# ESSAYS ON HOUSING, CONSUMPTION, AND ASSET PRICES

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# Abstract

This thesis is a collection of three essays that analyze the interplay between financial and mortgage markets, and household consumption.

In Chapter 1, we study the spillovers from government intervention in the mortgage market on households' consumption. After an expansionary mortgage market operation, the consumption response of homeowners with mortgage debt is large and significant, while the consumption response of homeowners without the mortgage debt is small and insignificant. Non-homeowners also increase their consumption but less than mortgagors. We also find that expansionary policy significantly increases consumption inequality of mortgagors. We explain these facts through the lens of a life-cycle model with incomplete markets and endogenous housing choice. Reduction in credit rates creates extra wealth for the mortgagors while the reduction in interest rates shifts this wealth towards consumption. Increase in wealth is bigger for those with a larger mortgage – this exacerbates consumption inequality.

In Chapter 2, we study the role of durable consumption in the context of long-run risk models. These models became a cornerstone in the macro-finance literature for their ability to capture key asset price phenomena. They are, however, known to entail implausibly high levels of timing and risk premia. In this chapter, we resolve this puzzle by considering the consumption of durable goods in addition to that of non-durable goods. In our estimated model, the timing premium is 11 percent and the risk premium is 16 percent of lifetime consumption. These values are about a third of the previously implied premia and are more consistent with empirical and experimental evidence.

In Chapter 3, using the Michigan Survey of Consumers, we provide evidence that a rise in consumers' beliefs about current and future aggregate durable expenditure predicts a rise in expected returns. We rationalize this finding through a consumption-based asset pricing model with recursive preferences over non-durable and durable goods and uncertainty about the underlying endowments. The model generates high equity premium, low and stable risk-free rate, and explains up to 60% of the volatility of equity premium, with calibrated parameters that are consistent with the macroeconomic literature (risk aversion of 2.1 and elasticity of intertemporal substitution of 1.09).

# Declaration

No portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

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# Dedication

*To Severyn and Daniela.*

# Chapter 1

# Introduction

This thesis is a collection of three essays that analyze the interplay between financial and mortgage markets, and household consumption. The first chapter of this thesis focuses on distributional consequences of economic policies and mortgage market interventions and the role of households' financial position in the propagation of such policies. The second and third chapters of this thesis explore the link between households' consumption and financial markets and how households' beliefs about future consumption growth affect the financial markets' returns.

**Spillovers from Mortgage Markets to Private Consumption.** By now it is well documented that activity in secondary mortgage markets boosts mortgage lending, lowers mortgage rates and influences prices on other financial markets (see, for example, Fieldhouse, Mertens and Ravn, 2018). Little is known, however, how this activity affects the largest component of GDP, household consumption. In the first chapter of this dissertation, "Heterogeneous Spillovers of Housing Credit Policy", we show that households' financial position is crucial in understanding the spillovers from the activity in secondary mortgage markets to private consumption. In the paper, we proxy households' financial position through housing tenure status. First, we show empirically, that following an expansionary policy change to the secondary mortgage markets, homeowners with mortgage debt increase their spending substantially, while homeowners without the mortgage debt do not react to policy change. Non-homeowners also increase their consumption but less than mortgagors. We also show that the same expansionary policy significantly increases consumption inequality of mortgagors. Second, in order to explain this empirical evidence, we present a life-cycle model with incomplete markets in the vein of Huggett (1996), which we extend to include endogenous housing choice. In our policy experiment, we reproduce the aggregate effects of mortgage market policy change and change both interest and mortgage rates as well as the spread between the two. Lower mortgage rates imply lower mortgage payments for the mortgagors and hence a rise in long-term permanent income for this group. Lower interest rates imply that part of this extra income goes to consumption rather than saving. In the model, wealth is a function of house size and thus the mortgage size. Lower mortgage payments generate a higher increase in wealth that in turn increases inequality among the mortgagors. This chapter is a vital contribution to the literature as evidence is scant why households' portfolio composition matters for the strength of spillovers from mortgage markets' activity to private consumption and what exactly is the propagation mechanism.

**Durable Consumption and Financial Markets.** The Long-Run Risk Model, first introduced by Bansal and Yaron (2004), is considered as one of the main theoretical pillars in financial macroeconomics. In its original version, the long-run risk model reconciled several key asset pricing phenomena in a unified framework by combining recursive preferences a la Epstein and Zin with a model of aggregate consumption growth that exhibits predictable low-frequency movements and time-varying volatility. However, despite its success, the long-run risk model suffers from a quantitative drawback similar to Mehra and Prescott's (1985) equity premium puzzle. When calibrated to financial and macroeconomic data, the long-run risk model implies unrealistically high levels of timing and risk premia, see Epstein, Farhi and Strzalecki (2014). A representative household with recursive preferences, a relative risk aversion of 7.5, and an elasticity of intertemporal substitution of 1.5 would give up around one-quarter of her lifetime consumption to resolve uncertainty one month earlier, and around half of her lifetime consumption to live in a world without consumption risk. Both percentage amounts seem implausibly high. To tackle this problem with the long-run risk model in the second chapter of this dissertation "The Resolution of Long-Run Risk" we introduce in an otherwise standard long-run risk model, durable consumption alongside consumption of non-durable goods. The main message of our study is that this simple modification can reduce by around a factor of three the timing and risk premia, without compromising (and possibly improving) the model's ability to match standard macroeconomic and financial moments. In our benchmark estimation exercise, our long-run risk model can rationalize key asset pricing facts, and deliver a timing premium of 11 percent and a cost of eliminating all consumption uncertainty of 16 percent of lifetime consumption. These results are consistent with both empirical and experimental studies regarding consumption risk.

In the third chapter if this dissertation we further explore the link between households' consumption of durable goods and financial markets. In "Consumer Sentiment, Durable Consumption, and Stock Returns", we provide novel empirical evidence that consumers' beliefs about aggregate durable expenditure predict future movements in financial markets. Using the Survey of Consumers from the University of Michigan we show that the aforementioned beliefs predict future excess returns in both short and long horizons as well as the future price-dividend ratio. This chapter introduces in an otherwise of classic consumption-based asset pricing model with recursive preferences of Epstein and Zin, consumption of durable goods, aggregate uncertainty about consumption growth and belief formation through Bayesian learning. These beliefs drive the price-dividend ratio and future expected returns through the intertemporal marginal rate of substitution. In order to discipline the asset-pricing model, we estimate the structural parameters of the model to match the levels and volatility of the equity premium and the risk-free rate. The risk aversion coefficient and elasticity of intertemporal substitution required to match key financial variables are much lower than previously suggested and are consistent with the real business cycle literature. We are, therefore, able to rationalize our empirical finding without compromising the model's ability to match standard financial moments.

# Chapter 2

# Heterogeneous Spillovers of Housing Credit Policy

## 2.1 Introduction

Activity in secondary mortgage markets boosts mortgage lending, lowers mortgage rates and influences prices on other financial markets (Fieldhouse, Mertens and Ravn, 2018). In this chapter we study how this activity affects the largest component of GDP, household consumption. We show that households' financial position is crucial in understanding the spillovers from the activity in secondary mortgage markets to private consumption. We proxy households' financial position through housing tenure status. First, we show empirically, that following an expansionary policy change to the secondary mortgage markets, homeowners with mortgage debt increase their spending substantially, while homeowners without the mortgage debt do not react to policy change. Non-homeowners also increase their consumption but less than mortgagors. We also show that the same expansionary policy significantly increases consumption inequality of mortgagors. Second, in order to explain this empirical evidence, we present a life-cycle model with incomplete markets in the vein of Huggett (1996), which we extend to include endogenous housing choice. In our policy experiment, we change both interest and mortgage rates as well as the spread between the two. Lower mortgage rates imply lower mortgage payments for the mortgagors and hence a rise in long-term permanent income for this group. Lower interest rates imply that part of this extra income goes to consumption rather than saving. In the model, the wealth is a function of house size and thus the mortgage size. Lower mortgage payments generate a higher increase in wealth that in turn increases inequality among the mortgagors.

In our empirical exercise, we explore the link between expansionary credit policy changes and an increase in households' expenditure. In particular, we focus on credit policy changes through exogenous governmental intervention in the mortgage markets via various federal housing agencies, and mortgage assets purchases of these agencies. For the most part, credit policy changes are a reaction to business cycle conditions (the most recent QE3 being the prime example). In order to analyze the response of consumption to any of these policy changes, it is, therefore, important to isolate the non-cyclically motivated policy changes that are free of any confounding influences of the business cycle (such as long-term objectives of increasing the homeownership). We combine the exogenous non-cyclically motivated events from Fieldhouse and Mertens (2017) with mortgage purchases of two largest federal housing agencies (Fannie Mae and Freddie Mac). We then use the former as an instrument in regressions of households' consumption on measures of agency purchase activity. We measure consumption using household-level data from the Consumer Expenditure Survey and the Survey of Consumer Finances. If credit market interventions were neutral (Greenspan, 2005; Lehnert, Passmore and Sherlund, 2008; Meltzer, 1974) an increase in agency purchases should have little impact on private consumption. Instead, we find that expansionary credit policy leads to an increase in private consumption of mortgagors and an increase in consumption inequality for this group.

In our theoretical exercise, we use a structural model to identify the transmission mechanism we found in our reduced-form analysis. We model the credit policy change experiment by replicating the aggregate macroeconomic effect of mortgage market interventions documented in Fieldhouse, Mertens and Ravn (2018). In particular, we focus on change in both interest and mortgage rates as well as on change in the spread between the two. Our first finding is that lower mortgage rates imply lower mortgage payments for the mortgagors and a rise in long-term permanent income for this group. Since the opportunity cost of saving goes down when the interest rates drop - mortgagors consume this extra income instead of saving. The results we find are in line with Cloyne, Ferreira and Surico (Forthcoming), who argue that the behavior of mortgagors resembles that of wealthy hand-to-mouth households and empirically document a similar response of individual consumption to expansionary monetary policy shock. Indeed, in the model, mortgagors hold little liquid wealth, outstanding mortgage debt and illiquid asset in the form of the house. We then analyze the response of other types of households: renters and outright homeowners. Similarly to mortgagors, renters' utility from consumption outweighs that of saving, and they consume more once the new credit policy is at hand. For outright homeowners, who are mostly older than renters and mortgagors, bequest motive outweighs that of dissaving one, and they barely increase consumption and save instead. Using the same policy experiment, we also reproduce the increase in consumption inequality. In the model, net wealth depends on assets and on mortgage outstanding (that is zero for both renters and outright homeowners). When the mortgage payments go down, the overall mortgage balance decreases and thus we observe the increase in wealth. This increase is larger for the households with a bigger mortgage (and therefore bigger house), generating a heterogeneous response of consumption increase within the mortgagors' group.

**Related Literature.** In exploring the link between exogenous credit policy changes and individual consumption our paper adds to both empirical and theoretical literature on housing and mortgage markets. From the empirical side, we relate to four strands of literature. Firstly, we analyze the US federal government interventions into the mortgage markets. For the most part the literature focused on governments' intervention in terms of tax policies. Recent studies include Chambers, Garriga and Schlagenhauf (2009); Floetotto, Kirker and Stroebel (2016); Hilber and Turner (2014); Sommer and Sullivan (2018), among others. Fieldhouse, Mertens and Ravn (2018) is the most recent study that instead analyzes the interventions to the federal housing agencies, rather than any tax policies. In this paper, we use exogenously identified policy interventions from Fieldhouse, Mertens and Ravn (2018); unlike Fieldhouse, Mertens and Ravn (2018), however, we analyze the transmission mechanisms through which the policy operates using the US household survey data.

Secondly, this paper is related to literature that analyzes the interaction between federal housing agencies and other markets. The most recent studies include Gonzalez-Rivera (2001); Hancock and Passmore (2014, 2011); Lehnert, Passmore and Sherlund (2008); Naranjo and Toevs (2002) as well as Fieldhouse, Mertens and Ravn (2018). We focus specifically on the effect of mortgage purchases of governmental housing agencies on consumption of different types of households using a novel identification strategy.

Thirdly, our paper is related to the literature on the role of household balance sheet channels in the transmission of monetary and fiscal policy shocks. These include Auclert (2017); Bilbiie (2017); Cloyne, Ferreira and Surico (Forthcoming); Eggertsson and Krugman (2012); Greenwald (2018); Hedlund et al. (2016); Iacoviello (2005); Kaplan, Moll and Violante (2018); Luetticke (2018); Motta and Tirelli (2010), to name a few. Coibion et al. (2017) also uses US household level data to study the effect of conventional monetary policy on income and consumption inequality. Like in Cloyne, Ferreira and Surico (Forthcoming), we use the households' housing tenure status to proxy their asset and debt position.

Finally, this paper is related to literature that analyzes the effects of monetary policy shocks on inequality. Coibion et al. (2017) uses US household level data to study the effect of conventional monetary policy on income and consumption inequality. We follow Coibion et al. (2017) methodology to construct the measure of expenditure inequality between all types of households as well as within each housing tenure group. Unlike Coibion et al. (2017) we focus on the effect of credit policy shocks on expenditure inequality.

From the theoretical side, our model resembles the recent literature that extends Huggett (1996) model to incorporate housing decision and aggregate housing and mortgage markets. To name a few, we build on the models of Favilukis, Ludvigson and Van Nieuwerburgh (2017); Kaplan, Mitman and Violante (2018); Sommer and Sullivan (2018), that analyze heterogeneous agents life-cycle economies with uninsurable income risk in which households make a housing and mortgage choice. Unlike these papers, however, we do not focus on the aggregate implications of different macroeconomic shocks but rather analyze the individual households' behavior.

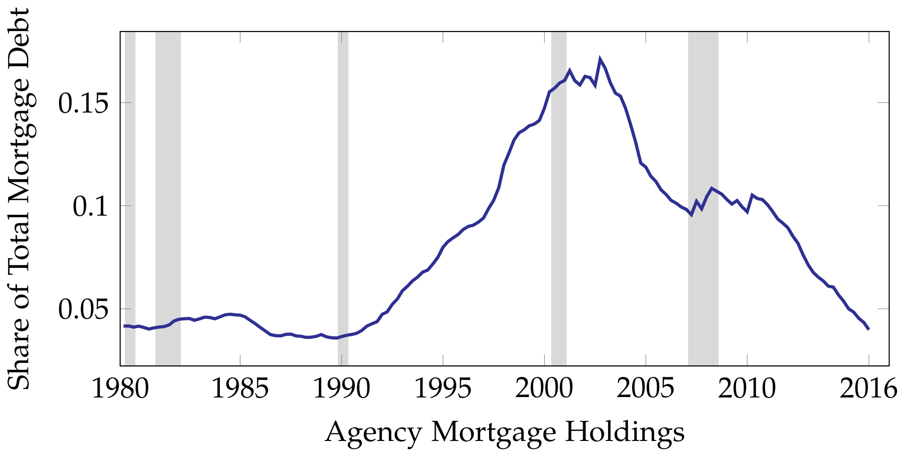
## 2.2 Empirical Framework

### 2.2.1 Institutional Background and Identification of Exogenous Policy Changes

US mortgage market is the largest capital market in the world and is the dominant source of credit for American households. It finances key component of household wealth and aggregate spending - housing. By the 3rd quarter of 2017, the total mortgage debt in the US was about $8.7 trillion. In comparison, auto, credit card and student debt combined was about $2.3 trillion.

