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Ride comfort enhancement for passenger vehicles using the structure-immittance approach

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ABSTRACT
This paper presents a novel approach to identify inerter-based suspension struts, which can provide significant performance enhancement for passenger vehicles. The inerter has been used on Formula 1 racing cars, and several beneficial devices incorporating inerters have also been identified for ride comfort enhancement. However, previous investigations either were limited to simple network configurations with moderate performance improvement, or resulted in complicated configurations with a large number of elements which are impractical for real-life applications. In addition, some important practical performance constraints have not been taken into consideration, such as high-frequency dynamic stiffness which influences the NVH performance, and frequency content consideration of the sprung mass acceleration which more directly relates to passenger perception. In this paper, a quarter-car model including top mount is studied, with the performance of a conventional suspension strut presented as baseline. The structure-immittance approach, which can cover all networks with pre-determined numbers of each element type, is adopted for the identification of the optimal suspension configurations. Several configurations with up to a 14.7% performance improvement are identified with all other practical performance indices to be no worse than the baseline. The suspension devices proposed in previous works are also considered for a sake of comparison, demonstrating significant advantages of the structure-immittance approach. Subsequently, a sensitivity analysis against the sprung and unsprung mass changes is carried out, which represents cargo and tyre weight variations, respectively. Time domain response and other reality checks are then conducted for the out-performing configurations, which reconfirm the ride comfort enhancement and ensure no unexpected behaviour occurs.

KEYWORDS
inerter; structure-immittance approach; secondary ride comfort; high-frequency dynamic stiffness; tyre load; suspension travel

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Nomenclature

Vehicle parameters

c_m	Top mount damping
c_t	Tyre damping
k_m	Top mount stiffness
k_s	Suspension stiffness
k_t	Tyre stiffness
m_s	Sprung mass
m_u	Unsprung mass

Suspension parameters

b	Inertance parameter
c	Damping parameter
k	Stiffness parameter
K_{dyn}	Dynamic stiffness
K_{static}	Static stiffness

Others

a_{m_s}	Weighted sprung mass accleration
f	Frequency in Hz
$f\|_{a_{m_s}=\max(a_{m_s})}$	Frequency of $\max(a_{m_s})$
J_r	Ride comfort parameter
J_s	Suspension travel parameter
J_t	Tyre load parameter
$\max(a_{m_s})$	Maximum weighted sprung mass accleration
n	Wavenumber in cycles/meter
$\bar{P}_{ct}$	Power dissipated by the tyre damping
$\bar{P}_{in}$	Input power
$\bar{P}_{sus}$	Power dissipated by the suspension device
V	Vehicle speed
$z_s - z_u$	Suspension movement
$z_u - z_r$	Tyre displacement
κ	Road roughness parameter

1. Introduction

When a vehicle travels on the road, it is always subjected to excitation from road irregularities, braking forces, acceleration forces, and inertial forces if on a curved track, which causes discomfort to the driver and influences manoeuvrability. Passive absorbers, viscous dampers in parallel with suspension springs, have been widely used to suppress these vibrations. To achieve better ride quality and road handling, semi-active and active suspensions have been explored by many researchers. Semi-active elements, such as the MR damper and the ER damper were proposed to be used in vehicle suspensions, of which the damping coefficient can be adjusted within a large actuation bandwidth [1,2]. Moreover, theoretical analysis and experimental validations have been carried out to investigate the advantages of the actively controlled passenger vehicles, such as in [3–5]. Despite the potential benefits of active or semi-active suspension struts, potential issues remain regarding the control-induced instability and larger control effort requirements.

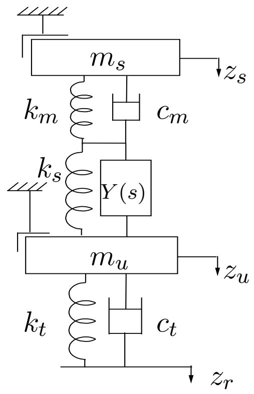
In the field of passive vibration suppression, the inerter is a relatively new element [6]. It has the property that the applied force is proportional to the relative acceleration between its two terminals. The introduction of the inerter completes the analogy between mechanical and electrical systems, allowing all the positive-real immittance functions to be realised by the passive networks consisting of inerters, dampers and springs. Performance benefits from employing inerters have been identified for various mechanical systems, including vehicle suspensions [7–12], motorcycle steering systems [13], train suspension [14,15], buildings [16,17] and landing gears [18]. The application of the inerter in passive suspension systems was first investigated in [7], where six networks were proposed as suspension candidate layouts. It has been shown that improvements in ride comfort, tyre grip and dynamic load carrying capability of about 10% or greater can be obtained for a quarter-car model. Later on, the analytical solusions of the optimum ride comfort and tyre grip were derived in [11] for the six simple networks studied in [9]. Using these analytical solutions, the obtained optimum results can be ensured to be global optima with no optimisation needed. Papageorgiou and Smith [8] proposed an approach, in which a fixed-order positive-real immittance functions with Linear Matrix Inequalities was optimised. This approach led to a further performance improvement for the same quarter car model. In [10], by considering suspension travel as a performance measure, it was illustrated that the suspension deflection is the more fundamental limitation for both ride comfort and tyre grip performance for passenger vehicles. An experimental study has been reported in [9], demonstrating the effectiveness of an inerter-based suspension device, on improving performance of passenger vehicles.

Despite the performance benefits that have been identified for passenger vehicles in previous studies, these potential benefits are yet to be realised in industry. This is because firstly, in previous investigations (e.g. [7,9]), some important practical performance constraints have not been taken into consideration, such as high-frequency dynamic stiffness which influences the Noise-Vibration-Harshness (NVH) performance and frequency content consideration of the sprung mass acceleration which more directly relates to passenger perception. Secondly, most of the previous works e.g. [9,10], were limited to simple passive configurations, which inevitably restrict the achievable performance of inerter-based suspension devices. The remaining works, e.g. [8] can cover a larger range of network possibilities by using the immittance functions, however they may result in complicated configurations with exceeded element numbers and parameter values. For example, some positive-real bicubic immittance require the Bott-Duffin synthesis [19], which corresponds to the series-parallel networks with thirteen elements.; and the analytical solutions derived in [11] do not hold when constraints, e.g. suspension travel, dynamic stiffness restrictions, are taken into consideration. In this paper, the structure-immittance approach, proposed in [20], is adopted for identifying the beneficial suspension configurations. This approach can both cover all network possibilities with pre-determined numbers of each element type, and provide explicit information of network topology and element values. By selecting the passenger ride comfort with frequency content consideration is set as the key performance index. Other important performance measures including the suspension travel, the tyre load and the high-frequency dynamic stiffness are imposed as hard constraints that the resulting suspension configurations must provide equivalent or better level of performance as the conventional one. In addition, a damper top mount, commonly used in vehicles for NVH performance [21], is also included in the considered quarter-car model.

This paper is structured as follows. In Section 2, a quarter-car model with damper top mount is introduced, together with the candidate suspension layouts formulated using the structure-immittance approach. The performance measures and the optimisation constraints are also provided and presented. In Section 3, the baseline performance is firstly obtained by considering a quarter-car model with a conventional suspension strut. For the ride comfort index, several inerter-based suspension configurations are then identified, with which the tyre load and suspension travel performance measures are then checked. In Section 4, constraints on both tyre load and suspension travel are implemented in the optimisation process, to make sure our analysis is in practical scenarios. Sensitivity analysis regarding the quarter-car parameter changes is then investigated for selecting the beneficial suspension struts. Time domain verification and reality check are finally carried out, to show the feasibility of the proposed suspension configurations. Conclusions are drawn in Section 5.

2. Quarter-car model, suspension strut and performance measure

2.1. Quarter-car model with damper top mount



Parameter	Value	Unit
m_s	250	kg
m_u	40	kg
k_s	30	kN/m
k_t	200	kN/m
c_t	270	Ns/m
k_m	200	kN/m
c_m	270	Ns/m

Figure 1.: A quarter model with a damper top mount and its corresponding parameter values.

