5.03 | 5.73 | 6.64 | 7.75 | 9.05 | 10.53 |
| $\kappa_{i,2}$     | 4.54 | 4.93 | 5.65 | 6.60 | 7.72 | 9.03 | 10.53 |
| $\kappa_{i,3}$     | 4.43 | 4.86 | 5.62 | 6.58 | 7.72 | 9.04 | 10.54 |
| $\kappa_{i,4}$     | 4.36 | 4.84 | 5.62 | 6.59 | 7.74 | 9.06 | 10.57 |
| $\kappa_{i,5}$     | 4.34 | 4.86 | 5.65 | 6.63 | 7.78 | 9.11 | 10.62 |
| $\kappa_{i,6}$     | 4.38 | 4.93 | 5.72 | 6.70 | 7.85 | 9.18 | 10.69 |
| $\kappa_{i,7}$     | 4.47 | 5.04 | 5.83 | 6.81 | 7.95 | 9.27 | 10.77 |
| $\kappa_{i,8}$     | 4.61 | 5.18 | 5.98 | 6.94 | 8.08 | 9.39 | 10.88 |

Panel B: Feasible debt capacity coupon ($$)
Table 4: **The range for feasible firm value under ambiguity aversion.** This table shows the range for feasible firm value based on a grid of $\kappa$ (the row index) and $\sigma$ (the column index). Firm value is debt value plus equity value, where the coupon argument takes the optimal value given by Equations (4.8) and (4.13), respectively. A firm value is feasible under a pair $(\kappa, \sigma)$, when $\kappa$ is in the positive leverage domain: $[0, \eta/\sigma]$. To find the range for feasible firm value for each $\sigma_i$, we construct an equally spaced grid $\{\kappa_{i,j}\}_{j=1}^N$ on $[0, \eta/\sigma_i]$. Other parameter values are from Table 1.

![Table 4](https://ssrn.com/abstract=3676905)
Table 5: The range for feasible optimal leverage (coupon) under ambiguity aversion and market integration. This table shows the range for feasible optimal leverage (coupon) based on a grid of $\kappa$ (the row index) and $\sigma$ (the column index). Optimal leverage is the ratio of debt over debt plus equity, where the coupon argument takes the optimal value given by Equation (4.13). An optimal leverage (coupon) is feasible under a pair $(\kappa, \sigma)$, when $\kappa$ is in the positive leverage domain: $[0, \eta/\sigma_i]$. To find the range for feasible optimal leverage (coupon) for each $\sigma_i$, we construct an equally spaced grid $\{\kappa_{i,j}\}_{j=1}^N$ on $[0, \eta/\sigma_i]$. Other parameter values are from Table 1.

<table>
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<tr>
<th>$(\kappa, \sigma_i)$</th>
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<th>0.1</th>
<th>0.15</th>
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<th>0.25</th>
<th>0.3</th>
<th>0.35</th>
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<td>$\kappa_{i,1} = 0$</td>
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<td>0.72</td>
<td>0.65</td>
<td>0.61</td>
<td>0.57</td>
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<td>0.53</td>
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<tr>
<td>$\kappa_{i,2} = 0.05$</td>
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<td>0.71</td>
<td>0.64</td>
<td>0.60</td>
<td>0.57</td>
<td>0.54</td>
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<td>0.59</td>
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<td>0.54</td>
<td>0.52</td>
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<td>$\kappa_{i,5} = 0.2$</td>
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<td>0.66</td>
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<td>0.57</td>
<td>0.55</td>
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<td>$\kappa_{i,8} = 0.35$</td>
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<td>0.54</td>
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Panel B: Feasible optimal coupon (\$)