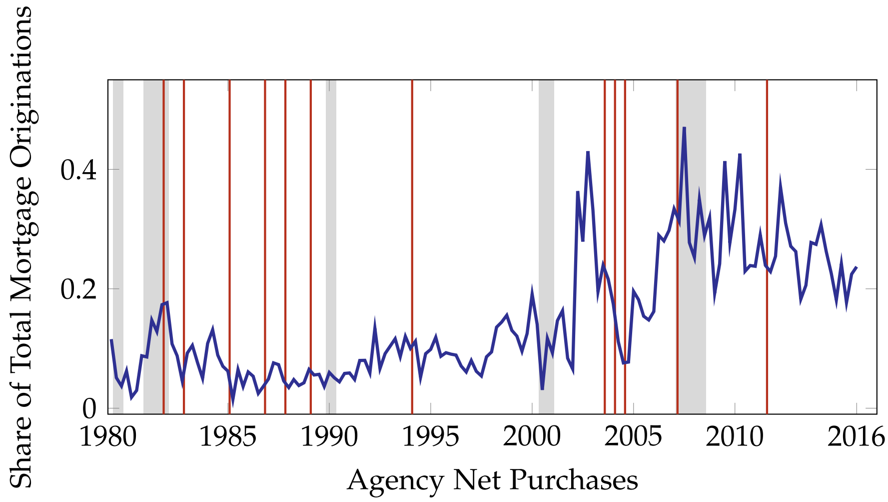
The US mortgage market is also quite unique. The US federal government is heavily involved in the mortgage market (especially in terms of residential mortgage purchases) though various agencies: Government-Sponsored Enterprises (GSEs) and Government Agencies. We focus on the involvement of the government through the GSEs. In particular, we focus on two largest GSEs: Fannie Mae, funded in 1938 and publicly traded since 1968, and Freddie Mac, funded in 1970. GSEs were chartered by Congress to support secondary mortgage markets and are subject to favorable tax and regulatory treatment. These agencies acquire mortgages through advance commitments to buy loans from mortgage lenders which are delivered once the loans are originated in the primary market; they are not allowed to do any direct lending. Over time, the agencies played and increasingly active role in the residential mortgage markets. As Figure 2.2.1 indicates, in 2004 Fannie Mae and Freddie Mac held almost 20% of all mortgage debt.

FIGURE 2.2.1: **Agency mortgage holdings.** Agency mortgage holdings as a percent of total mortgage originations. Data is between 1980 and 2016. Grey areas represent NBER recessions.




In the empirical section of this chapter we focus on the portfolio purchases of the housing agencies, shown in solid blue line in Figure 2.2.2, and how it affects expenditure of households with different debt position. Unfortunately, simply correlating measures of agency activity with households' expenditure ignores potential endogeneity problems. On one hand, Fannie Mae and Freddie Mac respond to market conditions, and thus act pro-cyclically. On the other hand, Fannie Mae and Freddie Mac have a public mission to provide stability on the mortgage markets, and thus act counter-cyclically. Ignoring these potential problems makes the causal inference invalid.

FIGURE 2.2.2: **FNMA & FHLMC net purchase for portfolio investment.** Data is between 1980 and 2016. Grey areas represent NBER recessions.




To account for the endogeneity in agency market activity we adopt narrative identification approach and use major regulatory policy events as an instrument for agency purchase activity. Fieldhouse and Mertens (2017) document significant policy changes that are expected to affect agency portfolios and isolate those events (which they call non-cyclical events) that are free of confounding influences in the spirit of Romer and Romer (2004) and Ramey (2011). These policy changes are indicated by vertical red lines in Figure 2.2.2. We quantify these changes as a percentage of the average annualized level of originations in the preceding year. As most of the policy interventions after 2006 were related to 2007/2008 financial crisis and were mostly cyclically motivated, we limit the analysis to pre-crisis sample.

### 2.2.2 Impulse Response Specification

To evaluate the effect of agency purchase activity on households' income and consumption we conduct an impulse response analysis of shock to agency mortgage purchase. We use a local projections instrumental variable approach where we use the narrative instrument identified in the previous section for identification.

We start with assessing whether the narrative policy changes do lead to significant changes in net agency purchases. Our first-stage regression specification is of the form

$$\frac{\sum_{j=0}^{h} p_{t+j}}{X_t} = \tilde{a}_h + \tilde{c}_h \frac{\tilde{m}_t}{X_t} + \tilde{d}_h(L) Z_{t-1} + \tilde{u}_{t+h}, \tag{2.2.1}$$

where $p_t$ is the agency's net purchase, $X_t$ trend in real mortgage originations, $\tilde{m}_t$ is non-cyclically motivated narrative measure in real dollars, and $Z_t$ is a set of controls (defined below). $\tilde{d}_h(L)$ denotes the polynomial of order 4. We pick the value of horizon $h$ for which out instrument is the strongest. For that, we run regression (2.2.1) for horizons $h = 1$ (one quarter) to $h = 20$ (five years) and pick $h$ that maximizes the robust F-statistics on the excluded instrument for each $h$. The results indicate that the narrative measure is a strong instrument for agency purchasing activity for horizons between 1 and 3 quarters after the policy events, with robust F-test statistics exceeding 10. The F-statistics are low for longer horizons. Given these results we restrict the analysis to horizons between 1 and 3 quarters. Specifically, we focus on the agency purchase activity 2 quarters after the shock, as the robust F-statistic is the highest and equal to 15. Figure 2.B.1 in Appendix 2.B shows the robust F-statistics on the excluded instrument in each of the first-stage regressions (2.2.1) for horizons $h = 1$ (one quarter) to $h = 20$ (five years).

We now proceed to identifying the effect of agency purchase activity on variable of interest. Our goal is to identify the response to shocks to expectations of future agency purchasing activity. For a given outcome variable $y_t$, we estimate the response at horizon $h$ using

$$\frac{y_{t+h} - y_{t-1}}{y_{t-1}} = a_h + b_h \left( \frac{4}{2} \times \frac{\sum_{j=0}^{2} p_{t+j}}{X_t} \right) + d_h(L) Z_{t-1} + u_{t+h}, \tag{2.2.2}$$

where

$$\frac{4}{2} \times \frac{\sum_{j=0}^{2} p_{t+j}}{X_t} \tag{2.2.3}$$

denotes annualized agency commitments made over a 2 quarter period expressed as a ratio of long-run trend in annualized originations $X_t$; we choose an 2 quarter horizon to measure expected future purchases because at this horizon the robust F-statistic associated with the narrative instrument in the first-stage regression is the largest.

The regression in (2.2.2) estimates the quarter $h \geq 0$ response to a time 0 news shock to agency purchases. Expected agency purchases are proxied by agency net purchases made over the next half a year. To address endogeneity, we use the indicator of non-cyclical policy events, deflated by the core PCE price index and scaled by trend originations $X_t$ , as the instrument. The IV estimates of $b_h$ in (2.2.2) can be interpreted as the response associated with a percent increase in the agency net purchase that becomes anticipated $h$ periods before.

The control variables $Z_t$ include the lagged growth rates of the core PCE price index, a nominal house price index, and total mortgage debt, the log level of real mortgage originations, housing starts, and lags of several interest rate variables: the 3-month T-bill rate, the 10-year Treasury rate, the conventional mortgage interest rate, and the BAA-AAA corporate bond spread. They also include lags of agency net purchases and commitments as a ratio of $X_t$ as well as the unemployment rate and the growth rate of real personal income. See Appendix 2.A for a detailed description of the data sources and definitions.

### 2.2.3 Measuring Expenditure Data

We use households' expenditure on non-durable goods and services as a response variable $y_t$ in equation (2.2.2). To construct our measure of expenditure we use the interview section of the Consumer Expenditure Survey (CEX) between 1980 and 2007.[1] We define *non-durable goods and services* as food, alcohol, tobacco, fuel, light and power, clothing and footwear, personal goods and services, fares, leisure services, household services, non-durable household goods, motoring expenditure and leisure goods. We adjust the food at home between 1982 and 1987 following Aguiar and Bils (2015). We also define households' income as a amount of income before tax in the past 12 months. After 2005, BLS started imputing missing income observations. Before 2004 we impute missing income observations as in Coibion et al. (2017). We exclude households that are in either top 1% or bottom 1% of either the non-durable expenditure or income level. We also exclude the households who report zero food expenditure. Finally, we exclude households who's household head is below 25 and over 74 years old. We also keep the households that do not change the housing tenure status between the interviews.

### 2.2.4 The Effect of Agency Purchases on Expenditure: Pseudo-Cohort Analysis

In this section we document the response of households' expenditure to news shock to agency purchases, proxied by agency net purchases made over next half a year.

As documented by Fieldhouse, Mertens and Ravn (2018), an increase in mortgage purchases by the agencies boosts mortgage lending and lowers mortgage rates. It is, therefore, important to distinguish between those households who own the house

[1] Data between 1980 and 1995 is obtained from ICPSR through UK Data Service. Post-1995 data is publicly available at the Bureau of Labor Statistics (BLS) website.

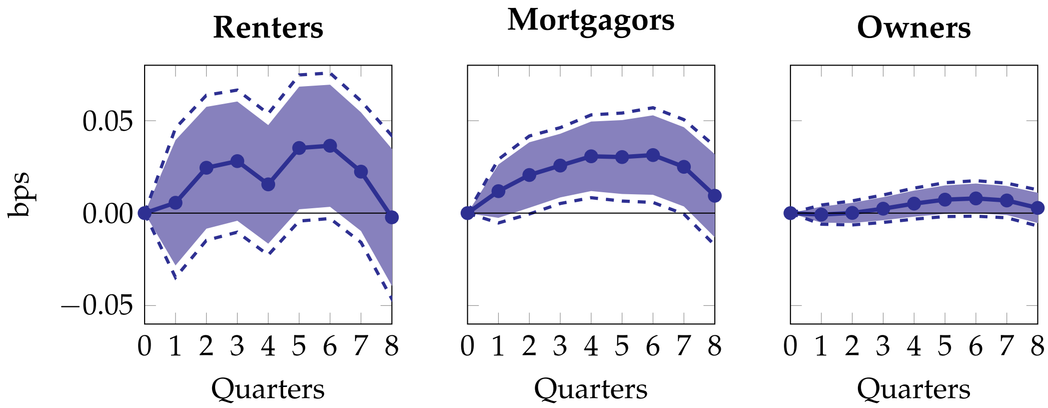
with a mortgage and those without. Agency purchases also influence house prices and expand homeownership, therefore the effect on those households who own the house and those who do not might be different. The CEX survey, on top of containing rich income and expenditure data, contains information on housing tenure status. We utilize this information and group the households into three categories based on their tenure status in the spirit of Cloyne, Ferreira and Surico (Forthcoming). The categories are *renters*, *mortgagors* and *outright owners*. Unfortunately, given the rotating panel nature of the CEX survey it is not possible to follow individual households for more than four quarters over which they are observed. We, therefore, employ a grouping estimator to aggregate individual observations into pseudo-cohorts by housing tenure as in Browning, Deaton and Irish (1985).

We then look at the response of households' expenditure, based on their housing tenure status,to a 1% increase in net purchase by the agencies, anticipated 2 quarters in advance, under the specification in (2.2.2) using non-cyclically motivated narrative measure as an instrument. Figure 2.2.3 plots the coefficients $b_h$ from equation (2.2.2) over the horizon $h = 1$ (one quarter) to $h = 8$ (two years) along with 90% and 95% confidence intervals. We see from the figure, that after a news shock to agency net purchases the only group that significantly increases their expenditure are the mortgagors, for horizon between three and seven quarters, while the change in expenditure for renters and owners is insignificant for all horizons. Moreover, a year after the shock we document a clear ranking of the responses: mortgagors react the most (about 0.03 basis points), followed by renters (about 0.015 basis points), and finally homeowners (close to zero).

### 2.2.5 Response of Expenditure Inequality

In Section 2.2 we documented the evidence that following a news shock to agency purchase activities there is a heterogeneous response between housing tenure groups.

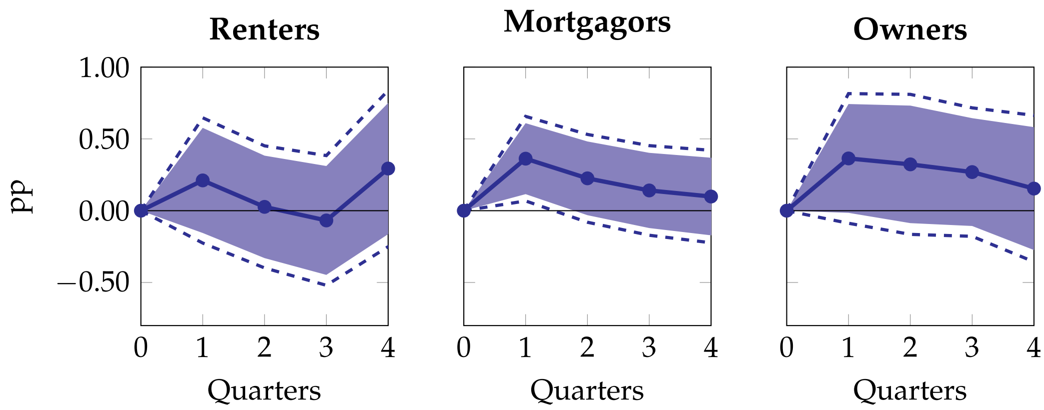
FIGURE 2.2.3: **Impulse response of expenditure.** Impulse response of expenditure to an additional to a 1% increase in net purchase by FNMA & FHLMC, anticipated 2 quarters before. Blue areas and broken lines represent 90% and 95% confidence intervals, respectively.




We now look at what happens with expenditure within each of the groups. For that we construct Gini coefficient of level of expenditure on non-durable goods and services in the spirit of Coibion et al. (2017). Our measure of inequality is raw, not controlling for any household characteristics like the number of household members, age, education, etc. The only control characteristic that we take is the housing tenure status.

Figure 2.2.4 plots the response of Gini coefficient (measured between 0 and 100) to a 1% increase in net purchase by agencies, anticipated 2 quarters before. Top left panel plots the response of Gini coefficient to a news shock for all the households in the data. We can see the positive and significant increase (at 90% significance level) in expenditure inequality one quarter after the shock by about quarter of percentage point. Expenditure inequality within renters group (top right panel) does not respond significantly. We can neither see a significant increase in expenditure inequality within the home-owners group (bottom right panel). With regards to expenditure inequality within the mortgagor group (bottom left panel), there is a positive and significant (both at 90% and 95% significance level) increase of inequality by almost half percentage point. This suggests that overall increase in expenditure inequality is mostly driven by increase within the mortgagors. In the next section we will analyze what characteristics of households (depending on their income level and their housing tenure status) and of mortgagors in particular (depending on the length of their mortgage) drives the heterogeneous response of expenditure and expenditure inequality between the three groups of households.

FIGURE 2.2.4: **Impulse response of expenditure.** Impulse response of expenditure Gini to a 1% increase in net purchase by FNMA & FHLMC, anticipated 2 quarters before. Blue areas and broken lines represent 90% and 95% confidence intervals, respectively.




## 2.3 A Life-Cycle Model with Housing Markets

In the previous section we documented the causal effect of news shock to agency mortgage purchases. Unfortunately, with the data available it is not possible to exactly identify the transmission mechanism through which the effects work. To understand which channel is exactly responsible for increase in expenditure for mortgagors only, and increase in the inequality for that group, in this section we develop a Huggett (1996) type of heterogeneous agent life-cycle model with uninsurable risk, endogenous housing choice and aggregate mortgage market shocks. Through the lens of this model we explain the empirical evidence we found in the previous sections and analyze which channels contribute to the results indicated.

### 2.3.1 Demographics, Preferences and Labor Income

**Demographics** Time is discrete and economy is populated with continuum of finitely-lived households. Age is indexed by $j = 1, \ldots, J$. Households work for first $J_r - 1$ periods and are retired until period $J$. Life span is certain and all households die after age $J$.