A standard model for developing suspension systems is the quarter-car model shown in Figure 1 with a sprung mass m_s, an unsprung mass m_u and a tyre modelled as a linear stiffness k_t and viscous damping c_t. The suspension strut consists of a suspension spring k_s and a passive absorber represented by a mechanical admittance $Y(s)$, where $Y(s) = F(s)/V(s)$ is the transfer function from the relative velocity $V(s)$ across the terminals to the force $F(s)$ exerted to the terminals in the Laplace domain. The suspension strut is attached on the unsprung mass and connected with the sprung mass through a damper top mount, which is commonly used in passenger vehicles to enhance the NVH performance. Here we consider the top mount as a linear spring k_m and a damper c_m. The quarter-car parameters used in this study are listed in Figure 1. The values for the sprung and unsprung mass and stiffness parameters of the quarter car model shown in Figure 1 are obtained from [22]. It has been shown in [23] that the tyre damping has quite a significant effect in tyre load, and the range of the tyre damping ratio was considered to be within 0 to 0.15. By selecting the tyre damping ratio as 0.1, the tyre damping value can be calculated as $c_t = 270$ Ns/m. The top mount damping is chosen to be the same value as the tyre damping in the model, in line with [24].

2.2. A traditional damper and Candidate layouts

Following [25], a traditional linear damper c_s with 0.25 damping ratio is selected as the default suspension strut, as shown in Figure 2(a), where c_s equals 1.37 kNs/m. The candidate layouts considered here are the networks consisting of inerters, dampers

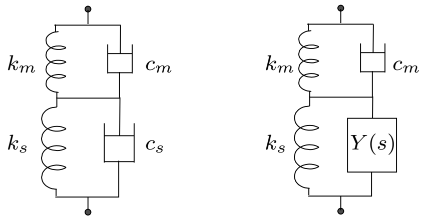


Figure 2.: A suspension strut with top mount of (a) the traditional susepsnion strut and (b) the candidate layouts.

and springs with pre-determined numbers of each element type. Given any element number included in the candidate layout, all the series-parallel network possibilities can be covered by the generic networks, formulated with the structure-immittance approach [20]. The force-velocity immittance functions of the generic networks are then derived as $Y(s)$ of Figure 2(b) for optimisation. A detailed introduction of the structure-immittance approach is provided in Appendix A. In this paper, we consider networks with up to six elements, divided into eight cases, which are (i) three-element networks, the 1b1c1k case representing all possible series-parallel suspension layouts consisting of one inerter, one damper and one spring (other cases are defined in the same manner), (ii) four-element networks with the 1b1c2k, 1b1k2c and 1c1k2b cases, (iii) five-element networks including the 1b2c2k, 1c2b2k and 1k2b2c cases, and (iv) six-element networks considering the 2b2c2k case.

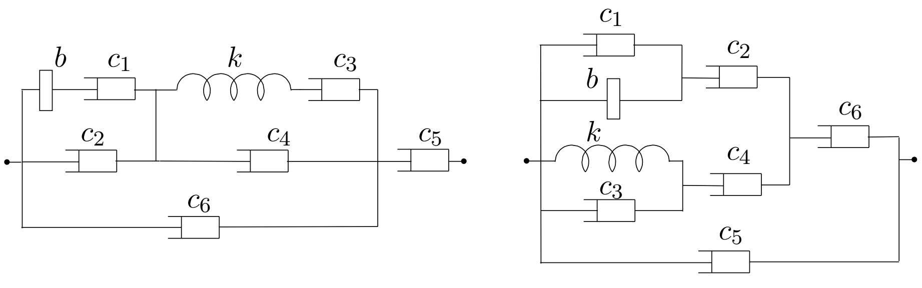


Figure 3.: Generic networks for the case with one inerter, one spring and two dampers.

Taken the 1b1k2c case as an example, based on the formulation procedure shown in Figure 5 of [20], two generic networks can be obtained as Figure 3. These two networks, along with the constraints that at most two dampers exist, cover all 18 network layout possibilities. Their corresponding force-velocity immittance functions can be calculated as

$$Y_i(s) = \frac{n_i(s)}{m_i(s)} \quad (i = 1,\ 2) \tag{1}$$

with

$$n_1(s) = b(c_4 + c_6)s^2 + (bk(\frac{c_2}{c_1} + \frac{c_6}{c_1} + \frac{c_4}{c_3} + \frac{c_6}{c_3} + 1) + c_2c_4 + c_2c_6 + c_4c_6)s + k(c_2 + c_6),$$

$$m_1(s) = b(\frac{c_2}{c_1}+\frac{c_4}{c_1}+\frac{c_4}{c_5}+\frac{c_6}{c_5}+1)s^2+(bk(\frac{1}{c_1}+\frac{1}{c_3}+\frac{1}{c_5})+c_2+c_4)s+k(\frac{c_2}{c_3}+\frac{c_2}{c_5}+\frac{c_4}{c_3}+\frac{c_6}{c_5}+1)$$

$$n_2(s) = b(\frac{c_3}{c_2}+\frac{c_5}{c_2}+\frac{c_3}{c_4}+\frac{c_5}{c_6}+1)s^2+(bk(\frac{1}{c_2}+\frac{1}{c_4})+c_1+c_3+c_5)s+k(\frac{c_1}{c_2}+\frac{c_1}{c_4}+\frac{c_5}{c_4}+\frac{c_5}{c_6}+1)$$

$$m_2(s) = b(\frac{1}{c_2} + \frac{1}{c_6})s^2 + (bk(\frac{1}{c_2c_4} + \frac{1}{c_2c_6} + \frac{1}{c_4}c_6) + \frac{c_1}{c_2} + \frac{c_1}{c_6} + \frac{c_3}{c_4} + \frac{c_3}{c_6} + 1)s + k(\frac{1}{c_4} + \frac{1}{c_6})$$

where $b \geq 0,\ k \geq 0$ and for $i = 1$, corresponding to the left-hand generic network in Figure 3, at least four of parameters $1/c_1,\ c_2,\ 1/c_3,\ c_4,\ 1/c_5,\ c_6$ equal zero, whereas with $i = 2$, corresponding to the right-hand network in Figure 3, at least four of parameters $c_1,\ 1/c_2,\ c_3,\ 1/c_4,\ c_5,\ 1/c_6$ equal zero. These two immittance functions with the imposed parameter conditions can then be used for further optimisation.

2.3. Performance measures and constraints

There are a number of practical design requirements for a suspension system such as passenger comfort, tyre load, suspension travel and NVH performance. Ride comfort, an important measure depending on the acceleration level, frequency, direction and position, is selected as the key performance index in this paper. The ISO 2631 standard specifies a method to evaluate the effect of exposure to vibration on humans by weighting the acceleration with human vibration-sensitivity curves. The frequency weighting curve for vertical acceleration (measured at the seat surface) has been provided in the ISO 2631 standard, and a second-order shape filter of the form

$$H_{2631}(s) = \frac{50s + 500}{s^2 + 50s + 1200} \tag{2}$$

has been used in [25] to approximate the ISO weighting curve, which is also adopted here for measuring the ride comfort index. In addition, it is pointed out in [26] that vehicle ride comfort can be evaluated by using the vertical acceleration of the body up to 20Hz, while the higher frequency response is more related to the NVH performance. Also based on [27], the secondary ride, vibrations in the frequency range from 4Hz to 20Hz, relates most to the ride comfort. Based on these observations, for improving the passenger ride comfort, we propose a cost function J_r, expressed by

$$J_r = \Delta\omega S_{\dot{z}_r} \sum_{w_1}^{w_2} |T_{\dot{z}_r \to \ddot{z}_s}(j\omega) H_{2631}(j\omega)|^2 \tag{3}$$

where $w_1 = 8\pi$, $w_2 = 40\pi$ represents the secondary ride frequency range of [4Hz, 20Hz], $T_{\dot{z}_r \to \ddot{z}_s}$ denotes the transfer function from the road velocity input $\dot{z}_r$ to the sprung mass acceleration $\ddot{z}_s$ and