<table>
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<tr>
<th>$(\kappa, \sigma_i)$</th>
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<th>0.1</th>
<th>0.15</th>
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<td>3.41</td>
<td>3.58</td>
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<td>3.51</td>
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<td>3.58</td>
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</table>
Table 6: The range for feasible debt capacity (coupon) under ambiguity aversion and market integration. This table shows the range for feasible debt capacity (coupon) based on a grid of $\kappa$ (the row index) and $\sigma$ (the column index). Debt capacity is the maximum amount a firm can borrow against its asset in place, and is given by Equation (5.1). However, in this case, we replace the default boundary in Equation (4.5) with the boundary given by Equation (4.12). Debt capacity coupon is the argument such that the maximum in Equation (5.1) is obtained. A debt capacity (coupon) is feasible under a pair $(\kappa, \sigma)$, when $\kappa$ is in the positive leverage domain: $[0, \eta/\sigma]$. To find the range for feasible debt capacity (coupon) for each $\sigma_i$, we construct an equally spaced grid $\{\kappa_{i,j}\}_{j=1}^N$ on $[0, \eta/\sigma_i]$. Other parameter values are from Table 1.

<table>
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<tr>
<th>$(\kappa, \sigma_i)$</th>
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<td>88.55</td>
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<td>87.37</td>
<td>86.67</td>
</tr>
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<td>94.99</td>
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<td>89.48</td>
<td>88.16</td>
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<td>86.81</td>
<td>86.28</td>
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<td>88.22</td>
<td>87.35</td>
<td>86.71</td>
<td>86.22</td>
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</tbody>
</table>

| Panel B: Feasible debt capacity coupon ($) |
| $\kappa_{i,1} = 0$   | 4.68  | 5.03  | 5.73  | 6.64  | 7.75  | 9.05  | 10.53 |
| $\kappa_{i,2}$       | 4.70  | 5.14  | 5.88  | 6.82  | 7.95  | 9.26  | 10.76 |
| $\kappa_{i,3}$       | 4.75  | 5.27  | 6.04  | 7.01  | 8.16  | 9.48  | 10.99 |
| $\kappa_{i,4}$       | 4.84  | 5.42  | 6.22  | 7.21  | 8.37  | 9.71  | 11.23 |
| $\kappa_{i,5}$       | 4.97  | 5.59  | 6.42  | 7.42  | 8.59  | 9.94  | 11.47 |
| $\kappa_{i,6}$       | 5.15  | 5.78  | 6.62  | 7.63  | 8.82  | 10.18 | 11.71 |
| $\kappa_{i,7}$       | 5.36  | 5.99  | 6.84  | 7.86  | 9.05  | 10.42 | 11.96 |
| $\kappa_{i,8}$       | 5.59  | 6.22  | 7.07  | 8.09  | 9.29  | 10.66 | 12.21 |
References


Electronic copy available at: https://ssrn.com/abstract=3676905


Appendix: Optimal default under ambiguity aversion and private negotiation

In our main analysis, we assume that default is settled through liquidation bankruptcy under absolute priority. However, liquidations are costly when accounting for direct and indirect costs associated with liquidation (Almeida and Philippon, 2007). In practice, investors often resort to alternative default settlement approaches, which avoid the deadweight losses due to liquidation. For example, default can be settled either through reorganization inside a bankruptcy court or through private negotiation outside.\footnote{The literature makes the distinction between the two types mainly through the allowed "grace" period before liquidation. In court-based reorganization, liquidation will take place after a finite grace period. In private negotiation, liquidation will never happen. See François and Morellec (2004) and Broadie et al. (2007) for discussions.} In this section, we extend our main model to a case where equity investors and debt investors settle default by debt-equity swap, a typical form of private negotiation.

For this debt-equity swap analysis, we maintain all the basic assumptions from Fan and Sundaresan (2000). In this case, equity and debt investors negotiate at an endogenously determined trigger point, where they stop operating the firm as an ongoing entity. After the negotiation, debt investors swap their debt for equity to become shareholders and sell the assets to outside investors with existing shareholders. The main implication is that the firm is not liquidated in this case, thereby the liquidation costs are not incurred by neither equity investors nor debt investors. Hence, optimal capital structure does not exist in this case, because there is only tax benefits of debt but no bankruptcy costs incurred.

