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Manuscript title: Modelling the soil desiccation cracking by peridynamics

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Abstract

Understanding the desiccation-induced cracking in soil has been improved over the last 20-30 years through experimental studies but the progress in predictive modelling of desiccation cracking has been limited. The heterogeneous structure of soils and the multi-physics nature of the phenomenon, involving emergence and propagation of discontinuities, make the mathematical description and analysis a challenging task. We present a non-local hydro-mechanical model for soil desiccation cracking capable of predicting crack initiation and growth. The model is based on the peridynamics (PD) theory. Attempts to model the soil desiccation cracking by PD are limited to a purely mechanical description of the process that involves calibration of the parameters. Differently, the model presented in this paper describes soil desiccation cracking as a hydro-mechanical problem, where moisture flow and deformation are coupled. This allows for investigating and explaining the mechanisms controlling the initiation and propagation of discontinuities. The model is applied and tested against two sets of experimental data to explain the typical features of drying-induced cracking of clays. The validations use experimental parameters (Young’s modulus, water retention characteristics) and avoid calibrations to test the accuracy of the model. We demonstrate the correlations between the shrinkage of soil clay, changes in displacement fields and crack growth. Crack initiation, propagation and ultimate crack patterns simulated by the model are found to be in very good agreement with experimental observations. The results show that the model can capture realistically key hydraulic, mechanical and geometry effects on clay desiccation cracking.

Keywords: clays; coupled problems; discontinuity; mathematical modelling; numerical modelling; peridynamics; shrinkage; shrinkage cracking; soil desiccation
1. Introduction

Soil desiccation is characterised by loss of moisture which can lead to shrinkage and potentially to cracking. Describing and analysing this phenomenon is challenging because it involves strongly coupled processes of moisture flow, deformation, emergence and evolution of cracks in a complex, heterogeneous soil structure, but is important for many applications. For example, desiccation accelerates infiltration in road embankments and soil slopes (Jamalinia et al., 2020) and causes anti-seepage failures of landfill barriers (Wan et al., 2018). Further, desiccation cracking can damage local ecologies as cracks increase the surface roughness and accelerate evaporation in soil, contributing to faster soil weathering and erosion leading to potential dust storms, landslides and debris flow (Zeng et al., 2020). Additionally, desiccation cracks can create pathways for water and gas transport, which will reduce the performance of clay barriers used for waste isolation (Wan et al., 2018; DeCarlo and Shokri, 2014).

The understanding of desiccation cracking in clay soils has progressed in the last few decades through experimental investigations and theoretical developments. Desiccation has been investigated in slurries (Li and Zhang, 2011), clay soils (Tang et al., 2011; Wei et al., 2016) and starch water mixture cakes (Kodikara et al., 2004). Desiccation cracking experiments have been focused on characterising the cracks’ geometries including depth, thickness, spacing and aperture of the cracks (Najm et al., 2009; Péron et al., 2007 & 2009a; Wan et al., 2018; Amarisiri et al., 2014; Zhao et al., 2020; Xu et al., 2020). The results of these studies have contributed to understanding of the processes, but their relevance is limited to the specific laboratory conditions (Tran et al., 2019). Predicting desiccation cracking under field conditions requires physically realistic models that capture the complex and coupled processes involved. The aim of this paper is to address the need for realistic modelling of soil desiccation.

Numerical methods for solving continuum physical or mechanical problems, such as the finite element method (FEM), cannot capture crack initiation without the introduction of special elements. These are typically cohesive elements inserted at the interfaces between continuum elements. They require specific calibration of their behaviour and predetermine crack initiation locations and paths.
Modelling crack propagation also requires specific calibration of a crack extension criterion, which is expensive and typically not universally valid. For example, calibration with experimental data for one specific stress state may not correctly predict the behaviour for other stress states. Some of the difficulties of FEM-based methods can be avoided by discrete methods, such as lattice methods (Zhang and Jivkov, 2016; Morrison et al., 2016; Dassios et al., 2018), lattice spring methods (Zhu et al., 2020) and discrete element methods (Amarasiri and Kodikara, 2013; Sima et al., 2014; Tran et al., 2020). Alternative approaches, including phase-field methods (Cajuhi et al., 2018) and smoothed-particle hydrodynamics (Tran et al., 2019), have also been adopted for soil desiccation problems. However, these non-FEM-based methods inherit a requirement from FEM – introduction and calibration of failure initiation and propagation criteria – dictated by the local (differential) nature of the underlying governing equations (Wang et al., 2017 & 2019).