**Preferences** Expected lifetime utility of the households is given by

$$\mathbb{E}_0 \left[ \sum_{j=1}^{J} \beta^{j-1} u(c_j, s_j) + \beta^J v(a_J) \right],$$

where $c_j$ denotes the consumption of non-durable goods at age $j$ and $s_j$ denotes the consumption of housing services at time $j$, $\beta$ is the discount factor and $a_J$ is the bequest. The only source of uncertainty in the economy is the idiosyncratic income shock (described below). We assume that utility function $u$ takes the following functional form

$$u(c, s) = \frac{\left[(1-\phi)c^{1-\gamma} + \phi s^{1-\gamma}\right]^{\frac{1-\vartheta}{1-\gamma}} - 1}{1-\vartheta}$$

while the bequest function $v$ is given as

$$v(a) = \psi \frac{(a + \underline{a})^{1-\vartheta} - 1}{1-\vartheta},$$

where $\phi$ denotes taste for housing, $1/\gamma$ measures the elasticity of substitution between non-durable consumption and housing services, $1/\vartheta$ is the IES, $\psi$ measures the strength of bequest motive while $\underline{a}$ measures how luxurious is the bequest.

**Labor Income** Working-age households receive exogenous income $y_j$ given by

$$y_j = \Theta \chi_j exp(\epsilon_j),$$

where $\Theta$ is the aggregate labor productivity, $\chi_j$ is the deterministic age profile and $\epsilon_j$ is the idiosyncratic component that follows first-order Markov process. Government runs a pay as you go social security system. After retirement, households receive social security benefits

$$y_j = \rho_{ss} y_{J_r}, \quad j > J_r,$$

where $\rho_{ss}$ is a replacement rate and $y_{J_r}$ are their earnings in the last working period. Finally, let $\Upsilon_j$ denote the age-dependent transition of earning from age $j$ to age $j+1$ conditional on income $y_j$.

### 2.3.2 Housing

Households can either rent or own the house. Houses are characterized by their size, which is given by a discrete set. Let $\widetilde{\mathcal{H}}$ denote the set of houses available for rent, while $\mathcal{H}$ denotes the set of owner-occupied houses. We assume that the per-unit price of house is equal to $p_h$ while the rental price of housing unit is denoted by $\rho_h$.

To distinguish house owners from house renters, we assume that housing generates service flow equal to the size of the house, i.e. $s = h$, where $h \in \widetilde{\mathcal{H}}$, while owning a house generates an extra utility for the household, such that $s = \omega h$, where $\omega > 1$ and $h \in \mathcal{H}$.

Owner-occupied housing carries a per-period maintenance cost $\delta_h p_h h$ that fully offsets physical depreciation of the house, and tax cost $\tau_h p_h h$. There is a transaction cost equal to $\kappa_h p_h h$ associated with buying or selling the house. Changing the size of the rented house does not incur any transaction costs.

### 2.3.3 Assets, Mortgages and Market Arrangements

**Liquid Assets** Households can save in one-period bonds, $a$, with a exogenous interest rate given by $r_a$. Non-homeowners are not allowed any unsecured borrowing and their borrowing constraint is given by

$$a \geq 0$$

Homeowners, on the other hand, have access to home equity line of credit (HELOC), that we model as a one-period non-defaultable bonds. They can borrow up to a fraction $\lambda_a$ of the value of the house at the interest rate equal to $r_a$ and their borrowing constraint is given by

$$a \geq -\lambda_a p_h h$$

In the baseline version of the model we set $\lambda_a = 0$, so that no borrowing is allowed for any type of households. Let $q_a$ denote the price of bond, such that $q_a = 1/(r_a + 1)$

**Mortgages** House purchase can be financed by a mortgage. A household that takes out a new mortgage with principal balance $m'$ receives from a lender $q_m m'$ units of the numeraire good. The mortgage price $q_m$ is such that $q_m < 1$. In the benchmark setting of the model we assume that all mortgages are long-term, subject to interest rate $r_m$ and have to be repaid over the remaining life of the borrower. We assume that mortgage rate $r_m$ is given by

$$r_m = (1 + \iota) r_a,$$

where $\iota$ controls the spread between $r_a$ and $r_m$. Down-payment for a borrower who takes out a mortgage of size $m'$ to buy a house of size $h'$ is

$$p_h h' - q_m m'$$

Mortgage origination is also subject to a fixed origination cost $\kappa_m$. When taking out a mortgage, households have to satisfy two constraints. The first one is the maximum loan-to-value constraint: the initial mortgage size must be less than a fraction $\lambda_m$ of the value of the house being purchased

$$m' \leq \lambda_m p_h h'$$

The second constraint is the maximum payment-to-income constraint: the first minimum mortgage payment must be less than a fraction $\lambda_\pi$ of the income at time of purchase

$$\pi_j^{min}(m') \leq \lambda_\pi y_j, \tag{2.3.1}$$

where we define the minimum payment function $\pi_j^{min}(m')$ using a constant amortization formula

$$\pi_j^{min}(m') = \frac{r_m(1+r_m)^{J-j}}{(1+r_m)^{J-j}-1} m' \tag{2.3.2}$$

that assumes that the borrower is required to make $J-j$ payments $\pi$ that exceed minimum payment requirement after mortgage origination. The remaining mortgage principle evolves according to

$$m' = m(1+r_m) - \pi$$

We also assume that households are allowed to refinance the existing mortgage. When refinancing (taking out a new mortgage), households have to repay the existing mortgage balance, pay the fixed mortgage origination cost, and satisfy both loan-to-value and payment-to-income constraints. Households are also allowed to sell the house, given that they repay the remained of the mortgage as well as transaction costs. Finally, households can default on the mortgage, if they cannot satisfy the minimum payment requirement. Households that choose to default incur the utility cost of $\xi$ and are forced to rent the smallest available dwelling that period.

### 2.3.4 Government

In the model, government receives revenues from the property tax $\tau_h$ and progressive income tax $\mathcal{T}(y,m)$ that depends on income $y$ and mortgage holdings $m$. It is assumed that households can deduct the interest payed on mortgages against their taxable income. We assume that tax function $\mathcal{T}$ takes the form

$$\mathcal{T}(y,m) = \tau_y^0 \left(y - r_m \min\{m, \bar{m}\}\right)^{1-\tau_y^1} \tag{2.3.3}$$

where $\tau_y^0$ and $\tau_y^1$ measure the progressivity of the tax system and $\bar{m}$ denotes the maximum allowed deductible mortgage. On the spending side, the government finances social security system for the households. The government runs a balanced budget, with services $G$ (not valued by the household) adjusting to absorb any difference between government income and spending.

### 2.3.5 Dynamic Problem of the Household

We now describe the dynamic problem of the households. There are two types of households in the economy: homeowners and non-homeowners. Let $V_j^n$ denote the value function of non-homeowner at age $j$ and let $V_j^h$ denote the value function of the homeowner at age $j$. When non-homeowner enters the economy at age $j$ he has two choices - either remain non-homeowner in the next period (rent a house) or become a homeowner next period (buy a house). Let $V_j^r$ and $V_j^o$ denote the value function of renters and buyers, respectively. Non-homeowners essentially solve the following problem

$$V_j^n(\mathbf{x}_j^n) = \max\left\{V_j^r(\mathbf{x}_j^n), V_j^o(\mathbf{x}_j^n)\right\}$$

where $\mathbf{x}_j^h$ denotes the vector of state variables of the non-homeowner, described below. When home-owner enters the economy he has four different choices. He can either continue paying the existing mortgage (let $V_j^p$ denote the value function of the mortgage payer), repay the existing mortgage and get a new mortgage (let $V_j^f$ denote the value function of the mortgage refinancer), repay the remaining mortgage and sell the house (let $V_j^s$ denote the value function of the seller) or default on the mortgage payments (let $V_j^d$ denote the value function of mortgage payer who defaults). Every period, the homeowner solves the following problem

$$V_j^h(\mathbf{x}_h) = \max\left\{V_j^p(\mathbf{x}_j^h), V_j^f(\mathbf{x}_j^h), V_j^s(\mathbf{x}_j^h), V_j^d(\mathbf{x}_j^h)\right\}$$

where $\mathbf{x}_j^h$ denotes the vector of state variables of the homeowner, described below.

Non-homeowners of age $j$ enter the period with holding of liquid assets $a_j$ and exogenous income $y_j$. Homeowners of age $j$, on the other hand, also enter the period with outstanding balance on the mortgage $m$ and house $h$. When $m > 0$ we refer to homeowners as the mortgagor, whereas when $m =$ we refer to them as outright owners. Thus

$$\begin{aligned}\mathbf{x}_j^n &= \left(a_j, y_j\right)\\ \mathbf{x}_j^h &= \left(a_j, m_j, h_j, y_j\right)\end{aligned}$$

We now describe in detail the problem of each household in a recursive form. From here on the state and control variables with no subscript denote the current age/period variables, i.e. $a_j = a$, while state and control variables with $'$ superscript denote the next period/age variables, i.e. $a_{j+1} = a'$.

**Renters** The households of age $j$ that enter the period as non-homeowners and decide to rent next period, choose the level of consumption today ($c$), the level of liquid savings next period ($a'$) and the size of the rented dwelling for the next period ($\tilde{h}'$). In recursive form, their problem can be written as

$$V^r(\mathbf{x}^n) = \max_{c,b',\tilde{h}'} u(c,s) + \beta\mathbb{E}_\epsilon\left[V^{n'}\left(\mathbf{x}^{n'}\right)\right] \tag{2.3.4}$$

where the expectation is taken with respect to next period idiosyncratic income shock $\epsilon'$. Renters solve the above problem subject to the following constraints:

$$
\begin{aligned}
& c + \rho_h \tilde{h}' + q_a a' \leq a + y - T(y, 0) \\
& a' \geq 0 \\
& s = \tilde{h}', \quad \tilde{h}' \in \widetilde{\mathcal{H}} \\
& y' \sim Y(y)
\end{aligned}
$$

where the equations above are budget constraint, borrowing constraint, housing services production and income evolution, respectively. Let $\mathbb{1}^r(\mathbf{x}^n)$ denote the decision of non-homeowner with state variables $\mathbf{x}^n$ to rent a house.

**Buyers** The households of age $j$ that enter the period as non-homeowners and decide to buy a house, choose the level of consumption today ($c$), the level of liquid savings next period ($a'$), the size of the house to buy ($h'$), and the level of mortgage to take out. In recursive form, their problem can be written as

$$
V^o(\mathbf{x}^n) = \max_{c, b', h', m'} u(c, s) + \beta \mathbb{E}_\epsilon \left[ V^{h'} \left( \mathbf{x}^{h'} \right) \right] \tag{2.3.5}
$$

where the expectation is taken with respect to next period idiosyncratic income shock $\epsilon'$. Renters solve the above problem subject to the following constraints:

$$
\begin{aligned}
& c + q_a a' + p_h h' + \kappa_m \leq a + y - T(y, 0) + q_m m' \\
& m' \leq \lambda_m p_h h' \\
& \pi^{\min}(m') \leq \lambda_\pi y \\
& a' \geq 0 \\
& s = \omega h', \quad h' \in \mathcal{H} \\
& y' \sim Y(y)
\end{aligned}
$$

where the equations are the budget constraint, LTV constraint, PTI constraint, borrowing constraint, housing services production, and income evolution, respectively. Let $\mathbb{1}^o(\mathbf{x}^n)$ denote the decision of non-homeowner with state variables $\mathbf{x}^n$ to buy a house, with

$$\mathbb{1}^r(\mathbf{x}^n) + \mathbb{1}^o(\mathbf{x}^n) = 1$$

**Mortgage payers** The households of age $j$ that enter the period as homeowners with a given level of mortgage $m$ and house size $h$, and decide to make the payment towards the mortgage balance, choose the level of consumption today ($c$), the level of liquid savings next period ($a'$), and the size of payment ($\pi$). In recursive form, their problem can be written as

$$V^p(\mathbf{x}^h) = \max_{c,b',\pi} u(c,s) + \beta \mathbb{E}_\epsilon \left[ V^{h'} \left( \mathbf{x}^{h'} \right) \right] \tag{2.3.6}$$

where the expectation is taken with respect to next period idiosyncratic income shock $\epsilon'$. Mortgage payers solve the above problem subject to the following constraints:

$$
\begin{aligned}
& c + q_a a' + (\delta_h + \tau_h) p_h h' + \pi \leq a + y - T(y,m) \\
& m' = (1 + r_m) m - \pi \\
& \pi \geq \pi^{\min}(m) \\
& a' \geq -\lambda_a p_h h \\
& s = \omega h', \quad h' = h \in \mathcal{H} \\
& y' \sim Y(y)
\end{aligned}
$$

where the equations are the budget constraint, mortgage balance evolution, PTI constraint, borrowing constraint, housing services production, and income evolution, respectively. Let $\mathbb{1}^p(\mathbf{x}^h)$ denote the decision of homeowner with state variables $\mathbf{x}^p$ to make a payment towards the mortgage.

**Mortgage refinancers** The households of age $j$ that enter the period as homeowners with a given level of mortgage $m$ and house size $h$, and decide to refinance the existing mortgage, choose the level of consumption today ($c$), the level of liquid savings next period ($a'$), and the level of new mortgage ($m'$). In recursive form, their problem can be written as

$$V^f(\mathbf{x}^h) = \max_{c,b',m'} u(c,s) + \beta \mathbb{E}_\epsilon \left[ V^{h'} \left( \mathbf{x}^{h'} \right) \right] \tag{2.3.7}$$

where the expectation is taken with respect to next period idiosyncratic income shock $\epsilon'$. Mortgage refinancers solve the above problem subject to the following constraints:

$$\begin{aligned}
& c + q_a a' + (\delta_h + \tau_h) p_h h' + (1 + r_m) m + \kappa_m \leq a + y - T(y,m) + q_m m' \\
& m' \leq \lambda_m p_h h' \\
& \pi^{\min}(m') \leq \lambda_\pi y \\
& a' \geq -\lambda_a p_h h \\
& s = \omega h', \quad h' = h \in \mathcal{H} \\
& y' \sim Y(y)
\end{aligned}$$

where the equations are the budget constraint, mortgage balance evolution, PTI constraint, borrowing constraint, housing services production, and income evolution, respectively. Let $\mathbb{1}^f \left( \mathbf{x}^h \right)$ denote the decision of homeowner with state variables $\mathbf{x}^p$ to refinance the existing mortgage.

**Sellers** The households of age $j$ that enter the period as homeowners with a given level of mortgage $m$ and house size $h$, and decide to sell their house in the current period, choose the level of consumption today ($c$), the level of liquid savings next period ($a'$) and the size of the rented dwelling for the next period ($\tilde{h}'$), as they will remain non-homeowners for the following period.

$$V^s(\mathbf{x}^n) = \max_{c,b',\tilde{h}'} u(c,s) + \beta \mathbb{E}_\epsilon \left[ V^{n\prime} \left( \mathbf{x}^{n\prime} \right) \right] \tag{2.3.8}$$

where the expectation is taken with respect to next period idiosyncratic income shock $\epsilon'$. House sellers solve the above problem subject to the following constraints:

$$\begin{aligned}
& c + \rho_h \tilde{h}' + q_a a' \leq a_s + y - T(y,0) \\
& a' \geq 0 \\
& s = \tilde{h}', \quad \tilde{h}' \in \widetilde{\mathcal{H}} \\
& y' \sim Y(y)
\end{aligned}$$

where $a_s$ denotes the current level of assets plus the proceedings from selling the house net of transaction costs and mortgage balance, given by

$$a_s = a + (1 - \delta_h - \tau_h - \kappa_h) p_h h - (1 + r_m) m.$$

Let $\mathbb{1}^s \left( \mathbf{x}^h \right)$ denote the decision of homeowner with state variables $\mathbf{x}^p$ to sell the house.

