Attitude controller design for reusable launch vehicles during reentry phase via compound adaptive fuzzy H-infinity control

DOI: 10.1016/j.ast.2017.10.012

Document Version
Accepted author manuscript

Link to publication record in Manchester Research Explorer

Citation for published version (APA):

Published in:
Aerospace Science and Technology

Citing this paper
Please note that where the full-text provided on Manchester Research Explorer is the Author Accepted Manuscript or Proof version this may differ from the final Published version. If citing, it is advised that you check and use the publisher's definitive version.

General rights
Copyright and moral rights for the publications made accessible in the Research Explorer are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

Takedown policy
If you believe that this document breaches copyright please refer to the University of Manchester’s Takedown Procedures [http://man.ac.uk/04Y6Bo] or contact uml.scholarlycommunications@manchester.ac.uk providing relevant details, so we can investigate your claim.
Attitude controller design for reusable launch vehicles during reentry phase via compound adaptive fuzzy H-infinity control

Qi Mao*a, Liqian Dou**, Qun Zonga, Zhengtao Dingb

aSchool of Electrical and Information Engineering, Tianjin University Tianjin, 300072, China
bSchool of Electrical and Electronic Engineering, The University of Manchester, Manchester, M13 9PL, U.K.

Abstract

In this paper, the attitude control problem of reusable launch vehicles (RLVs) during reentry phase is investigated by using compound adaptive fuzzy H-infinity control (CAFHC) strategy in the presence of parameter uncertainties and external disturbances. Firstly, the control-oriented attitude model is established by a model transformation based on the six-degree-of-freedom (6-DoF) dynamic model of the RLV. Secondly, a novel attitude control scheme is developed and the control strategy consists of two parts to achieve a stable and accurate attitude tracking during reentry flight process. An attitude tracking controller is designed using adaptive fuzzy H-infinity control approach combined with an identification model to improve the attitude tracking performance in the interior of fuzzy approximation region of attitude angle. Next, an attitude stabilization controller based on boundary adaptive technique is employed to assure the robustness of the closed-loop system in the exterior of fuzzy approximation region of attitude angle. Furthermore, the stability of the closed-loop system is guaranteed within the framework of Lyapunov theory and the attitude tracking error converges to a small neighborhood around origin. Finally, the simulation results are presented to demonstrate that the effectiveness of the proposed control scheme for reentry RLV, and its tracking performance performs better than the other control method.

Keywords: reusable launch vehicle; attitude control; fuzzy logic system; H-infinity control; reentry phase

1. Introduction

With the aim to develop a more cost-effective and reliable approach to the space, reusable launch vehicles (RLVs) have attracted intensive research interest in the field of aerospace engineering[1][2][3]. Mainly due to the unique advantages in reusability, flexibility as well as low operational cost, RLVs have a good prospect in aerospace activities and broad applications in civilian and military fields [4]. Whereas the philosophy sounds interesting and attracting, a major challenging problem posed in flight missions is that of atmospheric reentry. During the reentry phase, the altitude and velocity of the vehicle vary rapidly and drastically as a RLV goes through a wide range of flight envelope. At the same time, RLVs are subject to the poor flight conditions, severe parameter uncertainties and unknown external disturbances [5][6][7]. Thus, the reentry attitude control of RLVs is still a challenging problem [8], and it is critical for the RLV to track the expected commands rapidly and accurately.

In the recent past, a variety of control approaches have been proposed for designing the flight controllers of the vehicle. In the early studies, the flight control problem was investigated based on a linearized model of aerocrafts [9][10], whereas it is unsuitable for highly nonlinear and multivariable model of the RLV. Then, gain scheduling (GS) [11][12][13] as a popular control approach was applied for flight control system design. Although gain scheduling control method is verified to be effective to solve some control issues for reentry vehicles, the system robustness and global stability cannot be guaranteed especially under the circumstance of abrupt change of the control parameters. In the work of [14], model predictive control (MPC) method was combined with feedback linearization to develop a controller for reentry vehicle. After that, the trajectory linearization controller (TLC) [15][16][17] scheme was developed to improve the nonlinear performance of GS in flight control system for aircrafts. But the robust performance of this approach is finite as the tracking dynamics are linearized by the trajectory linearization controller. Analogously, dynamic inversion (DI) technique [18][19][20] was proposed to cope with the flight control problem of a reusable launch vehicle. However, the drawback of this strategy is poor in robustness for parameter uncertainties and modeling errors if improperly designed. Further, Lam and Krishnamurthi et.al. have investigated an adaptive controller using the state dependent Riccati equation (SDRE) approach [21] to a twin-rotor aircraft, but it is difficult for SDRE to deal with the control system with a high order. In [22][23], backstepping methodology was implemented to design the flight control system of the vehicle. Nevertheless, it is worth noting that the external disturbance was not taken into consideration in [22][23].

For the purpose of designing an attitude controller with a
good robust performance, Shih et al. [24][25] have explored the
application of sliding mode control (SMC) scheme to atti-
dude control system of RLV. SMC technique is well-known
as a robust method to design control law for uncertain system.
However, the boundary values of model uncertainties and
e external disturbances ought to be known a priori and it is hard
to satisfy this requirement in practical application. In [26][27],
robust control approach was used to tackle the flight control
issue in the presence of hard constrains, model uncertainties
and external disturbances for RLVs. Similarly, Jee and Yala-
gach et al. have addressed H-infinity control strategy [28] to
design flight control system under the highly uncertainties and
changing dynamics of aerodynamic coefficients for the RLV
during reentry process. However, robust control scheme cannot
achieve a good tracking performance easily since the robust
controller is designed in the worst case of the control system.
And then, adaptive control [29][30] technique was employed to
combine with robust control method in order to compensate
for its shortcomings and improve the tracking performance.
Recently, fuzzy logic system (FLS) [31][32][33][34]
was applied to design the attitude controller of reentry vehi-
cle for its good approximation to model uncertainties and un-
modeled dynamics. This control approach is an efficient way
to cope with the model uncertainties and external disturbances
even if the boundaries of uncertainties and disturbances are un-
known. Moreover, FLS are always combined with other control
strategies, such as adaptive control [35], robust control [36],
fault-tolerant control [37] and so on, with the goal to improve
the tracking performance of the flight control system and
achieve a good robustness simultaneously. Besides, adap-
tive fuzzy control method is often combined with robust or H-
infinity control method in many applications [38][39][40].

In this research, we will further focus on the attitude con-
troller design problem of RLVs during its reentry phase where
parameter uncertainties and external disturbances are consid-
ered, and propose a compound adaptive fuzzy H-infinity control
(CAFC) strategy. Wherein, the fuzzy logic system is intro-
duced to approximate the uncertainty term, and the H-infinity
controller is adopted to compensate for fuzzy modeling errors
and the external disturbances. However, the introduction of H-
infinity control term would degrade the approximation capabil-
ity of FLS, which would further weaken the tracking perfor-
ance of reentry RLV. In order to avoid the problem of "ap-
proximation capability weakening", a novel control scheme is
developed and this control strategy mainly consists of two parts.
An attitude tracking controller is designed utilizing adaptive
fuzzy H-infinity control approach combined with an identifica-
tion model to improve the attitude tracking performance, while
an attitude stabilization controller based on boundary adaptive
technique is employed to assure the system robustness and the
boundedness of approximation error. Moreover, the stability
analysis is carried out to demonstrate that the proposed stra-
gy can guarantee the semi-global stability of the closed-loop
system. Finally, the simulation results of 6-DoF dynamic mode
for the RLV are presented to illustrate the effectiveness of the
proposed control strategy.