Nonlocal formulations have been developed to address problems with discontinuities, large deformations and heterogeneities (Eringen and Edelen, 1972; Bažant and Chang, 1984). A recent non-local theory (Silling, 2000), referred to as the Peridynamics (PD), is receiving increasing interest among the mechanics community, as it overcomes the limitations of the classical theory particularly for problems with discontinuities. PD avoids calculations of spatial derivatives, which are mathematically undefined at sharp discontinuities such as cracks or shocks (Kilic and Madenci, 2009a; Oterkus et al, 2014). This allows for developing PD formulations for all physical phenomena beyond the original applications to solids mechanics proposed by Silling (2000). Such formulations remain mathematically consistent irrespective of the emergence and evolution of discontinuities. The PD theory has been applied in different areas, including fluid driven fractures (Ouchi et al., 2015; Zhou et al., 2020), thermal fracture (Oterkus et al., 2014; Wang et al., 2018b; Kilic and Madenci, 2009b), clay erosion (Sedighi et al., 2021) and pitting corrosion damage (Jafarzadeh et al., 2019). This paper presents the development of PD formulations for coupled hydro-mechanical processes to predict the desiccation cracking of soils. In Section 2 we present our PD formulations for water flow and mechanical deformation, including crack initiation and growth. In Section 3 we present...
validations by comparison of our predictions with three different soil desiccation tests and discuss the
results. Conclusions are summarised in Section 4.

2. Hydro-mechanical formulations of soil desiccation cracking

2.1. Classical (local/differential) formulation of soil deformation induced by drying

Soil desiccation results from the loss of moisture. As the water content in clay decreases, the suction
increases (total soil water potential decreases), leading to soil shrinkage. This is appropriately
described by Biot’s poroelasticity (Biot, 1941), which relates the stresses, strains and fluid mass or
pore pressure change (Péron et al., 2013). The total strain increment is

$$\text{d} \varepsilon_{ij} = \text{d} \varepsilon^m_{ij} + \text{d} \varepsilon^h_{ij}$$  \hspace{1cm} (1)

where $\varepsilon_{ij}$ are the components of the total strain tensor, $\varepsilon^m_{ij}$ are the components of the elastic strain
tensor work-conjugate to the mechanical stresses, and $\varepsilon^h_{ij}$ are the components of a strain tensor
arising from water content changes (hydration/dehydration) and referred to as the hydric strain tensor.
This is volumetric, i.e., $\varepsilon^h_{ij} = 0$, $i \neq j$, irrespective of coordinate system selection.

The mechanical stress increment is

$$\text{d} \sigma_{ij} = D_{ijkl} \text{d} \varepsilon^m_{kl} = D_{ijkl} (\text{d} \varepsilon_{kl} - \text{d} \varepsilon^h_{kl})$$  \hspace{1cm} (2)

where $D_{ijkl}$ are the components of the elasticity tensor of the solid material.

Assuming the response of soil to changes in water content is isotropic, the hydric strain tensor
increment can be expressed by (Péron et al., 2009a)

$$\text{d} \varepsilon^h_{kl} = \delta_{kl} \alpha \text{d} \theta$$  \hspace{1cm} (3)

where $\delta_{kl}$ is the Kroneker delta, $\text{d} \theta$ represents the water content change, and $\alpha$ is a shrinkage
coefficient, which can be represented as a function of void ratio $e$ and specific gravity $G_s$ of the soil
(Péron et al., 2009a)

$$\alpha = \frac{G_s}{1 + e}$$  \hspace{1cm} (4)

We assume that drying by moisture loss is a diffusion-like phenomenon. We neglect the effects of
mechanical deformation on the water transport due to the small mechanical strains involved (Péron et
al., 2009b). The mass conservation of water is described by Yan et al. (2020)

$$\frac{\partial \theta}{\partial t} = \nabla \cdot [D \nabla \theta]$$  \hspace{1cm} (5)
where, \( t \) is time and \( D \) is the diffusion coefficient of water in soil.

Three distinct stages can be identified during soil desiccation (Péron et al., 2009b; Tang et al., 2011). Stage I is characterised by shrinkage but the soil remains saturated. In Stage II air gradually replaces moisture in the soil pore system but the shrinkage is smaller than in Stage I. Stage III is characterised by continued loss of moisture, but without further shrinkage. We will limit our desiccation model to Stage I (Maarry et al., 2012) where significant shrinkage is expected. The implication of this decision is that the diffusion coefficient of water will be considered as constant.

2.2. Peridynamic formulations of coupled deformation and moisture flow

In peridynamics, a body occupying a region \( R \) is considered to be a collection of peridynamic particles, which can be placed in a regular grid, as illustrated in Fig. 1(a), or in any other required arrangement. One of the two classical PD approaches – the bond-based PD, which we utilise here – represents the interactions between the peridynamic particles by ‘bonds’. A particle at position \( \mathbf{x} \) interacts with (is connected to) all particles at positions \( \mathbf{x}' \) within a certain finite region \( H_x \), called the horizon of the particle at \( \mathbf{x} \). The horizon radius is denoted by \( \delta \). Further, \( \rho(\mathbf{x}) \) and \( \theta(\mathbf{x}) \) denote the soil density and water content at position \( \mathbf{x} \), respectively. The bond vector between particles at \( \mathbf{x} \) and \( \mathbf{x}' \) is \( \mathbf{\xi} = (\mathbf{x}' - \mathbf{x}) \), the change of water content along this bond is \( \mathbf{z} = \theta(\mathbf{x}') - \theta(\mathbf{x}) \), and the average bond water content is \( \bar{\theta} = [\theta(\mathbf{x}') + \theta(\mathbf{x})]/2 \).