**Defaulters** The households of age $j$ that enter the period as homeowners with a given level of mortgage $m$ and house size $h$, might decide to default on their mortgage if they aren't able to make the minimum payment towards the mortgage balance. If they default, they choose the level of consumption today ($c$) and the level of liquid savings next period ($a'$); they are forced to rent the minimum dwelling available for renting and are not allowed to buy a house for another period. In recursive form, their problem can be written as

$$V^d(\mathbf{x}^n) = \max_{c,b',\tilde{h}'} u(c,s) - \zeta + \beta \mathbb{E}_\epsilon \left[ V^{n\prime} \left( \mathbf{x}^{n\prime} \right) \right] \tag{2.3.9}$$

where $\zeta$ denotes the utility penalty and the expectation is taken with respect to next period idiosyncratic income shock $\epsilon'$. Renters solve the above problem subject to the following constraints:

$$\begin{aligned}
&c + \rho_h \tilde{h}_{min} + q_a a' \leq a + y - T(y, 0) \\
&a' \geq 0 \\
&s = \tilde{h}_{min}, \quad \tilde{h}_{min} \in arg \min \widetilde{\mathcal{H}} \\
&y' \sim Y(y)
\end{aligned}$$

Let $\mathbb{1}^d\left(\mathbf{x}^h\right)$ denote the decision of homeowner with state variables $\mathbf{x}^p$ to default on the mortgage, with

$$\mathbb{1}^p\left(\mathbf{x}^h\right) + \mathbb{1}^f\left(\mathbf{x}^h\right) + \mathbb{1}^s\left(\mathbf{x}^h\right) + \mathbb{1}^d\left(\mathbf{x}^h\right) = 1$$

### 2.3.6 Definition of Equilibrium

Our definition of equilibrium consists of households' consumption decision rules

$$\left\{c^r(\mathbf{x}^n), c^o(\mathbf{x}^n), c^p(\mathbf{x}^h), c^f(\mathbf{x}^h), c^s(\mathbf{x}^h), c^d(\mathbf{x}^h)\right\}$$

savings decision rules

$$\left\{a^r(\mathbf{x}^n), a^o(\mathbf{x}^n), a^p(\mathbf{x}^h), a^f(\mathbf{x}^h), a^s(\mathbf{x}^h), a^d(\mathbf{x}^h)\right\}$$

mortgage decision rules

$$\left\{m^o(\mathbf{x}^n), m^f(\mathbf{x}^h), \pi(\mathbf{x}^h)\right\}$$

and housing choice rules

$$\left\{\tilde{h}^r(\mathbf{x}^n), h^o(\mathbf{x}^n), h^p(\mathbf{x}^h), h^f(\mathbf{x}^h), \tilde{h}^s(\mathbf{x}^h)\right\}$$

and government expenditure $G$, such that

1. Households' policy function solve problems (2.3.4), (2.3.5), (2.3.6), (2.3.7), (2.3.8) and (2.3.9) given prices $p_h$ and $\rho_h$
2. Government expenditure $G$ clears governmental budget constraint

We next describe the value of the model parameters that we use to calculate the equilibrium.

## 2.4 Parametrization

We set the parameters of the model to be consistent with key cross-sectional features of the U.S. economy using the 2001 wave of SCF. A subset of parameters are set exogenously without the need to solve for the steady-state of model. The target model-implied and data moments are reported in Table 2.4.1.

Table 2.4.1: Targeted moments in the parametrization

| **Targeted Moments** | | |
|---|---|---|
| Moment | Model Value | Empirical Value |
| Net worth to income ratio | 5.8 | 5.5 |
| Ratio of net worth 75/50 | 1.6 | 1.5 |
| Homeownership rate | 0.63 | 0.66 |
| Default rate | 0.002 | 0.005 |
| House size of owners to renters | 1.5 | 1.5 |

**Demographics and Preferences** The model period is set to one year. Households enter the economy in age 21, retire at age 65 and live until age 81. This corresponds to $J_r = 44$ and $J = 60$. The elasticity of substitution between consumption and housing services is set to 1.25, corresponding to $\gamma = 0.8$ and is based on the estimates from

Piazzesi, Schneider and Tuzel (2007). We use the same strategy as Kaplan, Mitman and Violante (2018) set risk aversion parameter $\vartheta$ equal to 2 so that the EIS is 0.5 The properties of the baseline model are robust to change in $\vartheta$ as long as EIS is less than 1. The discount factor $\beta$ is set equal to 0.964, implying the average net worth to income ratio of 5.8, slightly above empirical value of 5.5 from SCF. To control to which extent bequest is perceived as luxury good, we set $\underline{a} = 7.7$. The strength of the bequest motive is controlled by $\psi$, which we set equal to match the ratio of net worth at age 75 to net worth at age 50 (to proxy the importance of bequests as a saving motive). For $\psi$ equal to 7, the model-implied ratio is 1.6, compared to 1.5 in the SCF. The extra utility from owned housing, $\omega$, is set to be equal to 1.015, to match the average homeownership rate. The model-implied homeownership rate is 63 percent compared to 66 percent in the data. The dis-utility from defaulting, $\zeta$, is set equal to 5. The model-implied default rate is about 0.2 percent, compared 0.5 percent in the data. Finally, we set the share of utility from housing $\phi$ equal to 0.16, that matches the share of housing in total consumption expenditure in NIPA. These are summarized in Table 2.4.2.

TABLE 2.4.2: Parameter values (demographics and preferences)

| **Demographics and Preferences** | | |
|---|---|---|
| $J$ | Length of life | 60 |
| $J_r$ | Working life | 44 |
| $\gamma$ | 1/EIS | 0.8 |
| $\vartheta$ | Risk aversion | 2 |
| $\beta$ | Discount factor | 0.964 |
| $\underline{a}$ | Bequest as luxury | 7.7 |
| $\psi$ | Strength of bequest | 7 |
| $\omega$ | Utility from homeownership | 1.015 |
| $\zeta$ | Disutility from default | 5 |
| $\phi$ | Share of housing in utility | 0.16 |

**Labor Income and Government Expenditure** The deterministic component of labor earnings, $\chi_j$, is calculated using the data on labor earnings from 2001 wave of the

SCF. The productivity parameter, $\Theta$, is set to be equal to 1. The stochastic component of earnings is modeled as an AR(1) process with mean 0.75 and standard deviation 0.08. Standard deviation of the initial distribution of income is set to 0.04. We set the social security replacement rate to 60 percent. This matches the initial distribution of income at age 21 as well as the rise in variance of log earnings of 2.5 between age 21 and 64 from 2001 wave of SCF. The parameters of the tax function (2.3.3), $\tau_y^0$ and $\tau_y^1$, are set to 0.75 and 0.151, respectively and are based on estimates from Heathcote, Storesletten and Violante (2017) for the US. Parameter $\tau_y^0$ measures the average level of taxation and parameter $\tau_y^1$ measures the degree of progressivity of the US tax and transfer system. The maximum level of tax-deductible mortgage, $\bar{m}$, is set to correspond to $1 million. The property tax $\tau_h$ is set to 1 percent, which is the median tax rate across the US. These are summarized in Table 2.4.3.

TABLE 2.4.3: Parameter values (labor income and government expenditure)

| | **Labor Income and Government Expenditure** | |
|---|---|---|
| $\chi_j$ | Deterministic life-cycle profile | — |
| $\Theta$ | Productivity | 1 |
| $\tau_y^0$ | Income tax parameter | 0.75 |
| $\tau_y^1$ | Income tax parameter | 0.151 |
| $\rho_{ss}$ | Replacement rate | 0.6 |
| $\bar{m}$ | Mortgage deduction limit | 20* |
| $\tau_h$ | Property tax | 0.01 |

* A unit of the final good corresponds to $50000, which is the median income in the 2001 wave in SCF.

**Housing** We fix the grid for the owner-occupied houses ($\mathcal{H}$) and rented houses ($\widetilde{\mathcal{H}}$), so that households are only allowed to choose to buy or rent of the dwellings from the grid. The minimum size of the owner-occupied dwelling is set to 1.5 to represent the ratio of the average house size of owners to renters (Chatterjee and Eyigungor, 2015). The depreciation rate of housing is set equal to 1.5 percent to match the annual depreciation rate of the housing stock from the BEA. Transaction cost of selling the house, $\kappa_h$, is set to 8 percent, which is the average value reported in Quigley (2002). These are summarized in Table 2.4.4.

**Liquid Assets and Mortgages** We set the interest rate $r_a$ exogenously equal to 3 percent, and the spread parameter $\iota$ equal to 33 percent. This implies the mortgage rate $r_m$ of about 4 percent. These values are consistent with the gap between the average rate on 30-year fixed-term mortgages and the 10-year T-Bill rate for the US. The implied price of bond, $q_a$ is equal to 0.97. The mortgage origination cost, $\kappa_m$, is set to equivalent of $ 2000, corresponding to the sum of application, attorney, appraisal and inspection fees. In the baseline version of the model we set the unsecured borrowing parameter, $\lambda_a$ equal to 0. The minimum down payment requirement $q_m$ is set to 15 percent and controls the overall market tightness. This number is consistent with recent estimates by Sommer and Sullivan (2018) and Kaplan, Mitman and Violante (2018). These are summarized in Table 2.4.4.

TABLE 2.4.4: Parameter values (liquid assets and mortgages)

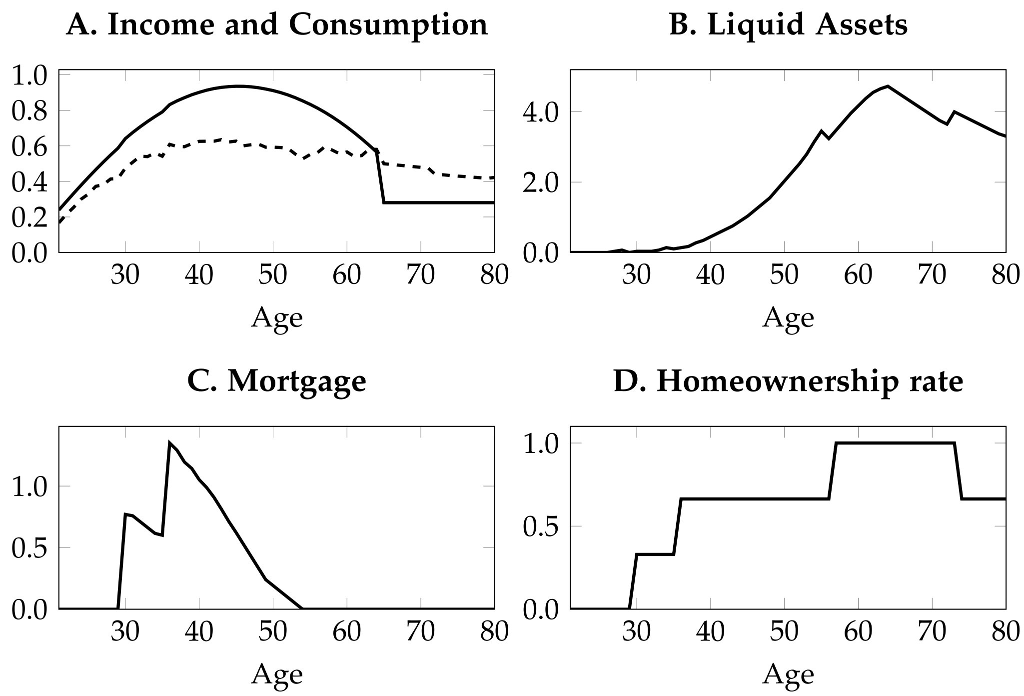
| | **Housing, Liquid Assets and Mortgages** | |
|---|---|---|
| $\delta_h$ | Depreciation rate | 0.015 |
| $\kappa_h$ | Transaction cost | 0.08 |
| $r_a$ | Interest rate | 0.03 |
| $\iota$ | Spread | 0.33 |
| $r_m$ | Mortgage rate | 0.04 |
| $q_a$ | Price of bond | 0.97 |
| $\kappa_m$ | Mortgage origination cost | 0.04* |
| $\lambda_a$ | Maximum borrowing limit | 0 |
| $q_m$ | Down payment requirement | 0.15 |

* A unit of the final good corresponds to $50000, which is the median income in the 2001 wave in SCF.

### 2.4.1 Properties of the Baseline Model

In this subsection we describe the life-cycle properties of the baseline model with parametrization specified in Tables 2.4.2-2.4.4. Figure 2.4.1 displays the lifetime profiles for several key model variables. Panel A plots the mean labor and pension income (solid black line) and non-durable consumption (dashed black line). Households increase their consumption until about age 30, and then keep it constant until the end of the lifetime. Panel B displays the mean lifetime savings profile of the households. As the households have the bequest motive - they do not dis-save towards the end of the lifetime and leave the portion of the savings as a bequest for the future generations. Panel C displays the mean mortgage balance in the economy. Households take out the mortgage later in life, when they are about 30 years old, so that the payment-to-income constraint (2.3.1) is satisfied. As the income is stochastic, some households do not take out the mortgage until later in life. Finally, Panel D displays the average homeownership rate in the economy. Some households (that receive good income shock early in life) buy house early, while the others postpone the purchase until later in life. Households that had a sequence of bad income shocks towards the end of the lifetime sell their house and choose to rent instead, and use the selling proceedings to smooth consumption and leave towards bequest.

FIGURE 2.4.1: **Mean life-cycle profiles in the baseline model.** Panel A displays mean income (black solid line) and consumption (black dashed line). Panel B displays mean holdings of liquid asset. Panel C displays mean mortgage balance. Panel D displays mean homeownership rate.



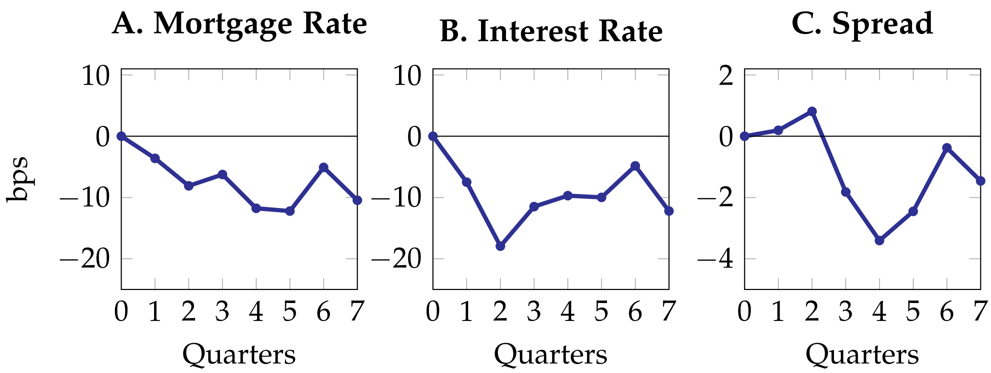
## 2.5 Mortgage Market Intervention Experiment

We next perform a mortgage market intervention experiment in the baseline model using the empirical evidence on the effects of governmental mortgage markets interventions on interest and mortgage rates.

### 2.5.1 Macroeconomic Effects of Mortgage Market Intervention

In their paper, Fieldhouse, Mertens and Ravn (2018) document the macroeconomic effects of news shock to agency mortgage purchases. They find that following a shock, the interest rates as well as the mortgage rates decrease, as does the spread between of mortgage rates over the interest rates. Panels A and B in figure 2.5.1 plots the response of mortgage and interest rates, respectively, along with one standard deviation confidence intervals. Panel C in figure 2.5.1 plots the response of spread between the two along with one standard deviation confidence intervals. We see that interest and mortgage rates (panels A and B) decline significantly immediately after the shock and remain low for at least two years. Spread between the two (panel C) declines significantly 3 quarters after the shock and remains negative and significant for half a year.

FIGURE 2.5.1: **Impulse responses of interest rates.** Impulse response of mortgage rate (Panel A), interest rate (Panel B), and spread (Panel C) to an additional to a 1% increase in net purchase by FNMA & FHLMC, anticipated 2 quarters before.



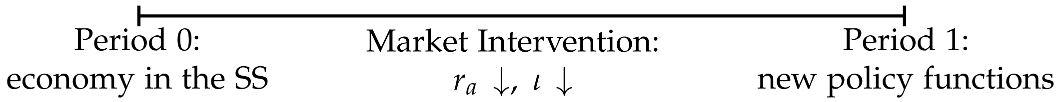

## 2.5.2 Transitional Dynamics and Transmission Mechanism

To understand the transmission mechanism through which mortgage market intervention operates, we perform the following policy experiment experiment. Suppose that in period 0 the economy is in the steady state, where interest rates and mortgage rates are fixed, and so is the spread between the two. Between period 0 and period 1 (a year in the model), there is an exogenous intervention to the mortgage markets such that interest rate $r_a$ goes down. To account for the fact that empirical evidence suggests the drop in mortgage rates as well as the drop in spreads, and using

$$r_m = (1 + \iota) r_a$$

we also assume that spread parameter $\iota$ also declines. In period 1, households enter the period with new interest and mortgage rates, and adjust their choice of consumption, mortgage balance, and liquid savings using the new policy functions. Figure 2.5.2 displays the simplified timeline of the policy experiment.