The rest of this paper is organized as follows. The 6-DoF
dynamic model and the control-oriented attitude model of reen-
try RLV are stated in Section 2. Next the attitude controller
design strategy via CAFC approach is developed, followed
by the stability analysis of the closed-loop control system in
Section 3. After that the numerical simulations applying the
proposed control scheme are conducted, and the simulation re-
results and discussions are presented in Section 4. Finally,
the conclusion of the paper is drawn in Section 5.

2. Problem formulation

In this section, the 6-DoF dynamic model of the RLV within
its reentry phase is described, and the control-oriented attitude
model is derived by a model transformation to design the atti-
dude controller.

2.1. 6-DoF dynamic model of RLV

For the sake of simplicity, it is reasonable to assume that
the impact of Earth’s rotation on the flight control system is not
taken into consideration in this work [5], and RLV is regarded
as an unpowered rigid body flight vehicle during its reentry
phase. The complete application model of RLV is given as de-
picted in Fig. 1. Generally, the 6-DoF dynamic model of RLV
can be separated into the 3-DoF translational kinematic equa-
tions, and the 3-DoF rotational kinematic equations, which can
be derived based on Ref. [41][42][43].

The 3-DoF translational kinematic equations are stated by six
first-order equations as follows:

\[
\begin{align*}
\dot{x} &= V \cdot \cos \gamma \cdot \cos \chi \\
\dot{y} &= V \cdot \cos \gamma \cdot \sin \chi \\
\dot{z} &= V \cdot \sin \gamma \\
\dot{V} &= \frac{1}{m} \left( -D - mg \sin \gamma \right) \\
\dot{\alpha} &= q - \tan \beta (p \cos \alpha + r \sin \alpha) \\
&\quad + \frac{1}{m V \cos \beta} \left[ -L + mg \cos \gamma \cos \mu \right] \\
\dot{\beta} &= p \sin \alpha - r \cos \alpha + \frac{1}{m V} \left[ Y + mg \cos \gamma \sin \mu \right]
\end{align*}
\]  

\[(2)\]

Fig. 1. The complete application model of RLV
where \( x, y, \) and \( z \) denote the location of RLV referenced to the flight-path coordinate frame. \( V \) represents the flight velocity, and \( \gamma, Y, \) and \( \mu \) are the flight path angle, heading angle and bank angle, respectively. \( \alpha, \beta \) are the angles of attack (AOA) and sideslip angle respectively. \( p, q, \) and \( r \) are the roll, pitch and yaw rates, respectively. \( L, D \) and \( Y \) are aerodynamic lift, drag and sides forces as described in (6).

The 3-DoF rotational kinematic equations are described by six first-order equations as follows:

\[
\begin{align*}
\dot{p} &= (c_1 r + c_2 p) q + c_3 L + c_4 N \\
\dot{q} &= c_5 pr - c_6 (p^2 - r^2) + c_7 M \\
\dot{r} &= (c_8 p - c_2 r) q + c_9 L + c_9 N
\end{align*}
\]

where \( L, M \) and \( N \) are the rolling, pitching and yawing aerodynamic moments described in (7), \( c_1, \ldots, c_9 \) are stated in (5). \( \phi, \theta, \) and \( \psi \) are the roll, pitch and yaw angles respectively,

\[
\begin{align*}
c_1 &= (J_{yy} - J_{zz}) J_{zz} - J_{zz}^2 \\
c_2 &= (J_{xx} - J_{yy}) J_{yy} + J_{zz}^2 \\
c_3 &= J_{zz} J_{yy} - J_{zz}^2 \\
c_4 &= J_{zz} J_{zz} - J_{zz}^2 \\
c_5 &= J_{zz} J_{zz} - J_{zz}^2 \\
c_6 &= J_{zz} J_{zz} - J_{zz}^2 \\
c_7 &= J_{zz} J_{zz} - J_{zz}^2 \\
c_8 &= J_{zz} J_{zz} - J_{zz}^2
\end{align*}
\]

(3)

\[
\begin{align*}
\phi &= p + (q \sin \phi + r \cos \phi) \tan \theta \\
\theta &= q \cos \phi - r \sin \phi \\
\psi &= q \sin \phi \sec \theta + r \cos \phi \sec \theta
\end{align*}
\]

(4)

where \( J, M \) and \( N \) are the rolling, pitching and yawing aerodynamic moments described in (7), \( c_1, \ldots, c_9 \) are stated in (5). \( \phi, \theta, \) and \( \psi \) are the roll, pitch and yaw angles respectively,

\[
\begin{align*}
c_1 &= (J_{yy} - J_{zz}) J_{zz} - J_{zz}^2 \\
c_2 &= (J_{xx} - J_{yy}) J_{yy} + J_{zz}^2 \\
c_3 &= J_{zz} J_{yy} - J_{zz}^2 \\
c_4 &= J_{zz} J_{zz} - J_{zz}^2 \\
c_5 &= J_{zz} J_{zz} - J_{zz}^2 \\
c_6 &= J_{zz} J_{zz} - J_{zz}^2 \\
c_7 &= J_{zz} J_{zz} - J_{zz}^2 \\
c_8 &= J_{zz} J_{zz} - J_{zz}^2
\end{align*}
\]

(5)

where \( c_1, \ldots, c_9 \) are the rolling, pitching and yawing aerodynamic moments described as follows

\[
\begin{align*}
C_L &= C_0 + C_1 \delta_e + C_2 \beta + C_3 \delta_r \\
C_D &= C_0 + C_1 \alpha + C_2 \beta + C_3 \delta_r \\
C_Y &= C_0 + C_1 \beta + C_2 \delta_r \\
C_1 &= C_1^0 \beta + C_1^0 \delta_e + C_1^0 \delta_r + C_1^0 (\frac{pb}{2V}) + C_1^0 (\frac{rb}{2V}) \\
C_m &= C_0 + C_1 \alpha + C_2 \delta_r + C_3 \delta_e + C_3 (\frac{qR}{2V}) \\
C_n &= C_0 + C_1 \beta + C_2 \delta_r + C_3 (\frac{pR}{2V}) + C_3 (\frac{rR}{2V})
\end{align*}
\]

where \( \delta_e, \delta_r, \delta_r \) represent the aileron, elevator and rudder deflections of the RLV, respectively. \( C_1 \) denotes the corresponding aerodynamic derivatives. All parameters are showed in Table 1 of the section 4.

2.2 Control-oriented attitude model

In general, the 3-DoF translational kinematic equations referenced to a flight-path coordinate system are used to generate flight trajectory and design the guidance system, whereas the 3-DoF rotational kinematic equations referenced to a vehicle-body-fixed coordinate system are employed to design the attitude controller. This paper aims at designing attitude controller for reentry RLV, and hence a control-oriented attitude model will be established by a model transformation on the basis of rotational kinematic equations (3) - (4) for facilitating the attitude controller design.