We formulate the analysis as a Nash bargaining game in a similar fashion to Fan and Sundaresan (2000). Under a debt-equity swap, the firm becomes an all-equity firm, making equity value the same as asset value $A_t$. Hence, equity holders and debt holders bargain over the asset value at the optimal negotiation boundary $A_S$ as

$$
\Pi^e_S = \omega A_S, \quad \text{and} \quad \Pi^d_S = (1 - \omega) A_S.
$$

\footnote{12}{The literature makes the distinction between the two types mainly through the allowed "grace" period before liquidation. In court-based reorganization, liquidation will take place after a finite grace period. In private negotiation, liquidation will never happen. See François and Morellec (2004) and Broadie et al. (2007) for discussions.}
Here $\omega$ characterizes equity holders’ share in the distressed firm. Let $\beta \in [0, 1]$ denote equity holders’ bargaining power and $1 - \beta$ be debt holders’ bargaining power. Then relative to the case of liquidation, the gain to equity holders is $\omega A_S - 0$, because in liquidation, the absolute priority rule dictates that equity holders recover nothing. Similarly, the gain to debt holders is $(1 - \omega) A_S - (1 - \alpha) A_S = (\alpha - \omega) A_S$, which implies that when liquidation cost $\alpha$ is sufficiently high, debt holders can have strong incentives to engage in private negotiation.

The Nash solution to this bargaining game is

$$\omega^* = \arg \max_{0 \leq \omega \leq \alpha} (\omega A_S)\beta \cdot ((\alpha - \omega) A_S)^{1-\beta}$$

$$= \beta \alpha. \quad (A.2)$$

Equation (A.2) suggests that when equity investors have the full bargaining power ($\beta = 1$), they gain the whole liquidation cost $\alpha A_S$. When they have no bargaining power ($\beta = 0$), they gain nothing as in our benchmark case, but in this case, debt holders recover all the assets, a departure from the benchmark case.

Using these results, we can write equity value under ambiguity aversion at time $t$ prior to default as

$$E_t = A_t - (1 - q) \frac{C}{r} + \sup_{\tau} \inf_{Q^\theta \in Q} \mathbb{E}_{Q^\theta} \left[ e^{-r(\tau-t)} \left( (1 - q) \frac{C}{r} - (1 - \beta \alpha) A_S \right) \right] \left[ F_t \right]. \quad (A.3)$$

Similar to Proposition 1, we have here the worst-case measure to be $Q^{-\kappa}$. Also similar to Proposition 2, we have $C^* = 0$ when $\kappa > \frac{r-\mu}{\sigma}$. Next, when $\kappa \leq \frac{r-\mu}{\sigma}$, we have the optimal negotiation boundary to be

$$A^*_S = \frac{(1 - q) C}{r} \frac{\gamma_2^{-\kappa}}{\gamma_2^{-\kappa} - 1} \frac{1}{(1 - \beta \alpha)}. \quad (A.4)$$

Consistent with the result of Fan and Sundaresan (2000), the optimal negotiation boundary $A^*_S$ is higher than the optimal liquidation boundary $A^*_D$ by a factor $1/(1 - \beta \alpha)$, implying that default can occur earlier in this case.
Appendix: Proofs

Proposition 1. For all admissible stopping times, the worst-case probability measure for equity holders is $Q^{-\kappa}$, under which the asset value dynamics have the drift $\mu + \kappa \sigma$.

Proof. To utilize the standard results from backward stochastic differential equation (BSDE), we first restrict the time horizon to a fixed and finite time $T$ and derive the results. Then, we extend the results to infinite horizon.

In the infinite horizon, under any $Q^\theta \in \mathcal{Q}$ and default time $\tau$, equity value at time $t \leq \tau$ is

$$E^\theta_t = A_t - (1-q)\frac{C}{r} + \mathbb{E}^{Q^\theta} \left[ e^{-r(\tau-t)} \left( (1-q)\frac{C}{r} - A_\tau \right) \right] | \mathcal{F}_t].$$  \hspace{1cm} (B.1)

It is sufficient to focus only on the term that contains the conditional expectation, and we term it as $Y$. Namely,

$$Y^\theta_t = \mathbb{E}^{Q^\theta} \left[ e^{-r(\tau-t)} \left( (1-q)\frac{C}{r} - A_\tau \right) \right] \left( 1_{\tau \leq T} \right) | \mathcal{F}_t].$$  \hspace{1cm} (B.2)