FIGURE 2.5.2: **Timeline of the policy experiment.**




We then analyze whether the policy experiment can reconcile the empirical evidence presented in section 2.2.4. Empirically, we found that exogenous intervention to mortgage markets makes households with the mortgage significantly increase their consumption expenditure, followed by a positive (but insignificant) increase in consumption expenditure of renters. The policy intervention has the smallest (and insignificant) increase of consumption expenditure for outright homeowners. We identify the same three groups of people in the model: renters (either renters that choose to rent, or homeowners that sell their house or default on the mortgage), mortgagors (either mortgagors who make payments towards positive mortgage balance or refinancers) and outright owners (household that own the house and have zero mortgage outstanding). We then calculate the change in consumption expenditures for these types of households. We report the results of policy experiment in Table 2.5.1.

TABLE 2.5.1: Response of consumption expenditure to mortgage market intervention

| Tenure | Change in Consumption |
|---|---|
| Renters | 0.7pp |
| Mortgagors | 1.3pp |
| Outright Owners | 0.2pp |

Following an exogenous change in interest rate and in spread parameter, the group that responds the most to policy change is the mortgagor group. After a cut in the interest and mortgage rates, they increase consumption by 1.3pp. Renters also respond positively to change in the interest rates, increasing their consumption by 0.7pp relative to initial steady state. Outright homeowners, on the other hand, react the least to the policy change, and increase their consumption by only 0.2pp.

Our second empirical result, reported in Section 2.2.5 states that expenditure inequality increases significantly for mortgagors while there is no significant increase for the other two groups of households. To compare the empirical results with those of the model, we calculate the model-implied Gini coefficient before and after the policy experiment took place for all three groups of households. We then look at the change of Gini coefficient after the policy. We report the results of policy experiment in Table 2.5.2.

TABLE 2.5.2: Response of expenditure Gini to mortgage market intervention

| Tenure | Change in Gini |
|---|---|
| Renters | 0.2pp |
| Mortgagors | 1.7pp |
| Outright Owners | -0.1pp |

Following an exogenous change in interest rate and in spread parameter, the expenditure Gini increases significantly for mortgagors. After a cut in the interest and mortgage rates, the consumption inequality measure increases by 1.7pp. For renters and outright homeowners the change in expenditure inequality is small, 0.2pp and -0.1pp, respectively. This response goes in line with the empirical evidence reported in Section 2.2.5.

We next analyze what is the transmission mechanism that policy operates through and what drives the increase in consumption reported in Table 2.5.1. The decrease in the interest rate has a straightforward effect on consumption of all households - as the interest rates drop, the opportunity cost of savings goes down and households choose to consume the extra income instead. For the outright homeowners (who are also older), the bequest motive plays a higher role, and those with high level of savings decide to keep the savings. The households with the lower savings instead to decide to sell their house and also keep it as a bequest. Both renters and mortgagors act as a typical hand-to-mouth consumers: lowering the interest rate makes them save less and consume more. So why do mortgagors and renters react differently? The mortgage market intervention also affects the mortgage rate and the spread between the mortgage and interest rate. Mortgagors minimum payment requirement, given by equation (2.3.2), depends on the mortgage rate $r_m$. Lowering the rate $r_m$ (due to lowering in $r_a$ and $\iota$) relaxes the payment constraint for the mortgagors. So on top of the effect coming directly from lower interest rates, they also receive extra income from lower minimum payment.

## 2.6 Conclusions

We study the heterogeneous impact of expansionary credit policies by combining exogenous policy changes in US federal housing agencies mortgage holdings with household level data from the Consumer Expenditure Survey and the Survey of Consumer Finances. We group households into pseudo-cohorts based on their housing tenure status: renters, mortgagors and homeowners. We show that following an increase in agency purchases, households with mortgage increase their spending, while outright homeowners and renters do not adjust their expenditure significantly. We explain this evidence through the lens of a Huggett (1996) type of heterogeneous life-cycle model with endogenous housing choice and idiosyncratic income risk. We calibrate the mortgage market intervention to be consistent with empirical evidence and show that lower interest rate partially explains small increase in expenditure of renters. We also show that bequest motive outweighs the effect of lower interest rate for outright homeowners Finally, and more importantly, we also show that lower mortgage rates as well as the change in spread between the rates explains the high observed increase in expenditure for mortgagors.

# Appendix

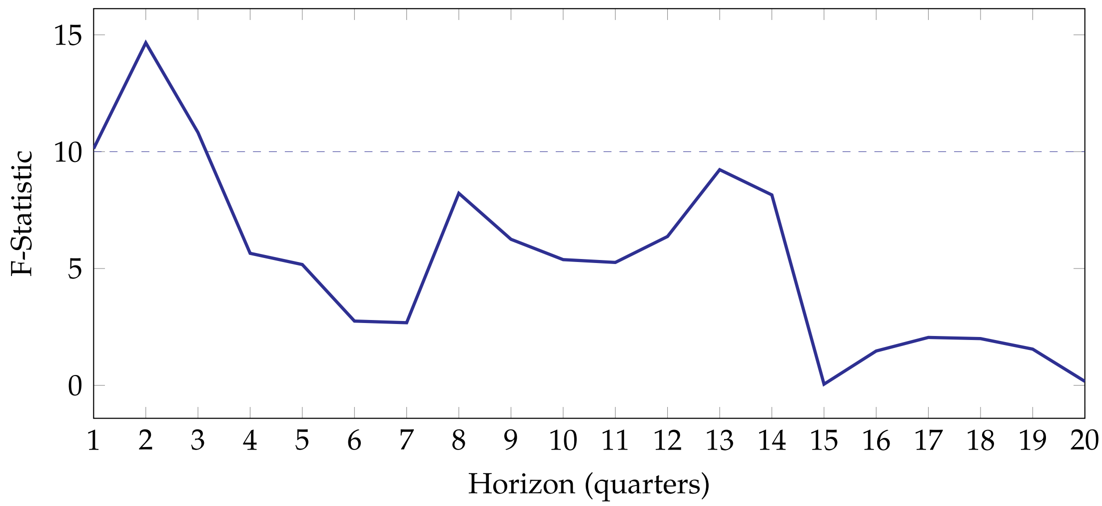
## 2.A Agency and Market Data

*Residential mortgage debt* is the sum of home mortgages and multifamily residential mortgages from the Federal Reserve's Financial Accounts of the United States. *Nominal GDP* is from the National Income and Product Accounts. *Agency mortgage holdings* is the sum of the retained mortgage portfolios of Fannie Mae and Freddie Mac. Between 1980 and 2003, the data on retained mortgage portfolio is available from various issues of Federal Reserve Bulletin. After 2003 the data is from monthly volume summaries combined with annual OFHEO/FHFA reports. *Residential mortgage originations* before 1997 is from monthly releases of the Survey of Mortgage Lending Activity from the HUD. After 1997 the data on originations is available from Datastream (series USMORTORA). *Net portfolio purchases* is the sum of corresponding series for Fannie Mae and Freddie Mac. Individual series before 2003 are available from various issues of Federal Reserve Bulletin. After 2003 the data is from Fannie Mae's and Freddie Mac's monthly volume summaries. *Conventional mortgage rate* is the 30-year fixed-rate conventional conforming mortgage rate, available at Freddie Mac mortgage market survey. *Housing starts* is obtained from FRED database at the Federal Reserve Bank of St. Louis (series HOUST). *House prices* is measured by the Freddie Mac house price index (FMHPI) available on Freddie Mac's website. *Nominal price level* is obtained from FRED database at the Federal Reserve Bank of St. Louis (series PCEPILFE). *Personal* *income* is obtained from FRED database at the Federal Reserve Bank of St. Louis (series PI). *Unemployment rate* is obtained from FRED database at the Federal Reserve Bank of St. Louis (series UNR). *Short- and long-term interest rates* are 3-month and 10-year Treasury rates, obtained from FRED database at the Federal Reserve Bank of St. Louis (series TB3MS and GS10). *BAA and AAA corporate bond rates* are the Moody's seasoned BAA and AAA yields, obtained from FRED database at the Federal Reserve Bank of St. Louis (series BAA and AAA).

## 2.B Testing for Exclusion Restrictions

Below we present the plot of robust F-statistics on the excluded instrument of the first-stage regressions of cumulative agency net purchases given by equation (2.2.1) for different horizons $h$. Horizontal dashed line represents the threshold level of 10.

FIGURE 2.B.1: **First Stage Robust F-statistic.** Figure displays robust F-statistics on the excluded instrument of the first-stage regressions of cumulative agency net purchases.



## 2.C Additional Impulse Response Analysis

### 2.C.1 SCF Data

We obtain the Survey of Consumer Finances from the Board of Governors of the Federal Reserve System website. We use nine surveys between 1983 and 2007. We apply the same data restrictions as for the Consumer Expenditure Survey. We collect information on households' date of birth, the housing tenure status and the length of mortgage remaining. We then construct five birth cohorts (see Table 2.C.1) and match the information on average mortgage length remaining with the CEX data.

### 2.C.2 Pseudo-Cohort Construction

For households with mortgage debt, the length of the mortgage debt that remains to be repaid is in important factor in determining their expenditures. For example, people with longer mortgage remaining might benefit more from the cut in mortgage rates, as their lifetime value of debt is now lower. The CEX survey, unfortunately, does not contain rich information on mortgage length and structure. The SCF survey, on the other hand, contains information on mortgage origination, value and length remaining. As SCF survey is a triennial survey, we again use synthetic panel techniques to group individual households into groups. To follow more or less homogeneous group over time and to merge the mortgage information contained in SCF with income and expenditure information in CEX, we define groups by the year of birth of the household head, or cohorts, similarly to Attanasio, Kovacs and Molnar (2018). We define cohorts over five year bands, using 1989 as a benchmark, as reported in Table 2.C.1. We then calculate the average length of mortgage remaining for each cohort for each wave in SCF survey and merge it with the CEX survey on a cohort basis.

Table 2.C.1: Cohort Definition

| Cohort | Year of Birth | Age in 1989 |
|---|---|---|
| 1 | 1965 - 1974 | 15 - 24 |
| 2 | 1955 - 1964 | 25 - 34 |
| 3 | 1945 - 1954 | 35 - 44 |
| 4 | 1935 - 1944 | 45 - 54 |
| 5 | 1925 - 1934 | 55 - 64 |

We divide households into two groups, based on length of mortgage remaining - those with mortgage remaining below 18 years, which we call a group with *short mortgage*, and those above, which we call a group with *long mortgage*. We choose these categories to maximize the number of households in each group.

Mortgage refinancing decision is another important factor in determining their expenditure. For example, those that decide to refinance their existing mortgage for the one with the lower rate can benefit from smaller mortgage payments and allocate extra cash to expenditure. On the other hand, the costs associated with refinancing might be too high for the household to decide to refinance existing mortgage, reaching almost 3% of the of the household's initial mortgage balance (see Hurst and Stafford (2004)). CEX survey includes information on mortgage refinancing decision starting from 1994. We extrapolate the information between 1980 and 1993 using the k-nearest neighbor algorithm, to classify the households into those who *refinance* and those who *don't refinance* using a set of household characteristics.

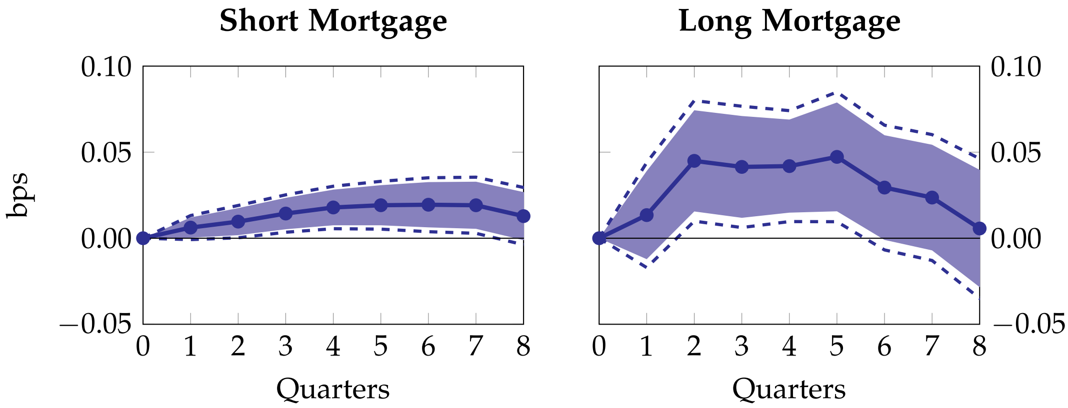
Similarly, we group the households by their decision to borrow against their house using a home equity line of credit (HELOC). While households that decide to use HELOC can enjoy extra cash, they also face another debt on top of their existing mortgage debt. Similarly to the refinancing decision described above, we extrapolate the household's decision to take out HELOC to the whole sample using k-nearest neighbor algorithm. We group households into two groups: those that *take HELOC* and those that *do not take HELOC*.

Finally, we also group households based on their pre-tax income. We define two groups - *poor* households, that are in the bottom 50% of the income distribution, and *rich* households, that are in the top 50% of the income distribution.

### 2.C.3 Response of Expenditure Along the Mortgage Length Distribution

We group the mortgagors into those with a *short* mortgage - where the remaining mortgage debt matures in less than 18 years - and with a *long* mortgage - where the remaining mortgage debt matures 18 or more years. The idea behind this classification is quite straightforward: following an expansion in agency portfolio activity, both the short term, the long term, and the mortgage rates fall (see Andrew Fieldhouse, Karel Mertens and Morten O Ravn, 2018); mortgagors with a long mortgage might anticipate a long-term effects of reduction in value of their mortgage, and thus benefit more. Indeed, we can confirm this intuition by looking at Figure 2.C.1. As figure indicates, following a shock, mortgagors with short mortgage (left panel) increase their expenditure slightly (reaching a peak of about 0.01 basis points), whereas the mortgagors with long mortgage exhibit a strong and significant increase between 3 and 6 quarters following a shock (reaching a peak of about 0.05 basis points).

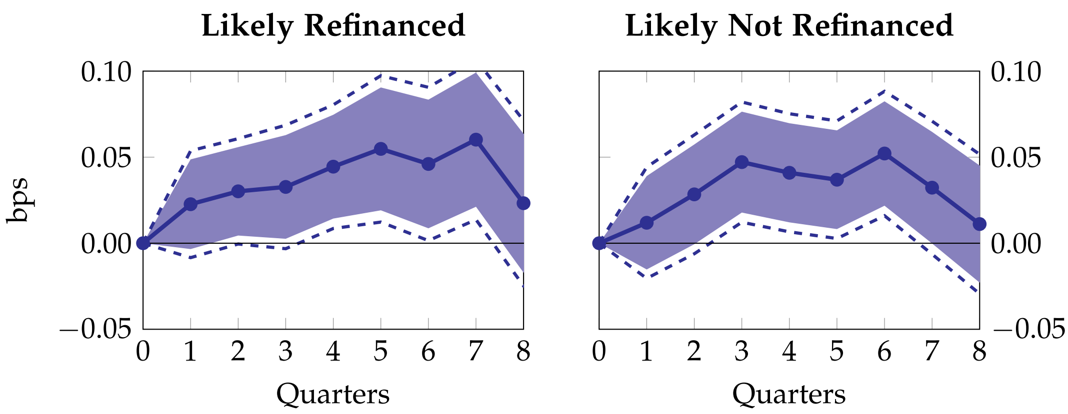
FIGURE 2.C.1: **Impulse response of expenditure.** Impulse response of expenditure to an additional to a 1% increase in net purchase by FNMA & FHLMC, anticipated 2 quarters before. Blue areas and broken lines represent 90% and 95% confidence intervals, respectively.