Substituting (6) and (7) into (3), then the Eq.(3) can be rewritten as follows,

\[
\begin{bmatrix}
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} = \begin{bmatrix}
f_p \\
f_q \\
f_r
\end{bmatrix} + \begin{bmatrix}
G_u \\
G_v \\
G_w
\end{bmatrix} \begin{bmatrix}
\delta_a \\
\delta_b \\
\delta_r
\end{bmatrix}
\]

(8)

where \( f_p, f_q, f_r \) and \( G_u \) are described respectively as,

\[
\begin{align*}
f_p &= (c_1 r + c_2 p) q + c_3 M_x + c_4 M_z \\
f_q &= c_5 pr - c_6 (p^2 - r^2) + c_7 M_y \\
f_r &= (c_8 p - c_2 r) q + c_9 M_x + c_9 M_z
\end{align*}
\]

(9)

and \( M_x, M_y, M_z \) are stated by

\[
\begin{align*}
M_x &= qS \delta bC_i^0 + C_i^0 (\frac{pb}{2V}) + C_i^0 (\frac{rb}{2V}) \\
M_y &= qS \delta C_m^0 + C_m^0 \alpha + C_m^0 \beta + C_m^0 (\frac{qR}{2V}) \\
M_z &= qS \delta bC_n^0 + C_n^0 (\frac{pb}{2V}) + C_n^0 (\frac{rb}{2V})
\end{align*}
\]

(10)

Taking time derivative of (4), the second-derivative of attitude-angle is derived as below,

\[
[\dot{\phi}, \dot{\theta}, \dot{\psi}]^T = L(\phi, \theta, \psi)[\dot{\phi}, \dot{\theta}, \dot{\psi}]^T + g(\phi, \theta, \psi)
\]

(12)

and substituting (8) into (12), the equation of motion (12) is converted to a second-order nonlinear dynamic model as,

\[
[\ddot{\phi}, \ddot{\theta}, \ddot{\psi}]^T = L(\phi, \theta, \psi)[f_p, f_q, f_r]^T + g(\phi, \theta, \psi)
\]

(13)

where \( L(\phi, \theta, \psi) \) and \( g(\phi, \theta, \psi) \) are depicted respectively as

\[
L(\phi, \theta, \psi) = \begin{bmatrix}
1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi \sec \theta & \cos \phi \sec \theta
\end{bmatrix}
\]

(14)

\[
g(\phi, \theta, \psi) = \begin{bmatrix}
b \psi \sec \theta + \dot{\phi} \theta \tan \theta \\
-\dot{\psi} \phi \tan \theta \\
\dot{\phi} \theta \sec \theta + b \psi \tan \theta
\end{bmatrix}
\]

(15)
However, it is worth noting that the wide range envelope of reentry RLV causes great uncertainties in model parameters and unknown external disturbances. Thus, Eq.(13) is not available to describe the actual model information exactly. To characterize the reentry flight process as specifically as possible, the parameter uncertainties and external disturbances should be considered. And hence a control-oriented attitude model with more practical information is formulated, which is the basis of the attitude controller design in this work,

\[
\begin{bmatrix}
\dot{\phi}, \dot{\theta}, \dot{\psi}
\end{bmatrix}^T = L(\phi, \theta)[f_\phi, f_\theta, f_\psi]^T + g(\phi, \theta, \psi) + [\Delta f_1, \Delta f_2, \Delta f_3]^T
\]

where \(\Delta d_i (i = 1, 2, 3)\) denote external disturbances; \(\Delta f_i\) and \(\Delta G_a\) denote parameter uncertainties, which can be stated detailedly as

\[
[\Delta f_1, \Delta f_2, \Delta f_3]^T = L(\phi, \theta)[c_1 \Delta M_1, c_4 \Delta M_3, c_2 \Delta M_2, c_1 \Delta M_3, c_2 \Delta M_3, c_5 \Delta M_3]^T
\]

where \(\Delta M_i\), \(\Delta M_i\), and \(\Delta M_i\) denote uncertainty terms induced by the perturbations of aerodynamic parameters, which are described respectively as

\[
\Delta M_x = \bar{q} \cdot \bar{b} [\Delta C_{d_1} \cdot \beta + \Delta C_{f_1} \cdot \gamma] + \Delta C_{l}\cdot \left(\frac{\bar{p}b}{2V}\right)
\]

\[
\Delta M_y = \bar{q} \cdot \bar{c} [\Delta C_{d_1} \cdot \alpha + \Delta C_{m} \cdot \beta + \Delta C_{o}] \cdot \left(\frac{\bar{q}c}{2V}\right)
\]

\[
\Delta M_z = \bar{q} \cdot \bar{b} [\Delta C_{d_1} \cdot \beta + \Delta C_{m} \cdot \left(\frac{\bar{p}b}{2V}\right) + \Delta C_{o} \cdot \left(\frac{\bar{r}f}{2V}\right)]
\]

where \(\Delta C_i\) represents the corresponding aerodynamic coefficient perturbation of RLVs.

For designing the attitude controller succinctly, the attitude control model in (16) can be simplified as a second-order nonlinear system as following,

\[
\begin{align*}
\dot{x} &= f(x) + \Delta f(x) + (g(x) + \Delta g(x))u + d \\
\dot{y} &= x
\end{align*}
\]

where \(x = [x_1, x_2, x_3]^T = [\phi, \theta, \psi]^T\) is the vehicle state vector, \(y = [y_1, y_2, y_3]^T\) is the system output vector, \(u = [u_1, u_2, u_3]^T = [\delta_a, \delta_e, \delta_i]^T\) is the control input vector generated by aerodynamic surface; \(f(x) = L(\phi, \theta)[f_\phi, f_\theta, f_\psi]^T + g(\phi, \theta, \psi)\) and \(g(x) = L(\phi, \theta)G_a = [g_\phi, g_\theta, g_\psi]^T\) denote the system function matrix; \(\Delta f(x) = [\Delta f_1, \Delta f_2, \Delta f_3]^T\) and \(\Delta g(x) = L(\phi, \theta)G_a\) denote parameter uncertainties; \(d = [d_1, d_2, d_3]^T\) represents external disturbance.

The following notation is adopted throughout this paper for the convenience of controller design and stability analysis.

**Definition 1:** \(\Delta f_1(x), \Delta f_2(x)\) and \(\Delta f_3(x)\) denote the uncertainty term of system function matrix \(f(x)\).

Further, the following assumption is taken into account in the flight controller design.

**Assumption 1:** The system parameter uncertainties \(\Delta f(x)\), \(\Delta g(x)\) and external disturbance \(d\) are bounded, i.e., there exist known bounded positive function matrix \(\Delta f_1(x), \Delta f_2(x) \in R^{3 \times 1}\), \(\Delta g_1(x), \Delta g_2(x) \in R^{3 \times 3}\) and \(d_1(x) \in R^{3 \times 1}\), such that for any state vector \(x\), \(0 < \Delta f_1(x) < \Delta f_2(x) \leq \Delta f_3(x), \Delta g_1(x) < \Delta g_2(x) \leq \Delta g_3(x)\) and \(|d_1(x)| < d_2(x)\) hold, where \(||\) denotes the absolute value, and \(i, j = 1, 2, 3\).