Hence, for a finite horizon $T$, we have at $t \leq \tau \leq T$

$$Y^\theta_{t,T} = \mathbb{E}^{Q^\theta} \left[ e^{-r(\tau-t)} \left( (1-q)\frac{C}{r} - A_\tau \right) \right] 1_{\tau \leq T} \left( 1_{\tau \leq T} \right) | \mathcal{F}_t].$$  \hspace{1cm} (B.3)

Next, Proposition 2.2 of El Karoui et al. (1997) indicates that $Y^\theta_{t,T}$ or $(Y^\theta_{t,T}, Z^\theta_{t,T})$ is the unique solution of the following linear BSDE

$$-dY^\theta_{t,\tau} = -Z^\theta_{t,\tau} dW^\theta_t, \quad Y^\theta_{\tau,\tau} = e^{-r\tau} \left( (1-q)\frac{C}{r} - A_\tau \right) 1_{\tau \leq T},$$  \hspace{1cm} (B.4)

under $Q^\theta$. Proposition 2.5 of El Karoui et al. (1997) ensures the extension of the terminal condition based on stopping times. Then, the Girsanov Theorem implies that $Y^\theta_{t,\tau}$ also solves

$$-dY^\theta_{t,\tau} = -\theta_t Z^\theta_{t,\tau} dt - Z^\theta_{t,\tau} dW_t, \quad Y^\theta_{\tau,\tau} = e^{-r\tau} \left( (1-q)\frac{C}{r} - A_\tau \right) 1_{\tau \leq T},$$  \hspace{1cm} (B.5)

under the reference measure $Q$. 

38
Denote $Y^\theta_{t,\tau} = u^\theta(t, A_t)$. By Itô’s Lemma, we have
\[
du^\theta(t, A_t) = (u^\theta_t(t, A_t) + \mu A_t u^\theta_A(t, A_t) + \frac{\sigma^2 A_t^2}{2} u^\theta_{AA}(t, A_t)) dt + \sigma A_t u^\theta_A(t, A_t) dW_t, \tag{B.6}
\]
under $Q$. Hence, we have $Z^\theta_{t,\tau} = \sigma A_t u^\theta_A(t, A_t)$.

Next, since the payoff function $\xi_{\tau} = e^{-r\tau}((1-q)\frac{C^*}{r} - A_{\tau})1_{T \leq \tau}$ is decreasing in the $A$ argument, we have $Y^\theta_{t,\tau}$ decreasing in $A_t$ or $u^\theta_A(t, A_t) < 0$. Since $\sigma A_t > 0$, we have $Z^\theta_{t,\tau} < 0$.

Applying the Comparison Theorem of BSDE, Theorem 2.2 of El Karoui et al. (1997), we know that the minimum $Y^\theta_{t,\tau}$ in Equation (B.5) is obtained at $\theta_t = -\kappa$ for all $t$. Hence, we have found
\[
Y^{-\kappa}_{t,T} = \inf_{Q^\theta \in \mathcal{Q}} Y^\theta_{t,T}. \tag{B.7}
\]
Alternatively, we can directly use Theorem 2.2 of Chen and Epstein (2002) to find $Q^{\theta^*} = Q^{-\kappa}$. Lastly, since default is not relevant when $A_{\tau}$ is sufficiently large, $Y^{-\kappa}_{t,T}$ is finite. Since $Y^{-\kappa}_{t,T}$ is increasing in $T$, by the Monotone Convergence Theorem we have $\lim_{T \to +\infty} Y^{-\kappa}_{t,T} = Y^{-\kappa}_{t}$.

**Proposition 2.** If equity holders’ $\kappa > \frac{r - \mu}{\sigma}$, then $C^* = 0$, i.e., it is optimal to have zero leverage.