$$S_{\dot{z}_r} = 2\pi\kappa V \tag{4}$$

is the power spectra of the road velocity input, same as [28]. In this study, we select $V = 25$ m/s and $\kappa = 5 \times 10^{-7}$ m^{3}cycle^{-1}, in line with [7,8]. Also note that in the following discussion, we denote $|T_{\dot{z}_r \to \ddot{z}_s}(j\omega) H_{2631}(j\omega)|$ as a_{m_s} for simplicity. A detailed derivation of J_r is provided in Appendix B. For the tyre load performance measure, the rms tyre load parameter J_t is defined as [7]:

$$J_t = 2\pi(V\kappa)^{1/2} ||T_{\dot{z}_r \to k_t(z_u - z_r)}||_2 \tag{5}$$

and the suspension travel performance measure can be represented by the maximum relative movement of the suspension strut [29], expressed by:

$$J_s = (2\pi V\kappa)^{1/2} ||T_{\dot{z}_r \to (z_s - z_u)}||_\infty \tag{6}$$

Considering the vehicle NVH performance relating to the high-frequency dynamic stiffness, it has been proposed in [30] that a higher dynamic stiffness of the suspension strut will provide a poorer NVH performance. Hence, in order to make sure the NVH performance will not be deteriorated, we impose a constraint that the dynamic stiffness K_{dyn} should be no larger than the default one in the frequency range above 20Hz. It has also been pointed out in [26] that the primary ride (frequency ranging from 0 Hz to 4 Hz) is generally related with rigid body movement and has prominent controdiction between comfort and body control. To address this point, another constraint is proposed, that within the primary ride frequency range, $\max(a_{m_s})$ and $f|_{a_{m_s}=\max(a_{m_s})}$ should be within the range [95%, 105%] of the default values for the traditional suspension strut. Note that many investigation made use of H2 norm of the sprung mass acceleration as the cost function for improving the ride comfort performance, which will result in significant reduction of the first resonance frequency peak, e.g. [7–9]. It is worth to point out that the optimum suspension identification approach presented in this work is directly applicable to alternative performance cost functions and constraints. Also note that to form a direct comparison with previous research on inertance-integrated suspension struts, for example, Smith&Wang [7] and Papageorgiou&Smith [8] which investigated single performance index without constraints, we first conduct the optimisation without tyre load and suspension travel constraints in Section 3 and then include these constraints in Section 4.

3. Optimisation results without tyre load and suspension travel constraints

In line with the previous research works [7,9–11] which considered the suspension layouts with up to four elements, this section takes the networks consisting of three and four elements as candidate layouts, i.e. the 1b1c1k, 1b1c2k, 1b1k2c and 1c1k2b cases. The response of the quarter-car model with the traditional damper is treated as the baseline. Optimisations are carried out for improving the ride comfort performance, satisfying the constraints on primary ride and suspension dynamic stiffness. The identified suspension configurations are then assessed using tyre load and suspension travel performance measures, suggesting the necessity of imposing constraints on both of these in designing the suspension devices.

3.1. Baseline performance and optimisation procedure

For the default suspension strut, shown in Figure 2(a), the frequency response of the weighted sprung mass acceleration is presented in Figure 4(a), where two short vertical dashed lines show the frequency boundaries of primary and secondary ride. It can be

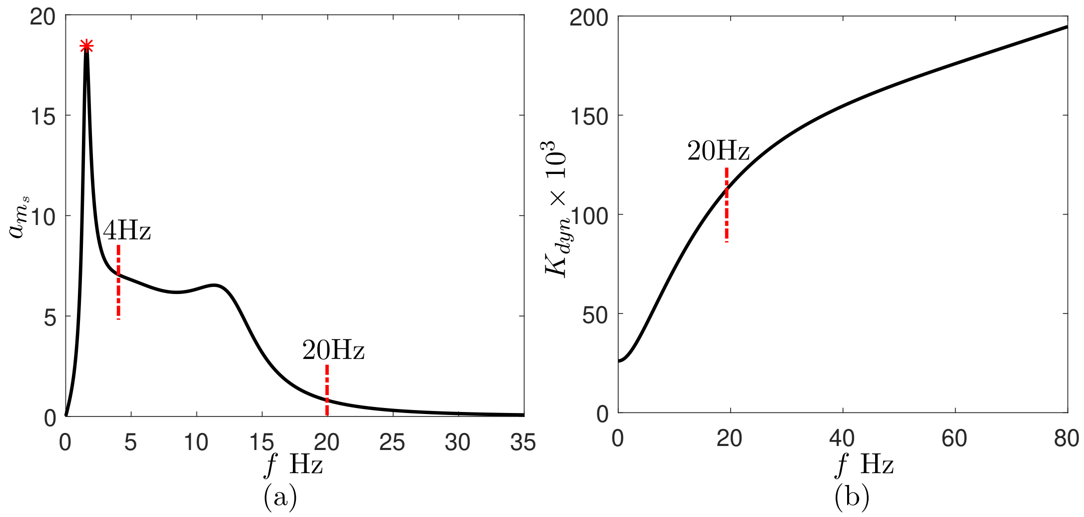


Figure 4.: Frequency response with the default suspension device of (a) weighted sprung mass acceleration, (b) dynamic stiffness.

calculated that with this traditional suspension strut, the value of the ride comfort objective function J_r (3) is 0.211. It can also be noted from Figure 4(a) that the default primary ride maximum weighted acceleration and corresponding frequency is $\max(a_{m_s}) = 18.46$ and $f|_{a_{m_s}=\max(a_{m_s})} = 1.58$ Hz, shown as red star in Figure 4(a). Hence the constraint considering the primary ride vibrations, discussed in the previous section can be mathematically expressed as:

$$\frac{|\max(a_{m_s}) - 18.46|}{18.46} \leq 5\%, \quad \frac{|f|_{a_{m_s}=\max(a_{m_s})} - 1.58|}{1.58} \leq 5\% \tag{7}$$

It should be noted that for another vehicle, an updated constraint with different primary peak frequency and magnitude will be applied. Figure 4(b) shows the dynamic stiffness of the traditional suspension strut of Figure 2(a), which is the magnitude of its force-displacement transfer function, represented as $|\frac{(k_m+c_m s)(k_s+c_s s)}{k_m+k_s+s(c_m+c_s)}|$. The frequency range of vibrations related to the vehicle NVH performance is also predicted by the short vertical dashed line in Figure 4(b). From it, we can express the dynamic stiffness constraint as (8) with $Y(s)$ representing the force-velocity transfer function of the suspension candidate layouts.

$$|\frac{(k_m + c_m s)(k_s + Y(s)s)}{k_m + k_s + s(c_m + Y(s))}| \leq |\frac{(k_m + c_m s)(k_s + c_s s)}{k_m + k_s + s(c_m + c_s)}| \text{ when } s \geq 20 \text{ Hz} \tag{8}$$

Note that the static stiffness of the traditional suspension strut (Figure 2(a)) is $K_{\text{static}} = 26.09$ kN/m, which equals $(k_m^{-1} + k_s^{-1})^{-1}$. This static stiffness is used to ensure the suspension strut is capable of supporting the car body. In this work, the optimisation will be conducted for the case that the static stiffness of each candidate layouts should be no smaller than this default value, while the suspension stiffness k_s can be optimised. For all the optimisations carried out in the present work, we use the MATLAB command *patternsearch* first and then *fminsearch* for fine-tuning of the parameters. It should be noted that the *patternsearch* and *fminsearch* functions require a set of initial values of the design variables, and their efficiency is dependent on the setting of the initial values. However, if the initial values are given properly, *patternsearch* and *fminsearch* functions are more accurate and faster than those global optimum design methodologies, which do not need to set any initial values, such as the genetic algorithm. To derive a global optimisation result by the *patternsearch* and *fminsearch* functions, a number of sets of initial values can be given in terms of random numbers in a considered value range, and then, the obtained values of the variables that would provide the objective function J_r with minimum value would be the global optimum parameters. Note that, during the optimization process, no restriction due to practical implementation consideration is placed on the parameter values. Instead, we consider whether the parameter values are practical after the optimization stage. Also note that all the suspension configurations discussed in the following sections have been tuned to their respective optimum.

