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A novel device to improve robustness of end plate beam-column connections: analytical model development

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Abstract

A novel device to increase rotational capacity of steel end plate beam-column connections is proposed. By inserting a steel sleeve with a designated length, thickness and wall curvature between the end plate and the washer, the load path between the end plate and the bolts can be interrupted, promoting a more ductile response. For a specific sleeve material and dimensions, the capacity of sleeve is controlled by the amplitude value. The sleeve develops severe plastic deformations and is crushed between the end plate and the washer at the plastic amplitude. This study proposes an analytical solution to predict the plastic amplitude for the sleeve with a circular wave form. The sleeve is mathematically represented using shell of revolution theories under axisymmetric load. To simplify the analysis, the sleeve material is analysed using the two-moment limited interaction yield condition and follows an idealized rigid-perfectly plastic material response. The comparison between the proposed analytical solution with validated FE models show that the plastic amplitude is accurately captured. However, the predicted rotational capacity is very conservative.

Key words: Analytical solution; Mathematical representation; End plate connection; Robustness; Rotational capacity; Ductility

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1. Introduction

The structural performance of steel structures subjected to extreme loading such as blast, seismic events, and progressive collapse has received increased attention in recent years [1–4]. These studies have illustrated the significance of connection configurations and the associated parameters for enhancing the overall structural performance under the extreme loading and in avoiding subsequent progressive collapse. Since energy dissipation mainly takes place in the connection zone, the connection should be capable of absorbing significant strain energy without rupture. For example General Services Administration (GSA) [5] requires rotational capacity of 0.2 rad (11 deg) to allow the beam to develop catenary action to avoid progressive collapse under accidental load. However, due to the limited ductility provided by the connection components, previous studies have focussed mainly on mobilising the plastic rotation away from the connection zone to a predefined location in the beam. This is feasible either by strengthening the connection region by using stiffeners and haunches [6] or weakening the beam by trimming away steel parts from the beam flanges or webs, reduced beam section (RBS) [7] and reduced web section (RWS) [8]. In case of a fire event, flexible end plate connections are proposed [9, 10] to accommodate high rotation and accordingly allow the beam to develop catenary action. Examples of these methods are illustrated in Fig. 1.

The load path shown in Fig. 1 reveals that the commonly adopted methods to improve ductility are either to change the stiffness of the beam, or the stiffness of the end plate. However, the rotational capacity of the connection is mainly controlled by the least ductile elements in the load path which are the bolts [11, 12]. The end plate connection is commonly analysed and designed using an equivalent T-stub [13], in which the bolts are the boundary conditions. The higher the bolt elongation, the higher the rotational capacity that can be achieved [14]. However, the most common bolts in engineering practice are frequently manufactured from high strength steel grades of 8.8 and 10.9 [15] which achieve their ultimate
strength at a strain of approximately 0.05 followed by a sudden fracture [16]; by contrast mild steel can achieve a strain of 0.2 without failure [13]. Considering the small length of the bolt, the contribution of the bolt to rotational capacity due to its elongation is generally minimal when compared with the end plate.

With a view to improving the ductility of end plate connections, the authors have recently proposed a novel method incorporating sleeve components around the bolt [17]. Fig. 2 schematically illustrates the proposed system. A sleeve with designated dimensions including length, thickness and wall curvature is inserted between the end plate and the washer. The sleeve is a shell of revolution that resists the applied load by a combination of membrane and bending stress, with the latter becoming more significant as the ratio between the sleeve thickness and radius of curvature is increased [18]. A curvature in the sleeve wall is introduced to promote failure by bending rather than instantaneous buckling. This curvature

![Diagram](image)

**Figure 1:** Various methods to increase the ductility of connections [17].
is defined based on the amplitude at the mid-length of the sleeve and the corresponding geometrical equation of the wave form. Examples of these are shown in Fig. 2. Positive and negative Gaussian curvatures are applicable for the same wave form, however, the latter requires a washer with very specific dimensions as the outer radius of the sleeve can be larger than the washer radius after introducing the amplitude. Furthermore, the bearing between the sleeve with negative Gaussian curvature and the washer can result in high internal forces in the washer which may require a non-standard thick washer. Therefore, only the sleeve with positive Gaussian curvature is considered. The authors previously conducted a numerical investigation using a validated finite element model to prove the concept of the proposed method [17]. It was concluded that the proposed system substantially enhances the rotational capacity up to 2.92 times that of a standard connection. Furthermore, the parameters of the sleeve (e.g. length and amplitude) can be selected such that achieving various ductile responses without altering connection strength or basic configuration.

