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What can we learn about the Average Treatment Effect of retirement on consumption?

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January 30, 2019

Abstract

We present a nonparametric bounds analysis of the Average Treatment Effect (ATE) of retirement on domestic expenditure. We consider identification under a wide catalogue of assumptions which restrict either the potential outcomes or the process of selection into treatment. We put forward new assumptions that exploit the longitudinal information in panels to refine existing results regarding the partial identification of ATEs. The tightest identification region suggest that retirement could lead to a drop of up to 7% in expenditure. However, the sign of the actual effect is not identified unless one restricts the variation in potential outcomes across households or one assumes that, on average, retirement is detrimental for consumption. The latter assumption might be suitable only in environments like the United States or the United Kingdom, where the pension replacement rate are low. We find that savings and education have mitigating effects on the magnitude of the maximum potential drop in expenditure.

Key Words: Partial Identification, Monotonicity Restrictions, Panel Data, Retirement, Consumption, Expenditure, Housework, Average Treatment Effect.

JEL Classification: C21, C30, C90, J26, I18.

Word count: 4514.

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1 Introduction.

This article presents a nonparametric bounds analysis of the Average Treatment Effect (ATE) of retirement on one of the principal determinants of consumption: domestic expenditure\textsuperscript{1}. Ageing is increasing the contribution of the older segments of the population to the economy, and this trend is expected to continue in the future. In the UK, for example, data from the Office for National Statistics reveals that 18% of the population was 65 years of age or older in 2017; this proportion is expected to increase to 24% by 2040. Most individuals in the older segments of society are retired or approaching retirement. Therefore, it is important to understand how demographic trends might affect consumption via retirement: not only is consumption a crucial determinant of individuals’ welfare (and thus decision taking), but it is also a crucial macroeconomic aggregate, accounting for over 60% of the Gross Domestic Product in modern economies. The question acquires particular relevance in countries like the United Kingdom (UK) and the United States (US), where pension replacement rates are low, with retirement often leading to a sizeable drop in personal income.

Existing research has systematically documented a one-off drop in expenditure following retirement, a regularity which has been reported for countries as diverse as Italy (Battistin et al., 2009), China (Li et al., 2015), Spain (Luengo-Prado and Sevilla, 2012), UK (Banks et al., 1998) and the US (see Bernheim et al., 2001; Ameriks et al., 2007; D. Hurd and Rohwedder, 2008; Hurst, 2008 ). Most of these sources have noted that observed declines in expenditure following retirement are limited to food and work-related expenses. However, whether this drop in expenditure reveals a significant drop in actual consumption is a hotly debated topic of research, not only because such a finding would contradict the predictions of well accepted models such as Modigliani’s life-cycle model (e.g Modigliani, 1966), but also because of the difficulty of characterising any substitution of food produced at home for market-produced goods—eating out in particular (see, for example, Banks et al., 1998; Aguiar and Hurst, 2005; Hurd and Rohwedder, 2013; Luengo-Prado and Sevilla, 2012; Olafsson and Pagel, 2018 to mention but a few).

\textsuperscript{1}Domestic expenditure refers to the household’s food expenditure together with estimates of non-food items such as paper products, home cleaning supplies and pet foods, but excluding alcoholic drinks.
When trying to measure the variation in expenditure implied by retirement, several complicating factors complicate estimation. These include pension replacement rates, trends in expenditure and life-styles, and expectations about post-retirement consumption or health. As a result, researchers have put forward a range of instrumental variable strategies to circumvent confounding and point-identify the effect of retirement\(^2\). An important limitation of these strategies is that, while policy-makers are interested in population-wide effects, instrumental variable methods identify the effect of retirement on a sub-population of compliers. The latter are those individuals whose retirement decisions can be explained by known exogenous factors, such as pension or retirement eligibility rules. In other words, where the parameter of interest for policy is the ATE, instrumental variable methods estimate the Local Average Treatment Effect (LATE; Imbens and Angrist, 1994 and Angrist, Imbens, and Rubin, 1996)\(^3\) Given the problems of confounding, and given the likely infeasibility of allocating individuals to retirement in a randomize control trial fashion, what can we credibly learn about the ATE of retirement?

This article explores what information about the consequences of retirement can be extracted from data under assumptions of varying credibility and strength. The strongest assumptions we consider point identify ATE, but their credibility is secondary to their convenience. The results thus obtained are very sensitive to minor variations in the information set. The weakest assumptions that we consider are virtually unquestionable and they partially identify the ATE of retirement. However, the ensuing identification regions are too large to be informative. Between these two polar scenarios, we consider a range of assumptions which restrict either the counterfactual potential outcomes or the process of selection into treatment. The former assumptions include Manski’s Monotone Treatment Response (Man-
ski, 1997) and Manski and Pepper’s Bounded Variation assumptions (Manski and Pepper, 2013, Manski and Pepper, 2017) but we also put forward a new set of assumptions (Mono-
tone Time Varying Response), which exploit longitudinal information in datasets to narrow
the no-assumption bounds in Manski (1990). Among the assumptions restricting the process
of selection into treatment, we include Manski and Pepper’s Monotone Treatment Selection
(Manski and Pepper, 2000) as well as a new dynamic version of the latter, which exploits lon-
gitudinal information in datasets to narrow the Monotone Treatment Selection bounds. The
new assumptions introduced in this paper complement, and can be combined with, previous

Our empirical analysis is based on ‘Understanding Society’ a longitudinal study representa-
tive of the UK population. The focus is on the years 2011-2016. The tightest identification
regions reported here suggest that retirement could have, at most, a small negative effect on
domestic expenditure of up to 7%. Critically, however, the sign of the effect of retirement is
only identified if the variation in potential outcomes across individuals is itself confined to a
discrepancy of ±7%. Otherwise, we can’t rule out increases in expenditure following retire-
ment, unless one among two alternative sets of additional assumptions also hold. The first set
of assumptions imposes (a) that the variation of potential expenditure levels across households
is confined to ±21% and (b) the independence of the retirement decisions and the potential
level of household expenditure in the absence of retirement. In other words, this first pair of
assumptions limits the relationship between pre-retirement consumption and the retirement
decision (which involves limiting the degree of myopic behaviour in people). The second set
of assumptions imposes that potential outcomes under retirement will be lower than potential
outcomes when retirement does not occur. Such and assumption might be suitable in context
such as the UK or the US were pension replacement rates are low, but it might fail to hold
in countries like Spain, where pension replacement rates are high (Luengo-Prado and Sevilla,
2012). Both sets of assumptions suggest that the large effects reported in the literature for the
sub-population of compliers would not apply more generally (for example, Battistin et al., 2009
report a drop in consumption of about 14%; Li et al., 2015 find a 11% drop; Luengo-Prado
and Sevilla, 2012 find drops of between 5 and 13%).
We also find that a household’s average savings and levels of educational attainment influence the width of the identification regions. All the weakest assumptions we consider coincide in endowing savings and education a mitigating effect: the identification region for households with above median savings or where educational attainment is higher have a higher lower bound for ATE. This does not imply that the actual effect of retirement is lower for these households, but it suggests that retirement can potentially lead to larger drops of expenditure in households where savings or educational attainment is lower. The effect of education is particularly stark: the largest lower bound for the ATE of retirement on domestic expenditure suggest a maximum drop of 2-3% in domestic expenditure in households with the highest educational attainment, compared to a 6-7% in other households.

The structure of the paper is as follows. Section 2 presents the data and discusses the problem of selection, introducing some notation. Section 3 studies identification under extreme assumptions. On the one hand, we present a family of parametric models which establish the magnitude and sign of the ATE with incredible certitude (Manski, 2018). On the other hand, we present identification regions under bounded potential outcomes which, though highly credible and weak, convey little information about ATE. Sections 4 to 6 consider alternative, middle of the way, assumptions. Section 4 considers assumptions that restrict the potential outcomes. Section 5 considers assumptions of bounded variation. Section 6 considers assumptions that restrict the process of selection. This section also looks at the role of education and savings on the identification regions. Finally, Section 7 concludes the paper with some remarks.

2 Data and the problem of selection.

The empirical analyses below rely on waves 3 to 6 (corresponding to years 2011-2016) of ‘Understanding Society’, a large panel representative of the UK population\(^4\). The unit of analysis is the household at a given point in time, where households are constituted by the reference person (the main respondent) and the partner when there is one. In accordance to this definition, the same household at two different points in time constitutes two different units (this is conventional in the literature on program evaluation; Imbens and Rubin, 2015).

\(^4\)Waves 1,2,7 and 8 are also used, but only to support estimation of the bounds described below
For the purposes of the analysis, a person is retired if that is her self-reported status. However, we make a number of imputations and exclusions. If an individual mentions ‘caring for a family member’ as the main occupation, but this individual had previously reported to be retired from the workforce, then we record the person’s work status as ‘retired’. In addition to this, we exclude from the analysis those households where a reference person or a partner returns to employment after a spell of retirement. We also exclude those households where the reference person or the partner moves into retirement from unemployment or after caring for a relative. Finally, we keep only households where the reference person is observed between the ages of 50 and 75 at least once.

The primary outcome of interest is the ratio of a unit’s domestic expenditure to the average domestic expenditure across units in the same wave of the panel. A unit’s self-reported ‘domestic expenditure’ refers to the household’s food expenditure during the four weeks before the interview, together with estimates of non-food items such as paper products, home cleaning supplies and pet foods, but excluding alcoholic drinks. By using the ratio of reported expenditure to average expenditure, we can interpret treatment effects in terms of proportional changes. We deflate expenditure using the annual Consumer Price Index including owner occupiers’ housing costs (CPIH), calculated by the UK Office for National Statistics. Treated units are those households in which a respondent or a partner is retired. The non-treated units are those households in which the respondent and the partner are not retired.

Table 1 shows some descriptive statistics of the final sample. There are 6,965 main respondents in the sample, contributing 38,390 valid observations (for which domestic expenditure levels have been recorded). Main respondents are on average 67.5 years of age, predominantly men (54%), and 25% of them completed high school. 69% of their observations are recorded under retirement and another 25% were recorded under employment. Half of these men are married or in a relationship (51.8%). We also have data from 3587 partners. These individuals are predominantly women (75%), with an average age of 64 years of age, 25% of whom completed high school. 60% of their observations were recorded under retirement and

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5 More precisely, respondents are asked to report ‘About how much has your household spent in total on food and groceries in the last four weeks from a supermarket or other food shop or market? Please do not include alcohol but do include non-food items such as paper products, home cleaning supplies and pet foods.’
### Table 1: Descriptive Statistics. (*) Domestic expenditure includes food and groceries from a supermarket or other food shop or market and non-food items such as paper products, home cleaning supplies and pet foods. It excludes alcoholic drinks. (†) Equals 1 if the highest educational qualification is a university degree, diploma of higher education, teaching qualification, nursing qualification or A level.