2.3. Control objective

Assume that the guidance command of attitude angular is \(y_d = [\phi, \theta, \psi]^T\) with a continuous and bounded and derivative \(y_d\). The tracking error of attitude angular is defined as \(e_i = y_d - \hat{y}_d\) and the attitude tracking error vector of flight attitude is denoted as \(E_i = (e_\phi, e_\theta, e_\psi)^T (i = 1, 2, 3)\). Noting that the uncertainty terms caused by the aerodynamic parameter perturbations will affect the practical control performance during the reentry phase and then reduce the actual attitude tracking performance of the reusable launch vehicle. Thus, the flight control objective in this paper aims at designing a proper control law \(u\) for attitude control system (16) and (20) to guarantee that the actual output \(\hat{y}\) can track the desired command \(y_d\) well in spite of parameter uncertainties and external disturbances while the close-loop control system of reentry RLV can achieve asymptotic stability simultaneously, i.e.,

\[
\lim_{t \to \infty} ||\phi_d - \phi_1|| = 0, \lim_{t \to \infty} ||\theta_d - \theta_2|| = 0, \lim_{t \to \infty} ||\psi_d - \psi_3|| = 0
\]

The overall control framework of attitude control system in this research is presented in Fig. 2. As depicted in Fig. 2, the control scheme is composed of two controllers designed via CAHC: the attitude tracking controller utilizing adaptive laws is employed to improve the attitude tracking performance, and the attitude stabilization controller using estimation laws is applied to guarantee the system robustness and the boundness of approximation error. And the detailed design of this control structure is presented in next section.

3. Attitude control strategy and stability analysis

This section focuses on developing attitude controller for reentry RLV to attenuate the adverse effects of various parameter uncertainties and unknown external disturbances and track the expected guidance commands stably and accurately.
3.1. Modeling of fuzzy logic system (FLS)

To develop the attitude controller for RLVs, the following lemma is necessary in this study:

**Lemma 1** [38]: Suppose the given f(x) is a real-continuous function on a compact set Ω, and for any small constant ε > 0, there exists a fuzzy logic system (FLS) θ*(x) such that

\[ \sup_{x \in \Omega} |f(x) - \theta^*(x)| \leq \varepsilon \]

where \( \varepsilon \) is approximation error, \( \theta \) denotes a bounded parameter vector, and \( \xi(x) \) is the fuzzy basis function vector. Ω denotes the fuzzy approximation region satisfying \( \Omega = \{x : ||x|| \leq R_{\Omega}\} \) and \( R_{\Omega} = R - \Omega \) is denoted as an area outside the fuzzy approximation region, where \( || \cdot || \) is Euclidean norm and \( R_{\Omega} > 0 \) is a finite constant.

Although the uncertainty terms \( \Delta f, \Delta g \) are unknown, the function vectors of \( \Delta f, \Delta g \) are continuous under the flight conditions considered in this study. Consequently, the dynamics of \( \Delta f, \Delta g \) can actually be caught by the FLS. To find the FLS, we use Lemma 1, the three-channel attitude angle \( \theta_i \) of the RLV, define \( N_i \) fuzzy sets \( F_i^j \) (\( i = 1, 2, ..., N_i \)) thus the terms \( \Delta f, \Delta g \) can be approximated by utilizing the FLS respectively, which are detailed as follows,

\[ \Delta f(x) = \Delta f(x|\theta_i) + \epsilon_f \]

\[ \Delta g(x) = \Delta g(x|\theta_i) + \epsilon_g \]

where \( \epsilon_f = [\epsilon_{f1}, \epsilon_{f2}, \epsilon_{f3}]^T \) and \( \epsilon_g = [\epsilon_{g1}, \epsilon_{g2}] \in \mathbb{R}^{3 \times 3} \) are vectors of approximation errors, \( \theta_i = [\theta_{i1}, \theta_{i2}, \theta_{i3}], \theta_g = [\theta_{g1}, \theta_{g2}] \in \mathbb{R}^{3 \times 3} \) are bounded vectors of corresponding adjusting parameters, and \( \theta_{i1} \in \mathbb{R}^{1 \times 3}, \theta_{i2} \in \mathbb{R}^{1 \times 3}, i = 1, 2, 3 \), in which \( W = \prod_{i=1}^{m} N_i \), and \( m = 3 \). \( \xi_f(x) \) and \( \xi_g(x) \) represent the fuzzy basis function vectors, where \( \xi_{f1}(x) = [\xi_1(x), ..., \xi_w(x)]^T \in \mathbb{R}^{w \times 1} \), and its \( l_1, ..., l_m \)-th function can be described as follows,

\[ \xi_{l_j}(x) = \prod_{i=1}^{m} \mu_{F_{ij}}(x) \]

where \( \mu_{F_{ij}}(x) \) represents the membership function of \( F_{ij} \) for flight attitude \( x_i \).
where $\dot{x}_i$ is the estimated value of attitude angle $x_i$, $\alpha_{Fi} > 0$ means the filter parameter pre-set by the designer. And $v_i$ denotes the compensation term of identification model, which can be stated by,

$$v_i = -\beta_i \operatorname{sgn}(\xi_{Fi})$$

(35)

where $\beta_i \geq \bar{v}_i$ denotes a finite constant pre-set by the designer, and $\bar{v}_i = \sup_{x \in \mathcal{X}} |\dot{\xi}_i|$.

Noting that $\dot{\varepsilon}_{Fi} = \ddot{x}_i - \dot{x}_i$, according to the expression of the filtering modeling error. And subtracting Eq.(27) from Eq.(34), then the dynamic of filtering modeling error can be derived as,

$$\dot{\varepsilon}_{Fi} = -\alpha_{Fi} \varepsilon_{Fi} - \bar{w}_i + \dot{\theta}_{ij} \xi_j(x) + v_i + \sum_{j=1}^{3} \dot{\theta}_{ijk} E_k(x) u_{Tj}$$

(36)

Accordingly, the adaptive laws of parameter vectors $\theta_{ij}$ and $\theta_{ijk}$ are developed on the basis of the Eq.(33) and above dynamic equation (36) respectively as below,

$$\dot{\theta}_{ij} = -\gamma_{ij} (E_i^T P_i B_i + \gamma_{ij} \varepsilon_{Fi}) \xi_j(x),$$

$$\dot{\theta}_{ijk} = -\gamma_{ijk} E_i^T P_i B_i + \gamma_{ijk} \varepsilon_{Fi} \xi_j(x) u_{Tj}.$$  

(37)

where $\gamma_{ij}, \gamma_{ij}, \gamma_{ic} \in \mathbb{R}^n$ denote the learning rates.

**Theorem 1:** Considering the attitude control system (16) and (20) for reentry RLV, given that the Assumption 1 and Lemma 1 are satisfied when the aircraft attitude $x \in \mathbb{X}$. The attitude tracking controller is chosen as (24) together with the identification model (34) and the adaptive laws are designed as (37), then the closed-loop control system can achieve semi-global asymptotic stable, and the attitude tracking error vector $E_i$ and filter modeling error vector $\varepsilon_{Fi}$ can both converge to a small neighborhood $D_{TE}, D_e$ around origin through tuning the designed controller parameters appropriately.

$$D_{TE} = \{E_i : \|E_i\| \leq \rho_1 \bar{w}_i (\lambda_{\min}(Q_i))^{-1/2}\},$$

$$D_e = \{\varepsilon_{Fi} : \|\varepsilon_{Fi}\| \leq \rho_1 \bar{w}_i (\gamma_{ij} \alpha_{Fi})^{-1/2}\}$$

(38)

where $\lambda_{\min}(Q_i)$ is the minimum eigenvalue of matrix $Q_i$.