## 2.C.4 Response of Expenditure Based on Refinancing Decision

We group households into those that decide to refinance and those that do not. We find that refinancing decision does no matter for the households. Indeed, as Figure 2.C.2 indicates, both households that decide to refinance (left panel) and those that do not (right panel) increase their expenditure by almost the same amount, reaching a peak of almost 0.05 basis points a year following a shock.

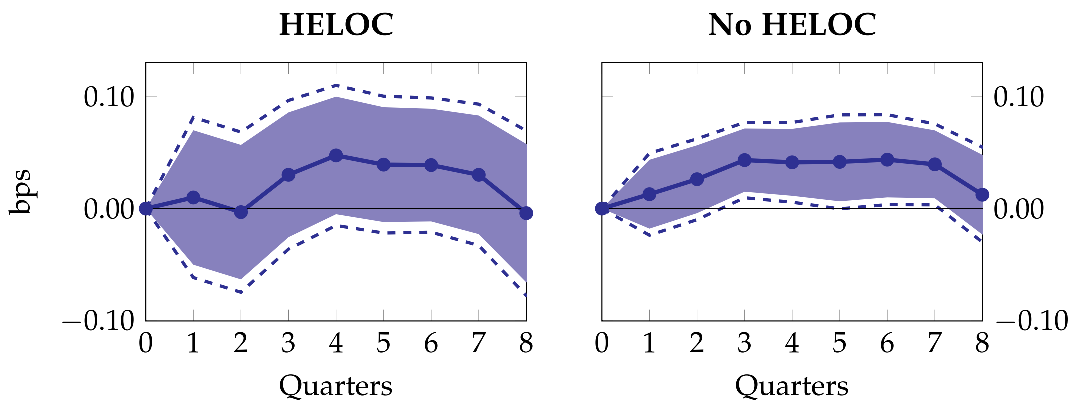
FIGURE 2.C.2: **Impulse response of expenditure.** Impulse response of expenditure to an additional to a 1% increase in net purchase by FNMA & FHLMC, anticipated 2 quarters before. Blue areas and broken lines represent 90% and 95% confidence intervals, respectively.




## 2.C.5 Response of Expenditure and HELOC

We now analyze whether a households' decision to take out a home equity line of credit matters. Figure 2.C.3 plots the response of expenditure of households that do take out HELOC (left panel) and those that do not (right panel). Following a shock, those households that do not take out an extra debt increase their expenditure significantly, again reaching a peak of about 0.05 basis points a year after, while for those that do take out HELOC the increase is insignificant.

FIGURE 2.C.3: **Impulse response of expenditure.** Impulse response of expenditure to an additional to a 1% increase in net purchase by FNMA & FHLMC, anticipated 2 quarters before. Blue areas and broken lines represent 90% and 95% confidence intervals, respectively.

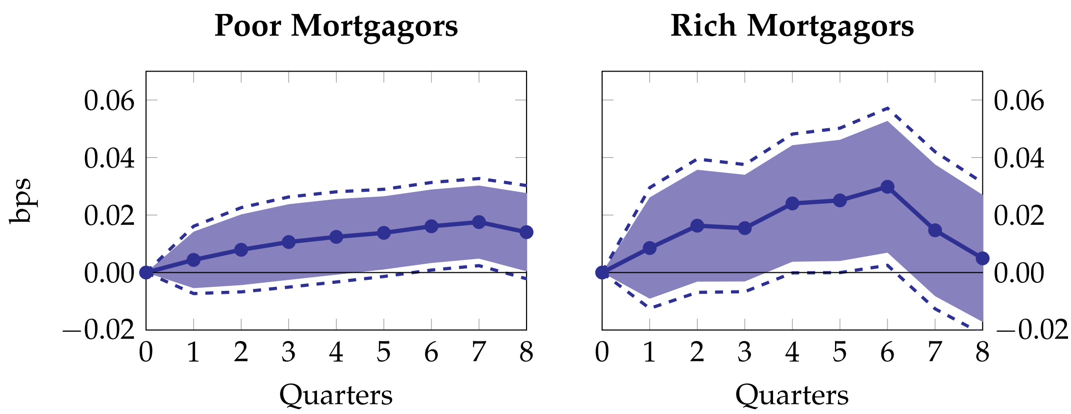



### 2.C.6 Response of Expenditure Along the Income Distribution

Finally, we use the definition of income from before and divide the households into two categories: *poor* households that are in the bottom 50% of income distribution, and *rich* households, that are in the top 50% of income distribution. As before, we also divide households by housing tenure status. This way we have six different groups of households: renters, mortgagors and homeowners, each of whom are either poor or rich.

The mortgagors, however, show a different result. As Figure 2.C.4 indicates, the increase in expenditure among rich mortgagors (right panel) is higher, following a shock, and is significant between 5 and 7 quarters (with a 90% confidence), while the response of the poor mortgagors is quantitatively lower, and is only significant after quarter 6 (with a 90% confidence). This indicates that there is indeed a heterogeneous within the mortgagors group that is consistent with an increase in expenditure inequality within the mortgagors group.

FIGURE 2.C.4: **Impulse response of expenditure.** Impulse response of expenditure to an additional to a 1% increase in net purchase by FNMA & FHLMC, anticipated 2 quarters before. Blue areas and broken lines represent 90% and 95% confidence intervals, respectively.



# Chapter 3

# The Resolution of Long-Run Risk

**(with Raffaele Rossi and Klaus Schenk-Hoppé)**

## 3.1 Introduction

The Long-Run Risk Model (LRRM) introduced by Bansal and Yaron (2004), is one of the main theoretical pillars in financial macroeconomics. In its original version, the LRRM reconciled several key asset pricing phenomena in a unified framework by combining recursive preferences á la Epstein and Zin (1989) with a model of aggregate consumption growth that exhibits predictable low-frequency movements and time-varying volatility. Despite its success, the LRRM suffers from a quantitative drawback similar to Mehra and Prescott (1985)'s equity premium puzzle. When calibrated to financial and macroeconomic data, the LRRM implies unrealistically high levels of timing and risk premia, see Epstein, Farhi and Strzalecki (2014). A representative household with recursive preferences, a relative risk aversion of 7.5, and an elasticity of intertemporal substitution of 1.5 would give up around one quarter of her lifetime consumption to resolve uncertainty one month earlier, and around half of her lifetime consumption to live in a world without consumption risk. Both percentage are difficult to reconcile with microeconomic evidence or introspection.

This chapter introduces in the standard LRRM durable consumption alongside the consumption of non-durable goods. The main message of our study is that this simple modification can reduce by about two-thirds the timing and risk premia, without compromising (and possibly improving) the model's ability to match standard macroeconomic and financial moments. In our benchmark estimation exercise, our LRRM can rationalise key asset pricing facts, and deliver a timing premium of 11 percent and a cost of eliminating all consumption uncertainty of 16 percent of lifetime consumption.

Regarding the cost of eliminating total consumption risk, our results are consistent with the empirical evidence provided by Alvarez and Jermann (2004), who find a cost of eliminating consumption risk around 16 percent of lifetime consumption. In connection to the timing premium, the empirical evidence presented in Schlag, Thimme and Weber (2017) imply a value of seven percent, while the experimental study of Meissner and Pfeiffer (2018) finds an average timing premium of around 5 percent of lifetime consumption. The timing premium implied by our model is larger, but much closer to the empirical and experimental findings than the original LRRM with only non-durable consumption.

The main driver behind our results is that durable goods yield utility over several periods as their service flow spans over a relatively long time horizon, see for instance Browning and Crossley (2009). In other words, in bad times households can cut their expenditure on durable goods, while benefiting from the service flow that their stock of owned durables provides. As such, durable consumption supplies partial insurance against future uncertainty, potentially mitigating the timing and risk premia.

Durable consumption makes up a substantial part of household expenditure. According to personal consumption data from the US National Income and Product Accounts, in the past three decades households spent three dollars on durable consumption for each dollar spent on non-durable consumption. Over the last 70 years, on average twice as much was spent on durable than on non-durable consumption.

Durable consumption is also known to improve substantially the quantitative performance of consumption-based asset pricing models. Yogo (2006) finds that including durable consumption in the standard CCAPM can explain the cross-sectional variation in expected stock returns as well as the time variation in the equity premium. Gomes, Kogan and Yogo (2009) show that durability of output is reflected in stock prices and accounts for differences in risk premia between durable goods producers and service providers. Yang (2011) emphasises the importance of long-run risk in durable consumption risk in understanding asset price phenomena such as pro-cyclical dividend yields, counter-cyclical equity premia and stock return predictability. Eraker, Shaliastovich and Wang (2016) find that LRRM with durable goods and inflation risk can explain the correlation between expected inflation and future real growth.

Conducting the quantitative analysis of our model poses several challenges. First, rather than calibrating the endowment processes for consumption and asset prices, we use a data-driven approach that estimates these processes in a non-linear fashion with a sequential Monte Carlo particle filter as in Schorfheide, Song and Yaron (2018). This method considerably complicates the evaluation of the likelihood function as well as the implementation of Bayesian inference. However it allows to be less restrictive about the role of the time-varying volatilities in the endowment processes as well as in the long-run components. Second, we solve and estimate the full non-linear LRRM in the spirit of Chen, Favilukis and Ludvigson (2013). This is particularly challenging from a numerical point of view, due to the presence of durable consumption acting as an extra endogenous state variable. However this technique permits to recover important non-linearities of the LRRM. This is crucial as using Campbell/Schiller linearisation methods can lead to wrong model predictions, see Pohl, Schmedders and Wilms (2018). The technique also reduces substantially the composition effect between non-durable and durable consumption and its impact on risk and timing premia. To the best of our knowledge, our quantitative analysis is the first one where non-linear solution and estimation techniques are applied jointly to the endowment processes and to the LRRM.

The estimated model provides a good fit of the data. The representative household has risk aversion of 1.86 and its elasticity of intertemporal substitution is 1.18. Crucially, and in line with the existent evidence, e.g. Yogo (2006), we find that durable and non-durable consumption goods are gross complements. We also find that the predictable component of durable consumption growth is more persistent than the predictable component of non-durable consumption growth, as in Yang (2011) and Eraker, Shaliastovich and Wang (2016). Finally, we show that the volatilities of both durable and non-durable long-run components are time-varying and have a strong impact on dividend growth. These results are interesting on their own as they provide further empirical evidence that durable and non-durable consumption do not follow random walk processes. Simulation of the model reveals a mean equity premium of 5.78 percent and an average return volatility of 17 percent. The mean risk-free rate is 1.01 percent. The main achievement of the model however is that these values are obtained with a timing premium of 11 percent and a risk premium of 16 percent.

Related to our research, Andries, Eisenbach and Schmalz (2018) addresses the same shortcoming of LRRM studied here. They show that an economy where agents have horizon-dependent risk aversion can mitigate (or even reverse) the implied preference for early resolution of uncertainty, thus reducing the term and risk premia of LRRM. This alternative explanation can be seen as complementary to ours, and based, instead, on durable consumption. It would be interesting to combine these two approaches in a unified framework. We leave this exercise to future research.

## 3.2 The Model

We consider an infinite-horizon, discrete-time endowment economy à la Lucas (1978) in which in every period $t$ a representative household derives utility from a bundle of non-durable and durable consumption represented by a Constant Elasticity of Substitution (CES) function

$$u(C_t, D_t) = \left((1-\alpha)C_t^{\frac{\rho-1}{\rho}} + \alpha D_t^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}. \tag{3.2.1}$$

$C_t$ is the non-durable consumption good that is non-storable and is entirely consumed in period $t$, $D_t$ is the service flow from durable consumption goods, $\alpha \in [0,1]$ is the relative importance of durable consumption whereas $\rho$ is the elasticity of substitution between non-durable and durable consumption. When $\rho = 1$, equation (3.2.1) collapses to the familiar Cobb-Douglas case, while for $\rho < 1$ ($\rho > 1$) durable and non-durable consumption goods are gross complements (substitutes). As in Yogo (2006), Lustig and Verdelhan (2007) and subsequent contributions, we assume that the service flow from durable consumption good is proportional to the stock of durable goods, which evolves according to the law of motion

$$D_t = (1-\delta)D_{t-1} + E_t,$$

where $\delta \in (0,1)$ is the depreciation rate and $E_t$ is the expenditure on durable consumption.

The utility function of the agent is recursive as in Epstein and Zin (1989, 1991) (see also Kreps and Porteus, 1978 and Weil, 1989), i.e.

$$\mathcal{U}_t = \left\{(1-\beta)u(C_t, D_t)^{\frac{1-\gamma}{\theta}} + \beta\left(\mathbb{E}_t[\mathcal{U}_{t+1}^{1-\gamma}]^{\frac{1}{\theta}}\right)\right\}^{\frac{\theta}{1-\gamma}}. \tag{3.2.2}$$

The parameters of the agent's utility function are the subjective discount factor $\beta \in$

$(0,1)$, the relative risk aversion coefficient $\gamma > 0$, and the elasticity of intertemporal substitution $\psi \geq 0$ with $\theta \equiv (1-\gamma)/(1-\frac{1}{\psi})$. Recall that the household with utility function in (3.2.2) is averse to volatility in future utility i.e. it prefers early resolution of risk, if $\gamma > \psi$, whereas the agent loves volatility in future utility, i.e. it prefers late resolution of risk, in the opposite case where $\gamma < \psi$. Thus, when $\gamma > \psi$, recursive utility implies a curvature with respect to future risks, a feature that is typically important for matching asset-pricing facts.[1]

In our endowment economy there are four assets: a non-durable consumption good, a durable consumption good, a stock (in positive net supply), and a risk-free discount bond (in zero net supply). In each period $t$, the agent chooses the level of consumption (both non-durable and durable) and asset holdings to maximize (3.2.2) subject to its budget constraint

$$C_t + P_t E_t + B_{b,t} + B_{s,t} = B_{b,t-1} R_{b,t} + B_{s,t-1} R_{s,t}, \tag{3.2.3}$$

where $P_t$ is the relative price of durable goods in terms of non-durable goods, $B_{b,t}$ is the $t-$period risk-free bond holdings, $B_{s,t}$ is the $t-$period stock holdings, $R_{b,t}$ is the return on risk-free bond, and $R_{s,t}$ is the return on stock.