3.2. Identified suspension configurations

Considering all the possible networks with up to four elements, the optimal configurations for the $Y(s)$ in Figure 2(b) are shown in Figure 5, together with the optimisation results summarised in Table 1. Note that the traditional suspension strut of Figure 2(a), denoted as Default in Table 1, is also optimised for the sake of comparison. The optimised traditional suspension strut is denoted as S1, with the value of the objective function J_r obtained as 0.1994. It can be seen that S1 can only provide limited performance advantage, approximately 5.2% better than the default one.

For the case where the suspension device includes one inerter, one damper and one spring, out of the eight possible layouts (enumerated in [20]), the optimisation indicates that two networks S2 and S3 shown in Figure 5(a) and (b) are optimal. With these two configurations, up to 46.3% performance improvement in ride comfort can be obtained. By allowing two springs in the suspension struts, the networks S4 and S5 are obtained as optimal configurations, providing 51.9% and 52.2% smaller value of ride comfort cost function J_r, respectively. The other two cases, i.e. 1b1k2c and 1c1k2b do not result in any better performance comparing with the 1b1c1k case. This means the additional damper in the 1b1k2c case or an inerter in 1c1k2b case always effectively

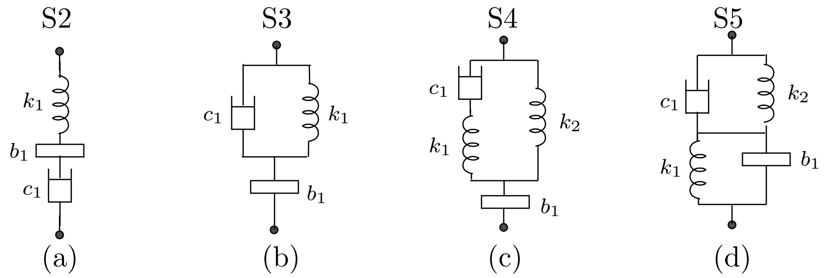


Figure 5.: Optimal configurations identified for improving the ride comfort performance, representing the $Y(s)$ in Figure 2(b).

Table 1.: Optimum results with the identified suspension configurations.

Configuration	Ride comfort cost function J_r (imp %)	Parameter values (kg, kNs/m, kN/m)
Default	0.2105 (-)	$c = 1.37,\ k_s = 30$
S1 ($\text{Default}_{\text{opt}}$)	0.1994 (5.2%)	$c = 1.29,\ k_s = 30$
S2	0.1442 (31.5%)	$b = 87.2,\ c = 1.19, k_1 = 9.6\ k_s = 30$
S3	0.1131 (46.3%)	$b = 95.7,\ c = 0.49, k_1 = 6.2\ k_s = 30$
S4	0.1011 (51.9%)	$b = 60.3,\ c = 0.29,\ k_1 = 792.02,\ k_2 = 4.99,\ k_s = 30$
S5	0.1005 (52.2%)	$b = 101.8,\ c = 0.39,\ k_1 = 2.29,\ k_2 = 5.35,\ k_s = 30$

disappears during the optimisation with its value turning to zero if it is connected in parallel or to infinity if connected in series. The frequency response of the weighted sprung mass acceleration with the identified configurations are shown in Figure 6(a), from which it can be seen that all the identified suspension configurations satisfy the constraints that the maximum sprung mass acceleration and its corresponding frequency should be similar to the default ones. Figure 6(b) shows the dynamic stiffness of the obtained configurations, suggesting that their dynamic stiffness is smaller than that of the default structure when the frequency is larger than 20Hz. Note that the configuration S5 has been proposed by Smith & Wang [7] for improving the ride comfort. However, the other three beneficial configurations of Figure 5(a-c), which can also provide significant performance benefits, have not been considered, previously.

3.3. Analysis on tyre load and suspension travel with identified configurations

Up to this point, we have considered the ride comfort using the objective function J_r, the tyre load and suspension travel performance measures are now checked for the identified configurations. For the default suspension strut, based on (5) and (6), the tyre load and suspension travel can be calculated as $J_t = 559$ and $J_s = 0.0025$, respectively.

Table 2 provides the values of J_t and J_s for the optimal configurations S1 to S5, showing that the tyre load and suspension travel of the obtained configurations are significantly larger than the default structure. Note that with configuration S2, the tyre load is almost twice the default value, and configuration S5 results in around a 50% larger value of suspension travel J_s. The frequency response of $z_u - z_r$ and $z_s - z_u$ is shown in Figure 7. It can be seen from Figure 7(a) that the suspension configurations S1-S5 result in larger tyre displacement around the second fundamental frequency of the quarter car model, 12 Hz. Also note from Figure 7(b), the maximum

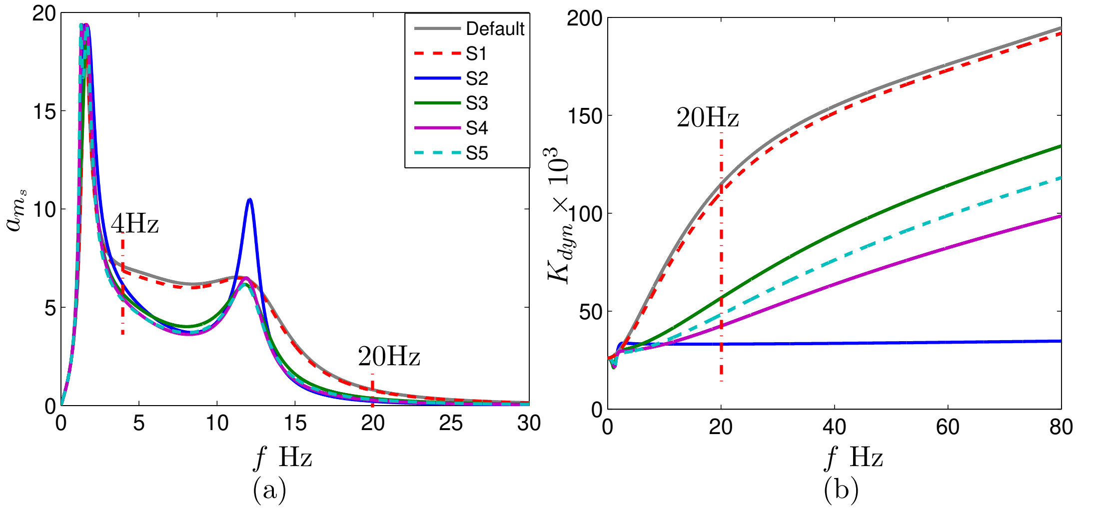


Figure 6.: Frequency response of (a) weighted sprung mass acceleration, (b) dynamic stiffness with configurations S1-S5 and the default suspension device.

Table 2.: Tyre load and suspension travel values of the identified suspension configurations.