For a specific wave form and length, developing plastic deformations depends primarily upon the amplitude value. The present study proposes an analytical solution to identify: (i) the plastic amplitude, the amplitude value that allows the sleeve to develop plastic deformation and eventually crushing between the end plate and the washer before failure of any component in the connection; and (ii) the corresponding axial deformation of the sleeve to calculate the increase in rotational capacity.
2. Analytical solution of the proposed sleeve system

Fig. 3 is a schematic illustration of the proposed sleeve under the bolt load. The sleeve is classified as deep shell of revolution as its rise \((L)\) to the shorter side \(2r_s\) is larger than \(0.2\) [18]. In such types of shells, the bending stress is more influential near to the edges resulting in formation of a plastic hinge at an angle \(\alpha\). The sleeve can sustain more post-buckling loads after the formation of the plastic hinge due to the strengthening effect of the membrane forces, a function of the bolt and sleeve geometry and material properties. The capacity of the sleeve corresponding to the formation of the plastic hinge is referred as the sleeve collapse load \(P\). The friction between the sleeve edges and the plate and the washer is ignored in the mathematical operations such that the sleeve edges are free to move laterally. Whilst this assumption underestimates the sleeve capacity, it will reduce the complexity of the mathematical operations. The consequences of eliminating friction on the analytical solution are discussed in Section 4 of this paper.
To predict the load-deflection behaviour of the sleeve, a complete elastic-plastic analysis taking into account the geometry changes is required. Complete solutions of shells with openings are quite rare in the open literature due to the associated complexity of their mathematical operations and their exact yield surface. As we are rather more interested in the plastic capacity of the sleeve and the corresponding deformation than predicting the full load-deflection curve, the problem is analysed using the two-moment limited interaction yield condition, which involves two separate yield hexagons, one for the direct forces and the other for the bending moments [19]. Furthermore, the idealized rigid-perfectly plastic material response is adopted. Thus, when the load is less than the collapse load, the shell will remain rigid and the strain will be zero everywhere.

Figure 3: Plastic collapse of the proposed sleeve centrally loaded through the bolt.
2.1. Plastic capacity of sleeve

The equilibrium equations for a shell of revolution subjected to axisymmetric loading are [20]:

\[
\frac{d}{d\phi}(rN_\phi) - r_1 N_\theta \cos(\phi) - rQ + r_1 r P_\phi = 0
\]
\[
\frac{d}{d\phi}(rQ) + r_1 N_\theta \sin(\phi) + r N_\phi + r_1 r P_\theta = 0
\]
\[
\frac{d}{d\phi}(rM_\phi) - r_1 M_\theta \cos(\phi) - r_1 r Q = 0
\]

where \(N_\phi\) and \(N_\theta\) are meridional and circumferential membrane forces; \(M_\phi\) and \(M_\theta\) are the meridional and circumferential bending moments; \(Q\) is the shearing force; \(P_\phi\) and \(P_\theta\) are the applied load per unit length in the directions of the tangent and normal to the meridian area; \(r_1\) is the meridional radius of curvature of the middle surface; \(r\) and \(r_2\) are shown in Fig. 3. Using simple geometry the following relations can be derived:

\[
r = r_2 \sin(\phi)
\]
\[
r_1 = \frac{a^2 + (0.5L)^2}{2a}
\]
\[
c = r_1 - r_s - a
\]
\[
r_2 = r_1 - \frac{c}{\sin(\phi)}
\]

where \(r_s\) is the end radius of sleeve; \(a\) is the amplitude value; and \(L\) is the sleeve length as shown in Fig. 3. The sign of \(c\) should be be negative when \(r_1 < r_s + a\) (i.e. \(c\) lies on the right hand side of the axis of revolution).

The loads applied on the sleeve are edge loads only, thus terms including \(P_\phi\) and \(P_\theta\) are eliminated. A comparison of Eqs. (1) and (2) shows that the \(N_\theta\) can be eliminated by multiplying
the first equation by $\sin(\phi)$ and the second equation by $\cos(\phi)$, then adding the resulting expression:

$$\frac{d}{d\phi}(rN_\phi \sin(\phi)) + \frac{d}{d\phi}(rQ \cos(\phi)) = 0 \quad (8)$$