In each period $t$, a non-durable good $C_t$, a durable good $D_t$, and a dividend from stock $S_t$ arrive. As originally introduced by Bansal and Yaron (2004), the growth rate of non-durable consumption, $\Delta C_{t+1} = \log(C_{t+1}/C_t)$, contains a small persistent predictable component $x_t$,

$$\begin{aligned} \Delta C_{t+1} &= \mu_c + x_t + \sigma_t \varepsilon^c_{t+1}, \\ x_{t+1} &= \rho_x x_t + \psi_x \sigma_t \varepsilon^x_{t+1}, \end{aligned} \tag{3.2.4}$$

where $\mu_c$ is the unconditional mean of non-durable consumption growth, $\rho_x$ is the

[1] Note that when $\theta = 1$, i.e. when $\gamma = 1/\psi$, the recursive preferences collapse to a standard Constant Relative Risk Aversion (CRRA) expected utility.

persistence of the predictable component and $\psi_x$ is the loading on the (time-varying) volatility of $x_t$. As in Eraker, Shaliastovich and Wang (2016), the growth rate of durable consumption, $\Delta D_{t+1} = \log(D_{t+1}/D_t)$, also contains a small persistent predictable component $y_t$ (potentially different from $x_t$),

$$\begin{aligned} \Delta D_{t+1} &= \mu_d + y_t + \psi_d \sigma_t \varepsilon^d_{t+1}, \\ y_{t+1} &= \rho_y y_t + \psi_y \sigma_t \varepsilon^y_{t+1}, \end{aligned} \tag{3.2.5}$$

where $\mu_d, \rho_y$ and $\psi_y$ are defined analogously to (3.2.4) but for durable consumption growth. Dividend growth, $\Delta S_{t+1} = \log(S_{t+1}/S_t)$, is exposed to low frequency risks in the aggregate economy, $x_t$ and $y_t$, and to high frequency shocks from $\Delta C_{t+1}$ and $\Delta D_{t+1}$,

$$\Delta S_{t+1} = \mu_s + \phi_x x_t + \phi_y y_t + \pi_c \sigma_t \varepsilon^c_{t+1} + \pi_d \sigma_t \varepsilon^d_{t+1} + \psi_s \sigma_t \varepsilon^s_{t+1}, \tag{3.2.6}$$

where $\phi_x$ and $\phi_y$ allow controlling for the correlation of stocks with both non-durable and durable consumption growth. All shock components have a time-varying term, $\sigma_t$, whose conditional volatility evolves according to

$$\begin{aligned} \sigma_t &= \bar{\sigma} \exp(h_t) \\ h_{t+1} &= \rho_h h_t + \sigma_h \sqrt{1 - \rho_h^2} \varepsilon^h_{t+1} \end{aligned} \tag{3.2.7}$$

with $\bar{\sigma}$ the unconditional mean of the standard deviation $\sigma_t$, $\rho_h$ the persistence parameter of the residual component $h_t$, and $\sigma_h$ the (constant) standard deviation of the shock to $\sigma_t$. Finally, shocks $\varepsilon^c_{t+1}, \varepsilon^d_{t+1}, \varepsilon^x_{t+1}, \varepsilon^y_{t+1}, \varepsilon^s_{t+1}$ and $\varepsilon^h_{t+1}$ are i.i.d., $\mathcal{N}(0,1)$ and mutually independent.

The solution of the model is characterized by first-order conditions that will be used in the empirical analysis. Let $W_t$ denote the period $t$ wealth of the agent given by

$$W_t = C_t + P_t E_t + B_{b,t} + B_{s,t}$$

while $W_{t+1}$ is given by

$$W_{t+1} = B_{b,t}R_{b,t+1} + B_{s,t}R_{s,t+1}.$$

Total wealth of the agent $\widetilde{W}_t$ is defined as the sum of his current wealth and the value of the stock of durable goods

$$\widetilde{W}_t = W_t + (1-\delta)P_t D_{t-1}.$$

Treating the durable consumption good as an asset, the holdings and the return on the durable consumption good are defined as

$$B_{d,t} = P_t D_t, \qquad R_{d,t+1} = (1-\delta)P_{t+1}/P_t.$$

Denoting the share of wealth net of non-durable consumption invested in asset $i$ by

$$\omega_{i,t} = B_{i,t}/(\widetilde{W}_t - C_t)$$

the agent's budget constraint can be written in recursive form:

$$\begin{aligned} \widetilde{W}_{t+1} &= (\widetilde{W}_t - C_t)\left(\omega_{b,t}R_{b,t+1} + \omega_{s,t}R_{s,t+1} + \omega_{d,t}R_{d,t+1}\right) \\ \omega_{b,t} + \omega_{s,t} + \omega_{d,t} &= 1. \end{aligned} \tag{3.2.8}$$

The consumption-portfolio choice problem of the agent can be expressed as follows. Given her current total wealth $\widetilde{W}_t$, she chooses consumption $C_t$ and investment shares $\omega_{b,t}, \omega_{s,t}$ and $\omega_{d,t}$ to maximize utility (3.2.2) subject to the budget constraint (3.2.8). The Bellman equation for the period-$t$ value function of this optimization problem can be written as

$$\mathcal{J}_t(\widetilde{W}_t) = \max_{\{C_t,\omega_{b,t},\omega_{s,t},\omega_{d,t}\}} \left\{ (1-\beta)u(C_t,D_t)^{\frac{1-\gamma}{\theta}} + \beta\left[\mathbb{E}_t\left(\mathcal{J}_{t+1}(\widetilde{W}_{t+1})\right)^{1-\gamma}\right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}. \tag{3.2.9}$$

The solution to this maximization problem yields to two optimality conditions. First, in any given period, the marginal rate of substitution between durable and non-durable consumption good equals their relative prices, i.e.

$$\frac{u_{D,t}}{u_{C,t}} = P_t - (1-\delta)\mathbb{E}_t\left[M_{t+1}P_{t+1}\right] = Q_t \tag{3.2.10}$$

where $M_{t+1}$ is the stochastic discount factor between period $t$ and $t+1$, and $Q_t$ is the user cost of the service flow for the durable good. Second, the intertemporal marginal rate of substitution (IMRS) between any two adjacent periods has to satisfy

$$M_{t+1} = \beta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\theta/\psi}\left(\frac{v(D_{t+1}/C_{t+1})}{v(D_t/C_t)}\right)^{\theta(1/\rho-1/\psi)} R_{W,t+1}^{\theta-1} \tag{3.2.11}$$

where the function $v(D_t/C_t)$ is defined as

$$v\left(D_t/C_t\right) = \left[1-\alpha+\alpha\left(D_t/C_t\right)^{1-1/\rho}\right]^{1/(1-1/\rho)} \tag{3.2.12}$$

and $R_{W,t+1} = \widetilde{W}_{t+1}/(\widetilde{W}_t - C_t - Q_tD_t)$ is the return on total consumption, which captures the return on the total wealth portfolio of the agent. Recall that in the one-good economy ($\alpha = 0$) of Bansal and Yaron (2004), equation (3.2.11) reduces to

$$M_{t+1}^{\text{non-durable}} = \beta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\theta/\psi} R_{W,t+1}^{\theta-1}. \tag{3.2.13}$$

In contrast to the non-durable consumption case, our model incorporates movements in the relative share of durable and non-durable goods (3.2.11) and adds the durable consumption good to the household's portfolio (3.2.9).

First-order conditions on non-durable consumption and portfolio choice imply (analogously to the derivation in Epstein and Zin (1989, 1991)) that the return on any tradable asset (risk-free bond $b$ and stock $s$) in the economy satisfies the Euler equation

$$\mathbb{E}_t\left[M_{t+1}R_{i,t+1}\right] = 1, \qquad i \in \{b, s\}. \tag{3.2.14}$$

Similarly first-order conditions on optimal durable consumption choice imply

$$\mathbb{E}_t \left[ M_{t+1}(R_{b,t+1} - R_{d,t+1}) \right] = \frac{u_{D,t}}{P_t\, u_{C,t}}. \tag{3.2.15}$$

As the Euler equation does not admit analytical solution, we rely on numerical methods to solve for the asset prices, see Appendix 3.A for a detailed description of our solution algorithm for both the linear and the non-linear case.

### 3.2.1 Timing and Risk Premia

The section details the implications for timing and risk premia which are defined analogously to Epstein, Farhi and Strzalecki (2014).

**Definition of timing and risk premia.** Suppose a consumer facing the endowment process described in Section 3.2, with $t = 0, 1, 2, \ldots$ where consumption and dividends risk is resolved gradually over time ($C_t, D_t, S_t, x_t$ and $y_t$ are realized at time $t$ only). Consider the alternative process in which all the risk is resolved in period 1. The consumer is allowed to chose the alternative endowment process over the original one at the cost of giving up a fraction $\pi$ of consumption today and in all subsequent periods. The maximum value $\pi^*$ for which the consumer is willing to accept this offer is the *timing premium*. Formally, we define it as follows. Let $\mathcal{U}_0$ be the utility with the original endowment process and $\mathcal{U}_0^*$ the utility of the alternative endowment process in which all risk is resolved at time 1. Then, $\pi^*$ is defined as

$$\pi^* = 1 - \frac{\mathcal{U}_0}{\mathcal{U}_0^*}.$$

Now, consider another alternative endowment process, in which the risk is resolved entirely, and the consumption and dividend processes are deterministic. The maximum fraction of current and future consumption $\bar{\pi}$ which you are willing to give up in favor of this deterministic process is the *risk premium* and is formally defined as

$$\bar{\pi} = 1 - \frac{\mathcal{U}_0}{\bar{\mathcal{U}}_0}$$

where $\bar{\mathcal{U}}_0$ is the utility associated with the deterministic endowment process.

**Calculating timing and risk premia.** We rely on numerical methods to calculate the value of $\mathcal{U}_0$. The value function $\mathcal{U}_0(C, D, x, y, \sigma^2)$ is the solution for the recursive functional equation

$$\mathcal{U}_t = \left\{(1-\beta)u(C_t, D_t)^{\frac{1-\gamma}{\theta}} + \beta\left(\mathbb{E}_t[\mathcal{U}_{t+1}^{1-\gamma}]^{\frac{1}{\theta}}\right)\right\}^{\frac{\theta}{1-\gamma}}.$$

Noting that value function $\mathcal{U}$ can be rewritten as $\mathcal{U}(C, D, x, y, \sigma^2) = C\,\mathcal{H}(z, x, y, \sigma^2)$, where $z = D/C$, the function equation above is

$$\mathcal{H}_t(z_t, x_t, y_t, \sigma_t^2) =$$
$$\left\{(1-\beta)\tilde{u}(z_t)^{\frac{1-\gamma}{\theta}} + \beta e^{\left(1-\frac{1}{\psi}\right)\left(\mu_c + x_t + \frac{\theta}{2}\sigma_t^2\right)}\left(\mathbb{E}_t[\mathcal{H}_{t+1}^{1-\gamma}(z_{t+1}, x_{t+1}, y_{t+1}, \sigma_{t+1}^2)]^{\frac{1}{\theta}}\right)\right\}^{\frac{\theta}{1-\gamma}}$$

where $\mathbb{E}_t$ is the expectation conditional on state variables $z_t, x_t, y_t$ and $\sigma_t^2$, and $\tilde{u}(z_t) = \tilde{u}(D_t/C_t) = u(C_t, D_t)/C_t$. We approximate $\mathcal{H}$ by Chebyshev polynomials and solve the functional equation using orthogonal collocation method; the expectation is approximated by Gauss-Hermite quadrature. We then run Monte-Carlo simulations with a fixed time horizon $T$ and pass $\mathcal{U}_0$ as the continuation value at time $T$ to obtain both $\mathcal{U}_0^*$ and $\bar{\mathcal{U}}_0$.

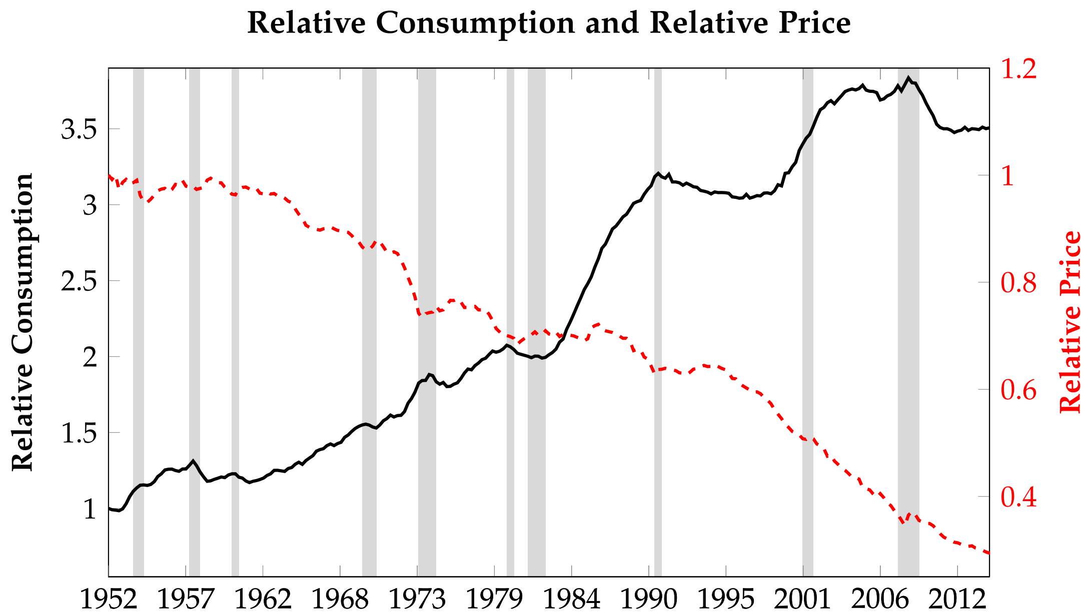
## 3.3 Empirical Analysis

**Data.** The sample period of all data is 1947:Q1–2014:Q4. Personal consumption data is from the US National Income and Product Accounts Bureau of Economic Analysis (BEA). We measure non-durable consumption as the sum of personal consumption expenditures on non-durable goods and services. This measure includes food, clothing items, housing and utilities, health care services, transportation.

Durable consumption includes motor vehicles and parts, furnishings and durable household equipment, recreational goods and services, jewelry and watches. Since the BEA reports only annual series for consumers stock of durable goods, we interpolate the quarterly series by assuming that the depreciation rate is constant within year, such that the implied value of the depreciation rate is consistent with annual stocks of durable goods both at the beginning and at the end of the year, and with quarterly series of personal consumption expenditure (PCE) on durable goods.

Figure 3.3.1 plots the durable consumption as a ratio of non-durable consumption (black solid line) from 1952:I to 2014:IV. The time series exhibits an upward trend during the sample period, with the value of durable consumption relative to non-durable consumption in 2014:IV being about 3.5 larger than corresponding value in 1952:I. The upward trend in the series is also consistent with the downward trend in price of durable goods relative to non-durable goods (red dashed line in Figure 3.3.1).

FIGURE 3.3.1: **Relative Consumption and Price.** Time series plot of durable consumption as a ratio of nondurable consumption (black solid line), and relative price of durable to nondurable consumption (red dashed line). The sample period is 1952:I - 2014:IV, 1952:I values are normalized to 1. The shaded areas indicate NBER recessions.




US Population data are retrieved from Federal Reserve Bank of St. Louis to obtain the per-capita quantities. The returns on the stock market and the short-term interest rate are from the Center for Research in Security Prices (CRSP). All asset returns are deflated with the PCE price index for non-durable consumption. The real dividend series are from Robert Shiller's website. We construct the ex-ante real risk-free as a fitted value from a projection of ex post real rate on the current nominal yield and inflation over the previous year (nominal yield is the CRSP Fama Risk Free Rate, inflation is CPI rate available from CRSP).