Another 33% were recorded under employment. The total number of observed retirements are 1280 among respondents and 662 among their partners. The average reported domestic expenditure during the four weeks prior to the interview is £256. This constitutes 10% of the average household gross monthly income of £2503. The average monthly pension income (£550) represents 21% of the average monthly gross household income. This simply reveals the well established fact that pension replacement rates are, on average, low in the UK and this is likely to impose substantial constraints on household consumption. Note, further, that domestic expenditure in the four weeks prior to the interview represents 46% of the average monthly pension income.
2.1 The problem of selection.

When studying the consequences of retirement for domestic expenditure in the UK (and elsewhere) we face the problem of selection, which arises because we cannot observe a unit’s non-retirement expenditure when some members of the household are retired (and vice versa). The problem can be formalised using Rubin and Holland’s potential outcomes framework (Rubin, 1974; Holland, 1986). Let $Y_{it}$ be the level of domestic expenditure revealed by household $i = 1, \ldots, N$ at $t = 1, \ldots, T$. The binary indicator $R_{it}$ equals 1 if the reference person, the partner or both are retired, otherwise $R_{it}$ equals 0. We assume throughout that $R$ is a permanent treatment. Therefore $R_{it} = 1 \Rightarrow R_{it+1} = 1$. From the perspective of the methods to be discussed below, this assumption covers a majority of empirical applications of interest, including the estimation of the causal effects of natural disasters (Maccini and Yang, 2009; Shah and Steinberg, 2017), changes to education policies (Oreopoulos, 2006), retirement (Battistin et al., 2009), serving in the military (Angrist, 1990), educational environment (Angrist and Lavy, 1999) or chronic diseases such as cancer, HIV, Type I diabetes, to mention but a few.

Let $Y_{it}(r)$ be the potential level of domestic expenditure that a unit would reveal when a respondent or a partner is retired ($r = 1$) or when neither the respondent nor the partner are retired ($r = 0$). We can then define a sequence of treatment effects

$$ATE_t = E\left(Y_{it}(1) - Y_{it}(0)\right), \text{ for } t = 1 \ldots, T.$$ (2.1)

where expectations are taken across the population of units. The parameters in the above sequence are not point identifiable from observational data alone because, for every $t$,

$$E(Y_{it}(r)) = E(Y_{it}(r)|R_{it} = 1)P(R_{it} = 1) + E(Y_{it}(r)|R_{it} = 0)P(R_{it} = 0)$$ (2.2)

where $E(Y(r)|R \neq r)$ is not revealed by data. More specifically, the difference in conditional

\footnote{Nonetheless, most of the results are applicable to non-permanent treatments (e.g. unemployment) without modification.}
does not have a causal interpretation, unless one could argue that \( Y_{it}(r) \) does not vary with \( R_{it} \). Under such an invariance assumption, the right hand side of (2.3) reduces to (2.1). However such assumption is strong and guarantees controversy in observational studies. First, retirement is not amenable to randomization and, second, there might be a process of non-random selection into treatment, captured by the second term in the right hand side of (2.3). For example, if people are not myopic about prospective levels of outcomes in short and mid term horizons, then varying expectations about expenditure power during retirement might condition retirement decision. Conversely, Parker et al. (2013) provide empirical evidence showing that lower cognitive ability workers are more likely to retire earlier. This could be due to various factors. These workers might be subject to less favourable working conditions or more physically demanding jobs, which might make staying at work less attractive. Similarly, employers have an incentive to keep higher ability individuals at work for longer as these are likely to be more productive. Furthermore, productivity and educational attainment are further correlated with income and expenditure and, ultimately might also correlated with retirement decisions.

Given the above problem of selection, what can we learn about the causal effect of retirement on domestic expenditure?

3 Identification under polar assumptions.

3.1 Parametric point identification.

One can point identify the ATE of retirement on expenditure under strong parametric assumptions. As is routine in the econometrics literature we could first assume that, in the absence of retirement, domestic expenditure follows a linear additive model such as \( E(Y(0)|X) = \)
\[ \alpha + \beta' X, \] where \( X \) is a \( k \times 1 \) vector of explanatory variables, which might include household income, the age of household members and additional measures of household wealth and individual characteristics. The vector \( X \) often includes a set of fixed effects capturing the influence of unobservable, time-invariant, individual specific traits. The linear specification is convenient, but there are not a priori obvious reasons (other than operational convenience) why such specification should be preferred to any other model.

In addition to the linear specification for the average \( Y(0) \) we could assume, as is often done, that retirement leads to a permanent, homogeneous shift in expenditure\(^7\), so that \( E(Y(1)|X) = E(Y(0)|X) + ATE \). Any remaining discrepancies between the expected and actual level of \( Y \) could be attributed to some stochastic process \( \varepsilon \), so that the generating mechanism governing \( Y \) is fully characterised in the following equation:

\[
Y = \alpha + \beta' X + ATE \cdot R + \varepsilon
\]

(3.1)

To characterise the properties of any estimator of \( ATE \) and \( \beta \) we need to further restrict the behaviour of \( \varepsilon \). Specifically, unconfounded least squares estimation requires that \( E(\varepsilon) = 0 \) and that the magnitude of \( \varepsilon \) is not associated with \( R \) or any of the elements in \( X \). The latter assumption is particularly strong, as it rules out, off hand, measurement errors and selection biases. If one wants to undertake statistical inferences regarding the magnitude or significance of \( ATE \) then it is customary to further characterise the distribution of \( \varepsilon \). We need to decide if \( \varepsilon \) has a constant variance across units (homoskedasticity) or it follows some (probably unspecified) pattern of variation (this would encompass situations with an linearly additive, albeit heterogeneous treatment effect, \( ATE_i = ATE + h(X) + \nu \) for some i.i.d. white noise process \( \nu \) and some affine transformation of \( X, h(\cdot) \)).

Finally, it is necessary to specify the relationship across units. A common assumption is that, whereas units are independent of each other, the error term might exhibit cross-sectional correlations in accordance to some variable. For example, given the availability of panel data, we could assume that the error term is correlated within individuals, but independent across

\(^7\)Alternatively, we could assume that \( ATE \) varies across individuals in accordance to specific schemes. For example, researchers often put forward a multi-level model with \( ATE_i = ATE + \nu_i \). This innovation has no bearing on the estimates fore \( E(ATE_i) \) but it will influence the structure and values of the covariance matrix.
### Table 2: Parametric estimates of the Average Treatment Effect of retirement on domestic expenditure.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retired(^1) ((ATE))</td>
<td>-0.066(^{***})</td>
<td>0.037(^{***})</td>
<td>-0.033(^{***})</td>
<td>-0.022(^{**})</td>
<td>-0.025(^{**})</td>
</tr>
<tr>
<td>Average age</td>
<td>0.052 (0.178)</td>
<td>0.101 (0.155)</td>
<td>-0.162 (0.134)</td>
<td>-0.183 (0.134)</td>
<td></td>
</tr>
<tr>
<td>Average age(^2)</td>
<td>-0.001 (0.003)</td>
<td>-0.002 (0.002)</td>
<td>0.002 (0.002)</td>
<td>0.002 (0.002)</td>
<td></td>
</tr>
<tr>
<td>Average age(^3)</td>
<td>0.000 (0.000)</td>
<td>0.000 (0.000)</td>
<td>-0.000 (0.000)</td>
<td>-0.000 (0.000)</td>
<td></td>
</tr>
<tr>
<td>Reference person in relationship</td>
<td>0.440(^{***})</td>
<td>0.172(^{***})</td>
<td>0.163(^{***})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finished high school(^1)</td>
<td>0.087(^{***})</td>
<td>0.114(^{**})</td>
<td>0.086(^*)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>1.048(^{***})</td>
<td>0.348 (3.994)</td>
<td>-1.018 (3.476)</td>
<td>4.977(^*)</td>
<td>6.234(^{**})</td>
</tr>
<tr>
<td>Household indicator</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Wave indicators</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
</tr>
</tbody>
</table>

Observations: 38,390

\(\dagger\) 1 if true for reference person or partner.

The dependent variable is the ratio of household domestic expenditure to average domestic expenditure in the sample. Estimation relies on linear additive models for potential outcomes, with linear additive homogeneous treatment effects, assuming independent units, subject to an error term which might be correlated within units (but not between units) and which is unexplained by the covariates in \(X\) or retirement itself. Clustered standard errors (at household level) are reported in parentheses. All models were estimated via Ordinary Least Squares. Models 4 and 5, which include household dummy variables, correspond to Fixed Effects linear panel data models.

Table 2 presents estimates of the \(ATE\) of retirement on domestic expenditure. To try to endow the analysis with some additional credibility, we consider five different nested structural specifications which range from a model with an intercept and an indicator capturing the retirement status of household members, to a model including a cubic polynomial in the average household age, marital status of the reference person, whether the respondent or the partner have a

...individuals. This assumption, which is maintained throughout this section, has not bearing on the consistency of the usual least squares estimator, but it has important consequences for the power and size of the associated inferential procedures (e.g. Moulton, 1986).
higher education degree and wave and household indicators (the latter corresponding to a fixed effects linear regression model). To take into account any potential cross-sectional correlation generated at individual level, we have estimated cluster-robust standard errors (at household level), which are reported in parentheses.

The main feature of the table is the large variation in the estimated effect of retirement across specifications. With the exception of model 2 (which suggest in increase in domestic expenditure following retirement), all models suggest that domestic expenditure falls with retirement. However, the magnitude and significance of the result varies enormously across models, ranging from a significant drop of 6% in the simplest model with just an intercept and the retirement indicator, to a statistically significant increase of 3.7%, following retirement and given a flexible third order polynomial on age. Domestic expenditure does not seem to vary with the average age of a household. Rather, expenditure is higher in households where the respondent is married or in a relationship (by about 16 to 40%, depending on the model under consideration -more comprehensive models suggesting lower effects) and expenditure is also positively correlated with higher educational attainment. Specifically, households were the respondent or the partner had a higher education degree reported between 8-11% higher expenditure in food.

Overall, the results are very sensitive to the parametric specification and, in particular, to the inclusion of household fixed effects. This latter finding would imply that there is substantial heterogeneity in consumption patterns across households. Once that heterogeneity is taken into account, retirement would seem to be innocuous for domestic expenditure. What this parametric analysis does not reveal, however, is which of the postulated specifications is closest to the unobservable process that generated the data. More critically, we remain in the dark in terms of how sensible the assumptions underpinning the structural models are. It is also unclear how results might vary if any potential effect of wealth on retirement were taken into consideration. If the latter form of endogeneity had any bearing on retirement decisions, then all the estimates in the table would lack of any meaning.