Integrating Eq. (8) with respect to $\phi$ gives:

$$r(N_\phi \sin(\phi) + Q \cos(\phi)) = C \quad (9)$$

where $C$ is the constant of integration and can be calculated from the equilibrium of forces in the vertical direction, $C = -\frac{P}{2\pi}$, where $P$ is the collapse load. If the two-moment limited interaction yield condition is considered, the yield conditions are:

$$N_\phi = -N_0, \quad M_\theta = M_0 \quad (10)$$

where $N_0$ is the axial capacity of the sleeve ($F_u t$), $M_0$ is the bending capacity of the sleeve ($F_u t^2 / 4$), and $t$ is the sleeve wall thickness. The ultimate strength $F_u$ is used instead of the yield strength $F_y$ to get the ultimate capacity of the sleeve rather than the plastic capacity. Substituting the value of $N_\phi$ and $C$ in Eq. (9) results in

$$Q = (N_0 - \frac{P}{2\pi r \sin(\phi)}) \tan(\phi) \quad (11)$$

Substituting Eq. (11) into Eq. (3) and applying the second yield condition in Eq. (10), results in

$$\frac{d}{d\phi}(rM_\phi) = r_1 M_0 \cos(\phi) + r_1 r (N_0 - \frac{P}{2\pi r \sin(\phi)}) \tan(\phi) \quad (12)$$

Integrating the above equation between $\beta$ and $\alpha$ results in distribution of the moment $M_\phi$. 

8
However, we are rather more interested in the capacity of the sleeve than the bending distribution. Considering that $M_\phi$ must vanish at $\beta$, the above equation therefore gives the relation

$$P = 2\pi M_0 \cos^2(\beta) + 2\pi N_0 r_s \sin(\beta)$$  \hspace{1cm} (13)$$

from which the plastic capacity of sleeve can be determined for any value of $\beta$ and the end radius $r_s$.

Before exploiting the above equation, we should check its kinematical admissibility. The generalized strain rate in terms of tangential $(u)$ and normal $(w)$ displacements are:

$$\dot{\lambda}_\phi = \frac{du}{d\phi} - w, \quad \dot{\lambda}_\theta = u \cot(\phi) - w$$  \hspace{1cm} (14)$$

$$\dot{k}_\phi = -\frac{d}{d\phi}(u + \frac{dw}{d\phi}), \quad \dot{k}_\theta = -\cot(\phi)(u + \frac{dw}{d\phi})$$  \hspace{1cm} (15)$$

According to the associated flow law $\dot{\lambda}_\theta = 0, \dot{k}_\phi = 0, \dot{\lambda}_\phi < 0$ and $\dot{k}_\theta > 0$. Thus it follows from Eqs. (14) and (15) that:

$$u = w \tan(\phi), \quad \frac{dw}{d\phi} + w \tan(\phi) = -A$$  \hspace{1cm} (16)$$

where $A$ is a positive constant. Integrating the above equation results in:

$$w = A \cos(\phi) \left[ \ln \left( \frac{\tan \left( \frac{\alpha}{2} \right) + 1}{1 - \tan \left( \frac{\alpha}{2} \right)} \right) - \ln \left( \frac{\tan \left( \frac{\phi}{2} \right) + 1}{1 - \tan \left( \frac{\phi}{2} \right)} \right) \right]$$  \hspace{1cm} (17)$$

Making use of Eqs. (14) to (17) one can check that $\dot{\lambda}_\phi < 0$ and $\dot{k}_\theta > 0$. Therefore, the
solution given by Eq. (13) is statically and kinematically admissible. This means that Eq. (13) presents the exact collapse load for the proposed sleeve.

Replacing the parameters in Eq. (13) with basic geometrical terms $L$, $t$ and $a$ and normalising the collapse load to the bolt capacity based on Eurocode 3 Part 1-8 [13] recommendation without the partial safety factor results in:

\[
P = \frac{2 \frac{t}{d_b} \frac{\sigma_u}{\sigma_{b,u}} \left[ \frac{L^2}{a^2} \frac{t}{d_b} + 2 \left( 1 + \frac{t}{d_b} \right) \left( 1 + 0.25 \frac{L^2}{a^2} \right) \sqrt{\left( 1 + 0.25 \frac{L^2}{a^2} \right)^2 - \frac{L^2}{a^2}} \right]}{0.9 \alpha_b A_b \sigma_{b,u}}
\]

(18)

where $A_b$ is the bolt area; $\sigma_{b,u}$ is the ultimate strength of the bolt material; $d_b$ is the nominal bolt diameter; $\alpha_b$ is ratio between nominal area and stressed area of the bolt; $\sigma_u$ is the strength of the sleeve material. Assume the collapse load parameters $q = \frac{P}{0.9 \alpha_b A_b \sigma_{b,u}}$, $\alpha_m = \frac{\sigma_u}{\sigma_{b,u}}$, $\alpha_L = \frac{L^2}{a^2}$ and $\alpha_t = \frac{t}{d_b}$, which results in:

\[
q = \frac{2.22 \alpha_t \alpha_m}{\alpha_b (1 + 0.25 \alpha_L)} \left( \alpha_L \alpha_t + 2 \left( 1 + \alpha_t \right) \left( 1 + 0.25 \alpha_L \right) \sqrt{\left( 1 + 0.25 \alpha_L \right)^2 - \alpha_L} \right) \alpha_{S,L}
\]