**Proof.** We prove the statement with contradiction by supposing $C^* > 0$.

Recall that under $Q^{-\kappa}$, for any default time $\tau > 0$, equity value right after debt issuance is
\[
E_0^{-\kappa} = \mathbb{E}^{Q^{-\kappa}} \left[ A_0 - (1-q)\frac{C^*}{r} + e^{-r\tau} \left( (1-q)\frac{C^*}{r} - A_{\tau} \right) \right] |\mathcal{F}_0]. \tag{B.8}
\]
We focus on the conditional expectation term that includes $\tau$ in the above and call it as
\[
u(A_0) = \mathbb{E}^{Q^{-\kappa}} [\xi(\tau, A_{\tau})] |\mathcal{F}_0], \tag{B.9}
\]
where $\xi(\tau, A_{\tau})$ denotes the payoff function at default.
Following Øksendal (2003), we introduce $B_t = (s+t, A_t)$. Hence $u(A_t)$ is a special case of $u_1(B_t) = E^{Q^{-\kappa}}[\xi(B_t)|\mathcal{F}_t]$ for $s = 0$. By Dynkin’s formula, we have

$$\mathbb{E}^{Q^{-\kappa}}[\xi(B_t)|\mathcal{F}_0] = \xi(s, A_0) + \mathbb{E}^{Q^{-\kappa}}\left[\int_0^T \mathcal{C}_B \xi(B_t) dl|\mathcal{F}_0\right],$$  \hspace{1cm} (B.10)

where $\mathcal{C}_B$, the characteristic operator of $B_t$, is

$$\mathcal{C}_B = \frac{\partial}{\partial s} + (\mu + \sigma \kappa)A \frac{\partial}{\partial A} + \frac{1}{2} \sigma^2 A^2 \frac{\partial^2}{\partial A^2}.$$  \hspace{1cm} (B.11)

Here we have replaced $\theta$ with $-\kappa$. The payoff function $\xi$ is given by

$$\xi(s, A) = e^{-rs} \left( (1-q) \frac{C^*}{r} - A \right).$$  \hspace{1cm} (B.12)

Hence, we have $\mathcal{C}_B \xi(s, A)$ as

$$\mathcal{C}_B \xi(s, A) = e^{-rs}((r - \mu - \sigma \kappa)A - (1-q)C^*).$$  \hspace{1cm} (B.13)

Therefore, (B.10) becomes

$$\mathbb{E}^{Q^{-\kappa}}[\xi(B_t)|\mathcal{F}_0] = \xi(s, A_0) + \mathbb{E}^{Q^{-\kappa}}\left[\int_0^T e^{-r(s+l)}((r - \mu - \sigma \kappa)A_l - (1-q)C^*) dl|\mathcal{F}_0\right].$$  \hspace{1cm} (B.14)

Accordingly, evaluating (B.14) at $s = 0$ and using (B.8) yield

$$E_0^{-\kappa} = \mathbb{E}^{Q^{-\kappa}}\left[\int_0^T e^{-rl}((r - \mu - \sigma \kappa)A_l - (1-q)C^*) dl|\mathcal{F}_0\right].$$  \hspace{1cm} (B.15)

So, if $r - \mu - \sigma \kappa < 0$ or $\kappa > \frac{r-\mu}{\sigma}$,

$$E_0^{-\kappa} < 0, \quad \text{for any } \tau > 0.$$  \hspace{1cm} (B.16)

This follows from the fact that $A_t$ can never be negative.
Hence, (B.16) contradicts limited liability that equity value cannot be negative. Thus, we have proven that \( C^* = 0 \), i.e., it is optimal to have zero leverage.

Proposition 3. If \( \kappa \leq \frac{r-\mu}{\sigma} \), for \( C > 0 \), equity value at time \( t \) under optimal default \( (E^*_t) \) given by Equation (3.1) is

\[
E^*_t = A_t - \frac{(1-q)C}{r} + \left( \frac{(1-q)C}{r} - A_D^* \right) \left( \frac{A_t}{A_D^*} \right)^{\gamma_2^{-\kappa}},
\]

where \( \gamma_2^{-\kappa} \) is

\[
\gamma_2^{-\kappa} = \sigma^{-2} \left[ \frac{1}{2} \sigma^2 - (\mu + \sigma \kappa) - \sqrt{(\mu + \sigma \kappa - \frac{1}{2} \sigma^2)^2 + 2r\sigma^2} \right],
\]

and \( A_D^* \) is

\[
A_D^* = \frac{(1-q)C}{r} \frac{\gamma_2^{-\kappa}}{\gamma_2^{-\kappa} - 1}.
\]

Proof. Since we have already found the \( Q^* \) for equity holders is \( Q^{-\kappa} \), then Equation (3.1) becomes

\[
E^*_t = \sup_{\tau} \mathbb{E}^{Q^{-\kappa}} \left[ A_t - (1-q) \frac{C}{r} + e^{-r(t-\tau)} \left( (1-q) \frac{C}{r} - A_{\tau} \right) \mid \mathcal{F}_t \right],
\]

which is a standard optimal stopping problem.