3.2 Partial identification under bounded outcomes.

In the preceding section, identification relied on a battery of parametric assumptions including a homogeneous linear effect, independence of treatment and the unobservable potential outcomes and independence of the distribution of the error term and the regressors. If any of these assumptions is wrong, then the preceding results will likely be misleading. Therefore, a natural question to ask is what can be learned about the relationship between retirement and expenditure without introducing substantive structural assumptions. Manski (1990) explored this issue finding that, although point identification under no assumptions is not achievable, it is possible to partially identify $ATE_t$ if the range of $Y$ has well defined bounds $y_{max}$ and $y_{min}$. Specifically, without any further assumption, (2.2) implies the following identification regions:

$$E(Y_{it}(0)) \in [\min, \max] = [E(Y_{it}\mid R_{it} = 0)P(R_{it} = 0) + y_{min}P(R_{it} = 1); \\
E(Y_{it}\mid R_{it} = 0)P(R_{it} = 0) + y_{max}P(R_{it} = 1)].$$

$$E(Y_{it}(1)) \in [\min, \max] = [E(Y_{it}\mid R_{it} = 1)P(R_{it} = 1) + y_{min}P(R_{it} = 0); \\
E(Y_{it}\mid R_{it} = 1)P(R_{it} = 1) + y_{max}P(R_{it} = 0)].$$

These bounds have often been referred to as the No-Assumption Bounds (and we adopt the acronym NAB hereafter to refer the them). Manski already anticipates that these bounds provide only limited information about $ATE_t$. Under the bounded outcome assumption, the identification region for $E(Y_{it}(0))$ is narrowest, and therefore most informative, when the proportion of retirees is smallest. However, the smallest the proportion of retirees, the largest is the identification region for $E(Y_{it}(1))$, which depends on the proportion of non-retirees. The consequence of this trade-off is that, for any $t$, (3.2) and (3.1) do not reveal much information.
about the ATE of retirement. Specifically,

\[
ATE_t \in \left[ \theta_{nab}; \bar{\theta}_{nab} \right] = \left[ E(Y_{it}|R_{it} = 1)P(R_{it} = 1) - E(Y_{it}|R_{it} = 0)P(R_{it} = 0) \\
+ y_{min}P(R_{it} = 0) - y_{max}P(R_{it} = 1); \\
E(Y_{it}|R_{it} = 1)P(R_{it} = 1) - E(Y_{it}|R_{it} = 0)P(R_{it} = 0) \\
+ y_{max}P(R_{it} = 0) - y_{min}P(R_{it} = 1) \right]
\]

(3.3)

The length of this interval ultimately depends on the admissible bounds around \(Y\). Therefore, as will be empirically verified in the following section, in the absence of further assumptions, data alone can normally only provide scant information about \(ATE_t\).

### 3.2.1 Estimation and results.

The preceding nonparametric bounds and those featuring in sections 4 and 6 below can be estimated using nonparametric regression methods. Specifically, the various conditional moments that require estimation can be approximated using the following Nadaraya-Watson estimator,

\[
\hat{E}(Y_{it}|R_{it} = r, R_{it'} = r') = \frac{\sum_{s=-\infty}^{+\infty} K(t - s/h) \hat{E}(Y_{is}|R_{is} = r, R_{is} = r')}{\sum_{s=-\infty}^{+\infty} K(t - s/h)}
\]

(3.4)

where \(K(.)\) is a kernel function (e.g. Normal, Epanechnikov, Triangular), \(h\) is the bandwidth parameter controlling the amount of smoothing (e.g. Hardle, 1990), and

\[
\hat{E}(Y_{is}|R_{is} = r, R_{is} = r') = \frac{\sum_{i=1}^{N} Y_{is} \mathbb{I}(R_{is-1} = r, R_{is} = r')}{\sum_{i=1}^{N} \mathbb{I}(R_{is-1} = r, R_{is} = r')}
\]

(3.5)

To obtain standard errors, one can resort to first order asymptotic theory. In so far asymptotic results critically hinge on sample sizes approaching infinity, in this article we advocate the use of re-sampling techniques. Specifically, we use a clustered-bootstrap algorithm where clusters, which are identified at household level, are re-sampled independently of each other with replacement. This takes into account inter-temporal correlations within individuals and maintains the panel structure of the data.

Once standard errors have been computed, it is possible to obtain confidence intervals
for the $ATE$. Following the now standard practice, we calculate 95% confidence intervals as described by Imbens and Manski (2004), with

$$CI_\alpha^\theta = \left[ \hat{\theta}_l - \bar{C}_N \frac{\hat{\sigma}_l}{\sqrt{N}}, \hat{\theta}_u + \bar{C}_N \frac{\hat{\sigma}_u}{\sqrt{N}} \right]$$

(3.6)

for some lower/upper bounds $\theta_l/\theta_u$ and where, in the above expression, $\bar{C}_N$ satisfies,

$$\Phi \left( \bar{C}_N + \frac{\hat{\theta}_u - \hat{\theta}_l}{\max(\hat{\sigma}_l, \hat{\sigma}_u)} \sqrt{N} \right) - \Phi(-\bar{C}_N)$$

(3.7)

The values of $\bar{C}_N$ can be calculated using the bisection method (Press et al., 1992). The above confidence interval asymptotically covers the true value of the intention to treat effect with fixed probability (95% in our applications).

The estimated NAB for each of waves 3 to 6 are reported in Table 3. As already anticipated in the previous section, the NAB is not informative, binding the effect of retirement on domestic expenditure between a drop of 190% to an increase of 133% on average. This range is too vast to obtain any meaningful conclusion about the parameter of interest.

Table 3: Partial identification region under the No Assumption Bound (NAB) for the ATE of retirement on domestic expenditure. Imbens and Manski’s 95% confidence interval for the true parameter value in parentheses.

<table>
<thead>
<tr>
<th>Wave 3</th>
<th>Wave 4</th>
<th>Wave 5</th>
<th>Wave 6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(-1.921 1.351)</td>
<td>(-1.941 1.329)</td>
<td>(-1.970 1.304)</td>
</tr>
<tr>
<td>Observations</td>
<td>5616</td>
<td>5208</td>
<td>4955</td>
</tr>
</tbody>
</table>
4 Partial identification assuming monotonicity of potential outcomes.

4.1 Weak cross-sectional monotonicity in potential outcomes.

We can take a first step towards obtaining more credible and informative estimates if we have any prior beliefs regarding the direction in which retirement might affect expenditure. In this sense, previous research suggests (and Table 1 confirms) that in the United Kingdom pension replacement rates are low, with retirement generally leading to a drop in personal income. Thus, on average, retirement implies a restriction on households expenditure power. It would follow that it is potentially uncontroversial to assume that domestic expenditure does not increase following retirement. More precisely, we could follow Manski (1997) and impose the Monotone Treatment Response (MTR) assumption that

\[
Y_{it}(1) \leq Y_{it}(0)
\]

leading to

\[
E(Y_{it}(0)) \in \left[ Y_{0mtr}; Y_{0nab} \right]
\]

\[
= \left[ E(Y_{it}|R_{it} = 0)P(R_{it} = 0) + E(Y_{it}|R_{it} = 1)P(R_{it} = 1); \right.
\]

\[
E(Y_{it}|R_{it} = 0)P(R_{it} = 0) + y_{max}P(R_{it} = 1) \right]
\]

(4.1)

\[
E(Y_{it}(1)) \in \left[ Y_{1nab}; Y_{1mtr} \right]
\]

\[
= \left[ E(Y_{it}|R_{it} = 1)P(R_{it} = 1) + y_{min}P(R_{it} = 0); \right.
\]

\[
E(Y_{it}|R_{it} = 1)P(R_{it} = 1) + E(Y_{it}|R_{it} = 0)P(R_{it} = 0) \right]
\]

(4.2)

and

\[
ATE \in \left[ \theta_{nab}; 0 \right]
\]

(4.3)

where \( \theta_{nab} \) is the bound defined in equation (3.3). Comparing the above interval with that obtained under the bounded outcome assumption alone, we observe that MTR assumption has considerable identifying power.

A limitation of the MTR assumption is that it needs to hold for all \( i,t \). This might be restrictive. Some households might see a reduction in work-related costs upon retirement (e.g. commuting costs). Furthermore, although the pension replacement rate is generally low in
the UK, some individuals might receive pensions which equal or even exceed their working wages. In addition to this, some other individuals might receive lump sums or annuities which might complement pension income. In all these instances, expenditure power might be left unaltered upon retirement (in which case MTR would remain a valid assumption). Occasionally, however, domestic expenditure might increase among those individuals. Given that caloric needs remain stable or decrease with age, as we discuss below, an increase in expenditure would come if some of the items consumed pre-retirement were of the Giffen type or retirees develop a heightened taste for luxury goods. Alternatively, retirees might increase the frequency with which they eat out -although upon retirement, work-related social events are likely to decrease or disappear. None of these seem particularly powerful reasons to justify an increase of expenditure upon retirement.

If researchers are still concerned about the strength of MTR, equation (2.2) signals that MTR could be replaced by a conditional version, namely

**Conditional Monotone Treatment Response**

\[ E(Y_{it}(1)|R_{it} = r) \leq E(Y_{it}(0)|R_{it} = r) \]

for \( r \in \{0, 1\} \), and \( t = 1, \ldots, T \).

where expectations are taken across \( i \), given \( t \). This conditional MTR leads to identical identification regions than MTR, however it rests on a qualitatively different presumption: regardless of actual retirement status, expenditure in food does not increase with retirement on average at any \( t \). Though this assumption is not refutable from the data it would be more harmonious and consistent with the evidence in Table 1, by allowing certain households to actually increase their expenditure in food.

### 4.2 Weak longitudinal monotonicity in potential outcomes.

Economic (and biological) outcomes are often subject to monotonic trends. This is the case of food intake needs (which is the main component of the domestic expenditure variable at the core of this paper). For example, the US Centre for Nutrition Policy and Promotion has issued guidelines about the calorie intake needs across age groups. As revealed by these guidelines, needs drop from an average of 2400 calories a day for younger adults to 2000 calories a day for adults over the age of 60 (Centre for Nutrition Policy and Promotion, 2018). Several factors
contribute to explain this reduction. First, as people age, their levels of activity decline, which further reduces their needs for energy in the form of food. Second, older people are more exposed to illness, which further increases the risk of cachexia, that is, weight loss due to an underlying illness. Third, ageing exposes humans to an involuntary loss of skeletal muscle mass and strength known as sarcopenia (Walston, 2012). This decline in skeletal muscle can start as early as the fifth decade of life and appears to progress in a linear fashion, at a drop of between 0.5-1% per year from the fifth decade. By the eighth decade of life, up to 50% of skeletal muscle can be lost (Metter et al., 1997) -this results in lower energy needs.