**Proof:**

Choose the Lyapunov candidate function $V_{Ti}$ about the flight attitude dynamic as

$$V_{Ti} = \frac{1}{2} E_i^T P_i E_i + \frac{1}{2} \gamma_{ij} \varepsilon_{Fi}^2 + \frac{1}{\gamma_{ij}} \dot{\theta}_{ij} \dot{\theta}_{ij} + \sum_{j=1}^{3} \frac{1}{2\gamma_{ijk}} \dot{\theta}_{ijk} \dot{\theta}_{ijk}$$

(39)

Taking the derivative of $V_{Ti}$ along Eqs.(24) and (33) yields,

$$V_{Ti} = -\frac{1}{2} E_i^T Q_i E_i + \gamma_{ij} \varepsilon_{Fi} \dot{\theta}_{ij} \xi_j(x) + \frac{1}{\gamma_{ij}} \dot{\theta}_{ij} \dot{\theta}_{ij} - \gamma_{ij} \alpha_{Fi} \varepsilon_i^2$$

(40)

$$+ E_i^T P_i B_i [\dot{\theta}_{ij} \xi_j(x) + \sum_{j=1}^{3} \dot{\theta}_{ijk} \xi_j(x) u_{Tj} - u_{bi} - \bar{w}_i] + \frac{1}{\gamma_{ijk}} \sum_{j=1}^{3} \dot{\theta}_{ijk} \dot{\theta}_{ijk} + \gamma_{ij} \varepsilon_{Fi} [v_i - \bar{w}_i^*]$$

Substituting (25), (35) and the Riccati-equation (26) into (40), $V_{Ti}$ can be rewritten as,

$$V_{Ti} = -\frac{1}{2} E_i^T Q_i E_i + \gamma_{ij} \varepsilon_{Fi} \dot{\theta}_{ij} \xi_j(x) + \frac{1}{\gamma_{ij}} \dot{\theta}_{ij} \dot{\theta}_{ij} - \gamma_{ij} \alpha_{Fi} \varepsilon_i^2$$

(41)

$$+ E_i^T P_i B_i [\dot{\theta}_{ij} \xi_j(x) + \sum_{j=1}^{3} \dot{\theta}_{ijk} \xi_j(x) u_{Tj} - u_{bi} - \bar{w}_i] + \frac{1}{\gamma_{ijk}} \sum_{j=1}^{3} \dot{\theta}_{ijk} \dot{\theta}_{ijk} + \gamma_{ij} \varepsilon_{Fi} [v_i - \bar{w}_i^*]$$

$$+ \gamma_{ij} \varepsilon_{Fi} \xi_j(x) u_{Tj} + \sum_{j=1}^{3} \frac{1}{\gamma_{ijk}} \dot{\theta}_{ijk}$$

$$+ \gamma_{ij} \varepsilon_{Fi} \xi_j(x) u_{Tj} + \sum_{j=1}^{3} \frac{1}{\gamma_{ijk}} \dot{\theta}_{ijk} (E_i^T P_i B_i + \gamma_{ij} \varepsilon_{Fi}) \xi_j(x) + v_i + \sum_{j=1}^{3} \frac{1}{\gamma_{ijk}} \dot{\theta}_{ijk}$$

(42)

As $Q_i$ is a positive-definite symmetric matrix, it can be restated as $Q_i = U_i^T \Lambda_i U_i$ based on Ref.[45], where $U_i$ denotes a unitary-matrix fulfilling $U_i^T U_i = I$, and $\Lambda_i$ is a diagonal-matrix including all eigenvalues of $Q_i$. Consequently, the following inequity is formulated,

$$\lambda_{\min}(Q_i) \|E_i\|^2 \leq E_i^T Q_i E_i$$

(43)

Using the above inequality, then expression in (42) becomes,

$$V_{Ti} \leq -\frac{1}{2} \lambda_{\min}(Q_i) \|E_i\|^2 + \frac{1}{2} \rho_1 \bar{w}_i^2 - \gamma_{ij} \alpha_{Fi} \varepsilon_i^2$$

(44)

$$\leq -\frac{1}{2} \lambda_{\min}(Q_i) \|E_i\|^2 + \frac{1}{2} \rho_1 \bar{w}_i^2 - \gamma_{ij} \alpha_{Fi} \varepsilon_i^2$$

(45)

According to the inequality (44), $V_{Ti} < 0$ holds for $\|E_i\| > \rho_1 \bar{w}_i (\lambda_{\min}(Q_i))^{-1/2}$. Thence the tracking error vetor of attitude angle $E_i$ converges to an arbitrarily small neighborhood around origin $D_{TE}$ presented in (38) through tuning the designed parameters and selecting the matrix $Q_i$. Properly, similarly. It is easy to demonstrate that $\varepsilon_{Fi}$ can converge to the region $D_e$ given in (38) on the basis of the formulation (44).

Assuming $E_i = [E_i^T, \varepsilon_{Fi}]^T, Q_{Ei} = diag(Q_i, \gamma_{ij} \alpha_{Fi}/2)$, thus inequality (44) can be further transformed as,

$$V_{Ti} \leq -\frac{1}{2} \dot{E}_i^T Q_{Ei} E_i + \frac{1}{2} \rho_1 \bar{w}_i^2$$

(46)

Inspired by the similar proof in Ref.[46], then the above inequality is derived as follows,

$$V_{Ti} \leq -\kappa_{min} V_{Ti} + \kappa_{min} V_{Ti}$$

(47)

in which $\kappa_{min} = \lambda_{\min}(Q_{Ei})/\lambda_{\min}(P_i), V_{Ti} \in \mathbb{R}^n$ represents a finite constant. Then solving (47), we have

$$V_{Ti}(t) \leq (V_{Ti}(0) - V_{Ti}(0)) e^{-\kappa_{min} t} + V_{Ti}(0)$$

(48)

then $V_{Ti}(t) \leq \max(V_{Ti}(0), V_{Ti}(0))$ can be obtained according to (48). Consequently, $V_{Ti} \in \mathcal{L}_\infty$ and it indicates that the states
3.2.2. Attitude stabilization controller design

Then the proof completes.

The designed attitude tracking controller can guarantee that the closed-loop control system of reentry RLV is semi-global asymptotic stable, and all related signals of this system are uniformly ultimately bounded. Then the proof completes.

Theorem 2 [46]: For any state vector \( x \in \mathbb{R}^3 \), the system uncertainty terms \( \Delta f_i(x) \) and \( \Delta g(x) \) of reentry RLV satisfy the following inequalities,

\[
- f_i'(\theta_i'|x) \leq \Delta f_i(x) \leq f_i'(\theta_i'|x),
- g_i'(x) \leq \Delta g_i(x) \leq g_i'(\theta_i'|x)
\]

where \( f_i'(\theta_i'|x) = m_{ij}, \ g_i'(\theta_i'|x) = m_{ijg}, \ m_f \in \mathbb{R}^{3 \times 1} \) and \( m_{ijg} \in \mathbb{R}^{3 \times 3} \) denote the vectors of boundary parameters.

Since the fuzzy logic systems in (21) and (22) and H-infinity controller term (25) are invalid when \( x \in \Omega \), the robust stabilization terms are introduced to design the attitude controller. Then the attitude stabilization controller is designed as

\[
u_{ii} = [g(x) + v_y]^{-1}[-f(x) - v_f + \hat{y}_d + KE]
\]

where \( v_f = [v_{1f}, v_{2f}, v_{3f}]^T, \ v_y = [v_{ijg}] \in \mathbb{R}^{3 \times 3} \) represent the robust stabilization terms. And \( v_{ijg} = -\text{sgn}(E_i^TP_iB_i)\hat{m}_{ijg} \) in which \( u_a = -f(x) - v_f + \hat{y}_d + KE, \ u_a = [u_{1a}, u_{2a}, u_{3a}]^T \) and \( m_{ij}, \ m_{ijg} \) are the estimated values of \( m_{ij}, \ m_{ijg} \) respectively.