Configuration	Tyre load J_t	Suspension travel J_s
Default	559	0.0025
S1(Default$_{\text{opt}}$)	579.4	0.0026
S2	1040	0.0031
S3	720	0.0026
S4	820	0.0036
S5	780	0.0039

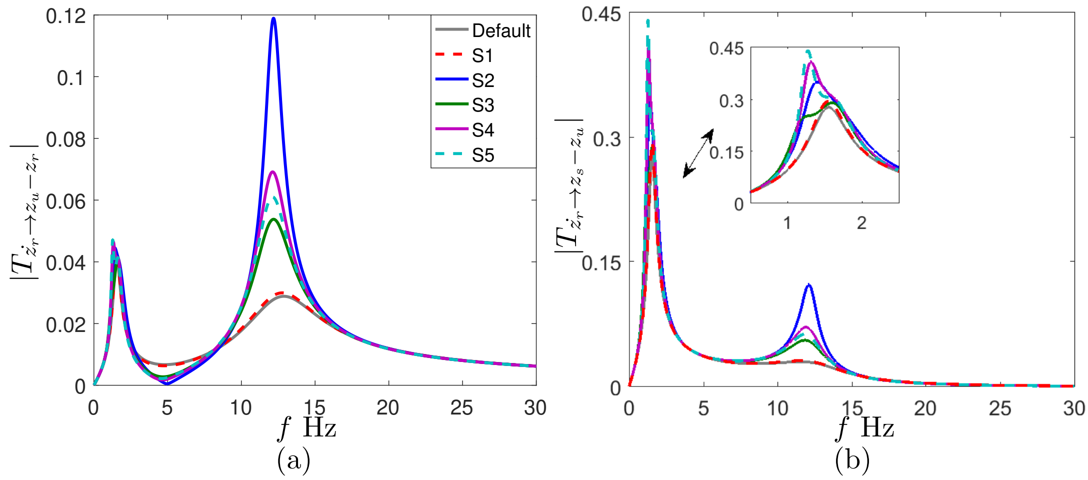


Figure 7.: Frequency response of (a) $z_u - z_r$, (b) $z_s - z_u$, showing the necessity to include tyre displacement and suspension travel constraints.

suspension displacement of the default suspension is significantly smaller than those of the identified configurations, as expected from Table 2. These inferior behaviours in tyre load and suspension travel with the obtained optimal suspension configurations will pose challenge for practical applications. Hence, constraints on these two performance measures will be implemented in the next section.

4. Beneficial suspension configuration identification

Further investigations with two extra constraints on tyre load and suspension travel are discussed in this section. It will be shown that, with these constraints, limited performance improvement in ride comfort is obtained when considering the suspension devices proposed in previous works [7,8]. By allowing up to six element numbers included in suspension strut, significant performance advantages can be identified using the structure-immittance approach. With the obtained suspension configurations, a sensitivity analysis is conducted considering changes in vehicle parameter values, to identify which beneficial configurations are robust. Note that while a 6 element device would be complicated to implement purely mechanically, it has been shown that the use of fluid-based inerter can provide integrated device solutions [31] with much reduced complexity.

4.1. Optimisation results with tyre load and suspension travel constraints

The tyre load J_t and the suspension travel J_s of the candidate layouts are constrained to be no larger than the default strut, i.e. $J_t \leq 559$ and $J_s \leq 0.0025$, which are implemented in the optimisation procedure. By minimising the ride comfort objective function J_r with the two additional imposed constraints, the optimum strut configurations and the corresponding optimal results are summarised in Figure 8 and Table 3(a), for the candidate network layouts including up to six elements (the subscript k is used to indicate optimisation with both tyre load and suspension travel constraints). For comparison, the strut devices proposed in previous works, i.e. S6 of [7] and a positive-real biquadratic function in [8] are also considered, of which the optimisation results are also shown in Table 3(b).

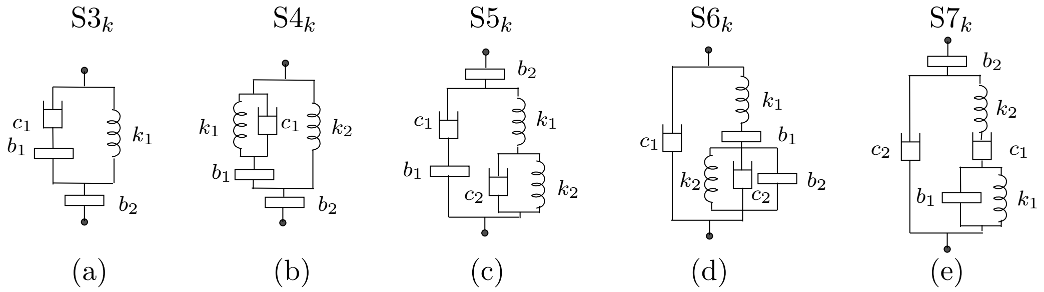


Figure 8.: The identified optimal configurations S3$_k$ - S7$_k$, representing the $Y(s)$ in Figure 2(b).

It can be seen from Table 3(a) that when satisfying the constraints on tyre load and suspension travel, the performance improvement in terms of ride comfort index has been significantly reduced and different layouts are now optimised, compare the 4 element device S3$_k$ to those in Figure 5, S4 and S5. It can be noted that the suspension strut with a traditional damper, denoted as S1$_k$ is unable to provide a better performance than the default one. The configuration S2$_k$ has the same topology as absorber S3, shown in Figure 5(c). It can still provide a better ride comfort than the default even when required to satisfy the tyre load and suspension travel constraints, but the performance improvement is reduced from 46.3% to 2.7%. For the case where the suspension device includes four elements, i.e. the 1b1c2k, 1b1k2c and 1c1k2b cases, out of the 54 possible layouts obtained using the structure-immittance approach, network S3$_k$, consisting of two inerters, one damper and one spring is ob-

Table 3.: Optimal results considering the tyre load and suspension travel constraints

(a): The configurations identified with the structure-immittance approach, see Figure 8

Configurations	Ride comfort cost function J_r (imp %)	Parameter values (kg, kNs/m, kN/m)	Tyre load J_t	Suspension travel J_s
Default	0.211 (-)	$c = 1.37,\ k_s = 30$	559	0.0025
$S1_k$(Default$_{\text{opt}}$)	0.211 (0%)	$c = 1.37,\ k_s = 30$	559	0.0025
$S2_k$	0.205 (2.7%)	$b = 87.2,\ c_1 = 1.19,$ $k_1 = 9.6\ k_s = 30$	559	0.0025
$S3_k$	0.190 (9.7%)	$b_1 = 167.6,\ b_2 = 141.7$ $c_1 = 1.34,\ k_1 = 12.89,$ $k_s = 30$	559	0.0025
$S4_k$	0.189 (10.1%)	$b_1 = 123.8,\ b_2 = 140.1,$ $c_1 = 1.34,\ k_1 = 2.67,$ $k_2 = 13.9,\ k_s = 30$	559	0.0024
$S5_k$	0.187 (11.4%)	$b_1 = 29.5,\ b_2 = 175.4,$ $c_1 = 1.27,\ c_2 = 1.08,$ $k_1 = 71.8,\ k_2 = 10.4,$ $k_s = 30$	559	0.0022
$S6_k$	0.185 (12.1%)	$b_1 = 40.6,\ b_2 = 6.08,$ $c_1 = 0.79,\ c_2 = 0.23,$ $k_1 = 57.8,\ k_2 = 3.68,$ $k_s = 30$	559	0.0023
$S7_k$	0.180 (14.7%)	$b_1 = 6.97,\ b_2 = 320.9,$ $c_1 = 0.994,\ c_2 = 0.972,$ $k_1 = 7.26,\ k_2 = 54.68,$ $k_s = 30.0$	559	0.0025

(b): The previously proposed suspension layouts [7,8]

Configurations	Ride comfort cost function J_r (imp %)	Element numbers required
S6 in [7]	0.205 (2.7%)	4
Biquadratic immittance in [8]	0.204 (3.1%)	9

tained as the optimum configuration, shown in Figure 8(a). With this strut, the value of J_r is obtained as 0.190, around 9.7% smaller than that of the default one. Note that the previously obtained beneficial configuration S4 and S5 in Section 3.1 both reduce to the S3 ($S2_k$) layout during the optimisation with the additional constraints applied, with k_1 of S4 turning to infinity and that of S_5 to 0. This means that with the additional spring, the ride comfort can be improved but its presence has a deleterious effect on the performance in tyre load and suspension travel. Consider the network possibilities consisting of five elements, configuration $S4_k$ is optimised, with a slightly improved performance over that of $S3_k$. By allowing two inerters, two dampers and two springs in the suspension device, the optimisation results indicate that three networks $S5_k$, $S6_k$ and $S7_k$ are all near optimal. Figure 8(c), (d) and (e) show these layouts. The strut $S7_k$ provides the best ride comfort performance, resulting in 14.7% smaller value of the objective function J_r comparing with the default structure. Considering the suspension devices proposed in previous works [7,8], it can be seen from Table 3(b) that the S6 in [7] provides very limited performance benefit, approximately 2.7% better than the default one. However, for the network with similar level of complicity (the included element number is the same, i.e. 4), the $S3_k$ identified in this paper can provide 9.7% performance advantage, almost five times that of S6 in [7]. Using the positive-real biquadratic immittance as a candidate layout [8], the performance improvement is also very limited, around 3.1%, while its realisation requires 9 elements.