Again, we focus on the conditional expectation term that includes \( \tau \) in the above and call it as

\[
u(A_t) = \sup_{\tau} \mathbb{E}^{Q^{-\kappa}} \left[ \xi(\tau, A_{\tau}) \mid \mathcal{F}_t \right],
\]

where \( \xi \) denotes the payoff function at default. We introduce \( B_t = (s + t, A_t) \). Hence \( \nu(A_t) \) is a special case of \( u_1(B_t) = \mathbb{E}^{Q^{-\kappa}}[\xi(B_{\tau}) \mid \mathcal{F}_t] \) for \( s = 0 \).

Denote \( C_B \) be the characteristic operator of \( B_t \), and an open set \( D \) be the continuation region. Theorem 10.4.1 of Øksendal (2003) indicates that \( u_1(B) \) solves the following free-boundary problem

\[
C_B u_1(B) = 0, \quad B \in D,
\]
\[ u_1(B) = \xi(B), \quad B \in \partial D, \]  
(B.20)

and

\[ u_1(B) \in C^1(D \cup \partial D). \]  
(B.21)

Using the expressions for \( C_B, \xi(B) \) and \( D \), we can explicitly write the above as

\[
\left( \frac{\partial}{\partial s} + (\mu + \sigma \kappa)A \frac{\partial}{\partial A} + \frac{1}{2} \sigma^2 A^2 \frac{\partial^2}{\partial A^2} \right) u_1(s, A) = 0, \quad A \in (A_D, \infty),
\]  
(B.22)

\[ u_1(s, A_D) = e^{-rs}((1 - q) \frac{C}{r} - A_D), \]  
(B.23)

and

\[
\left. \frac{\partial}{\partial A} u_1(s, A) \right|_{A = A_D} = -e^{-rs}.
\]  
(B.24)

Note that here the horizon is infinite, hence the continuation region does not depend on the \( s \) argument.

Following standard approach, we can solve the above and find the solution as

\[ u_1(s, A) = e^{-rs} \left( (1 - q) \frac{C}{r} - A_D^* \right) \left( \frac{A}{A_D^*} \right)^{\gamma_2^\kappa}, \]  
(B.25)

where

\[ \gamma_2^\kappa = -\mu + \frac{1}{2} \sigma^2 - \sigma \kappa - \sqrt{2r \sigma^2 + (\mu - \frac{1}{2} \sigma^2 + \sigma \kappa)^2}, \]  
(B.26)

and

\[ A_D^* = \frac{(1 - q)C}{r} \frac{\gamma_2^\kappa}{\gamma_2^\kappa - 1}. \]  
(B.27)

Hence,

\[ u(A_t) = u_1(0, A_t) = \left( (1 - q) \frac{C}{r} - A_D^* \right) \left( \frac{A}{A_D^*} \right)^{\gamma_2^\kappa}. \]  
(B.28)

\[ \square \]

Lemma 1. Let \( \tau_D = \inf_{s > t} \{ A_s \leq A_D \} \) for \( A_D \in (0, A_t) \). Then, \( \tau_D \) is stochastically smallest.
under $Q^\kappa$ for all $Q^\theta \in Q$.

**Proof.** It is equivalent to show that on $\{\tau_D > t\}$ we have for all $t < u \leq T$, and for all $Q^\theta$

$$Q^\kappa[\tau_D > u] \leq Q^\theta[\tau_D > u]. \quad (B.29)$$

Since under $Q^\theta$, $W_t^\theta = W_t + \int_0^t \theta_s ds$ is a standard Brownian motion. By Definition, we have

$$A_u = A_t e^{\sigma(W_u - W_t) + (\mu - \sigma^2/2)(u-t)}$$

$$= A_t e^{\sigma(W_u^\theta - W_t^\theta) + (\mu - \sigma^2/2)(u-t) - \int_t^u \theta_s ds}.$$  \hspace{1cm} (B.30)