As the body deforms, a particle at \( \mathbf{x} \) is displaced by \( \mathbf{u}(\mathbf{x}) \) and the relative displacement between particles at \( \mathbf{x} \) and \( \mathbf{x}' \) is \( \mathbf{\eta} = \mathbf{u}(\mathbf{x}') - \mathbf{u}(\mathbf{x}) \). The deformed configuration is illustrated in Fig. 1(b). Note, that \( \mathbf{\eta} + \mathbf{\xi} \) represents the current distance vector between particles. With these notations the quantities \( \epsilon \) and \( \mathbf{n} \) represent a micro-strain (relative change of bond length) and a unit vector along the bond between particles at \( \mathbf{x} \) and \( \mathbf{x}' \) in the deformed configuration, respectively:

\[
\epsilon = \frac{||\mathbf{\eta} + \mathbf{\xi}|| - ||\mathbf{\xi}||}{||\mathbf{\xi}||} \quad (6)
\]

\[
\mathbf{n} = \frac{\mathbf{\eta} + \mathbf{\xi}}{||\mathbf{\eta} + \mathbf{\xi}||} \quad (7)
\]

The micro-strain includes mechanical and hydric contributions, so that \( \epsilon = \epsilon_m + \epsilon_h \), where \( \epsilon_h = a \bar{\theta} \).
The peridynamic version of the conservation of linear momentum is the equilibrium of forces acting on a particle at position $\mathbf{x}$:

$$\rho(\mathbf{x})\ddot{\mathbf{u}}(\mathbf{x}) = \int_{H_x} f(\xi, \eta) \, dV_x + \mathbf{b}(\mathbf{x}) \quad (8)$$

where $\ddot{\mathbf{u}}(\mathbf{x})$ is the acceleration of the particle at $\mathbf{x}$, $f(\xi, \eta)$ is a pairwise force density vector representing the interaction of the particle at $\mathbf{x}$ with the particles in its horizon, $V_x$ is the horizon volume (area in 2D, length in 1D) of the particle at $\mathbf{x}$, and $\mathbf{b}(\mathbf{x})$ is a body force on the particle at $\mathbf{x}$.

A micro-elastic material (Silling 2000) is defined by postulating a micro-elastic potential, $\omega(\xi, \eta)$, from which the pairwise force density is derived by

$$f(\xi, \eta) = \frac{\partial \omega(\xi, \eta)}{\partial \eta} \quad (9)$$

We consider a linear micro-elastic material, for which the micro-elastic potential is a quadratic function of the mechanical micro-strain, similarly to continuum elasticity

$$\omega(\xi, \eta) = \frac{1}{2} C_p(\mathbf{x}, \mathbf{x}') \epsilon_m^2 \quad (10)$$

where $C_p(\mathbf{x}, \mathbf{x}')$ is a peridynamic micro-modulus. Considering that $\epsilon_m = \epsilon - \alpha \bar{\theta}$, this gives the following expression for the force density vector

$$f(\xi, \eta) = C_p(\mathbf{x}, \mathbf{x}') (\epsilon - \alpha \bar{\theta}) \mathbf{n} \quad (11)$$

The relationships between $C_p(\mathbf{x}, \mathbf{x}')$ and the macroscopic material properties have been proposed by Oterkus et al. (2014); for 2D problems considered here this reads:

$$C_p(\mathbf{x}, \mathbf{x}') = \frac{9E}{\pi \delta^3} \quad (12)$$

where $E$ is the Young’s modulus of the soil; details of derivation of Eq. (12) are provided in supplementary material (S1).

The peridynamic version of the conservation law for water content at position $\mathbf{x}$ is given by (Yan et al., 2020)

$$\frac{\partial \theta(\mathbf{x})}{\partial t} = \int_{H_x} g(\xi, \zeta) \, dV_x \quad (13)$$

where, $g(\xi, \zeta)$ is the pairwise mass exchange function. This is given by
where \( D_p(x, x') \) is a peridynamic micro-diffusivity (micro-transport coefficient). The relation between \( D_p(x, x') \) and the macroscopic diffusivity \( D \) for 2D moisture flow is given by Yan et al. (2020) and Sedighi et al. (2021)

\[
D_p(x, x') = \frac{4D}{\pi\delta^2}
\]

Details of derivations of Eq. (15) are provided in supplementary material (S2).