For the positive-real bicubic function, its potential performance improvement can be larger, however, 13 elements are required for its realisation, which is arguably too complicated for practical implementation.

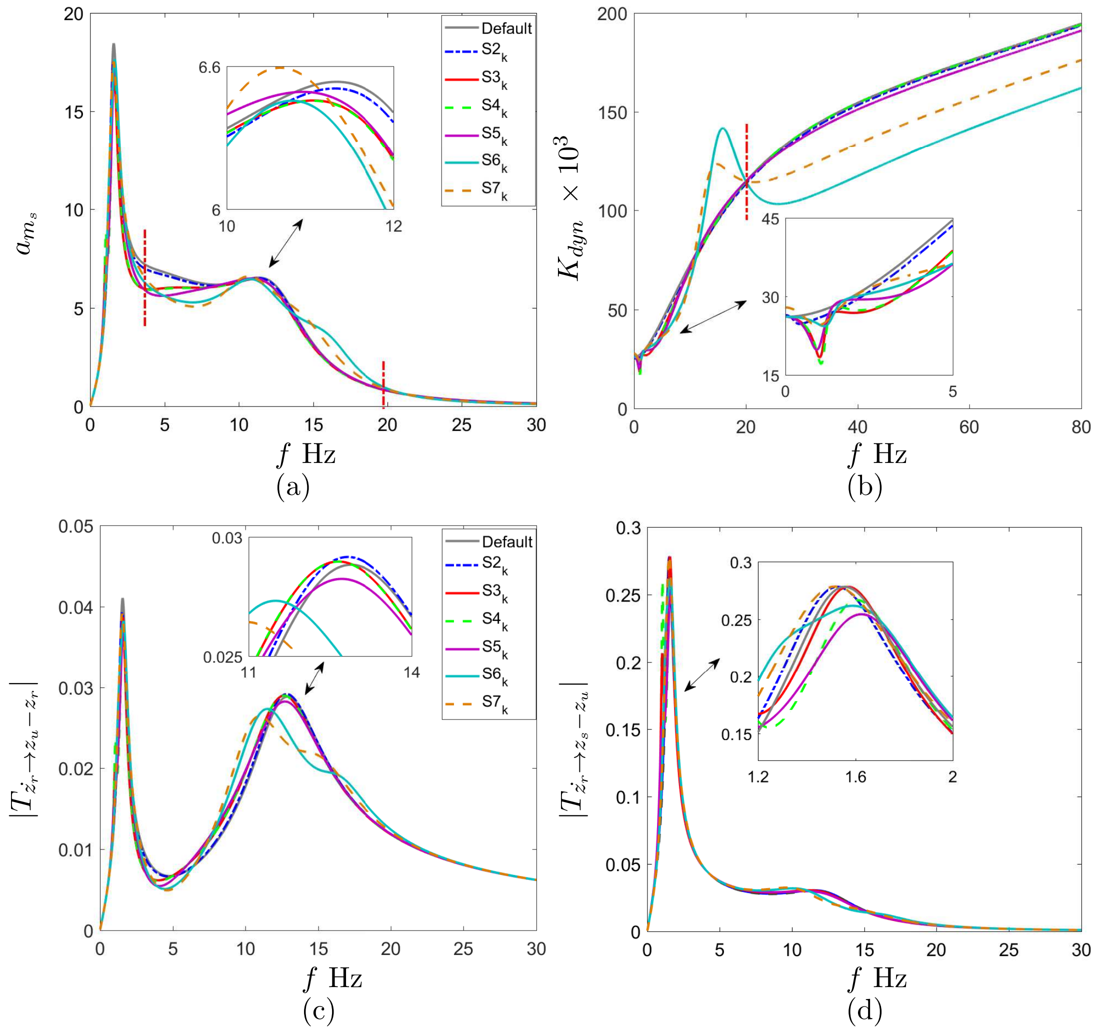


Figure 9.: Frequency response of (a) a_{m_s}, (b) K_{dyn}, (c) $z_u - z_r$ and (d) $z_s - z_u$ for the configurations S2$_k$-S6$_k$ and the default suspension strut.

The frequency response of the weighted sprung mass acceleration using these configurations is presented in Figure 9(a), and the dynamic stiffness is shown in Figure 9(b). It can be seen that configuration S7$_k$ results in the smallest maximum value of sprung mass acceleration across the frequency range approximately from 5Hz to 14Hz, while for the frequency range of [13Hz, 20Hz] it results in larger sprung mass accelerations than those using the default device. Also from Figure 9(b), we notice that the dynamic stiffness of S6$_k$ and S7$_k$ is larger than the default strut when the frequency is in the range of [11Hz, 20Hz], and for the frequency exceeds 20 Hz, its dynamic stiffness is significantly smaller than the default one. This predicts the superior NVH performance of the quarter car model with the suspension struts S6$_k$ and S7$_k$.

The tyre load and suspension travel of the obtained configurations are also provided in Table 3(a), suggesting the constraints have been satisfied. Figure 9(c) and (b) shows the frequency response of the tyre displacement $z_u - z_r$ and the suspension movement

$z_s - z_u$ of the quarter car model with these obtained suspension configurations. It can be seen from Figure 9(c) that with all these configurations, the maximum tyre displacement is smaller than that of the default structure, and S7$_k$ provides the smallest tyre displacement around the second fundamental frequency. In addition, it can also be seen from Figure 9(d) that comparing with the default structure, all the obtained suspension configurations provide equivalent or smaller maximum value of suspension movement, as expected from Table 3(a).

4.2. Sensitivity analysis and reality checks

Several suspension configurations have been identified for enhancing the passenger ride comfort while satisfying the constraints on high-frequency dynamic stiffness, tyre load and suspension travel performance measures. An important step in identifying a beneficial suspension configuration is to assess the robustness of the device to vehicle parameter changes via a sensitivity analysis. The quarter-car model of Figure 1 is varied to evaluate the suspension strut sensitivity to changes in the following parameters: sprung mass m_s and unsprung mass m_u, where the sprung mass m_s can be easily effected by the weight of passenger, cargo and the amount of gasoline, and the unsprung mass m_u is highly related with the tyre weight. The suspension robustness is evaluated using the three considered performance measures, i.e. the ride comfort J_r (3), the tyre load J_t (5) and the suspension travel J_s (6), for which, the performance changes with respect to either the sprung mass m_s or the unsprung mass m_u are shown in Figure 10.