(19)

For connections that are controlled by end plate failure, the bolts are not fully stressed. Thus Eq. (19) must be multiplied by a factor $\alpha_{S,L} = \frac{0.9 \sigma_{b,u}}{\sigma_b}$, where $\sigma_b$ is the bolt axial stress at failure. Furthermore, the presence of combined axial and shear stress and secondary stress in the bolt that is generated at high deformation affects the required amplitude. These result in the failure of the bolt at a lower axial stress level; thus a higher amplitude value than that is calculated based on Eq. (19) may be required. The factor $\alpha_{S,L}$ is also introduced to account for the reduction in bolt capacity due to the secondary stress.
Fig. 4 shows the graphical representation of the proposed solution (Eq. (19)). The collapse load parameter $q$ has to be less than unity so that the sleeve can develop sufficient plastic deformation before connection failure. For a sleeve with a specific $\alpha_m$ and $\alpha_t$, the required amplitude ($a$) can be defined based on the $\alpha_L$ parameter. It should be pointed out that the collapse load parameter $q$ depends not only on the sleeve capacity, but also on the stress level in the bolt at the point of connection failure.

![Graph showing the collapse parameter of proposed sleeve system with $\alpha_m = 0.638$.](image)

Figure 4: Collapse parameter of proposed sleeve system with $\alpha_m = 0.638$.

Fig. 4 shows that there are various thicknesses that could be used for the same $q$ parameter. However, it has been previously shown [17] that sleeves with different thicknesses provide similar connection behaviour providing that the minimum sleeve thickness is defined such that the yield capacity of a perfect sleeve (i.e. $a = 0$) is higher than the bolt yield capacity,
which can be represented as:

\[ t \geq \frac{0.72 A_s \sigma_{b,u}}{2 \pi r_s \sigma_y} \]  

(20)

where: \( A_s \) is the bolt stressed area; \( \sigma_y \) is the yield strength of the sleeve material; \( r_s \) is the end radius of sleeve \( (= \frac{d_b + t}{2}) \); the factor 0.72 is a product of 0.9 \( \frac{\sigma_{b,y}}{\sigma_{b,u}} \), and \( \sigma_{b,y} \) is the yield stress of the bolt material.

2.2. Plastic deformation of the sleeve

The basic displacements of an axially symmetric shell of revolution are \( v \), in the direction of the tangent to the meridian, and \( w \), in the direction of the normal to the middle surface. These displacements can be written as [20]:

\[ \frac{dv}{d\phi} - v \cot(\phi) = \frac{1}{Et} \left[ N_\phi (r_1 + \mu r_2) - N_\theta (r_2 + \mu r_1) \right] \]  

(21)

\[ w = v \cot(\phi) - \frac{r_2}{Et} (N_\theta - \mu N_\phi) \]  

(22)

The general solution of the differential equation of \( v \) is

\[ v = \sin(\phi) \left[ \int \frac{N_\phi (r_1 + \mu r_2) - N_\theta (r_2 + \mu r_1)}{\sin(\phi)Et} \, d\phi + c_1 \right] \]  

(23)

where \( c_1 \) is a constant of integration to be determined from the boundary conditions; \( E \) is the elastic modulus of material. The displacement \( w \) is readily obtained from Eq. (22).
The axial displacement, parallel to the bolt axis, can be calculated from the following relation:

$$\delta = w \cos(\phi) + v \sin(\phi)$$

(24)

The sleeve deformation can be calculated after substituting the value of \( v \) and \( w \) in the above equation, then determining the constant of integration considering \( \delta = 0 \) at the end of the sleeve on the washer side. Considering bending moment in the deflection equations results in an impractical expression. Alternatively, deflection of the sleeve is approximately calculated using the membrane forces only with plastic conditions of \( N_\phi = -N_0 \) and \( N_\theta = N_0 \).

Furthermore, due to the fact that the elastic displacement is negligible compared with the plastic deformation, a rigid-plastic material model was considered with strain hardening modulus of \( E_t = \frac{\sigma_u - \sigma_y}{\varepsilon_u - \varepsilon_y} = 1045 \) MPa. Introducing these two assumptions in Eqs. (21) to (23), results in the following simple expression for the vertical deflection

$$\delta = \frac{(\sigma_u - \sigma_y)(1 + \mu)}{E_t} \left[ a + \frac{L^2}{4a} \right] \left[ \cos \beta + 2 \ln \left( \tan \frac{\beta}{2} \right) \right]$$

(25)

A more conservative expression, but extremely simple, can be derived based on the failure strain of the material as follows

$$\delta = L_s(\varepsilon_u + \varepsilon_{bu})$$

(26)

where \( \varepsilon_u \) and \( \varepsilon_{bu} \) are the material strain at failure for sleeve and bolt, respectively. Values of \( \varepsilon_u = 0.15 \) for carbon steel and \( \varepsilon_{bu} = 0.07 \) for high strength bolt can be used.