### 2.3. Failure initiation and damage parameter

The bond between particles at \( x \) and \( x' \) can be equipped with an indicator function \( \mu(x, x') \) that encodes the state of the bond as either being intact or broken (Wang et al., 2018a; Menon and Song, 2019). Mathematically, the indicator \( \mu(x, x') \) is a Heaviside step function of the difference between a critical value of the micro-strain, \( \varepsilon_f \), and the mechanical micro-strain in the bond \( \varepsilon_m \), as follow:

\[
\mu(x, x') = \begin{cases} 
1 & \varepsilon_m < \varepsilon_f \\
0 & \varepsilon_m \geq \varepsilon_f 
\end{cases}
\]

This means that a bond is broken when the micro-strain exceeds the critical value. The critical micro-strain \( \varepsilon_f \) can be related to the fracture energy release rate of the material \( G_f \), measured in \( N/m \), which is the energy required for creating a unit of new surface area (Silling and Askari, 2005; Lehoucq and Silling, 2008). For example, the critical micro-strain for plane stress problem is related to the energy release rate by

\[
\varepsilon_f = \frac{4}{9E\delta} G_f
\]

Failure of bonds associated with particle at \( x \) redistributes the forces on this particle to the remaining intact bonds within its horizon (Gu et al., 2017). The change experienced by the particle can be described by a damage parameter \( \varphi(x) \), defined as the ratio of the number of broken bonds and the number of all initially present bonds in its horizon (Kilic and Madenci, 2009a and 2009b)

\[
\varphi(x) = \frac{\int_{H_x} \mu(x, x') \, dV_x}{\int_{H_x} \, dV_x}
\]
The damage ranges from zero, when all bonds associated with particle at \( x \) are intact, to one, when all bonds are broken.

2.4. Numerical implementation

The PD formulation for coupled water flow and mechanical deformation with cracking is summarised by

\[
\frac{\partial \theta(x)}{\partial t} = \int_{H_x} \mu(x,x') D_p(x,x') \frac{\theta(x') - \theta(x)}{||\xi||^2} \text{d}V_x \tag{19}
\]

\[
\rho(x) \ddot{u}(x) = \int_{H_x} \mu(x,x') C_p(x,x') (\varepsilon - \alpha \dot{\theta}) \text{d}V_x + b(x) \tag{20}
\]

The spatial discretization of Eqns. (19) and (20), using the mid-point rule is (Yan et al., 2020)

\[
\frac{\partial \theta(x_i)}{\partial t} = \sum_p \mu(x_i,x_p) D_p(x_i,x_p) \frac{\theta(x_p) - \theta(x_i)}{||x_p - x_i||^2} V_{ip} \tag{21}
\]

\[
\rho(x) \ddot{u}(x) = \sum_p \mu(x_i,x_p) C_p(x_i,x_p) \left[ \varepsilon(x_i,x_p) - \alpha \dot{\theta}(x_i,x_p) \right] V_{ip} + b(x) \tag{22}
\]

Iterative coupling approaches have been used in multiphysics problems (e.g. Sedighi et al. 2018).

Time integration of the system is performed by a weakly coupled method which deals with large displacements and evolving discontinuities. This method has been introduced in PD to solve thermo-mechanical problems (Wang et al., 2019; Antuono and Morandini, 2017). It uses different time stepping strategies for different physical or mechanical processes. Details of our implementation are provided in Supplementary material (S3).

3. Results and discussion

We have selected two sets of experimental data from literature (Najm et al., 2009; Péron et al., 2009a and 2013) to validate the model. The first applications, presented in sub-section 3.1, deal with the dessication cracking of clay rings of different geometries. The second application, presented in sub-section 3.2, deals with the dessication cracking of a long clay bar. The data reproduced and replotted from the literature is obtained by using the function of Digitizer in Origin Pro 2016 although there may have been slight deviations as the results of digitalisation.
3.1. Desiccation cracking of clay rings

Restrained ring tests are convenient for studying soil desiccation cracking as they allow for obtaining clearly isolated cracks (Najm et al., 2009). In these tests, a soil sample with inner and outer radii $R_{is}$ and $R_{os}$, respectively, is placed around a steel ring with inner and outer radii $R_{ir}$ and $R_{or}$, respectively (i.e., the sample thickness is $R_{or} - R_{ir}$). The sample is not subjected to external mechanical loads, but experiences dimensional changes (shrinkage) induced by drying. The shrinkage is constrained by the steel ring, leading to the development of stresses and strains in the ring and in the sample. If the strain at the inner surface of the steel ring is recorded by strain gauges, the hoop, $\sigma_{\phi}(r)$, and radial, $\sigma_r(r)$, stresses in the soil can be calculated analytically as functions of radial distance, $r$, (Najm et al., 2009):

\[
\sigma_{\phi}(r) = \varepsilon_r E_r \left( \frac{R_{os}^2 - R_{ir}^2}{2R_{or}^2} \right) \left( \frac{R_{is}^2}{R_{os}^2 - R_{ir}^2} \right) \left( 1 + \frac{R_{os}^2}{r^2} \right)
\]

\[
\sigma_r(r) = \varepsilon_r E_r \left( \frac{R_{os}^2 - R_{ir}^2}{2R_{or}^2} \right) \left( \frac{R_{is}^2}{R_{os}^2 - R_{ir}^2} \right) \left( 1 - \frac{R_{os}^2}{r^2} \right)
\]

where $\varepsilon_r$ is the strain at the inner wall of the steel ring and $E_r$ is Young’s modulus of the steel.