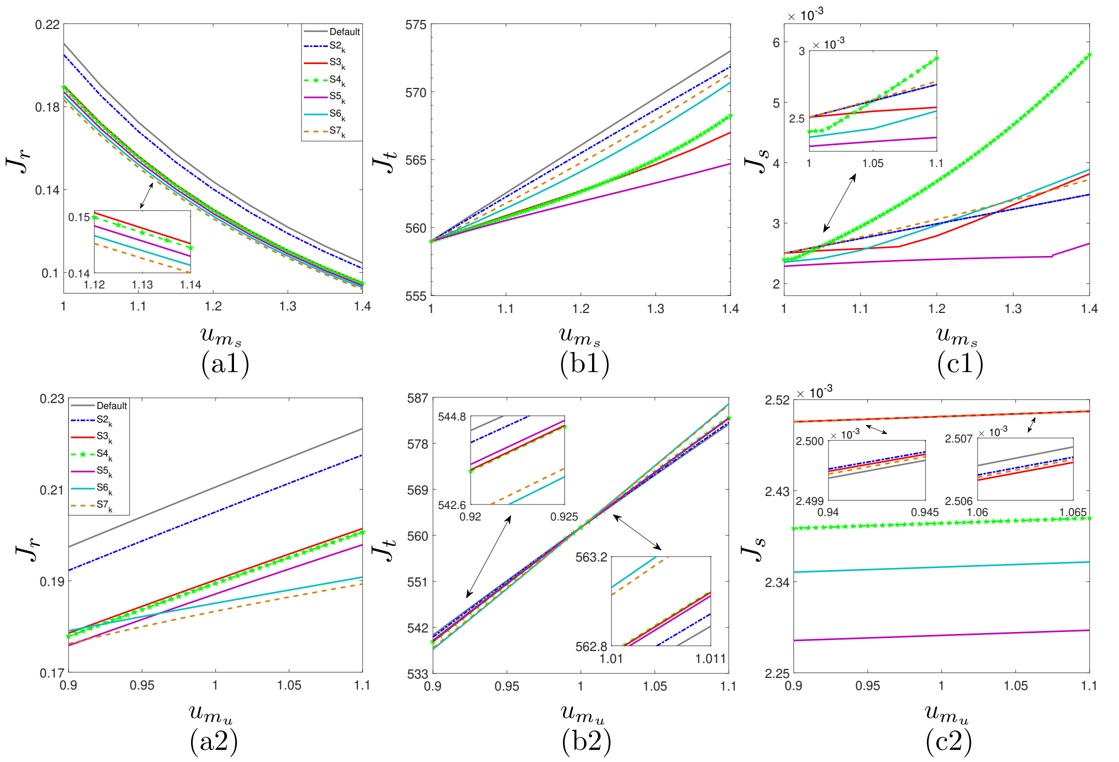


Figure 10.: Objective function values with respect to the sprung mass ratio u_{m_s} and unsprung mass ratio u_{m_u} for the ride comfort J_r, the tyre load J_t and the suspension travel J_s.

Considering the sprung mass m_s, the range of considered values are $u_{m_s} \times 250$ kg, where $u_{m_s} \in [1,\ 1.4]$ is the ratio between the changed sprung mass and the default value. The values of the considered performance measures are shown in Figure 10(a1-c1) with respect to the sprung mass ratio u_{m_s}, for all the proposed suspension configurations (without re-optimisation). It can be seen from Figure 10(a1) that the changing trend of the ride comfort cost function J_r with the obtained suspension configurations $S2_k$ - $S7_k$ is similar to that with the default suspension strut - a larger sprung mass results in a lower value of J_r, which means a better ride comfort performance. Across the whole range of sprung mass values, the configuration $S7_k$ always provides the best ride comfort performance however the performance improvement is slightly reduced with larger values of sprung mass. Figure 10(b1) shows that the value of tyre load J_t becomes larger with the increasing sprung mass and all the six proposed beneficial configurations provide better tyre load performance, comparing with the default strut, where the $S5_k$ outperforms all the other structures. It can also be seen that when the value of sprung mass increases, the beneficial configurations outperform the default one in terms of tyre load. The changing trend of the suspension travel versus the sprung mass is provided in Figure 10(c1). The suspension travel J_s for $S4_k$ is significantly larger than the default one when the weight of sprung mass increases, hence we reject this configuration. In contrast, configuration $S5_k$ provides the smallest suspension travel value across all configurations over the full range of the sprung mass values considered. If a 5% degradation of the value of J_s is acceptable then in addition to $S5_k$, configurations $S2_k$, $S3_k$, $S6_k$ and $S7_k$ can all be considered as robust suspension struts to the change of the sprung mass, in the performance of suspension travel.

For the unsprung mass m_u, we take its value to be changed within the range $m_u \in [0.9,\ 1.1] \times 40$ kg, where we define the ratio between its changed value to the default one as u_{m_u}. The other parameter values of the quarter car model are kept unchanged. Figure 10(a2-c2) presents the values of the considered performance measures $J_r,\ J_t$ and J_s with respect to the unsprung mass weights. From Figure 10(a2), it can be seen that comparing with the default suspension strut, all the six identified configurations $S2_k$-$S7_k$ provides better ride comfort performance in the whole range of the unsprung mass values. Also configurations $S6_k$ and $S7_k$ result in larger performance improvement when the unsprung mass becomes heavier. The values of tyre load J_t and suspension travel J_s are shown in Figure 10(b2) and (c2), respectively. Figure 10(b2) suggests that the proposed beneficial suspension struts have similar tyre load performance to that of the default structure over the range of unsprung mass values considered. Also, from Figure 10(c2), the suspension travel of the proposed configurations are only slightly effected by the change of the unsprung mass, and comparing with the default strut, the identified configurations still provide better suspension travel performance across the unsprung mass range.

In summary, five of the identified configurations, i.e. $S2_k$, $S3_k$, $S5_k$, $S6_k$ and $S7_k$ are the robust suspension strut designs, and because of the limited performance improvement achieved by $S2_k$, the other four configurations are considered as the beneficial suspension struts, and the $S7_k$ provides the best performance in passenger ride comfort. Time domain verification and reality checks are then carried out for the $S7_k$, using a random surface road input and a cobblestone road input. The power dissipation of this configuration is also analysed and checked. Results are included in Appendix C, based on which, it can be further confirmed that $S7_k$ can provide robust performance.

5. Conclusions

This paper has investigated the potential passenger comfort performance benefits in secondary ride using inerter-based suspension devices for a quarter car model with a damper top mount. Multiple performance requirements including primary ride performance and high-frequency dynamic stiffness have also been taken into consideration, formulated as optimisation constraints. Using the structure-immittance approach, up to 52% performance improvement has been identified with the optimal inerter-based configurations including at most four elements. An analysis on the tyre load and suspension travel was then carried out, showing significant undesirable behaviour occurs in these performance measures. Hence, extra constraints on tyre load and suspension travel were implemented for further optimisation, considering the networks with up to six elements as candidate suspension layouts. Six optimal configurations that provide up to 14.7% performance improvement were then obtained and presented making use of the structure-immittance approach. It needs to be pointed out that the optimum layouts identified in previous works, e.g. S6 in [7] and the biquadratic function in [8], deliver 2.7% and 3.1% performance advantages, respectively. For these obtained configurations, sensitivity analysis against quarter-car model parameter changes have further been investigated. Four of the six optimal configurations were finally identified as beneficial suspension configurations. By selecting a random surface as road input, effectiveness of the proposed suspension configuration was verified in time domain analysis. Furthermore, energy dissipation of the proposed beneficial configurations and the quarter-car model response subjected to a cobblestone road input were studied, suggesting the feasibility of the identified suspension configurations. This paper contributes to the design of vehicle suspension devices in practical scenarios, where not only the key performance index but also other performance constraints are considerd. Furthermore, it demonstrates the superiority of the structure-immittance approach for suspension system design, providing a motivation for industry to conduct experiment studies to develop the concept further.

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Appendix A. Structure-immittance approach

The structure-immittance approach is a newly proposed approach for designing passive vibration absorbers [20], which can both consider a full set of absorber layouts together, and restrict the complexity, topology and element values of the candidate layouts. Consider absorbers with a pre-determined number of inerters, dampers and springs. Using the structure-immittance approach, first the generic networks covering all the possible absorber layouts are formulated, and second the structural immittances (which are the force-velocity transfer functions of the generic networks) are obtained. These structural immittances are then used for the next step optimisation for a given vibration suppression problem.

Taking the absorber layouts consisting of one inerter, one damper and one spring (1b1c1k case) as an example, two generic networks are firstly formulated as Figure 6-Step 5 [20], which cover all 8 network possibilities (see Table 2 in [20]). The corresponding structural immittances can then be expressed by [20]:

$$Y_1(s) = \frac{bcs^2 + b(k_4 + k_6)s + c(k_2 + k_6)}{bc(1/k_3)s^3 + bs^2 + cs + k_2 + k_4}, \tag{A1}$$

$$Y_2(s) = \frac{bc(1/k_1 + 1/k_2)s^3 + bs^2 + cs + k_3}{b(1/k_1 + 1/k_5)s^3 + c(1/k_2 + 1/k_5)s^2 + s}. \tag{A2}$$

where $b \geq 0$ and $c \geq 0$, and for the function (A1), at least three of the parameters $k_2,\ 1/k_3,\ k_4,\ k_6$ must equal zero, whereas for (A2), at least three of the parameters $1/k_1,\ 1/k_2,\ k_3,\ 1/k_5$ must equal zero. These two structural immittances (A1, A2) can realise all the 8 network layouts shown in Table 2 in [20], for example, when k_2, k_4 and k_6 of (A1) equal 0, an absorber layout with an inerter, a damper and a spring connected in series can be obtained from $Y_1(s)$. For a given vibration suppression problem, these two structural immittances can then be used for optimisation, to allow the optimum absober configuration with one inerter, one damper and one spring to be obtained.