According to the Eqs. (20) and (49), yields the two dynamic equations respectively as follows,

\[
\ddot{y} = f(x) + \Delta f_i(x) + [g(x) + \Delta g(x)]u_T
\]

(50)

\[
\ddot{y}_d = f(x) + v_f + (g(x) + v_y)u_T - KE
\]

(51)

Similarly, subtracting (50) from (51) and carrying out some transformation, the dynamic of attitude tracking error vector can be obtained as,

\[
\dot{E}_i = A_i E_i + B_i[v_{ij} - \Delta f_i(x) + \sum_{j=1}^3 v_{ijg} \Delta g_{ijg}(x)]u_{Tj}
\]

(52)

Due to the proper selection of the parameter vector \( K_i \), given in Eq. (24), the matrix \( A_i \) is obviously stable. Therefore, there must exist a unique positive-definite matrix \( P_i = P_i^T \) satisfying the following Lyapunov-equation,

\[
A_i^TP_i + P_iA_i = -Q_i
\]

(53)

where \( Q_i \) represents an arbitrarily given positive-definite symmetric matrix.

For the purpose of averting the great control effect, the boundary adaptive technique is employed to stabilize the attitude control system (16) of reentry RLV and the estimation laws of boundary parameter vectors are constructed as follows,

\[
\dot{m}_{ij} = \eta_{ij}(E_i^TP_iB_i) - \epsilon_{ij}\hat{m}_{ij},
\]

(54)

where \( \epsilon_{ij}, \eta_{ij} > 0 \) are small constants pre-set by the designer, \( \eta_{ij}, \eta_{ijg} > 0 \) denote the learning rates.

Theorem 2: For the attitude control system (16) and (20) of reentry RLV, suppose that the Assumptions 1-2 are fulfilled when the aircraft attitude \( x \in \Omega \). The attitude stabilization controller is designed as (49) together with the estimation laws of boundary parameter vectors selected as (54), then the closed-loop control system can achieve global asymptotic stable, and the attitude tracking error vector \( E_i \) converges to a small neighborhood \( D_{ijg} \) around origin by tuning the controller parameters appropriately.

\[
D_{ijg} = \{E_i : |E_i| \leq [2|\max\{Q_i\}|]^{-1}((\epsilon_{ij}m_{ij}^2 + 3 \sum_{j=1}^3 \epsilon_{ijg}m_{ijg}^2))^2\}
\]

(55)

Proof:

Consider the Lyapunov candidate function \( V_{Hi} \) about the flight attitude dynamic as

\[
V_{Hi} = \frac{1}{2}E_i^TP_iE_i + \frac{1}{2\eta_{ij}}\hat{m}_{ij}^2 + \sum_{j=1}^3 \frac{1}{2\eta_{ijg}}\hat{m}_{ijg}^2
\]

(56)

Differentiating (56) with respect to time along the attitude tracking error dynamic (52) and utilizing (53), then

\[
\dot{V}_{Hi} = \frac{1}{2}E_i^TP_iE_i - E_i^TP_i\dot{B}_i[\text{sgn}(E_i^TP_iB_i)\hat{m}_{ij} + \Delta f_i(x)] + \sum_{j=1}^3 [sgn(E_i^TP_iB_i)u_{Tj}]\Delta g_{ijg}(x) + \sum_{j=1}^3 [sgn(E_i^TP_iB_i)u_{Tj}]\Delta g_{ijg}(x)u_{Tj}
\]

(57)

Substituting the estimation laws (54) into (57), \( V_{Hi} \) becomes,

\[
\dot{V}_{Hi} = \frac{1}{2}E_i^TP_iE_i - E_i^TP_i\dot{B}_i[\text{sgn}(E_i^TP_iB_i)\hat{m}_{ij} + \Delta f_i(x)] + \sum_{j=1}^3 [sgn(E_i^TP_iB_i)u_{Tj}]\Delta g_{ijg}(x)\hat{m}_{ij} + \sum_{j=1}^3 [sgn(E_i^TP_iB_i)u_{Tj}]\Delta g_{ijg}(x)\hat{m}_{ijg} + \hat{m}_{ij}^2(E_i^TP_iB_i)
\]

(58)

Therefore, the attitude stabilization controller design guarantees that the closed-loop control system of reentry RLV is semi-global asymptotic stable.
According to Assumptions 2, $V_{Hi}$ can be changed into,
\[
V_{Hi} \leq -\frac{1}{2}E_i^T Q E_i - e_{ij}\hat{m}_{ij}(\hat{m}_{ij} - m_{ij}) \\
- \sum_{j=1}^{3} e_{ij}\hat{m}_{ijk}(\hat{m}_{ijk} - m_{ijk})
\] 
(59)

By adopting the same inequality presented in (43), yields the following inequality,
\[
V_{Hi} \leq -\lambda_{\text{max}}(Q_i)||E_i||^2/2 - e_{ij}\hat{m}_{ij}(\hat{m}_{ij} - m_{ij}) \\
- \sum_{j=1}^{3} e_{ij}\hat{m}_{ijk}(\hat{m}_{ijk} - m_{ijk}) \\
\leq -\lambda_{\text{max}}(Q_i)||E_i||^2/2 - e_{ij}[(\hat{m}_{ij} + m_{ij})/2]^2 - \\
m_{ij}^2/4 - \sum_{j=1}^{3} e_{ij}\hat{m}_{ijk}(\hat{m}_{ijk} + m_{ijk})/2)^2 - m_{ijk}^2/4 \\
\leq -\lambda_{\text{max}}(Q_i)||E_i||^2/2 + [e_{ij}m_{ij}^2 + \sum_{j=1}^{3} e_{ij}m_{ijk}^2]/4
\] 
(60)

Similarly, $V_{T_i} < 0$ holds for $||E_i|| > \sqrt{2\lambda_{\text{max}}(Q_i)^{-1}[(e_{ij}m_{ij}^2 + \sum_{j=1}^{3} e_{ij}m_{ijk}^2)]^2}$ according to the inequality (60). Therefore, the tracking error vetor of attitude angle $E_i$ converges to an arbitrarily small neighborhood around origin $O_{Hi}$ given in (55) by tuning the designed parameters properly and $E_i \in \Lambda_{\text{oo}}$. On the basis of the definition of $E_i$, it is obvious that $x \in \Lambda_{\text{oo}}$. From the estimation laws in (54), $\hat{m}_{ij}, \hat{m}_{ijk} \in \Lambda_{\text{oo}}$ hold as well. In the light of Assumption 1, $f(x), g(x) \in \Lambda_{\text{oo}}$ can be obtained. Moreover, considering $y_{d} \in \Lambda_{\text{oo}}$, it can be concluded that all items on the right side of Eq.(49) are bounded. Then, it is easy to know that $u_{Hi} \in \Lambda_{\text{oo}}$. Accordingly, we can draw the conclusion that the closed-loop control system of reentry RLV is semi-global asymptotic stable, and all related signals are both uniformly ultimately bounded.

Then the proof completes.

With the goal to guarantee the smooth input of the control actuator for reentry RLV, the overall control law is developed based on the two attitude controllers designed in Eqs.(24) and (49),
\[
u = \tau u_{Hi} + (1 - \tau)u_{T},
\] 
(61)

where $\tau$ denotes a weight coefficient, which is described as,
\[
\tau = \begin{cases} 
1, & x \in \bar{\Omega} \\
(R_x - ||x||)/(R_x - R_0), & x \in \Omega - \bar{\Omega} \\
0, & x \in \Omega_0
\end{cases}
\] 
(62)

where $\Omega_0 = \{x : ||x|| \leq R_0\} \subset \Omega$, and $R_0$ is a finite constant pre-set by the designer.