Notably, the hoop stress is tensile, while the radial stress is compressive as the last term in Eq. (24) is negative at all locations in the sample. The analytical solution provided in Eqns. (23) and (24) is applicable to homogeneous isotropic linear elastic materials, hence approximate for the real experimental setup.

We consider two sets of experimental data obtained with constrained ring tests for model validation. These tests are illustrated in Fig. 2 (Najm et al., 2009) and their geometrical characteristics and the steel ring elastic modulus are given in Table 1. The figure shows the effect of test geometry on the cracking behaviour. The ring with the smaller inner radius, $R_{is} = 2.11$ cm, and smaller thickness, $3.19$ cm, referred to as Test 1, developed a single crack. The ring with the larger inner radius, $R_{is} = 15$ cm, and larger thickness, $7$ cm, referred to as Test 2, developed multiple cracks. The initial water content in Test 1 was 44.95%. In this validation round, we aim to assess the accuracy of the model to reproduce such effects.
In the experiments water was free to evaporate, evaporation rate was recorded, and evaporation was assumed to be uniform across the sample. Cracking was observed when the soil moisture content was reduced to around 35.9%. The tensile stress in the sample, calculated at the moment of crack initiation, was 65.9 kPa. This was taken to be the tensile strength, $\sigma_t$, of the soil. The parameters measured during Test 1, including moisture content, evaporation rate, and strain at inner steel surface, are not available for Test 2. Therefore, we assume that they are the same as in Test 1 for our validations.

For the simulation of Test 1, the soil sample was represented by a ring of inner and outer radii of 2.1 cm and 5.3 cm, respectively, populated by PD particles. The spacing between particles was $\Delta x = 0.5$ mm. An important parameter for the numerical implementation of PD is the non-local ratio $m = \delta/\Delta x$. As described by Dipasquale et al. (2014) the dependence of the crack path on the particle spacing is reduced by selecting an appropriate $m$ in the range 5-8. We used $m = 5$ and $\delta = 2.5$ mm. Young’s modulus of the soil was assumed to remain constant $E_i = 1.2$ MPa (Jabakhanji, 2013). We note that the experiments indicated changes of Young’s modulus with decreasing moisture content of clay, but consider our assumption as a reasonable approximation for the purposes of this study. Based on the experiments (Najm et al., 2009), a constant shrinkage coefficient $\alpha =0.065$ was adopted for this sub-section. The fracture energy release rate was taken from Jabakhanji (2013): $G_0 = 5.7N/m$. We note that this parameter has been reported in the literature in the range 1-10 N/m. The clay heterogeneity was accounted for by considering that the fracture energy release rate followed a Weibull distribution with shape parameter 20 (Tang et al., 2016; Oterkus and Madenci, 2017). The inner boundary of the soil was constrained, i.e., Dirichlet boundary condition was equal to zero which was achieved by adding a layer of fixed PD particles with a thickness equal to the horizon, $\delta$ (Oterkus et al.,2014). This neglects the deformability of the steel ring, but is a reasonable approximation considering the large difference in the elastic moduli. The outer boundary of the soil was left free, i.e., Neumann boundary condition equal to zero. The moisture content in the soil was reduced by assuming a continuous uniform water loss, $1.43 \times 10^{-3}$ wt. (%) /s, across the model at each time step. The total number of time steps was $10^5$ with a time step of 1 s.
Figure 3 shows a comparison between the hoop stress $\sigma_{\theta}(r)$ at $r = 2.1\text{cm}$ (inner soil surface), calculated from our PD simulations and the hoop stress calculated by Eq. (23) using the gauge strain recorded during the experiments. There is a good agreement between the experimental and modelling values. Observable differences can be attributed to two factors: (i) Young’s modulus of soil is changing with moisture content; and (ii) the clay is not a homogeneous isotropic linear elastic material for which Eq. (23) is strictly applicable. We can influence only the first factor, and this is a subject of ongoing work.

Figure 4 shows a comparison between the hoop stress $\sigma_{\theta}(r)$ along the radius, calculated by the PD simulations ($\theta = 36\%, 39\%$ and $42\%$) and those reported from the experimental data ($\theta = 36\%$). The curve at $\theta = 36\%$ corresponds to the moisture content at which the crack initiation was experimentally observed. The close agreement of the calculated and experimental values provides confidence that the peridynamic simulations predict accurately the stress distribution during desiccation under constrained shrinkage.

Figure 5 shows the crack evolution from moisture content $\theta=35\%$ to the eventual separation of the ring at $\theta=30.7\%$. This is illustrated by contours of the: (1) hoop displacement, $u_\theta$, showing the displacement jumps across the growing crack; (2) radial displacement, $u_r$, showing the soil shrinkage; and (3) damage parameter, marking the crack path.