The rotational capacity of the connection \( R \) in degrees can be simply calculated using the
following equation:

\[ R = R_{std} + \tan^{-1} \left( \frac{\delta}{d_c} \right) \]  

(27)

where: \( R_{std} \) is the rotational capacity of standard connections; \( d_c \) is the distance between the centre of the compression flange and the top bolt in tension. \( \delta \) can be calculated using either Eq. (25) that considers membrane theory or Eq. (26) that considers the strain limit of the material.

3. Verification of the proposed analytical solution

The proposed analytical approach is verified by comparing the predictions to results from a comprehensive finite non-linear element (FE) analysis conducted using ABAQUS/Standard [21]. The comparison covers common loading configurations of bolts including (i) a single bolt under pure tensile force, (ii) an end plate connection with bolt failure, and (iii) an end plate connection with plate failure. In each of the following sections, the FE model is briefly introduced followed by the comparison. A detailed description of the FE model has been given in previous papers [cite stripping paper and sleeve].

3.1. Single bolt

An axisymmetric FE model of a single bolt with sleeve was employed. Isotropic metal plasticity was used to define the material model parameters. A ductile damage criterion was considered to model the progressive degradation of the bolt material stiffness. A bilinear material model of steel grade S355 was adopted for the sleeve. A 4-node bilinear axisymmetric quadrilateral, reduced integration element type CAX4R was adopted. The FE model was discretized using mesh size of 2 mm. The bolt without a sleeve was validated against
experimental tests carried out by Hu et al. [22] on M20 Gr 8.8 bolts as shown in Fig. 5. The scalar stiffness degradation variable (SDEG) was used to visualise the damaged parts. SDEG measures the residual stiffness of an element and takes a value from zero (undamaged material) to one (fully damaged material). A qualitative comparison shows that the FE model accurately captured the force-displacement response of the bolt within the elastic, plastic, and descending parts (Fig. 5). The thread part of the bolt is not modelled in this study, thus the bolt stripping failure mode is not applicable [23].

![Graph](image)

Figure 5: Validation of single bolt model.

Fig. 6 compares the analytical predictions of plastic amplitude values for various $L/d$ ratios with the FE results. In the FE models, the amplitude value was increased in 0.5 mm increments until the plastic amplitude is observed. Eq. (19) is used to calculate the plastic amplitude value with $q = 0.98$ to insure the sleeve capacity is less than the bolt capacity. The considered loading configuration eliminates developing the secondary stress in the bolt;
thus the bolt stress level factor $\alpha_{SL}$ in Eq. (19) is assigned to unity. Furthermore, a frictionless formulation is considered between the sleeve and the plates to avoid enhancing to the sleeve capacity that would otherwise be generated from the partial lateral restraint of sleeve ends. Overall, the proposed analytical equation predicts the plastic amplitude value with an average difference of 10% for the considered $L/d_b$ range. It was observed that the analytical equation cannot predict the plastic amplitude value when the $L/t < 4.0$, since exceeding this limit results in compact sleeve dimensions requiring amplitudes significantly higher than those predicted (see Fig. 7a). This limitation is generally violated with small $L/d_b$ ratios for example 0.75.

![Graph](image.png)

Figure 6: Comparison of plastic amplitude values obtained from analytical and FE results.

There are three distinct failure modes observed during the analysis:

1. *The complete failure of the sleeve* (Fig. 7b): this failure mode is observed when $4.0 \leq L/t \leq 8.0$ resulting in complete crushing of the sleeve between the end plate and
the washer before the bolt necking occurs. In this failure mode the plastic hinge is
developed at approximately the mid-point of the sleeve.

2. *The partial-failure of the sleeve* (Fig. 7c): the plastic hinge is formed at a small angle
of $\phi$ which fails at the sleeve edges. The sleeve capacity is then increased due to the
strengthening effect of the membrane forces. The deformed configuration results in a
new sleeve supported by the outer edge of the washer generating severe plastic strain
in the washer as it is shown in Fig. 6c. This failure mode is observed when $L/t > 8.0$.

3. *Premature failure of the washer* (Fig. 8): this failure mode is observed when the outer
end diameter of the sleeve $r_o$ exceeds the outer nut diameter $r_n$. The washer develops
severe plastic strain before the complete failure of the sleeve.