Given all these factors and the fact that we are restricting our sample to mature individuals, a reasonable and seemingly innocuous assumption would be to impose that, for any given \( t \), \( Y_{it}(r) \geq Y_{it+1}(r) \) for all \( r \). The availability of panel data would allows us to transform this restriction into a new set of estimable bounds which, as shown below, will considerably narrow the no-assumption-bounds. Specifically, we can formally define the following Monotonically Time-Varying Response assumption.

**Monotonically time-varying response (MTVR)** \( Y_{i(t-1)}(r) \geq Y_{it}(r) \geq Y_{i(t+1)}(r) \) for \( r \in \{0,1\} \).

Note, that in MTVR the trend would operate regardless of actual retirement status. The assumption does not restrict the process of selection into treatment either. Thus, this assumption is, strictly speaking, neither a monotone treatment assumption nor a monotone instrumental variable assumption.

By projecting the counterfactual moments in (2.2) over \( m = 1, \ldots, M \) leads and lags of an unit’s retirement history, we can then incorporate the MTVR assumption to obtain (see Online Appendix),

\[
E(Y_{it}(0)) \in \left[ Y^{0}_{nab}; Y^{0}_{mter} \right] \quad (4.4)
\]
\[
E(Y_{it}(1)) \in \left[ Y^{1}_{mter}; Y^{1}_{nab} \right] \quad (4.5)
\]
\[
ATE_t \in \left[ \theta_{mter}; \theta_{nab} \right] \quad (4.6)
\]
where $\theta_{\text{mtvr}} = \bar{Y}_{\text{mtvr}} - \bar{Y}^0_{\text{mtvr}}$.

\[
\bar{Y}^0_{\text{mtvr}} = E(Y_{it}|R_{it} = 0)P(R_{it} = 0) + \sum_{m=1}^{M} E(Y_{it-m}|R_{it-m} = 0, R_{it-m+1} = 1, \ldots, R_{it} = 1)P(R_{it-m} = 0, R_{it-m+1} = 1, \ldots, R_{it} = 1) + y_{\text{max}}P(R_{it-m} = 1, R_{it-m+1} = 1, \ldots, R_{it} = 1)
\]

and

\[
\bar{Y}^1_{\text{mtvr}} = E(Y_{it}|R_{it} = 1)P(R_{it} = 1) + \sum_{m=1}^{M} E(Y_{it+m}|R_{it} = 0, \ldots, R_{it+m-1} = 0, R_{it+m} = 1)P(R_{it} = 0, \ldots, R_{it+m-1} = 0, R_{it+m} = 1) + y_{\text{min}}P(R_{it} = 0, \ldots, R_{it+M} = 0)
\]

The MTVR considerably improves the NAB lower bound obtained under the support condition alone because the terms $y_{\text{max}}P(R_{it} = 1)$ in (3.1) and $y_{\text{min}}P(R_{it} = 0)$ in (3.2) are replaced by averages of conditional means where the contributions of $y_{\text{min}}$ and $y_{\text{max}}$ are diluted in proportion to the number of lags and leads, $M$. Specifically, the larger the proportion of individuals in the sample who are observed to retire at $t$ (that is, the more information we have about retirement), the smaller the contribution of the terms $y_{\text{min}}$ and $y_{\text{max}}$. This results in a smaller upper bound for $Y(0)$, a bigger lower bound for $Y(1)$ and an overall larger lower bound for $ATE$. The magnitude of the gains depend on the extent to which MTVR holds in the data.\(^8\)

\[^8\text{This can be better seen when } M = 1, \text{ in which case,}\]

\[
\theta_{\text{mtvr}} = \Delta + \left[y_{\text{min}}P(R_{it} = 0, R_{it+1} = 0) - y_{\text{max}}P(R_{it-1} = 1, R_{it} = 1)\right] + \left[E(Y_{it+1}|R_{it} = 0, R_{it+1} = 1)P(R_{it} = 0, R_{it+1} = 1) - E(Y_{it-1}|R_{it-1} = 0, R_{it} = 1)P(R_{it-1} = 0, R_{it} = 1)\right]
\]

The first term, $\Delta = E(Y_{it}|R_{it} = 1)P(R_{it} = 1) - E(Y_{it}|R_{it} = 0)P(R_{it} = 0)$, is common to the no-assumption bound $\theta_{\text{nab}}$. The second term captures the uncertainty brought about by individuals whose treatment status remains unchanged within the horizon of the analysis. However this terms contributes less to $\theta_{\text{mtvr}}$ than to $\theta_{\text{nab}}$ because, whereas $P(R_{it} = 0)$ and $P(R_{it} = 1)$ provide a full description of the population,
MTVR does not provide additional refinements over the upper bound of ATE. However, it is possible to combine MTRV with MTR to obtain, $ATE \in \left[ \theta_{mtvr}, 0 \right]$. As with MTR, the main limitation surrounding MTVR is that this condition must hold over $t$ for all $i$. One could criticise this assumption on the basis that some individuals -e.g. people in their early twenties- might increase their levels of activity (and thus their needs for food intake) over a few years or that people might see real increases in income through, for example, promotion at work. This concern is empirically easy to address by restricting the age range under consideration -as we do in a later section. The extent to which additional income might result in increased food -and thus domestic- expenditure remains unclear, however. As explained above, food needs are either stable or decrease through adult life and, although preferences for eating out will vary across individuals, those are likely to be fairly stable over time (see, for example, Carlsson et al., 2014). However, as with MTR, it would be possible to replace MTVR with a conditional version along the lines of the Conditional Monotone Treatment Response above -and which would result in identical bounds.

We estimated the identification regions under MTVR and NAB as described in section 3.2.1. The results are reported in Table 4. The table reveals that MTVR is indeed informative, increasing the lower bound estimated under NAB alone by 10% on average, when a single lag and lead of $Y$ is used in the calculation of the bounds. As expected from the theoretical discussion, when two lags and leads are considered, we reach further refinements of 15% with respect to the NAB. These gains notwithstanding, the resulting identification regions remain uninformative. For example, under MTVR-2, the ATE is expected to lie between a drop of 160% and an increase of 133%. We can only improve on this up to this point if we impose MTR, in which the sign of the treatment effect is trivially identified to be negative.

Thus, overall, most information comes from a priori beliefs regarding the sign of the treatment effect (in the form of either MTR or its conditional version). The combination of

$P(R_{it} = 0, R_{it+1} = 0)$ and $P(R_{it} = 1, R_{it-1} = 1)$ do not. Finally, the third term uses the information of those individuals who are observed to retire in order to refine the lower bound. For example, assume that $P(R_{it} = 0, R_{it+1} = 1) = P(R_{it-1} = 0, R_{it} = 1) = \pi$, $Y_{it} = \delta t (\delta < 1)$ and $ATE = \tau$. Then, the third term above equals $2\delta \pi$, implying that the lower bound $\theta_{mtvr}$ will increase more (and thus, the identification region will be refined) the larger the effect of the time trend on $Y$ and the larger the proportion of individuals observed to retired around $t$ (that is, the more information we have about the effect of retirement and the more MTVR is supported by the data).

$\text{Unless the treatment is non-permanent. See Appendix.}$
Partial Identification under Monotone Time Varying Response (MTVR).

<table>
<thead>
<tr>
<th></th>
<th>Wave 3</th>
<th>Wave 4</th>
<th>Wave 5</th>
<th>Wave 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>MTVR-1</td>
<td>-1.706</td>
<td>-1.714</td>
<td>-1.737</td>
<td>-1.758</td>
</tr>
<tr>
<td></td>
<td>(-1.723)</td>
<td>(-1.731)</td>
<td>(-1.755)</td>
<td>(-1.777)</td>
</tr>
<tr>
<td>MTVR-2</td>
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<td>-1.613</td>
<td>-1.639</td>
<td>-1.663</td>
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<td></td>
<td>(-1.624)</td>
<td>(-1.633)</td>
<td>(-1.659)</td>
<td>(-1.685)</td>
</tr>
<tr>
<td>Observations</td>
<td>5616</td>
<td>5208</td>
<td>4955</td>
<td>4671</td>
</tr>
</tbody>
</table>

Table 4: Lower bound of the partial identification region under the Monotone Time Varying Response (MTVR) with 1 lag and lead (MTVR-1) and with two lags and lead (MTVR-2). MTVR leaves the upper bound of the NAB identification region unchanged. Lower limit of Imbens and Manski’s 95% confidence interval for the true parameter value in parentheses.

trends in $Y$ with longitudinal data contributes, surprisingly little information about the causal effect of retirement, as defined in 2.1. This points to the fact that stronger assumptions are required in order to obtain more concluding results.

5 Partial identification under bounded variation.

Model (3.1) allows domestic expenditure to vary in accordance to the elements in $X$ (which might include individual fixed effects). However, the model implies that $\tau$ is common across units (or varies around a mean which is common to all units). This type of ‘invariance’ assumption point identifies $ATE_t$ but, while it is reasonable to believe that there are some similarities in $ATE$ across units, there is no obvious reason why we should expect $ATE$ to be identical across units. To circumvent this restrictive premise, we can draw from Manski and Pepper (2013) and consider a series of more credible ‘bounded variation’ assumptions which allow us to contemplate truly heterogeneous treatment effects.
The $ATE_t$ in equation (2.1) can be rewritten as

$$ATE_t = \left[ E(Y_{it}(1)|R_{it} = 1) - E(Y_{it}(0)|R_{it} = 1) \right] P(R_{it} = 1)$$
$$\quad \quad + \left[ E(Y_{it}(1)|R_{it} = 0) - E(Y_{it}(0)|R_{it} = 0) \right] P(R_{it} = 0)$$
$$= ATT_t \cdot P(R_{it} = 1) + ATU_t \cdot P(R_{it} = 0) \quad (5.1)$$

where $ATT$ and $ATU$ are the average treatment effects on the treated and untreated units respectively. Neither of these parameters are identified from the data alone, unless one is willing to assume that $R$ is independent of the potential outcomes (in that case, $ATU = ATT = ATE$). That invariance assumption, however, goes against the grain of the selection problem and will generally have little justification in practice, particularly in observational studies. On the contrary, as noted by Manski and Pepper (2017), it might be reasonable to assume that the variation in potential outcomes is bounded above by a certain constant, $\delta$. Specifically, we could consider the follow ‘Bounded Variation’ assumptions,