The results in Fig 5 indicate a correlation between soil shrinkage and crack growth. Variations of strains and stresses as crack propagates are clearly visible. The hoop stress gradient causes the crack to emerge at the inner surface and to extends towards the outer surface. The crack path deviates from a straight line due to the heterogeneity introduced in the model by the Weibull distribution of the fracture energy release rate.

Figure 6 shows a comparison between the predicted final crack and the experimental outcome, including the damage contour from the PD simulations and a geometric representation of the ring that facilitates comparison with the experimental figure. There is a close agreement between the predicted and observed configurations, including the non-straight crack path due to soil heterogeneity.
The simulation was repeated for Test 2 with the corresponding change of ring geometry. Figure 7 shows the final cracked configuration (numerical prediction and experimental result) for Test 2. It can be seen that the multiple cracks observed experimentally in this thicker specimen with larger inner radius are correctly predicted by the model. The emergence of multiple cracks can be explained by the larger inner radius in Test 2. One crack in Test 1 appears to be sufficient for relaxing the hoop stress along the entire inner soil surface to levels that do not allow for damage to initiate elsewhere. In contrast, the emergence of one crack in Test 2 does not relax the hoop stress along the inner entire soil surface, which allows for initiation of multiple cracks, approximately equally spaced. This is an interesting outcome, which is affected not only by the size of the inner surface but also by the thickness of the soil ring and deserves further investigation. Presently, we note the excellent agreement between the predicted and experimentally observed cracks in terms of their number and approximate spacing.

3.2. Desiccation cracking of clay bars

A series of clay (Bioley silt) desiccation tests on long bars were conducted by Péron et al. (2009a) to study the mechanism of desiccation cracking induced by swelling/shrinkage processes. In these experiments, a clay slurry at a gravimetric water content of 1.5 times the liquid limit has been subject to drying. The clay slurry was poured into a plate mold with dimensions 295mm(length)×49mm(width)×12mm(height). The sample was not subjected to mechanical loads. The bottom of the sample was constrained by a notched metallic base. The initial water content of sample was 50% (Péron et al., 2013). Experimental results showed that the water content decreased linearly in the initial drying stage (lasting approximately 2000 min), which corresponded to a constant water evaporation rate of approximately 3 g/h, and accompanied by a decrease of void ratio. This was followed by a drying stage with non-linear water content reduction due to reaching a structural lower limit of the void ratio of approximately 0.4. Our simulations aimed to reproduce the initial drying stage.

To facilitate comparison with our PD results, the experimentally observed desiccation cracking at $t = 16.5h, 17.2h$ and $18h$, and the final crack pattern are shown in Fig. 8. These cover moments from the
first crack observation at $t = 16.5\text{h}$, through the middle of the linear regime at $t = 18\text{h}$, to the final pattern realised prior to the end of the linear regime.

The PD model was created using a 2D domain of 295mm(length)$\times$12mm(height), populated with PD particles. The spacing and horizon values were identical to the ones in Section 3.1. The material properties are listed in Table 2. Young’s modulus of clay $E = 32 \text{MPa}$ was taken from Péron et al. (2007), where it was determined by triaxial compression tests. The fracture energy release rate $G_0 = 1.8\text{N/m}$ was taken from Ayad et al. (1997), and as in Section 3.1 this material property was assumed to follow Weibull distribution with shape parameter 20 in order to represent the heterogeneity of clay. The drying shrinkage coefficient $\alpha = 1.56 \times 10^{-2}$ was taken from the shrinkage experiments by Péron et al. (2013). The diffusivity of moisture in clay $D = 2.8 \times 10^{-10}$ m$^2$/s was taken from Hirobe and Kenji (2017).

The displacements at the bottom side of the model were fixed, i.e. Dirichlet boundary condition equal to zero, to represent the effect of the notched metallic base, while the other three sides were left free, i.e. Neumann boundary condition equal to zero. Moisture flow was prohibited through the bottom, right and left sides of the sample, i.e. Neumann boundary condition equal to zero, while the top of the sample was subjected to the experimentally measured evaporation rate of 3 g/h (Péron et al., 2013). Figures 9-12 present the results for water content, horizontal displacement, vertical displacement and damage parameter at four instances: $t=17\text{h}$, 18.3h, 20.8h and 25h. Based on experimental observations, soil desiccation has been described by three main phases: (i) evaporation of moisture from the sample surface due to water potential difference between soil and atmosphere; (ii) water flow from the interior to the surface due to pore-water pressure gradient created in phase (i); and (iii) shrinkage of the sample with decreasing water pressure leading to increasing internal compression of the matrix/soil aggregate (Coussy et al., 1998; Kowalski 2003). The results presented show that the proposed PD model captures these main phases with acceptable accuracy. The surface water content decreases as the evaporation increases with time (see Fig. 9). This leads to a hydraulic gradient between the interior and the surface and moisture flow from the bottom to the top of the sample, with a subsequent reduction of the overall water content in the sample with time (Fig. 9). The contours of
water content are not uniform due to the emergence and growth of cracks, which disrupt fluid flow locally (Yan et al., 2020). The shrinkage of the sample with increasing evaporation is captured by the displacement plots in Figs. 10 and 11. The horizontal shrinkage is represented by the opposite horizontal displacements at the two ends of the sample in Fig. 10, while the maximal vertical shrinkage is observed between the growing cracks in Fig. 11.