Appendix B. Derivation of J_r

Consider road disturbances z_r. Following [7], its spectral density can be expressed as:

$$S^{z_r}(f) = \frac{\kappa}{V}|n|^{-2}$$

where f equals nV. Now consider the sprung mass acceleration $\ddot{z}_s$ which is related to $z_r(t)$ by the weighted transfer function $T_{z_r \to \ddot{z}_s}(s)H_{2631}(s)$ with the shape filter $H_{2631}(s)$ shown in (2). Then the expectation of $\ddot{z}_s$ is given by:

$$\begin{aligned} E[\ddot{z}_s^2] &= \int_{-\infty}^{\infty} |T_{z_r \to \ddot{z}_s}(j2\pi f)H_{2631}(j2\pi f))|^2 S^{z_r}(f)df \\ &= 2\pi\kappa V \int_{-\infty}^{\infty} |T_{\dot{z}_r \to \ddot{z}_s}(j\omega)H_{2631}(j\omega)|^2 d\omega \end{aligned} \tag{B1}$$

For the secondary ride (vibrations in the frequency range from 4Hz to 20Hz), based on (B1), the objective function capturing the ride comfort, J_r, can then be defined as (3).

Appendix C. Time domain verification and reality check on beneficial suspension configuration

Following [32], we chose a road velocity input $\dot{z}_r$, defined in (C1)

$$\dot{z}_r(t) = -0.111[Vz_r(t) + 40\sqrt{\kappa V}\tau(t)] \tag{C1}$$

to capture a random road with roughness coefficient as $\kappa = 5 \times 10^{-7}\text{m}^3/\text{cycle}$, same as that used in previous optimisations. Here V is the vehicle velocity, taken as 25 m/s and $\tau(t)$ is the Gaussian white noise with mean value zero.

Table C1.: Average and maximum value of a_{m_s}, $z_u - z_r$ and $z_s - z_u$, subjected to a random road input (C1).

Configurations	Average			Maximum		
	a_{m_s} m/s^2	$z_u - z_r$ mm	$z_s - z_u$ mm	a_{m_s} m/s^2	$z_u - z_r$ mm	$z_s - z_u$ mm
Default$_{\text{opt}}$	0.20	0.65	1.9	0.85	2.8	6.4
S7$_k$	0.18	0.64	2.0	0.78	2.8	6.5

When subjecting the quarter-car model with S7$_k$ and the default device to this road input, the time-domain response of sprung mass acceleration, the tyre displacement and the suspension movement can be obtained, of which the rms and maximum values are then calculated and summarised in Table C1. Note that the default device adopted here has been tuned to its optimum for comparsion. From it, we can see that both rms and maximum sprung mass acceleration values for the S7$_k$ are approximately 10% smaller than the default values. This reflects the performance improvement of the S7$_k$ configuration over that of the default as obtained in previous analysis. From Table C1, it can also be seen that for the configuration S7$_k$, the rms value of tyre displacement is

0.64 mm and the maximum suspension travel is 6.5 mm. Considering the default structure, their values can be obtained as 0.65 mm and 6.4 mm, respectively. From these, we can obtain that the proposed beneficial configuration $S7_k$ have similar tyre load and suspension travel performance with the default one, suggesting the effectiveness of implementing the constraints for frequency-domain optimisation.

The reality check on this beneficial configuration $S7_k$ is then carried out, to make sure that no unexpected behaviour occurs, including the power dissipation check and the cobblestone road input analysis. The dissipated power of the proposed configuration is firstly calculated, for which the power flow method proposed in [33] is adopted with a brief introduction provided here. The power flow is defined as $P = f(t) \cdot v(t)$ with $f(t)$ being the force loaded at a structure point and $v(t)$ as the velocity response of a point under the load $f(t)$. In frequency domain, a time average vibration power defined as [33]:

$$\bar{P} = \frac{1}{2}\mathrm{Re}(F \cdot V^*) \tag{C2}$$

is used, where F, V represent the frequency response of the loaded force and the relative velocity, and the subscript $*$ denotes the complex conjugate. This equation is obtained based on an assumption that the dynamic system is forced by a harmonic excitation and the averaging time is a cycle of the periodic response. Using the equation (C2), the input power $\bar{P}_{in}$ of the quarter car model is defined as

$$\bar{P}_{in} = \frac{V_r}{2}\mathrm{Re}((k_t + c_t s)(Z_u - \frac{V_r}{s})) \tag{C3}$$

where $V_r = \sqrt{2\pi V \kappa}$ denotes the constant road velocity input $\dot{z}_r$ in frequency domain and Z_u is the displacement of the unsprung mass m_u. The power dissipated by the tyre damping, $\bar{P}_{ct}$ can be expressed as:

$$\bar{P}_{ct} = \frac{c_t}{2}\mathrm{Re}([(Z_u - \frac{V_r}{s})s] \cdot [(Z_u - \frac{V_r}{s})s]^*) \tag{C4}$$

The power dissipated by the suspension devices, $\bar{P}_{sus}$ can also be calculated based on (C2), which also depends on their topological connection. For quantitatively representing the power flow distribution, the average and the maximum value of the power flow is calculated and summarised in Table C2(a). It can be seen that the average input power of the $S7_k$ and the conventional suspension strut is as the same as each other and for the maximum input power, the configuration $S7_k$ results in a smaller value. Also we notice that for the configuration $S7_k$, the tyre dissipated power $\bar{P}_{ct}$ is smaller, and the suspension dissipated power $\bar{P}_{sus}$ is larger than that of the default suspension. This can be regarded as beneficial since less tyre dissipated power means lower rolling resistance and higher fuel efficiency, as reported in [34].

Subsequently, a cobblestone road, one of the extreme surface irregularities encountered is also considered. This is to ensure that while the beneficial suspension configurations are optimised for good road class profiles (i.e. $\kappa = 5 \times 10^{-7}$ m^3/cycle), they will not result in inferior performance when subjected to poorer road surfaces. Consider the cobblestone road input [35], taking the expression of (C1), with the road roughness value selected as $\kappa = 1.7 \times 10^{-5}$m^3/cycle and the vehicle velocity V being 25 m/s. Subjecting the quarter car model to this road surface, Table C2(b) summarises the average and maximum values of the considered performance measures. It can be noted that the quarter-car model with configuration $S7_k$ will not experience any unexpected behaviour when subjected to a cobblestone road.

Table C2.: Reality check for the proposed optimal configuration $S7_k$ and the default device.

(a): The power flow

Configurations	Average			Maximum		
	$\bar{P}_{in}$	$\bar{P}_{ct}$	$\bar{P}_{sus}$	$\bar{P}_{in}$	$\bar{P}_{ct}$	$\bar{P}_{sus}$
$\text{Default}_{\text{opt}}$	0.072	0.020	0.052	0.311	0.060	0.308
$S7_k$	0.072	0.019	0.053	0.300	0.044	0.296

(b): The cobblestone road input

Configurations	Average			Maximum		
	a_{m_s} m/s^2	$z_u - z_r$ mm	$z_s - z_u$ mm	a_{m_s} m/s^2	$z_u - z_r$ mm	$z_s - z_u$ mm
$\text{Default}_{\text{opt}}$	1.20	3.8	11.2	4.93	16.2	37.1
$S7_k$	1.10	3.8	11.4	4.59	16.4	37.4