**Theorem 3:** For the attitude control system (16) and (20) of reentry RLV, the overall attitude control law is designed as (61) together with the adaptive laws chosen as (37) and estimation laws of boundary parameter vectors selected as (54), then the closed-loop control system can achieve semi-global asymptotic stable, and the attitude tracking error vector $E_i$ converges to a small neighborhood $D_e$ around origin through tuning the controller parameters properly.

\[
D_e = \{E_i : ||E_i|| \leq \frac{1}{3}[\lambda_{\text{max}}(Q_i)]^{-1}[2(\rho_i\hat{w_i})^2 + (e_{ij}m_{ij}^2 + \sum_{j=1}^{3} e_{ij}m_{ijk}^2)]}\}
\] 
(63)

**Proof:**

Since the coefficients $\tau$ and $1 - \tau$ are non-negative, thus chose the Lyapunov candidate function $V_i$ about the flight attitude dynamic as
\[
V_i = V_{T_i} + V_{Hi}
\] 
(64)

According to the proofs in *Theorem 1* and 2, we obtain the derivative of $V_i$ as
\[
\dot{V}_i = \frac{1}{2}(\rho_i\hat{w_i})^2 - E_i^T Q E_i - \left(\frac{E_i^T P B_i}{\rho_i} + \rho_i\hat{w_i})^2\right] - g_{\alpha}(\hat{w}_{d}) - \\
- \frac{1}{2}E_i^T Q E_i - E_i^T P B_i[s_{\alpha}(E_i^T P B_i)^{\frac{1}{2}}x] - \\
E_i^T P B_i g_{\alpha} \sum_{j=1}^{3} \frac{1}{2}[s_{\alpha}(E_i^T P B_i)^{\frac{1}{2}}x] - \\
e_{ij}\hat{m}_{ij}(\hat{m}_{ij} - m_{ij}) - \sum_{j=1}^{3} e_{ij}\hat{m}_{ijk}(\hat{m}_{ijk} - m_{ijk})
\] 
(65)

Based on the inequalities (45) and (60), yields $V_i$
\[
\dot{V}_i \leq -\lambda_{\text{max}}(Q_i)||E_i||^2 + (\rho_i\hat{w_i})^2/2 + [e_{ij}m_{ij}^2 + \sum_{j=1}^{3} e_{ij}m_{ijk}^2]/4
\] 
(66)

Thus, $V_i < 0$ holds for $||E_i|| > \sqrt{\frac{1}{2}[\lambda_{\text{max}}(Q_i)]^{-1}[2(\rho_i\hat{w_i})^2 + (e_{ij}m_{ij}^2 + \sum_{j=1}^{3} e_{ij}m_{ijk}^2)]}$ according to the inequality (66). According to the similar proofs in *Theorem 1* and 2, we could conclude that the closed-loop control system of reentry RLV can achieve semi-global asymptotic stability under the overall attitude control law (61), and all related signals are both uniformly ultimately bounded.

Then the proof completes.

**Remark 1:** The adaptive laws in (40) and (41) combining with the *Theorem 1* indicate that the boundedness of adjusting parameter vectors $\theta_{ij}$ and $\theta_{ijk}$ can be guaranteed by the developed control strategy. Similarly, the estimation laws in (54) with the *Theorem 2* demonstrate that the boundedness of boundary parameter vectors $m_{ij}$ and $m_{ijk}$ can be guaranteed by the designed control scheme as well.

**Remark 2:** As can be seen from (44) and (60), the system stability and convergence rate depend on the appropriate selection of designed parameters. Generally, for the parameters $\gamma_{ij}, \gamma_{ijk}, \gamma_{ie}$ in (37) and $\eta_{ij}, \eta_{ijk}$ in (54), larger values of these parameters would prompt the flight states of the RLV converge to the origin in a faster speed, while too large $\gamma_{ij}, \gamma_{ijk}, \gamma_{ie}$ and
is feasible to adopt the sine function as expected command in with the command of sine function in practical flight process, it difficult to track the attitude angles for reentry RLV in accordance performance of the proposed control strategy more clearly. In this paper to show the attitude tracking simulation are presented as,

4. Simulation results and discussions

In this section, numerical simulations of attitude maneuver based on the 6-DoF dynamic model of reentry RLV are conducted to illustrate the performance and effectiveness of the attitude controller developed in the previous section.

4.1. Simulation parameters setting

The aircraft parameters adopted in the attitude controller design are presented in Table 1, where the parameters are obtained by curve fitting approximations in MATLAB. In this simulation, the controlled variables are roll angle $\phi$, pitch angle $\theta$ and yaw angle $\psi$, respectively. The controller inputs are aileron deflection $\delta_{a}$, the elevator deflection $\delta_{e}$ and the rudder deflection $\delta_{r}$. Moreover, the parameter uncertainties and external disturbances are added to demonstrate the robustness of the proposed control scheme. And the uncertainty terms $\Delta f(x)$, $\Delta g(x)$ and disturbance term $d$ added to each channel of reentry RLV in the simulation are presented as,

$$\Delta f(x) = 20\% f(x), \Delta g(x) = 20\% g(x)$$

$$d = \begin{bmatrix} 0.35 + 0.45 \sin(0.30t) \\ 2.25 + 0.75 \sin(0.50t) \\ 0.55 + 0.60 \sin(0.25t) \end{bmatrix}$$

Due to the fact that the flight states of RLVs vary with the flight height during its reentry process, thus two flight cases are taken into account in this paper to show the attitude tracking performance of the proposed control strategy more clearly. In case 1, the initial altitude is 50km and the expected outputs of $\theta$, $\phi$, $\psi$ are set to be zeros, and the initial aerodynamic angle $\alpha$, attitude angles $\phi$, $\psi$ and angular rates are all set to be zeros, and $\alpha = \theta = 25\degree$. Furthermore, all the initial actuator deflections $\delta_{a}, \delta_{e}, \delta_{r}$ are set to be zeros as well, and choose $R_{f} = 19, R_{b} = 13$. The simulation results for this case are presented in Fig. 3-Fig. 5, and some necessary curve explanations are shown in the figures as well for the better demonstration of flight process. In addition, an adaptive fuzzy H-infinity control (AFHC) method is chose to compare the attitude tracking performance of the vehicle.