Figure 8 shows that the desiccation cracking generally starts from the top of the soil sample. The PD results Fig. 12 are consistent with this observation. For example, the first substantial crack in the PD model is observed at $t=17h$ and starts from the top, which is very close to the initiation of cracking recorded in the experiment at 16.5h. The second substantial crack is observed at $t=18.3h$, followed by a number of further cracks developing as the desiccation process proceeds. The final crack pattern is also close to the experimental observation. For example, the total number of primary cracks developed in experiments was between 7 and 9 (Péron et al., 2009a) and our simulations lead to 8 primary cracks (Fig. 12). The spacings between the developed cracks differ between model and experiment, which is to be expected considering the soil heterogeneity. The number of cracks and the average spacing, however, should depend on the length and thickness of the specimen, similarly to the case considered in Section 3.1, and this dependence deserves further investigation.

4. Conclusions

A model for analysis of soil desiccation cracking based on peridynamics was presented. It integrated bond-based formulations of moisture flow and mechanical deformation/failure. The coupled formulation was implemented within a computational framework Pyramid (Yan et al. 2020; Sedighi et al., 2021). The model was tested against two sets of experimental data. It was shown that the simulations reproduced the main phases of soil desiccation successfully. The results captured key features, such as stress distribution, sample geometry effects, crack initiation and propagation, including the number and the non-uniform spacing of cracks due to clay heterogeneity. Specifically, crack initiation was studied by investigating the correlation between the shrinkage of soil clay, changes in displacement fields and crack growth in ring specimens with different dimensions. In the specimen with small inner radius, a single crack emerged at the inner surface of the ring, due to the
larger hoop stresses there, and extended towards the outer surface as experimentally observed. For this geometry, it appears that one crack is sufficient to relax the hoop stress along the inner surface, so that other cracks cannot be realised. In the specimen with large inner radius, multiple cracks emerged at the inner radius and propagated towards the outer surface to cause fragmentation as experimentally observed. For this geometry, it appears that the initiation of one crack is not sufficient to relax the hoop stress along the entire inner surface, which allows for initiation of multiple cracks. The simulations of desiccation cracking of long clay bars showed cracks emerging from the free surface of the specimen where evaporation was allowed in accordance with experiments. The cracks propagation and final crack patterns were found to be in close agreement with the experimental observations. The results provide confidence that the proposed modelling approach is physically realistic and can be used to investigate physical, mechanical, and geometric effects on clay desiccation cracking.

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Nomenclature

\[ D \] \quad \text{Diffusion coefficient}
\[ D_{ijkl} \] \quad \text{The (fourth-order) elasticity tensor of the solid material}
\[ e \] \quad \text{Void ratio}
\[ E \] \quad \text{Young’s modulus of the soil}
\[ E_r \] \quad \text{Restraining ring Young’s modulus}
\[ G_f \] \quad \text{The fracture energy release rate}
\[ G_s \] \quad \text{Specific gravity}
\[ R_{is} \] \quad \text{The inner radius of soil ring}
\[ R_{os} \] \quad \text{The outer radius of soil ring}
\[ R_{ir} \] \quad \text{Restraining ring inner radius}
\[ R_{or} \] \quad \text{Restraining ring outer radius}
\( t \) | Time
---|---
\( \alpha \) | Shrinkage coefficient
\( \delta_{kl} \) | Kroneker delta
\( \theta \) | Water content
\( \varepsilon_{ij} \) | Total strain tensor
\( \varepsilon^m_{ij} \) | Elastic strain tensor work-conjugate to the mechanical stresses
\( \varepsilon^h_{ij} \) | Hydric strain tensor
\( \varepsilon_r \) | Strain at the inner wall of the steel ring
\( \zeta \) | Change of water content \( \dot{\theta} \)
\( \delta \) | The radius of the horizon
\( \varepsilon \) | The micro-strain
\( \varepsilon_f \) | The critical micro-strain
\( \sigma_\theta(r) \) | Hoop stress
\( \sigma_r(r) \) | Radial stress
\( \rho(x) \) | Soil density
\( \omega(\xi, \eta) \) | Micro-elastic potential
\( C_p(x, x') \) | Peridynamic micro-modulus
\( g(\xi, \zeta) \) | Pairwise mass exchange function
\( D_p(x, x') \) | Peridynamic micro-diffusivity
\( \mu(x, x') \) | Damage function
\( \varphi(x) \) | Damage parameter
\( n \) | The unit vector along the bond
\( f_h \) | Mass bond
\( H_\xi \) | The horizon of material point \( x \)
\( \xi \) | The distance vector between the two material points
\( u(x) \) | The particle displacement
\( b(x) \) | Body force
\( \eta \) | Relative displacement
\( f(\xi, \eta) \) | Pairwise force density
\( V_{x'} \) | Horizon volume of particle \( x' \)