Bounded cross-sectional variation: $\left| E(Y_{it}(r)|R_{it} \neq r) - E(Y_{it}(r)|R_{it} = r) \right| \leq \delta(r)$

Bounded temporal variation: $\left| E(Y_{it}(r)|R_{it} \neq r) - E(Y_{it-1}(r)|R_{it-1} = r) \right| \leq \delta(r)$

where $\delta(r)$ are constants chosen by the researcher. Unless $\Delta(r) = 0$, neither set of bounded variation assumptions point identify $ATE$ however it is not difficult to show that, under bounded cross-sectional variation,

$$ATE_t \in \left[ \underline{\theta}_{bcv}, \bar{\theta}_{bcv} \right] \quad (5.2)$$
$$\underline{\theta}_{bcv} = \underline{\theta}_{bcv}(\delta(r)) = \left[ \hat{\tau} - \delta(1) \right] \cdot P(R_{it} = 1) + \left[ \hat{\tau} - \delta(0) \right] \cdot P(R_{it} = 0) \quad (5.3)$$
$$\bar{\theta}_{bcv} = \bar{\theta}_{bcv}(\delta(r)) = \left[ \hat{\tau} + \delta(1) \right] \cdot P(R_{it} = 1) + \left[ \hat{\tau} + \delta(0) \right] \cdot P(R_{it} = 0) \quad (5.4)$$

22
where $\hat{\tau} = [E(Y_{it}\mid R_{it} = 1) - E(Y_{it}\mid R_{it} = 0)]$. Similarly, under bounded temporal variation,

\begin{align*}
ATE_t & \in \left[\theta_{btv}, \bar{\theta}_{btv}\right] \\
\theta_{btv} &= \theta_{btv}(\delta(r)) = \left[\hat{\tau} - \delta(1)\right] \cdot P(R_{it} = 1) + \left[\hat{\tau} - \delta(0)\right] \cdot P(R_{it} = 0) \\
\bar{\theta}_{btv} &= \bar{\theta}_{btv}(\delta(r)) = \left[\hat{\tau} + \delta(1)\right] \cdot P(R_{it} = 1) + \left[\hat{\tau} + \delta(0)\right] \cdot P(R_{it} = 0)
\end{align*}

where $\hat{\tau} = E(Y_{it}\mid R_{it} = 1) - E(Y_{it-1}\mid R_{it-1} = 0)$ and $\bar{\tau} = E(Y_{it-1}\mid R_{it-1} = 1) - E(Y_{it}\mid R_{it} = 0)$.

Despite of the weakness of the bounded variation assumptions, these new bounds have considerable identifying power, as the results below will illustrate. It is, however, possible to further narrow these bounds if one is willing to assume that $Y(0)$ alone is invariant to $R$. This assumption would imply, for example, that pre-retirement consumption is independent of the proximity of the moment of retirement - which would further suggest that saving behaviour might be tangential to perceptions about retirement (that is, individuals are myopic savers). It would also imply that although there may be a cross-over from ‘non-assigned’ individuals towards treatment (so that people might self-select into early retirement), there is not a cross-over from ‘assigned’ individuals towards non-treatment (and so, pre-consumption retirement has no bearing on decisions to stay at work beyond normal or statutory retirement ages). Under this invariance assumption, $E(Y_{it}(0)\mid R_{it} = 1) = E(Y_{it}(0)\mid R_{it} = 0)$ and, therefore $ATT_t = \hat{\tau}$.

This assumption does not identify $ATU$. We can, however, further assume that

\begin{equation}
|ATT_t - ATU_t| \leq \delta
\end{equation}

and from this it follows that

\begin{align*}
ATE & \in \left[\theta_{inv}, \bar{\theta}_{inv}\right] \\
\theta_{inv} &= \theta_{inv}(\delta) = \hat{\tau} - P(R_{it} = 0) \cdot \delta \\
\bar{\theta}_{inv} &= \bar{\theta}_{inv}(\delta) = \hat{\tau} + P(R_{it} = 0) \cdot \delta
\end{align*}
Estimates of the preceding bounds for $\delta(0) = \delta(1) = \delta$ are presented in Figure 1. The left panel shows the identification region in wave 3 of ‘Understanding Society’, whereas the right panel shows the corresponding region for wave 5. In each panel, the blue line delimits the region under bounded temporal variation; the red line delimits the region under bounded cross-sectional variation and the black line delimits the identification region under invariance of $Y(0)$ and bounded variation in ATU. The bounded cross-sectional and temporal variation assumptions yield similar results. Focusing on wave 3, for example, both set of assumptions identify the sign of the treatment effect when $\delta < 0.069$ (in which case $ATE$ is negative). When $\delta = 0$, the $ATE$ is point identified and equals the different in means among the treated and untreated. This difference is located at -0.069 (see Table 5). As $\delta$ increases, the uncertainty around this bound increases. Given the simplifying assumption that $\delta(0) = \delta(1) = \delta$ the bounded cross-sectional bounds are $-0.069 \pm \delta$.

As expected, the additional information imposed by the invariance assumption further reduces the length of the identification regions (which are now contained within the regions obtained under bounded cross-sectional variation). Specifically, under invariance of $Y(0)$, the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1}
\caption{Bounds on $ATE_t$ under bounded cross-sectional variation (red line), bounded temporal variation (blue line) and invariance of $Y(0)$ and bounded variation in ATU (black line). Results are presented for waves 3 (left panel) and 5 (right panel). Similar results were obtained for the remaining waves.}
\end{figure}

5.1 Results.

The regions obtained for the remaining waves were qualitatively and quantitatively similar. The results are available upon request.

Note the small discrepancy between the result for MTS (-0.066), which is obtained after smoothing over the lower bounds of each wave, and the raw difference in outcomes conditional on treatment, -0.069.
identification regions are defined by the equation $-0.069 \pm 0.31 \cdot \delta$. These later bounds identify the sign of the treatment effect for any $\delta < 0.212$.

6 Partial identification under monotonicity in the selection process.

The assumptions considered so far put an emphasis on the properties of the potential outcomes, however they are agnostic about the motivation behind individuals’ retirement decisions. It will often reasonable to assume that individuals self-select into treatment (or are selected into treatment) on the basis of perceived gains/losses. Indeed, the decision to retire from the workforce is likely to result from the confluence of three factors.

First, people will weight their levels of health, predicted post-retirement income and nature of the current job in order to motivate retirement. Individuals in poor health and in the worse occupations have strong incentives to retire. In contrast, low levels of post-retirement income will motivate individuals to stay at work. Therefore, it is unclear a priori how to characterise the selection process on the basis of individuals’ preferences for retirement.

Second, the institutional setting around retirement has an influence on the timing of retirement. Specifically, it is well documented that the state pension age is a strong predictor of about 10-20% of retirement decisions in western economies (e.g. Gruber and Wise, 2002). The state pension age is an exogenous factor and is thus uncorrelated to individuals’ potential outcomes. But, set at ages above 65, the existence of a state pension age implies that at any point in time, those retiring will be older on average -and thus will have, on average, no higher food consumption needs than those staying at work. This latter fact presents additional identification challenges (see Fé, 2018) and does provides no clarity regarding the mechanism of selection.

Third, employers have an incentive to keep the most productive individuals at work. Although in the UK there is legislation protecting workers against discrimination on the basis of age, health and other traits, employers can incentive the retirement of the least productive workers through early retirement packages or other means. The most productive employers
are likely to be also those on higher salaries (and thus higher capacity for expenditure, before
and after retirement) and those with higher levels of educational attainment.

The above discussion suggests that, although a full and uncontroversial characterisation of
the selection process is unattainable, there are stronger reasons to expect that those ‘selected’
into retirement will have lower capacity for expenditure anyway (particularly in view of point
3 above). Together with the fact that they will also be, on average, older, this suggests the
following Monotone Treatment Selection (MTS) assumption (Manski and Pepper, 2000)

$$E(Y_{it}(r)|R_{it} = 1) \leq E(Y_{it}(r)|R_{it} = 0)$$  \hspace{1cm} (6.12)

for every $r$, conditional on $t$. Under MTS, for any given period, the average potential weekly
household expenditure of households whose members are retired is not larger than the average
potential weekly household expenditure of households whose members are not retired. Under
MTS, Manski and Pepper (2000) show that

$$E(Y_{it}(0)) \in \left[\overline{Y}_{nab}^0, \overline{Y}_{mts}^0\right]$$

$$= \left[E(Y_{it}|R_{it} = 0)P(R_{it} = 0) + y_{min}P(R_{it} = 1); E(Y_{it}|R_{it} = 0)\right].$$  \hspace{1cm} (6.13)

$$E(Y_{it}(1)) \in \left[\overline{Y}_{mts}^0, \overline{Y}_{nab}^0\right]$$

$$= \left[E(Y_{it}|R_{it} = 1); E(Y_{it}|R_{it} = 1)P(R_{it} = 1) + y_{max}P(R_{it} = 0)\right]$$  \hspace{1cm} (6.14)

and so,

$$ATE_t \in \left[\theta_{mts}^t; \theta_{nab}^t\right]$$  \hspace{1cm} (6.15)

where, $\theta_{t}^t = E(Y_{it}|R_{it} = 1) - E(Y_{it}|R_{it} = 0)$ is the Ordinary Least Squares (OLS) estimator
of the slope in a regression of $Y$ on a constant and the treatment indicator $R_{it}$. That is, under
non-random selection, OLS can only be interpreted as a lower bound on the $ATE_t$. MTS
leaves the upper bound unaffected.

MTS and MTR refer to different aspects of treatment response and selection and they may
or not hold simultaneously. However, if they do, the ensuing identification region for $ATE_t$ is
can be narrowed further to:

\[
ATE_t \in \left[\theta_{mts}, 0\right]
\]  

(6.16)

Interestingly, under MTS and MTR, it follows that \( E(Y|Z = 1) \leq E(Y|Z = 0) \). This condition can be tested with data by, for example, using a standard one-sided t-statistic of the null hypothesis \( H_0 : E(Y|R = 1) \leq E(Y|R = 0) \). A rejection of the null hypothesis would suggest that at least one of the monotonicity assumptions is not supported by the data (but it will fall sort of asserting that would be the case -in other words, MTS and MTR combined imply \( E(Y|Z = 1) \leq E(Y|Z = 0) \), but the converse need not be true).

### 6.1 Dynamic Monotone Treatment Selection.

The MTS has considerable identifying power despite being relatively weak (it only need to hold on average). However, it relies on cross-sectional comparisons across units. The availability of panel data allows us to shift the focus on the process of selection into treatment within individuals. Specifically, we can consider the following Dynamic Monotone Treatment Selection assumption\(^{12}\).