The FE analysis shows no sign of detrimental effect on the behaviour when the latter two
modes (the partial-failure of sleeve and the premature failure of washer) are observed. How-
ever, it should be noted that the FE model overlooked the material damage for the washer;
which may lead to disengagement of the sleeve from washer side. Thus, the authors recom-
mand that the thickness, the amplitude and the length of the sleeve are selected such that
Eqs. (28) and (29) are satisfied:

\[ 4.0 \leq \frac{L}{t} \leq 8.0 \]  
\[ r_{o} = \sqrt{(r_{1} + 0.5t)^2 - (0.5L)^2} - c \leq r_{n} \]  

(to avoid the partial failure of sleeve) (28)

(to avoid the premature failure of washer) (29)

3.2. End plate connection with bolt failure

A numerical model of the connection shown in Fig. 9a is developed using ABAQUS/Standard [21] featuring eight-node linear brick elements with reduced integration (C3D8R). Surface-to-surface interactions are selected to model tangential behaviour between contact surfaces. Since the analytical model is derived assuming that the sleeve edges are free to move laterally, a frictionless formulation between the sleeve and the plates is used in the FE model to reflect this. The effect of friction on the predicted amplitude is discussed in Section 4. A constitutive model based on that of EC 3: Part 1.2 for carbon steel is used. Yield \( F_y = 356 \text{ MPa} \) and ultimate \( F_u = 502 \text{ MPa} \) stress are defined based on the tensile tests carried out on coupon specimens [24]. For high strength steel, the nominal material properties of bolt grade 8.8 is used \( F_y = 640 \text{ MPa} \) and \( F_u = 800 \text{ MPa} \). Ductile damage models featured in ABAQUS are used to account for material damage and fracture of the end plate and bolts. The FE model is validated against the specimen tested by Yu et al. [24] at ambient
temperature, see Fig. 9a. The applied force is inclined by an angle \( \alpha \) with respect to the beam’s axis to produce different combinations of shear and tying force. Due to symmetry, only half of the connection is modelled, with symmetric boundary conditions assigned at the plane of symmetry, which passes through the beam’s web. Fig. 9b depicts a comparison of the total force-rotation behaviour for the FE and experimental tests. It is clear from Fig. 9b that the FE captures the connection behaviour accurately within the elastic, plastic and post peak regions. Furthermore, the FE can accurately predict the failure mode of the connection.

The same connection configuration as in the validated model is re-analysed using the proposed sleeve system. However, the load is applied in the vertical direction rather than inclined with the beam axis. The end plate thickness is increased to 15 mm so that the connection fails by bolt necking rather than end plate failure. Two connection configurations with three and two bolt rows are considered to investigate the effect of bolt spacing and bolt numbers on the plastic amplitude value. In practice, the sleeve should be inserted between the end plate and the washer for every bolt in the connection. However, due to the nature of the applied load considered in this study, the bolt adjacent to the compression flange is modelled without the sleeve to reduce the computational effort.

Due to the displacement compatibility between the bolts and the holes of the end plate, minor bending stresses are observed in the top bolts, which results in the initiation of failure at stress less than the bolt capacity. Calibration of \( \alpha_{S,L} \) requires quantifying the reduction in bolt axial capacity in the presence of bending and shear stress, otherwise the bolt may fail before the sleeve develops the required level of plastic deformation. However, this is impractical and results in cumbersome calculations. Alternatively, the force in the bolt is limited to the code-prescribed bolt strength when exploiting the analytical equations. Providing that the failure is limited to bolt necking, \( \alpha_{S,L} \) is equal to the partial safety factor of the bolts,
(a) Geometry of tested connections.

(b) Force-rotation comparison between FE and experimental test.

Figure 9: FE model validation [17].

which according to Eurocode 3 Part 1.8 is 1.25.
Connections with two and three bolt rows record the same plastic amplitudes indicating that the plastic amplitude can be defined irrespective of the bolt spacing and the bolt distances. Fig. 10 depicts the predicted plastic amplitude compared with the FE results for sleeve thickness of 4 mm and 5 mm. It should be noted that a thickness of 4 mm is less than that proposed by Eq. (20), however it is considered to verify the proposed equation. It is clear that the proposed analytical equation predicts the plastic amplitude value with acceptable accuracy, the difference is ranging between 0.2 mm and 0.9 mm (i.e. about 10% to 20%).