References


Table 1. Geometry of the experimental tests and material properties

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Test 1</th>
<th>Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>The inner radius of soil ring, $R_{is}$ (cm)$^a$</td>
<td>2.11</td>
<td>15</td>
</tr>
<tr>
<td>The outer radius of soil ring, $R_{os}$ (cm)$^a$</td>
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<td>22</td>
</tr>
<tr>
<td>Restraining ring inner radius, $R_{ir}$ (cm)$^a$</td>
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<td>14.84</td>
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<tr>
<td>Restraining ring outer radius, $R_{or}$ (cm)$^a$</td>
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</tr>
<tr>
<td>Restraining ring Young's modulus, $E_r$ (GPa)$^a$</td>
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<td>2.9</td>
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<tr>
<td>The Young modulus of the soil, $E$ (MPa)$^b$</td>
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<td>1.2</td>
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<tr>
<td>Shrinkage coefficient, $\alpha$ (-)$^a$</td>
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<td>0.065</td>
</tr>
<tr>
<td>Fracture energy release rate, $G_0$ (N/m)$^b$</td>
<td>5.7</td>
<td>5.7</td>
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</tbody>
</table>

$^a$(Najm et al., 2009); $^b$(Jabakhanji, 2013);

Table 2. Soil properties of Bioley silt

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus of soil, $E$ (MPa)$^a$</td>
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</tr>
<tr>
<td>Shrinkage coefficient, $\alpha$ (-)$^b$</td>
<td>$1.56 \times 10^{-2}$</td>
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<tr>
<td>The fracture energy release rate, $G_0$ (N/m)$^c$</td>
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<tr>
<td>The diffusivity of moisture, $D$ ($m^2/s$)$^d$</td>
<td>$2.8 \times 10^{-10}$</td>
</tr>
</tbody>
</table>

$^a$(Péron et al., 2007); $^b$(Ayad et al., 1997); $^c$(Péron et al., 2013); $^d$(Hirobe and Kenji, 2017).

**Figure captions**

Figure 1. Illustration of interaction between peridynamic particles during deformation: (a) Reference configuration; and (b) Deformed configuration.

Figure 2. Desiccation cracking results of clay in restrained ring tests: (a) $R_{is}=2.11$ cm; and (b) $R_{is}=15$ cm. It is noted that Fig 2(b) is a recreation of the image presented by Najm et al., 2009 (Fig 6 in the original reference) which was digitised and plotted by OriginPro 2016 with enhanced quality and for quantitative comparison purposes with numerical results.

Figure 3. Hoop stress evolution with moisture content at the constrained soil surface of Test 1.

Figure 4. Hoop stress distribution in radial direction at different moisture content for Test 1.

Figure 5. Contours of hoop (left) and radial (middle) displacements, and damage parameter (right) from Test 1 PD simulations at moisture content: (a) $\theta = 35\%$; (b) $\theta = 33.6\%$; and (c) $\theta = 30.7\%$. 
Figure 6. Desiccation crack in Test 1: (a) damage contour from PD simulations; (b) reconstructed cracked configuration; and (c) experimental results. It is noted that Fig 6(c) is a recreation of the image presented by Najm et al., 2009 (Fig 6a in the original reference) which was digitised and plotted by OriginPro 2016 with enhanced quality and for quantitative comparison purposes against numerical results.

Figure 7. Desiccation cracks in Test 2: (a) damage contour from PD simulations; (b) reconstructed cracked configuration; and (c) experimental results. It is noted that Fig 7(c) is a recreation of the image presented by Najm et al., 2009 (Fig 6b in the original reference) which was digitised and plotted by OriginPro 2016 with enhanced quality and for quantitative comparison against numerical results.

Figure 8. Experimentally observed evolution of cracks with time (a) and final crack pattern (b) during desiccation cracking of long clay bar. In (b), the top figure is a top view, while the bottom figure is a side view. It is noted that these are recreations of the Fig.17 in Péron et al., 2009a and Fig.2 in Péron et al., 2013; digitised and plotted by OriginPro 2016 for enhanced quality and comparison purpose against numerical results.

Figure 9. Water content contours predicted by PD simulations at $t=17h$, $18.3h$, $20.8h$ and $25h$ (from top to bottom)

Figure 10. Horizontal displacement contours predicted by PD simulations at $t=17h$, $18.3h$, $20.8h$ and $25h$ (from top to bottom)

Figure 11. Vertical displacement contours predicted by PD simulations at $t=17h$, $18.3h$, $20.8h$ and $25h$ (from top to bottom)

Figure 12. Damage contours, illustrating cracks development, predicted by PD simulations at $t=17h$, $18.3h$, $20.8h$ and $25h$ (from top to bottom)
Fig. 3
Fig. 4
Fig. 9
Fig. 10
Fig. 11
Fig. 12