**Dynamic Monotone Treatment Selection** Let \( R_{it} = 1 \rightarrow R_{it+1} = 1 \). If \( k_{t-1} < k'_{t-1} \),

\[
E(Y_{it}(r)|R_{it-1} = k_{t-1}, R_{it} = k_t) \geq E(Y_{it}(r)|R_{it-1} = k'_{t-1}, R_{it} = k_t)
\]

for every \( t \).

The Dynamic Monotone Treatment Selection (DMTS) imposes that, at a given point in time, the average potential expenditure of households whose members have just retired will be weakly lower than the average potential expenditure of households whose members remain at work. As with MTS, the role of age and the incentives faced by employers make this assumption palatable. As with MTS, the assumption only needs to hold on average (and it could thus be violated within some units). Note that if the lag on \( R \) is ignored, DMTS reduces to the original MTS assumption in Manski and Pepper (2000) -so DMTS is a stronger assumption.

\(^{12}\)This assumption can be extended to encompass \( m > 1 \) lags in the history of treatment. However, this imposes more stringent conditions on the data generation process and it does not necessarily result in more informative identification regions.
It is straightforward to show that, under DMTS,

\[ E(Y_{it}(1)) \geq E(Y_{it}|R_{it} = 1)P(R_{it} = 1) + E(Y_{it}|R_{it-1} = 0, R_{it} = 1)P(R_{it} = 0) \]  \hspace{1cm} (6.17)

\[ E(Y_{it}(0)) \leq E(Y_{it}|R_{it} = 0). \]  \hspace{1cm} (6.18)

The extent to which this bound improves the MTS bound depends on whether \( E(Y_{it}|R_{it} = 1) \leq E(Y_{it}|R_{it-1} = 0, R_{it} = 1) \). This follows by noting that \( E(Y_{it}|R_{it} = 1) \) (the lower bound of \( Y_{it}(1) \) under MTS) can be written as

\[ E(Y_{it}|R_{it} = 1) = E(Y_{it}|R_{it} = 1)P(R_{it} = 1) + E(Y_{it}|R_{it} = 1)P(R_{it} = 0). \]

Specifically, DMTS will provide refinements above those achievable with MTS alone whenever the average expenditure of just-retired individuals (for whom \( R_{it} = 1, R_{it-1} = 0 \)) weakly dominates the expenditure of individuals who have been retired for longer. As with MTS, by conditioning on \( t \) the fact that retirees will be older on average makes this condition less controversial. In that case, \( \theta_{dmts} \geq \theta_{mts} \), where

\[ \theta_{dmts} = \Delta + E(Y_{it}|R_{it-1} = 0, R_{it} = 1)P(R_{it} = 0) - E(Y_{it}|R_{it} = 0)P(R_{it} = 1) \]  \hspace{1cm} (6.19)

Importantly, the condition under which DMTS ensures refinements over those provided by MTS is empirically verifiable. As with MTS, we can combine DMTS and MTR to bind the \( ATE_t \) above, which would result in the identification region \( ATE_t \in \left[ \theta_{dmts}; 0 \right] \). Interestingly, the combination of both assumptions implies that

\[ E(Y_{it}|R_{it-1} = 0, R_{it} = 0) \geq E(Y_{it}(0)|R_{it-1} = 0, R_{it} = 1) \geq E(Y_{it}(1)|R_{it-1} = 0, R_{it} = 1) \]

\[ \geq E(Y_{it}|R_{it-1} = 0, R_{it} = 1) \]  \hspace{1cm} (6.20)

This condition can be tested empirically (using a standard t-test of the difference to two means) in order to evaluate the joint suitability of MTR and DMTS.
### Lower bounds.

<table>
<thead>
<tr>
<th></th>
<th>Wave 3</th>
<th>Wave 4</th>
<th>Wave 5</th>
<th>Wave 6</th>
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</thead>
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<td>-0.064</td>
<td>-0.064</td>
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<td>(-0.090)</td>
<td>(-0.091)</td>
<td>(-0.094)</td>
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<tr>
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<tr>
<td></td>
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<td>(-0.082)</td>
<td>(-0.082)</td>
<td>(-0.087)</td>
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<td>Observations</td>
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<td>5208</td>
<td>4955</td>
<td>4671</td>
</tr>
</tbody>
</table>

Table 5: Lower bound of the partial identification region under the Monotone Treatment Selection (MTS) and Dynamic Monotone Treatment Selection assumptions. MTS and DMTS leave the upper bound of the NAB identification region unchanged. Lower limit of Imbens and Manski’s 95% confidence interval for the true parameter value in parentheses.

#### 6.2 Results

The lower bounds of the identification regions under either MTS or DMTS are provided in the top two panels of Table 5, together with the lower bound of the 95% confidence interval. Both set of assumptions substantially qualify the bounds obtained under NAB and MTVR. First, note that the lower bound under MTS coincides with the upper/lower bounds under Bounded Variation and $\delta = 0$. Therefore, MTS is more informative than the bounded variation assumption in so far it sets a larger lower bound. The assumption leaves the value of the ATE unrestricted between $\theta_{mts}$ and 0 (if MTR also holds) or $\theta_{nab}$. Focusing on wave 3\textsuperscript{13}, the MTS assumption suggests that the retirement of any member of the household may lead to a drop in expenditure of, at most, 6.6% drop over the average household domestic expenditure. The DMTS further refines this bound and suggests that, on average, retirement of a household member leads to, at most, a drop of 5.9% with respect the average household expenditure in food. Note that, across the whole sample, DMTS strictly dominates MTS, thus providing tighter lower bounds around $ATE_t$.

We can finally combine the bounds under (D)MTS and the bounded variation assumptions

\textsuperscript{13}Similar results were obtained elsewhere.
in order to characterise the ATE of retirement. Under these assumptions, retirees will have, on average, lower domestic expenditure potential outcomes than non-retirees, and the effect of retirement will be determined by the expected variation in potential outcomes across groups/over time (left panel, Figure 2). Again focusing on wave 3, the introduction of bounded variation assumptions identifies the sign of the treatment effect. If we consider bounded cross-sectional variation and MTS, the sign of the effect is identified if $\delta < 0.069$. The ATE is further qualified if we impose invariance of $Y(0)$ and bounded variation in $ATU$, in which case its sign is negative, provided that $\delta < 0.212$ (see left panel in the Figure). We could finally further introduce MTR, which will result in trivial identification of the sign of the treatment effect, as shown in the right panel of the Figure.

Overall, then, if MTS holds and the variation in potential outcomes across individuals holds within $\pm 0.069$, then we can conclude that retirement has a small negative effect on domestic expenditure of up to 7%. When the variation in potential outcomes across individuals exceeds that value, then we won’t expect retirement to lead to drops in consumption beyond 7% but, critically, we can’t rule out increases in expenditure following retirement, unless we can also assume that (a) either the retirement decisions and potential expenditure in the absence of retirement are independent -provided that the variation across individuals is confined to $\pm 21\%$- or (b) we find reasonable grounds to assume that potential outcomes under retirement will

---

**Figure 2:** Bounds on $ATE_t$ under bounded cross-sectional variation (red line), Monotone Treatment Selection (blue line) invariance of $Y(0)$ and bounded variation in $ATU$ (black line) and Monotone Treatment Response (purple line). Results are presented for waves 3.
be lower than potential outcomes when retirement does not occur. These set of assumptions would rule out some of the largest effects reported in the literature (For example, Battistin et al. (2009) report a drop in consumption of about 14%; Li, Shi and Wu find a 11% drop; Luengo-Prado and Sevilla find drops of between 5 and 13%; all these studies, however, report a different parameter -the Local Average Treatment Effect).

6.3 The role of savings and education.

We explore next how the preceding bounds might vary across two dimensions: savings and education. Regarding savings, the hypothesis is that higher levels of savings will enable households to, at least, partially compensate any drop in income following retirement. As a result, one would expect that these households will be subject to smaller drops in domestic expenditure following retirement. Regarding education, and although this is still hotly debated issue, it seems reasonable to think that higher educational attainment might be associated with higher productivity, more stability at work and, consequently, a better ability to foresee future income streams. The latter would imply that those households were educational attainment is higher will, on average, enjoy higher levels of income and, probably, savings. Similarly, households were educational attainment is higher will be able to better anticipate cuts in income following retirement and devise contingency plans to smooth consumption. These associations are investigated next.

For this analysis we split the sample into categories. First, we divide the sample in accordance to a household’s average savings across waves 1 to 8 of ‘Understanding Society’. Given average savings across all 8 waves, we differentiate between households with average savings above or below the median. The second classification is done in accordance to whether at least on person in the household (main respondent or partner) had completed high school or not. Table 6 shows the bounds of the identification regions under NAB, MTVR, MTS and DMTS across each category and wave. Focusing on the most informative bounds (MTVR, MTS and DMTS) we observed that, irrespectively of their magnitude, they all coincide in attributing savings and education a mitigating effect: higher savings and higher education result in higher lower bounds, from which we conclude that the impact of retirement on expenditure
is potentially bigger in households where savings are lower and households where educational attainment is lower. It is important to emphasise that the difference in lower bounds only implies a wider range of variation in specific sub-classes; the actual effect is not point identified and, given the overlapping regions, that effect could be bigger, smaller or identical across the specified classes. More specifically, the lowest bound for the identification region of ATE (under DMTS) indicates a maximum drop of 5-6.8% among households with above median savings. The same drop for households with below median savings ranges between 7.5-9.1%. The effect of education is stronger yet. Among households whose members did not finish high school, the highest lower bound on ATE indicates a drop in domestic expenditure ranging between 6.3-6.9%, however among households where at least one member finished high school, this lower bound indicates maximum drops in expenditure of 2-3.1%.

Finally, we can combine the MTS with the cross-sectional bounded variation assumption or MTR in order to try to identify the sign of the treatment effect. Then, the results for wave 3 in 'Understanding Society' are depicted in Figures 3 and 4. Figure 3 shows the wider identification region in households with savings below the median. In these households the sign of the treatment effect is identified if MTS and bounded variation are combined, provided that the variation in potential outcomes between households falls below 10% (for households with savings above the median, the sign of the effect is identified only if the variation in potential outcomes across households is limited to around 6%). Above these limits, MTR is required to identify the sign of the effect. Figure 3 shows the wider identification region in households where educational attainment is lowest. In these households the sign of the treatment effect is identified if MTS and bounded variation are combined, provided that the variation in potential outcomes between households falls below 8% (for households where at least one member finished high school, the sign of the effect is identified only if the variation in potential outcomes across households is limited to around 3%). Above these limits, MTR is required to identify the sign of the effect.
Table 6: Lower bound of the partial identification region under the Monotone Treatment Selection (MTS) and Dynamic Monotone Treatment Selection assumptions. MTS and DMTS leave the upper bound of the NAB identification region unchanged. Lower limit of Imbens and Manski’s 95% confidence interval for the true parameter value in parentheses.