A further series of FE analyses are conducted to check the sensitivity of connection behaviour to the variation in the amplitude value with reference to the predicted plastic amplitude. Two further amplitudes are considered: (i) amplitude 1.0 mm less than that of predicted by Eq. (19); and (ii) maximum allowed amplitude based on Eq. (29). The results are summarised in Tables 1 and 2 for connection with two and three bolt rows, respectively. On the one hand, an amplitude of 1.0 mm less than that of the predicted reduces the rotational capacity; the bolt failed before the sleeve developed appreciable plastic deformation. On the other hand, increasing the amplitude to the maximum allowed value slightly enhanced the
rotational capacity. It should be pointed out that the rotational capacity of sleeved connections is significantly higher than that of the standard connections for all amplitude values. Accordingly, amplitude values higher than those predicted by Eq. (19) are on the safe side.

(a) Sleeve with $L/d_b = 1.5$.

(b) Sleeve with $L/d_b = 2.0$.

Figure 11: Effect of sleeve amplitude variation on the behaviour of connection.

Tables 1 and 2 also summarise and compare FE with the predicted rotational capacity based on Eq. (27), when the sleeve deformation is calculated based on membrane theory (Eq. (25)) and the material strain limit (Eq. (26)). For the two bolt row connections, the predicted rotation capacity is lower than that of the FE by roughly 5% for short sleeve and 30% for long sleeve when Eq. (25) is used to calculate the sleeve deflection. For the three bolt row connections, the load redistribution between bolt rows allows the connection to record significantly higher rotation than the two bolt rows. This is not accounted for in the analytical solution resulting in more conservative rotation capacity predictions. Eq. (26) predicts comparable results with Eq. (25) but is more conservative; it can nevertheless be used to provide initial estimate for the required sleeve length. Overall, the proposed analytical model conservatively estimates the plastic amplitude and the corresponding rotational capacity when the mode of connection failure is limited to bolt necking.
Table 1: Comparison of rotational capacity of connections with two bolt rows

<table>
<thead>
<tr>
<th>( L/d_b )</th>
<th>( a (\text{mm}) )</th>
<th>( R/ R_{std}^{**} )</th>
<th>( \text{Eq. (25)} )</th>
<th>( \text{Eq. (26)} )</th>
<th>( \text{Eq. (25)} )</th>
<th>( \text{Eq. (26)} )</th>
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*Amplitude based on Eq. (19).
** \( R/R_{std} \) is the ratio between the rotational capacity of sleeved connection to that of standard configuration.

Table 2: Comparison of rotational capacity of connections with three bolt rows

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<th>( L/d_b )</th>
<th>( a (\text{mm}) )</th>
<th>( R/ R_{std}^{**} )</th>
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<th>( \text{Eq. (26)} )</th>
<th>( \text{Eq. (25)} )</th>
<th>( \text{Eq. (26)} )</th>
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<td></td>
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<td>( \text{Eq. (25)} )</td>
<td>( \text{Eq. (26)} )</td>
<td>( \text{FE} )</td>
<td>( \text{FE} )</td>
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</table>

*Amplitude based on Eq. (19).
** \( R/R_{std} \) is the ratio between the rotational capacity of sleeved connection to that of standard configuration.

3.3. End plate connection with plate failure

The FE model in Section 3.2 is used to verify the proposed analytical equations when the end plate failure is the controlling mode. An end plate thickness of 10 mm is considered
during the analysis. The top bolt force corresponding to the plate failure based on Eurocode
calculations is 86.4 kN (α\textsubscript{S,L} = 2.05). The bolt force at plate failure is significantly less than
the bolt capacity which requires a sleeve with a high amplitude value if sleeve thickness of
5 mm is used. The minimum sleeve thickness defined based on Eq. (20) is not applicable
with the plate failure mode of the considered configuration as the limit posed by Eq. (29) is
exceeded; the washer is expected to fail before the complete failure of the sleeve, see Fig. 12a.
Despite the behaviour of the connection not being detrimentally affected by the premature
washer failure as shown in Fig. 12b, the authors recommend to strictly follow Eq. (29) as
the premature washer failure can lead to disengagement of the sleeve at an early stage of
loading; this failure mode is not accounted for in the FE model used in this study.

![Diagram](image)

(a) Premature failure of washer when Eq. (29) limit is exceeded.

(b) Behaviour of connection with sleeve \( L/d_b = 1.5 \) and \( t = 5 \) mm.

Figure 12: Connection with the premature failure of washer.