The curves of roll angle, pitch angle and yaw angle tracking for the RLV are provided in Fig. 3 (a), (b) and (c) respectively, where the red solid line denotes the desired tracking commands, the blue solid line denotes the actual output signal of attitude an-

<table>
<thead>
<tr>
<th>Terms</th>
<th>Values</th>
<th>Terms</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>7500kg</td>
<td>$C_{0}^{f}$</td>
<td>4.63e-04</td>
</tr>
<tr>
<td>$\bar{b}$</td>
<td>3.12m</td>
<td>$C_{0}^{e}$</td>
<td>0.0039</td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td>1.30m</td>
<td>$C_{0}^{l}$</td>
<td>9.80e-28</td>
</tr>
<tr>
<td>$S$</td>
<td>4.17m$^2$</td>
<td>$C_{0}^{b}$</td>
<td>0.0023</td>
</tr>
<tr>
<td>$J_{xx}$</td>
<td>885kg$ \cdot m^2$</td>
<td>$C_{0}^{o}$</td>
<td>0.0028</td>
</tr>
<tr>
<td>$J_{yy}$</td>
<td>8110kg$ \cdot m^2$</td>
<td>$C_{0}^{p}$</td>
<td>0.2201</td>
</tr>
<tr>
<td>$J_{zz}$</td>
<td>7770kg$ \cdot m^2$</td>
<td>$C_{0}^{q}$</td>
<td>-0.0151</td>
</tr>
<tr>
<td>$J_{xg}$</td>
<td>-17.40kg$ \cdot m^2$</td>
<td>$C_{m}^{g}$</td>
<td>0.0117</td>
</tr>
<tr>
<td>$C_{0}^{0}$</td>
<td>-0.404</td>
<td>$C_{m}^{b}$</td>
<td>1.134e-10</td>
</tr>
<tr>
<td>$C_{0}^{L}$</td>
<td>0.0699</td>
<td>$C_{m}^{o}$</td>
<td>-0.008</td>
</tr>
<tr>
<td>$C_{0}^{e}$</td>
<td>0.0026</td>
<td>$C_{m}^{p}$</td>
<td>0.417</td>
</tr>
<tr>
<td>$C_{0}^{D}$</td>
<td>-0.239</td>
<td>$C_{n}^{b}$</td>
<td>7.95e-04</td>
</tr>
<tr>
<td>$C_{0}^{r}$</td>
<td>0.035</td>
<td>$C_{n}^{o}$</td>
<td>4.997e-04</td>
</tr>
<tr>
<td>$C_{0}^{L}$</td>
<td>-8.30e-10</td>
<td>$C_{n}^{p}$</td>
<td>0.0041</td>
</tr>
<tr>
<td>$C_{0}^{e}$</td>
<td>0.0021</td>
<td>$C_{n}^{q}$</td>
<td>0.142</td>
</tr>
</tbody>
</table>

4.2. Simulation discussions

Case 1: In this flight condition, the initial flight altitude and velocity are chosen as 50km and 15 Mach respectively. And the initial aerodynamic angle $\alpha$, attitude angles $\phi$, $\psi$ and angular rates are all set to be zeros, and $\alpha = \theta = 25\degree$. Furthermore, all the initial actuator deflections $\delta_{a}, \delta_{e}, \delta_{r}$ are set to be zeros as well, and choose $R_{f} = 19, R_{b} = 13$. The simulation results for this case are presented in Fig. 3-Fig. 5, and some necessary curve explanations are shown in the figures as well for the better demonstration of flight process. In addition, an adaptive fuzzy H-infinity control (AFHC) method is chose to compare the attitude tracking performance of the vehicle.

The curves of roll angle, pitch angle and yaw angle tracking for the RLV are provided in Fig. 3 (a), (b) and (c) respectively, where the red solid line denotes the desired tracking commands, the blue solid line denotes the actual output signal of attitude an-

Remark 3: From the flight control laws (24),(49) as well as the overall control law (61) of reentry RLV, it can be found that the proposed control approach does not need the boundary information of parameter uncertainties and external disturbances. The CAFHC scheme can adjust the designed parameters adaptively relying on the attitude tracking error vector $E$, and filter modeling error $E_{f}$. Thus, it can guarantee that the attitude control system can realize a good tracking performance and have a good robustness performance simultaneously.
ingle under the CAFHC strategy and the green solid line denotes the actual output under the AFHC strategy. It can be seen from the figures that the attitude angles are able to track the desired commands stably and smoothly in the presence of the parameter uncertainties and external disturbances during its reentry phase. In addition, the attitude control scheme designed in this paper performs better than the AFHC strategy.

To verify the tracking performance of attitude angle more clearly, the curves of roll angle, pitch angle and yaw angle tracking error are presented in Fig. 4 (a), (b) and (c). From the simulation results, it is obvious that the attitude controller developed in this study can track the desired commands within a short period of time. Since the airborne condition and flight state in the height of 50km are more complicated, thus there is about 10 deg changing magnitude within 0.25 seconds showed in the curve of pitch angle tracking error. And then the tracking error of three-channel attitude angles converge to a small neighborhood rapidly, and the tracking error curves fluctuate around zero in a small range. Besides, the attitude tracking errors by the CAFHC method are smaller than by the AFHC method as shown in the figures.

In the meantime, the time histories of the deflections of control actuators are given in Fig. 5 (a), (b) and (c). From the simulation, we can observe that the control actuators vary within a reasonable range (i.e., all the control actuators’ deflections vary between -20deg and 20deg), which ensures that the attitude control system has a good tracking performance. As the “coupling problem” exists between the roll angle channel and yaw angle channel, so the responses of aileron deflection and rudder deflection show some small fluctuations, while these fluctuations are within the control range and they do not affect the attitude tracking for reentry RLV.

Case 2: In this case, the initial flight altitude and velocity are selected as 30km and 8 Mach respectively. Since the atmospheric density at this altitude is thicker than the atmospheric density at 50km, hence it unnecessary to use the high angle of attack and pitch for control input in this simulation.
\[ \alpha = \theta = 15 \text{ deg}, \] and the initial values of the other parameters setting of the RLV are set as same as the simulation in case 1. And choose \( R_x = 12, R_0 = 10. \) The simulation results are provided in Fig. 6-Fig. 8, and several curve explanations are marked in the figures as well for the better illustration of dynamic process. Besides, the attitude tracking performance under the AFHC strategy is compared with the proposed scheme in this paper.

The flight attitude tracking curve of the RLV during the reentry phase is provided in Fig. 6, where the red solid line denotes the desired tracking commands, the blue solid line denotes the actual output of attitude angle by the CAFHC strategy and the green solid line denotes the actual output by the AFHC strategy. As shown in Fig. 6, it is obvious that the roll angle, pitch angle and yaw angle can track the desired commands well under the parameter uncertainties and external disturbances in this flight condition as well, and the actual attitude angle of reentry RLV by the CAFHC scheme can respond to the guidance commands with a higher accuracy than by the AFHC scheme.

The tracking errors of attitude angulars are shown in Fig. 7 (a), (b) and (c) respectively, which implies that the reentry vehicle can realize a stable tracking in a short time under the action of attitude control law proposed in this paper. And the maximum tracking error of roll angle, pitch angle and yaw angle does not exceed 0.65 deg, which demonstrates that the developed attitude controller in this paper not only tracks the guidance commands with a high accuracy, but also outperforms the AFHC approach.

Meanwhile, with the aim to verify the validity and capability of the designed attitude controller, the time histories of control actuators are presented in Fig. 8. From the simulation results, we can come to a conclusion that the deflections of control actuators response timely in the permitted range as the guidance commands change.

5. Conclusion

In this paper, a novel attitude control scheme based on CAFHC is developed to achieve a stable and reliable flight process for RLVs during reentry phase where the parameter un-
of parameter uncertainties and external disturbances. Meanwhile, the simulation results and analysis have illustrated that the proposed control strategy performs better and track the desired attitude angles for reentry RLV with a higher accuracy than the AFHC method.

### Acknowledgments

This work was partially supported by the Natural Science Foundation of China (NSFC) under Grant (61273092, 61673294, 91016018) and the Natural Science Foundation of Tianjin under Grant 12JCZDJC30300.

### References


[34] X. Luo, J. Li, Fuzzy dynamic characteristic model based attitude control of hypersonic vehicle in gliding phase, Science China Information Sciences 54 (3) (2011) 448–459.