MTS, MTR AND BOUNDED VARIATION (SAVINGS ABOVE/BELLOw MEDIAN)

Figure 3: Wave 3. Left panel: households where average savings are above the median. Right panel: households where average savings are below the median. Bounds on $ATE_t$ under bounded cross-sectional variation (red line), Monotone Treatment Selection (blue line) and Monotone Treatment Response (purple line).
Figure 4: Wave 3. Left panel: households where at least one member finished high school; right panel: households where no one finished high school. Bounds on $ATE_t$ under bounded cross-sectional variation (red line), Monotone Treatment Selection (blue line) invariance of $Y(0)$ and Monotone Treatment Response (purple line).

7 Summary of the results.

The preceding analysis can be summarised as follows. First, we considered a collection of flexible parametric models to point identify the ATE of retirement. These models, however, rely on very strong assumptions, many of which are not refutable (like, for example, the usual exclusion restriction by which the regression error term is uncorrelated to the retirement indicator). This collection of models indicates that retirement leads to a drop in domestic expenditure. However, there is no consensus among the models regarding the sign of this effect (some models revealed a positive effect) not its magnitude (which varied vastly). When it comes to identifying the sign of the treatment effect, we could take two avenues. First, following Manski, 1997, we could restrict the relationship between potential expenditure under retirement and in the absence of retirement. Specifically, in some settings (such as the UK or the US) it might be reasonably uncontroversial to assume that the low pension replacement rates impose substantial restrictions on households expenditure capabilities (so that average increases in expenditure following retirement would be discarded). In some settings, where pension replacement rates are high (such as in Spain), this assumption might be overly restrictive.

In order to identify the sign of the ATE, an alternative avenue seems to be to restrict the admissible range of variation for the potential outcomes. Here, there are two competing set of assumptions. First, the sign of the effect is identified (and negative) if the variation in
potential outcomes across individuals is itself confined to a discrepancy of ±7%. Otherwise, we can’t rule out increases in expenditure following retirement, unless (a) the variation of potential expenditure levels across households is confined to ±21% and (b) the independence of the retirement decisions and the potential level of household expenditure in the absence of retirement. This pair of assumptions limits the relationship between pre-retirement consumption and the retirement decision (which involves limiting the degree of myopic behaviour in people).

Unless any of the above assumptions hold, our data is not informative about the upper bound of the ATE of retirement. Regarding the lower bound, this can be set to -7% under a monotone treatment selection by which retired units would not have average levels of expenditure exceeding the average level of expenditure of non-retirement units. This assumption would be plausible if, for example, employers would be actively incentivising their least productive workers to retire or if workers in the lower categories of income had an incentive to retire earlier (as would be the case in occupations where the main task is physically demanding, psychologically taxing or subject to insalubrious conditions). If one is willing to hold this assumption, then we would conclude that the large effects reported in the literature for the sub-population of compliers would not apply more generally (for example, Battistin et al., 2009 report a drop in consumption of about 14%; Li et al., 2015 find a 11% drop; Luengo-Prado and Sevilla, 2012 find drops of between 5 and 13%).

Finally, we find that a household’s average savings and levels of educational attainment influence the width of the identification regions. All the weakest assumptions we consider coincide in endowing savings and education a mitigating effect: the identification region for households with above median savings or where educational attainment is higher have a higher lower bound for ATE. This does not imply that the actual effect of retirement is lower for these households, but it suggests that retirement can potentially lead to larger drops of expenditure in households where savings or educational attainment is lower. The effect of education is particularly stark: the largest lower bound for the ATE of retirement on domestic expenditure suggest a maximum drop of 2-3% in domestic expenditure in households with the highest educational attainment, compared to a 6-7% in other households.
Conclusion.

Consumption is an important determinant of households’ utility functions and a critical macroeconomic aggregate, accounting for more than half of a nation’s GDP. Against the background of generally ageing populations, understanding the implications of retirement for consumption is of critical importance for scholars and policy makers alike. Consumption, however, is difficult to measure (e.g. Olafsson and Pagel, 2018) and thus researchers have paid particular attention to the consequences of retirement for food and non-durable expenditure (which are critical components of household consumption). Despite the differences in terms of methods and models, the abundant received contributions have systematically found that retirement leads to a drop in food and non-durable expenditure. Because consumption, expenditure and retirement are likely to be simultaneously determined, instrumental variable methods have sat at the core of these earlier contributions. This implies, then, that existing results refer to a local average treatment effect and the associated sub-population of individuals whose retirement decision is solely driven by some pension or early retirement rule (that is, the compliers).

The most relevant parameter for policy is, however, the population average treatment effect. Therefore, this article explores what can data reveal about that parameter. To this end, we consider estimation under assumptions of varying strength and credibility. Our strongest assumptions point identify the treatment effect with incredible certitude (Manski, 2018), whereas our weakest assumptions partially identify the effect of retirement (but provide no useful information). In between these polar cases, we contemplate a range of assumptions that restrict either households’ potential level of expenditure or the process of selection into treatment. Some of the assumptions we consider are now well known (Monotone Treatment Selection and Response, Bounded Variation), whereas others are new to the literature (Monotone Time Varying Response and Dynamic Monotone Treatment Selection). The latter try to exploit information in longitudinal data to tighten the identification regions.

Our tightest identification regions suggests that retirement could have, at most, a small negative effect on domestic expenditure of up to 7%. This suggests that the larger effects reported in the earlier literature (indicating drops of around 14%) are specific to the population of compliers and do not apply more generally.
The sign of the effect, however, is not identified unless the variation in potential outcomes across individuals is itself confined to a discrepancy of $\pm 7\%$. Otherwise, we can’t rule out increases in expenditure following retirement, unless we either limit the relationship between pre-retirement consumption and the retirement decision (which involves limiting the degree of myopic behaviour admissible among people) or directly impose what the sign of the effect might be on average. The latter assumption might be suitable in context such as the UK or the US were pension replacement rates are low, but it might fail to hold in other countries (like Spain; Luengo-Prado and Sevilla, 2012), where pension replacement rates are higher.

We also find that a household’s average savings and levels of educational attainment influence the width of the identification regions. All the weakest assumptions we consider coincide in endowing savings and education a mitigating effect: the identification region for households with above median savings or where educational attainment is higher have a higher lower bound for ATE. This does not imply that the actual effect of retirement is lower for these households, but it does suggest that retirement can potentially lead to larger drops of expenditure in households where savings or educational attainment is lower.

This paper contributes a valuable guide to inform discussions around the effects of retirement for expenditure at a population level. Our results rely on transparent assumptions which have equally transparent effects on the length of the identification regions reported. As a result, scholars and policy makers can evaluate each result in this paper on the merits of each assumption, rather than complex parametric models which fail to reveal the factors or assumption driving each specific result. A potential limitation of our analysis, however, arises from the fact that, whereas the catalogue of assumptions we contemplate is vast, there might be other assumptions which, being credible and weak, might reveal additional features of the ATE of retirement. For example, it might be conceivable to find a very weak assumption identifying the effect of retirement. However, this is not a weakness of the analysis. Rather, if such new assumption would be found, it would be straightforward to compare the ensuing identification region with the regions presented in our analysis. From this point of view, this paper simply inherits a general advantage of the nonparametric bounds literature.
Compliance with ethical standards.

The author declares that there were no conflict of interests in conducting this research. Specifically, not funding was received to conduct this research.
References


Supplementary Web Appendix

Derivation of the bounds under MTVR.

Consider first the case with three waves, \( t - 1, t \) and \( t + 1 \). Then,

\[
E(Y_{it}(1)|R_{it} = 0) = E(Y_{it}(1)|R_{it} = 0, R_{it+1} = 0)P(R_{it+1} = 0|R_{it} = 0)
+ E(Y_{it}(1)|R_{it} = 0, R_{it+1} = 1)P(R_{it+1} = 1|R_{it} = 0)
\]

(1)

\[
E(Y_{it}(1)|R_{it} = 0) = E(Y_{it}(1)|R_{it-1} = 0, R_{it} = 0)P(R_{it-1} = 0|R_{it} = 0)
+ E(Y_{it}(1)|R_{it-1} = 1, R_{it} = 0)P(R_{it-1} = 1|R_{it} = 0)
\]

(2)

Note that, under MTVR,

\[
E(Y_{it}(1)|R_{it} = 0, R_{it+1} = 1) \geq E(Y_{it+1}|R_{it} = 0, R_{it+1} = 1)
\]

(3)

\[
E(Y_{it}(1)|R_{it-1} = 1, R_{it} = 0) \leq E(Y_{it-1}|R_{it-1} = 1, R_{it} = 0)
\]

(4)

From this, it follows that \( E(Y_{it}(1)) \in [c, d] \) where

\[
c = E(Y_{it}|R_{it} = 1)P(R_{it} = 1) + y_{\min}P(R_{it} = 0, R_{it+1} = 0)
+ E(Y_{it+1}|R_{it} = 0, R_{it+1} = 1)P(R_{it} = 0, R_{it+1} = 1)
\]

(5)

\[
d = E(Y_{it}|R_{it} = 1)P(R_{it} = 1) + y_{\max}P(R_{it-1} = 0, R_{it} = 0)
+ E(Y_{it-1}|R_{it-1} = 1, R_{it} = 0)P(R_{it-1} = 1, R_{it} = 0)
\]

(6)

Note, however, that bound \( d \) will not contribute any refinements beyond those provided by the no-assumption bounds if, for example, \( R \) is a permanent treatment (or an absorbing estate) as in the case of retirement, chronic diseases or exposure to an environmental shock during...
earliest life. Proceeding similarly, we obtain

\[
E(Y_{it}(0)|R_{it} = 1) = E(Y_{it}(0)|R_{it} = 1, R_{it+1} = 0)P(R_{it+1} = 0|R_{it} = 1) \\
+ E(Y_{it}(0)|R_{it} = 1, R_{it+1} = 1)P(R_{it+1} = 1|R_{it} = 1)
\]  

(7)

\[
E(Y_{it}(0)|R_{it} = 1) = E(Y_{it}(0)|R_{it-1} = 0, R_{it} = 1)P(R_{it-1} = 0|R_{it} = 1) \\
+ E(Y_{it}(0)|R_{it-1} = 1, R_{it} = 1)P(R_{it-1} = 1|R_{it} = 1)
\]

(8)

where, by MTVR,

\[
E(Y_{it}(0)|R_{it} = 1, R_{it+1} = 0) \geq E(Y_{it+1}|R_{it} = 1, R_{it+1} = 0) 
\]  

(9)

\[
E(Y_{it}(0)|R_{it-1} = 0, R_{it} = 1) \leq E(Y_{it-1}|R_{it-1} = 0, R_{it} = 1).
\]  

(10)

It follows that \(E(Y_{it}(0)) \in [a, b]\)

\[
a = E(Y_{it}|R_{it} = 0)P(R_{it} = 0) + y_{\min}P(R_{it} = 1, R_{it+1} = 1) \\
+ E(Y_{it+1}|R_{it} = 1, R_{it+1} = 0)P(R_{it} = 1, R_{it+1} = 0)
\]

(11)

\[
b = E(Y_{it}|R_{it} = 0)P(R_{it} = 0) + y_{\max}P(R_{it-1} = 1, R_{it} = 1) \\
+ E(Y_{it-1}|R_{it-1} = 0, R_{it} = 1)P(R_{it-1} = 0, R_{it} = 1)
\]

(12)

Note that bound \(a\) will not contribute any refinements beyond those provided by the no-assumption bounds if \(R\) is a permanent.