Alternatively, a sleeve wall thickness less than that proposed by Eq. (20) can be used. A
thickness of 4.0 mm is selected for further analysis. Fig. 13a depicts a comparison between
the predicted and FE plastic amplitude values. The FE plastic amplitude values are sign-
ificantly less (3 mm) than that predicted by the analytical solution. The first reason for
this is that the flexibility of the end plate results in uneven load distribution on the sleeve

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as shown in Fig. 13b, resulting in the sleeve failing at a lower load than expected. Second, Eurocode calculations are conservative for end plate failure [25]. The failure loads evaluated by Eurocode are based on the first stage of failure in the elastic phase of the response which significantly underestimates the force in the bolt; the FE bolt force is two times higher than that predicted from the Eurocode analysis. Fig. 14 shows behavior of the sleeved connection with an end plate failure mode. The amplitude value calculated based on Eq. (19) is noted in the legend. It is clear that the behavior of the connections is only marginally affected by the amplitude value. The predicted plastic amplitude allows the sleeve to develop into the plastic range resulting in higher rotational capacity. Overall, despite the predicted plastic amplitude being significantly higher than that of FE models when the plate failure is observed, it can still be used in design as it is on the conservative side.

![Graph](image)

(a) Comparison of predicted and FE plastic amplitude values. (b) Uneven load on the sleeve.

Figure 13: Connection with plate failure mode.
4. Limitations

The proposed analytical solution is derived considering that the sleeve edges are free to move laterally, however the friction between the sleeve edges and the steel plate can poses partial lateral restraint. This may increase the capacity of the sleeve and accordingly the proposed equation may underestimate the required plastic amplitude. A friction coefficient $\mu$ ranging between 0.2 and 0.5 depending on the surface treatment CEN2005 Fig. 15a depicts effect of $\mu$ on the rotational capacity of connection scaled to that of standard configuration $R/R_{std}$. The behaviour of connections is not affected by $\mu$ value up to 0.4, which may be attributed to the analytical solution overestimating the plastic amplitude. Fig. 15b shows that the sleeves with $\mu \leq 0.4$ are crushed between the end plate and the washer before bolt failure. However, when $\mu$ increases to 0.6, the sleeve develops limited plastic strain and the rotational capacity of the connection is reduced. Further increase in $\mu$ lead to a higher sleeve capacity, which results in the failure of the bolt while the plastic strain of the sleeve is minimal. When $\mu = 0.8$, the rotational capacity of the connection reduces by about 36% compared to the frictionless condition, however it remains 1.75 times the standard configuration (see Fig. 15a). It should be pointed out that the rotational capacity as predicted by the analytical
solution is 1.55 times the standard configuration (see Table 2), which is still on the safe side when \( \mu = 1.0 \). This is attributed to the fact that the proposed analytical solution is very conservative. However, further analytical and experimental work considering various connection configurations are required to account for the partial restraint of the sleeve edges due to friction. Although following similar trends, the differences observed between the FE and analytical predictions suggest further optimisation of the analytical solution is required.

The FE models developed in this study do not consider the material damage of the sleeve and the washer. Failure of the washer may result in the sleeve disengaging from the washer, which may result in the premature failure of the bolt. Threshold limits are proposed in Eqs. (29) and (28) to avoid the failure of washer before the sleeve. Future work will involve an extensive experimental study in order to verify the sleeve performance and provide data sets for further model validation. This will inform the development of analytical models and design methodologies for practical applications.

Figure 15: Effect of friction between sleeve edges and the plates on the behaviour of connection.
5. Conclusion

This study presents an analytical solution for prediction the behaviour of a novel device for enhancing the rotational capacity of beam-column end plate connections. The proposed system constitutes a sleeve with designated dimensions including length, thickness and wall curvature that is inserted between the end plate and the washer. Providing that the ultimate capacity of the sleeve is lower than the force in the bolt at failure, the sleeve develops a severe bending deformation before the failure of any connection components. The capacity of the sleeve can be reduced by increasing the amplitude value of the wall curvature. While there are various wave forms that are applicable including circular, sinusoidal and conical, this study presents an analytical solution for the sleeve with a circular wave form. This is used to predict the plastic amplitude at which the sleeve fails before the failure of any other connection components. Furthermore, the analytical solution predicts the increase in the rotational capacity of connection at the plastic amplitude.

The sleeve is analytically represented using the shell of revolution theory subjected to axisymmetric loading. To simplify the mathematical operations, the sleeve is analysed using the two-moment limited interaction yield condition and follows an idealized rigid-perfectly plastic material response. The friction between the sleeve edges and the steel plates is ignored to simplify calculations, however the consequences of eliminating friction on the accuracy of the proposed solution are discussed.

The proposed analytical solution is verified against FE models for the sleeve mated with a single bolt under tensile force and for sleeved end plate connections with various failure modes and different configurations. The results show that the proposed solution can accurately predict the plastic amplitude value, however, the predicted rotational capacity of the connection is very conservative.
Subsequent work will include experimental investigations, so that the analytical solution will be validated. This will inform the development of analytical models and design methodologies for practical applications. Furthermore, further analytical investigations will be carried out to develop and optimise the proposed equations while considering the limitations discussed in the present study.

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References