Given $\tau_D > t$, the event that we reach $A_D$ later than $u$ can be written as

$$\{\tau_D > u\} = \{v \in [t,u] : A_v \geq A_D\}$$

$$= \{v \in [t,u] : W_v^\theta - W_t^\theta - \int_t^v \theta_s ds \geq L\},$$ \hspace{1cm} (B.31)

where we write

$$L = \frac{1}{\sigma}(\log(A_D/A_t) - (\mu - \sigma^2/2)(v-t)).$$ \hspace{1cm} (B.32)

Since $\theta_s \leq \kappa$, this set contains

$$\{v \in [t,u] : W_v^\theta - W_t^\theta - \kappa(v-t) \geq L\}.$$ \hspace{1cm} (B.33)

Since $W_t^\theta$ is a standard Brownian motion under $Q^\theta$ and $W_t^\kappa = W_t + \kappa t$ a standard Brownian motion under $Q^\kappa$, we have

$$Q^\theta[v \in [t,u] : W_v^\theta - W_t^\theta - \kappa(v-t) \geq L|\mathcal{F}_t] = Q^\kappa[v \in [t,u] : W_v^\kappa - W_t^\kappa - \kappa(v-t) \geq L|\mathcal{F}_t].$$ \hspace{1cm} (B.34)
Hence, we have

\[
Q^\theta[\tau_D > u | \mathcal{F}_t] = Q^\theta[v \in [t, u] : W^\theta_v - W^\theta_t - \int_t^v \theta_s ds \geq L] \\
\geq Q^\theta[v \in [t, u] : W^\theta_v - W^\theta_t - \kappa(v - t) \geq L] \\
= Q^\kappa[v \in [t, u] : W^\kappa_v - W^\kappa_t - \kappa(v - t) \geq L | \mathcal{F}_t] = Q^\kappa[\tau_D > u | \mathcal{F}_t].
\]

(B.35)

**Proposition 4** When \( \kappa \leq \frac{r - \mu}{\sigma} \), anticipating equity holders’ default policy \( A^*_D \), debt holders’ valuation of debt at time \( t \) prior to default is

\[
D^\kappa_t = C + \left( (1 - \alpha) A^*_D - \frac{C}{r} \right) \left( \frac{A_t}{A_D} \right) \gamma^\kappa_2,
\]

where \( \gamma^\kappa_2 < 0 \) is given by

\[
\gamma^\kappa_2 = \sigma^{-2} \left[ \frac{1}{2} \sigma^2 - (\mu - \sigma \kappa) - \sqrt{(\mu - \kappa \sigma - \frac{1}{2} \sigma^2)^2 + 2 r \sigma^2} \right].
\]

Proof. Following Lemma 1, we know immediately by the first-order stochastic dominance that

\[
E^{Q^\kappa} \left[ e^{-r(\tau-t)} \left( (1 - \alpha) A^*_D - \frac{C}{r} \right) | \mathcal{F}_t \right] \leq E^{Q^\theta} \left[ e^{-r(\tau-t)} \left( (1 - \alpha) A^*_D - \frac{C}{r} \right) | \mathcal{F}_t \right].
\]

(B.36)

Note that \( (1 - \alpha) A^*_D - \frac{C}{r} < 0 \), based on the expression of \( A^*_D \). Hence, we have \( D^\kappa_t = \inf_{t} D^\theta_t \).

Since under \( Q^\kappa \), the drift of \( A_t \) is \( \mu - \kappa \sigma \), we can use the formula for the Laplace transform of the first-passage time to have

\[
E^{Q^\kappa} [e^{-r(\tau-t)} | \mathcal{F}_t] = \left( \frac{A_t}{A_D} \right) \gamma^\kappa_2,
\]

where

\[
\gamma^\kappa_2 = \sigma^{-2} \left[ \frac{1}{2} \sigma^2 - (\mu - \sigma \kappa) - \sqrt{(\mu - \kappa \sigma - \frac{1}{2} \sigma^2)^2 + 2 r \sigma^2} \right].
\]

(B.37)

(B.38)