When the number of waves before and after \(t\) increases, it is possible to obtain a number
Similarly, note first that,

\[ E(Y_{il}(1)|R_{il} = 0) \]

\[ \geq E(Y_{il+1}|R_{il} = 0, R_{il+1} = 1)P(R_{il+1} = 1|R_{il} = 0) \]

\[ + E(Y_{il}(1)|R_{il} = 0, R_{il+1} = 0)P(R_{il+1} = 0|R_{il} = 0) \]

\[ \geq E(Y_{il+1}|R_{il} = 0, R_{il+1} = 1)P(R_{il+1} = 1|R_{il} = 0) \]

\[ + E(Y_{il+2}|R_{il} = 0, R_{il+1} = 0, R_{il+2} = 1)P(R_{il+2} = 1|R_{il} = 0, R_{il+1} = 0)P(R_{il+1} = 0|R_{il} = 0) \]

\[ + E(Y_{il}(1)|R_{il} = 0, R_{il+1} = 0, R_{il+2} = 0)P(R_{il+2} = 0|R_{il} = 0, R_{il+1} = 0)P(R_{il+1} = 0|R_{il} = 0) \]

\[ \ldots \]

Continuing for \( M \) periods, and multiplying by \( P(R_{il} = 0) \) (given equation ??), we obtain:

\[ E(Y_{il}(1)|R_{il} = 0)P(R_{il} = 0) \]

\[ \geq \sum_{m=1}^{M} E(Y_{il+m}|R_{il} = 0, \ldots, R_{il+m-1} = 0, R_{il+m} = 1)P(R_{il} = 0, \ldots, R_{il+m-1} = 0, R_{il+m} = 1) \]

\[ + E(Y_{il}(1)|R_{il} = 0, \ldots, R_{il+M} = 0)P(R_{il} = 0, \ldots, R_{il+M} = 0) \]

\[ \geq \sum_{m=1}^{M} E(Y_{il+m}|R_{il} = 0, \ldots, R_{il+m-1} = 0, R_{il+m} = 1)P(R_{il} = 0, \ldots, R_{il+m-1} = 0, R_{il+m} = 1) \]

\[ + y_{min}P(R_{il} = 0, \ldots, R_{il+M} = 0) \]

(13)

Similarly,

\[ E(Y_{il}(1)|R_{il} = 0) \]

\[ \leq E(Y_{il-1}|R_{il-1} = 1, R_{il} = 0)P(R_{il-1} = 1|R_{il} = 0) \]

\[ + E(Y_{il}(1)|R_{il-1} = 0, R_{il} = 0)P(R_{il-1} = 0|R_{il} = 0) \]

\[ \leq E(Y_{il-1}|R_{il-1} = 1, R_{il} = 0)P(R_{il-1} = 1|R_{il} = 0) \]

\[ + E(Y_{il-2}|R_{il-2} = 1, R_{il-1} = 0, R_{il} = 0)P(R_{il-2} = 1|R_{il-1} = 0, R_{il} = 0)P(R_{il-1} = 0|R_{il} = 0) \]

\[ + E(Y_{il}(1)|R_{il-2} = 0, R_{il-1} = 0, R_{il} = 0)P(R_{il-2} = 0|R_{il-1} = 0, R_{il} = 0)P(R_{il-1} = 0|R_{il} = 0) \]

\[ \ldots \]
Continuing in this fashion for \( M \) periods and multiplying by \( P(R_{it} = 0) \) we obtain,

\[
E(Y_{it}(1)|R_{it} = 0)P(R_{it} = 0) \\
\leq \sum_{m=1}^{M} E(Y_{t-m}|R_{it-m} = 1, R_{it-m+1} = 0, \ldots, R_{it} = 0)P(R_{it-m} = 1, R_{it-m+1} = 0, \ldots, R_{it} = 0) \\
+ E(Y_{it}(1)|R_{it-m} = 0, R_{it-m+1} = 0, \ldots, R_{it} = 0)P(R_{it-m} = 0, R_{it-m+1} = 0, \ldots, R_{it} = 0) \\
\leq \sum_{m=1}^{M} E(Y_{t-m}|R_{it-m} = 1, R_{it-m+1} = 0, \ldots, R_{it} = 0)P(R_{it-m} = 1, R_{it-m+1} = 0, \ldots, R_{it} = 0) \\
+ y_{\text{max}}P(R_{it-m} = 0, R_{it-m+1} = 0, \ldots, R_{it} = 0) \tag{14}
\]

From this the identification region for \( E(Y_{it}(1)) \) follows. Similar steps are taken for \( E(Y_{it}(0)|R_{it} = 1) \).

\[
E(Y_{it}(0)|R_{it} = 1) \\
\geq E(Y_{it+1}|R_{it} = 1, R_{it+1} = 0)P(R_{it+1} = 0|R_{it} = 1) \\
+ E(Y_{it}(0)|R_{it} = 1, R_{it+1} = 1)P(R_{it+1} = 1|R_{it} = 1) \\
\geq E(Y_{it+1}|R_{it} = 1, R_{it+1} = 0)P(R_{it+1} = 0|R_{it} = 1) \\
+ E(Y_{it+1}|R_{it} = 1, R_{it+1} = 1, R_{it+2} = 0)P(R_{it+2} = 0|R_{it} = 1, R_{it+1} = 1)P(R_{it+1} = 1|R_{it} = 1) \\
+ E(Y_{it}(0)|R_{it} = 1, R_{it+1} = 1, R_{it+2} = 1)P(R_{it+2} = 1|R_{it} = 1, R_{it+1} = 1)P(R_{it+1} = 1|R_{it} = 1) \\
\ldots
\]

Continuing with this recursion for \( M \) periods and multiplying by \( P(R_{it} = 1) \) we obtain,

\[
E(Y_{it}(0)|R_{it} = 1) \\
\geq \sum_{m=1}^{M} E(Y_{it+m}|R_{it} = 1, \ldots, R_{it+m-1} = 1, R_{it+m} = 0)P(R_{it} = 1, \ldots, R_{it+m-1} = 1, R_{it+m} = 0) \\
+ E(Y_{it}(0)|R_{it} = 1, \ldots, R_{it+m} = 1)P(R_{it} = 1, \ldots, R_{it+m} = 1) \\
\geq \sum_{m=1}^{M} E(Y_{it+m}|R_{it} = 1, \ldots, R_{it+m-1} = 1, R_{it+m} = 0)P(R_{it} = 1, \ldots, R_{it+m-1} = 1, R_{it+m} = 0) \\
+ y_{\text{min}}P(R_{it} = 1, \ldots, R_{it+m} = 1) \tag{15}
\]

Supplementary Web Appendix, p. 4
Note that, in practice, this upper bound is unlikely to be informative beyond the no-assumption bound. Finally, similar calculations yield,

$$E(Y_{it}(0)|R_{it} = 1) \leq \sum_{m=1}^{M} E(Y_{it-m}|R_{it-m} = 0, R_{it-m+1} = 1, \ldots, R_{it} = 1) P(R_{it-m} = 0, R_{it-m+1} = 1, \ldots, R_{it} = 1)$$

$$+ y_{\text{max}} P(R_{it-m} = 1, R_{it-m+1} = 1, \ldots, R_{it} = 1)$$

The full set of recursions lead to the following bounds for $E(Y_{it}(0)) \in [a', b']$,

$$a' = E(Y_{it}|R_{it} = 0) P(R_{it} = 0)$$

$$+ \sum_{m=1}^{M} E(Y_{it+m}|R_{it} = 1, \ldots, R_{it+m-1} = 1, R_{it+m} = 0) P(R_{it} = 1, \ldots, R_{it+m-1} = 1, R_{it+m} = 0)$$

$$+ y_{\text{min}} P(R_{it} = 1, \ldots, R_{it+m} = 1)$$

$$b' = E(Y_{it}|R_{it} = 0) P(R_{it} = 0)$$

$$+ \sum_{m=1}^{M} E(Y_{it-m}|R_{it-m} = 0, R_{it-m+1} = 1, \ldots, R_{it} = 1) P(R_{it-m} = 0, R_{it-m+1} = 1, \ldots, R_{it} = 1)$$

$$+ y_{\text{max}} P(R_{it-m} = 1, R_{it-m+1} = 1, \ldots, R_{it} = 1)$$

The bounds for $E(Y_{it}(1)) \in [c', d']$ are given by

$$c' = E(Y_{it}|R_{it} = 1) P(R_{it} = 1)$$

$$+ \sum_{m=1}^{M} E(Y_{it+m}|R_{it} = 0, \ldots, R_{it+m-1} = 0, R_{it+m} = 1) P(R_{it} = 0, \ldots, R_{it+m-1} = 0, R_{it+m} = 1)$$

$$+ y_{\text{min}} P(R_{it} = 0, \ldots, R_{it+M} = 0)$$

$$d' = E(Y_{it}|R_{it} = 1) P(R_{it} = 1)$$

$$+ \sum_{m=1}^{M} E(Y_{it-m}|R_{it-m} = 1, R_{it-m+1} = 0, \ldots, R_{it} = 0) P(R_{it-m} = 1, R_{it-m+1} = 0, \ldots, R_{it} = 0)$$

$$+ y_{\text{max}} P(R_{it-m} = 0, R_{it-m+1} = 0, \ldots, R_{it} = 0)$$