### 3.3.1 Quantitative Assessment

**State-Space Representation and Bayesian Inference.** The non-linear state-space system consists of a measurement and a transition equation, determined by (3.2.4)–(3.2.7). The measurement equation can be written as

$$y_{t+1} = M + Zs_{t+1} \tag{3.3.1}$$

with

$$y_{t+1} = \begin{pmatrix} \Delta C_{t+1} \\ \Delta D_{t+1} \\ \Delta S_{t+1} \end{pmatrix}, \; s_{t+1} = \begin{pmatrix} x_t \\ y_t \\ \sigma_t \varepsilon^c_{t+1} \\ \sigma_t \varepsilon^d_{t+1} \\ \sigma_t \varepsilon^s_{t+1} \end{pmatrix}, \; M = \begin{pmatrix} \mu_c \\ \mu_d \\ \mu_s \end{pmatrix}, \; Z = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & \psi_d & 0 \\ \phi_x & \phi_y & \pi_c & \pi_d & \psi_s \end{pmatrix}.$$

The transition equation is

$$\begin{aligned} s_{t+1} &= \Phi s_t + v_{t+1}(h_t) \\ h_{t+1} &= \Psi h_t + \Sigma_h \varepsilon^h_{t+1} \end{aligned} \tag{3.3.2}$$

where

$$v_{t+1}(h_t) = \begin{pmatrix} \psi_x \sigma_t \varepsilon^x_{t+1} \\ \psi_y \sigma_t \varepsilon^y_{t+1} \\ \sigma_t \varepsilon^c_{t+1} \\ \sigma_t \varepsilon^d_{t+1} \\ \sigma_t \varepsilon^s_{t+1} \end{pmatrix}, \quad \Phi = \begin{pmatrix} \rho_x & 0 & 0 & 0 & 0 \\ 0 & \rho_y & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \Psi = \rho_h, \quad \Sigma_h = \sigma_h \sqrt{1 - \rho_h^2}.$$

We extend the approach of Schorfheide, Song and Yaron (2018) to estimate the parameter vector

$$\Theta = \left(\rho_x, \psi_x, \psi_d, \rho_y, \psi_y, \phi_y, \pi_c, \pi_d, \psi_s, \rho_h, \sigma_h\right).$$

of the system (3.3.1)-(3.3.2). To generate draws from the posterior distribution of $\Theta$ given the data $Y$ (growth rates of non-durable consumption, durable consumption, and dividends), $\mathbb{P}(\Theta|Y)$, we specify the prior distribution $\mathbb{P}(\Theta)$ and numerically evaluate the likelihood function $\mathbb{P}(Y|\Theta)$. As the volatility processes affect the conditional mean and the volatility of asset prices, one would have to carry out a non-linear estimation of the state space model. Fortunately, one can avoid applying non-linear filtering because, conditional on the volatility state $h_t$, the state-space model can be recast in linear form and is Gaussian. This approximation can be done using a computationally efficient particle filter in which the particle values of $s_t$ are replaced by the mean and covariance matrix of the conditional distribution $s_t|(h_t, Y_{1:t})$ which we obtain by linear Kalman filtering. We then insert the approximation $\hat{\mathbb{P}}(Y|\Theta)$ into a standard Metropolis-Hastings algorithm to generate $\mathbb{P}(\Theta|Y)$ (Andrieu, Doucet and Holenstein, 2010, show that use of the approximation in the MCMC algorithms still delivers draws from the true posterior distribution).

**Parameter Estimates.** Uninformative priors are chosen for the estimation. All parameters have a uniform distribution as a prior except for the volatility of the volatility parameter $\sigma_h^2$ where the Inverse-Gamma distribution is chosen and for loading parameters where highly dispersed priors between 0 and 10 (or 0 and 20) are chosen. For the persistence coefficients we also choose dispersed priors: the 90 percent credible interval for $\rho_x$ and $\rho_y$ ranges from 0.71 to 0.99 and covers the values reported in Bansal and Yaron (2004); Bansal, Kiku and Yaron (2009); Eraker, Shaliastovich and Wang (2016); Schorfheide, Song and Yaron (2018); Yang (2011).

Table 3.3.1 reports the 90 percent credible intervals for the priors of the parameters as well as the percentiles of the posterior distribution for the estimated parameters.

TABLE 3.3.1: Estimated Coefficients of the Endowment Process.

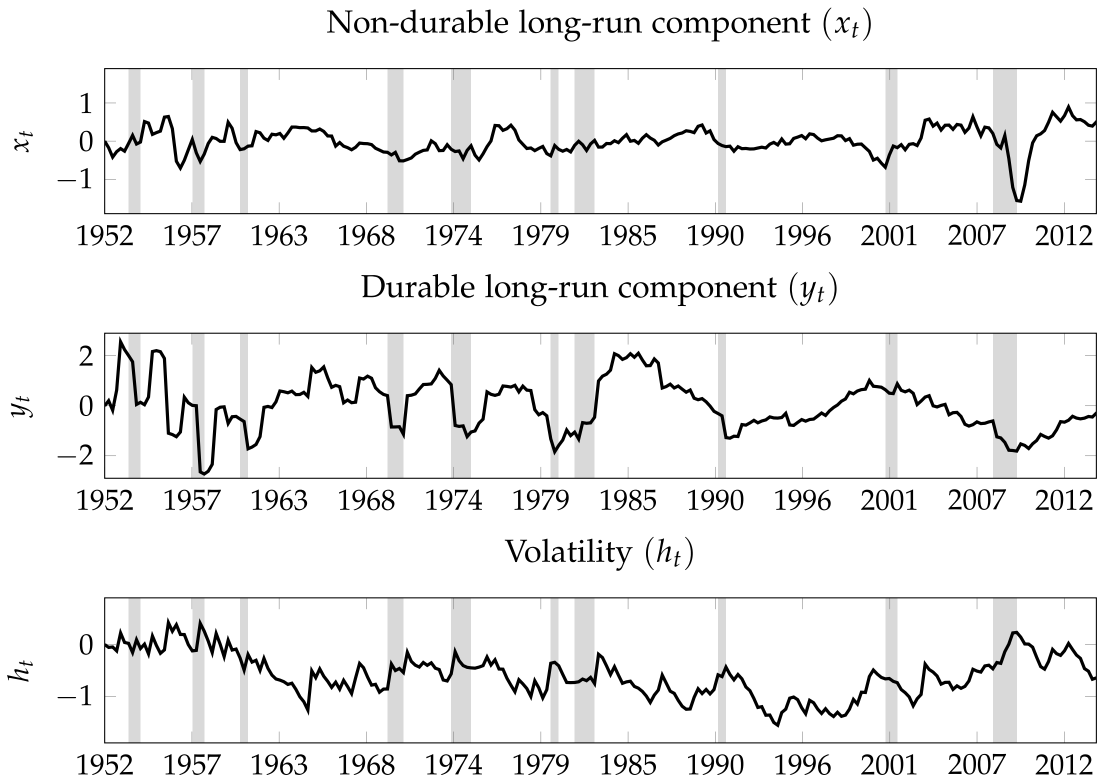
| **Parameter** | **Distribution** | **Prior** | | **Posterior** | | |
|---|---|---|---|---|---|---|
| | | 5% | 95% | 5% | 50% | 95% |
| $\rho_x$ | Uniform | 0.71 | 0.99 | 0.79 | 0.85 | 0.91 |
| $\psi_x$ | Uniform | 0.05 | 9.95 | 0.25 | 0.31 | 0.38 |
| $\psi_d$ | Uniform | 0.05 | 9.95 | 0.01 | 0.05 | 0.13 |
| $\rho_y$ | Uniform | 0.71 | 0.99 | 0.88 | 0.91 | 0.95 |
| $\psi_y$ | Uniform | 0.05 | 9.95 | 0.60 | 0.69 | 0.79 |
| $\phi_y$ | Uniform | 1 | 19 | 0.40 | 0.69 | 0.80 |
| $\pi_c$ | Uniform | 1 | 19 | 0.00 | 0.03 | 0.12 |
| $\pi_d$ | Uniform | 1 | 19 | 0.08 | 0.60 | 1.22 |
| $\psi_s$ | Uniform | 0.05 | 9.95 | 0.13 | 0.83 | 1.26 |
| $\rho_h$ | Uniform | 0.90 | 0.99 | 0.93 | 0.96 | 0.98 |
| $\sigma_h^2$ | Inverse-Gamma | 0.06 | 0.37 | 0.24 | 0.40 | 0.67 |

Estimation results are based on quarterly consumption and dividend data from 1952:Q1 to 2014:Q4. Parameter values $\mu_c = 0.0049, \mu_d = 0.0083$ and $\mu_s = 0.0016$ are set at their sample averages, further $\bar{\sigma} = 0.0096$ and $\phi_x = 4$.

The posterior estimates of the persistence of the long-run components are $\rho_x = 0.85$ and $\rho_y = 0.91$. The long-run component of the durable good is more persistent but also more volatile ($\psi_y = 0.69$) than that of the non-durable good ($\psi_x = 0.31$). Dividends depend on both non-durable and durable long-run components, with the larger effect coming from the non-durable long-run component (setting $\phi_x = 4$, we estimate $\phi_y = 0.69$, similar to Yang, 2011). We also find a non-zero loading on dividend growth from the noise to non-durable and durable consumption, with $\pi_c$ and $\pi_d$ both being strictly positive. Finally, the volatility process is highly persistent, with $\rho_h = 0.96$.

Figure 3.3.2 depicts filtered estimates of the predictable long-run components $x_t$ (top panel) and $y_t$ (middle panel) and of the implied volatility state $h_t$ (bottom panel). Both $x_t$ and $y_t$ tend to fall sharply during the recessions and tend to recover immediately after the recession. Sudden increases in volatility $h_t$ are often associated with NBER recessions.

FIGURE 3.3.2: **Filtered Mean and Volatility States.** The figure depicts filtered mean (top and middle panel) and volatility (bottom panel) states. Shaded areas represent NBER recessions.




### Estimating the Elasticity of Substitution

Equation (3.2.10) allows estimating the elasticity of substitution $\rho$ directly from the data. Taking logarithms of equation (3.2.10) we get

$$\log\left(\frac{\alpha}{1-\alpha}\right) + \frac{1}{\rho}(c_t - d_t) - p_t = q_t - p_t$$

The lowercase variable denotes the logarithm of the corresponding uppercase variable. Assuming that the user cost and the spot price of durable goods are cointegrated (so that $q_t - p_t$ is stationary) implies that $c_t - d_t$ and $p_t$ are cointegrated with the cointegrating vector equal to $(1, -\rho)$. Hence, we can estimate the elasticity of substitution without observing the user cost of durable goods (see Ogaki and Reinhart, 1998, where $\rho$ is estimated by regressing $c_t - d_t$ on $p_t$). We estimate the elasticity of substitution by a dynamic ordinary least square regression of $c_t - d_t$ on $p_t$ with four leads and lags as proposed by Stock and Watson (1993):

$$c_t - d_t = const. + \rho\, p_t + \sum_{s=-4}^{4} b_{p,s} \Delta p_{t-s} + \varepsilon_t.$$

For the full sample 1952:I - 2014:IV our estimate of $\rho = 0.78$ with standard error of 0.03. We test *the null hypothesis of no composition* $H_0 : \ \rho = 1$. The t-statistics is $t = -6.85$ and thus we *reject the hypothesis of no composition* on 1 percent significance level.

### Estimating the Linear Model

An analytical solution is derived through a linear approximation to the conditional volatility process (3.2.7) and assuming that volatility is given by a process that has a Gaussian distribution:

$$\begin{aligned}\sigma_{t+1}^2 &\approx \bar{\sigma}^2(1-\rho_h) + \rho_h \sigma_t^2 + 2\bar{\sigma}^2 \sigma_h \sqrt{1-\rho_h^2} w_{t+1} \\ &= \hat{\sigma} + \rho_h \sigma_t^2 + \sigma_w w_{t+1}.\end{aligned}$$

We derive the asset prices using the standard asset pricing condition

$$\mathbb{E}_t \left[ \exp(m_{t+1} + r_{i,t+1}) \right] = 1$$

for any asset $r_{i,t+1} = \log\left(R_{i,t+1}\right)$, where the log-pricing kernel of the economy is

$$m_{t+1} = \theta \log \beta - \frac{\theta}{\psi} \Delta c_{t+1} + \theta \left( \frac{1}{\rho} - \frac{1}{\psi} \right) \Delta f_{t+1} + (\theta - 1)\, r_{w,t+1}.$$

$r_{w,t+1}$ is the log return on the consumption claim, and $r_{m,t+1}$ is log market return. We use the approximation of Campbell and Shiller (1988*b*) for the returns:

$$r_{w,t+1} = z_{w,t+1} - \kappa_0 - \kappa_1 z_{w,t} - z_t + \Delta c_{t+1}$$
$$r_{m,t+1} = \kappa_0^m + \kappa_1^m z_{m,t+1} - z_{m,t} + \Delta s_{t+1}$$

where $z_t = \log(D_t/C_t)$, $z_{w,t}$ is the log-wealth-consumption ratio, and $z_{m,t}$ is the log-price-dividend ratio. The solution to the log-wealth-consumption ratio and to the log-price-dividend ratio is linear in states:

$$\begin{aligned} z_{w,t} &= A_0 + A_1 x_t + A_2 y_t + A_3 z_t + A_4 \sigma_t^2 \\ z_{m,t} &= B_0 + B_1 x_t + B_2 y_t + B_3 z_t + B_4 \sigma_t^2 \end{aligned} \qquad (3.3.3)$$

with functions $A_k, B_k$, $k = 0, ..., 4$ that depend on the preference parameters (see Appendix 3.B for their derivation). Given the solution, we can derive analytical expressions for both the market return and for the risk-free rate.

We use the analytical solution of the linear model estimate the set of preference parameters

$$\Lambda = (\gamma, \psi, \beta, \alpha)$$

where $\gamma$ is the risk aversion coefficient, $\psi$ is the elasticity of the intertemporal substitution, $\beta$ is the subjective discount factor, and $\alpha$ is the share of durable consumption in the intraperiod utility function. We estimate $\Lambda$ by solving a sample minimum distance problem with the identity weighting matrix. We simulate 100,000 samples of length equal to our sample size and use those to calculate the market and the risk-free returns and estimate $\Lambda$ to reflect values that are required to match first two unconditional moments of the market and risk-free returns.

Table 3.3.2 (Panel A, Linear Model) reports the values of the estimated parameters. The estimate of risk aversion coefficient $\gamma$ is around 3 and the estimate of the elasticity of intertemporal substitution $\psi$ is around 1.3. We also find that the subjective discount factor is estimated at $\beta = 0.9985$ and the share of durable consumption in the intraperiod utility function $\alpha$ is about 30 percent. Table 3.3.2 (Panel B, Linear Model) reports the simulated moments. The simulated mean of the risk-free rate and of the risky return is about 1 and 6 percent, which is close to the values observed in the data. The linear model also generates a high volatility of the risky return (about 20 percent compared to the 19 percent that is observed in the data) and a high value of price-dividend ratio. The linear model fails to reproduce the standard deviation of risk-free rate and the standard deviation of price-dividend ratio.

### Semi-parametric Estimation of the Preference Parameters

Pohl, Schmedders and Wilms (2018) argue that numerical errors that are introduced in the LRRM using the Campbell-Shiller linearization are economically and statistically significant and could lead to wrong model predictions. We re-estimate the model taking into account all possible nonlinear effects. We employ a semiparametric estimation methodology similar to that of Chen, Favilukis and Ludvigson (2013) and we proceed in two steps. On the first step, for a fixed value of preference parameters, we approximate the unknown wealth-consumption and price-dividend ratios as a series of Chebyshev polynomials and we nonparametrically estimate these functions using the wealth-Euler and the Euler equation. On the second step, given the estimate of these functions, we estimate the preference parameters by a sample minimum distance estimator (analog of GMM). The further details are in Appendix 3.A.

TABLE 3.3.2: Estimated Preference Parameters and Unconditional Moments of Returns.

| **A.** Estimated Preference Parameters | | |
|---|---|---|
| | Linear Model | Full Model |
| Risk aversion | $\gamma = 2.78$ | $\gamma = 1.86$ |
| IES | $\psi = 1.29$ | $\psi = 1.18$ |
| Subjective discount factor | $\beta = 0.998$ | $\beta = 0.991$ |
| Share of durable consumption | $\alpha = 0.30$ | $\alpha = 0.15$ |

| **B.** Unconditional Moments of Returns | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Data | Linear Model | | | Full Model | | |
| | | 5% | 50% | 95% | 5% | 50% | 95% |
| Mean$(r_f)$ | 0.95 | 0.69 | 1.08 | 1.62 | -0.59 | 1.01 | 8.16 |
| StdDev$(r_f)$ | 1.61 | 0.26 | 0.36 | 0.49 | 1.08 | 1.61 | 3.40 |
| Mean$(r_m)$ | 5.57 | 2.50 | 6.21 | 9.84 | 1.66 | 5.78 | 10.89 |
| StdDev$(r_m)$ | 18.94 | 16.16 | 20.52 | 25.44 | 13.03 | 16.85 | 20.97 |
| Mean$(p-d)$ | 4.93 | 4.20 | 4.27 | 4.35 | 3.83 | 4.60 | 5.20 |
| StdDev$(p-d)$ | 0.38 | 0.12 | 0.17 | 0.23 | 0.18 | 0.31 | 0.58 |

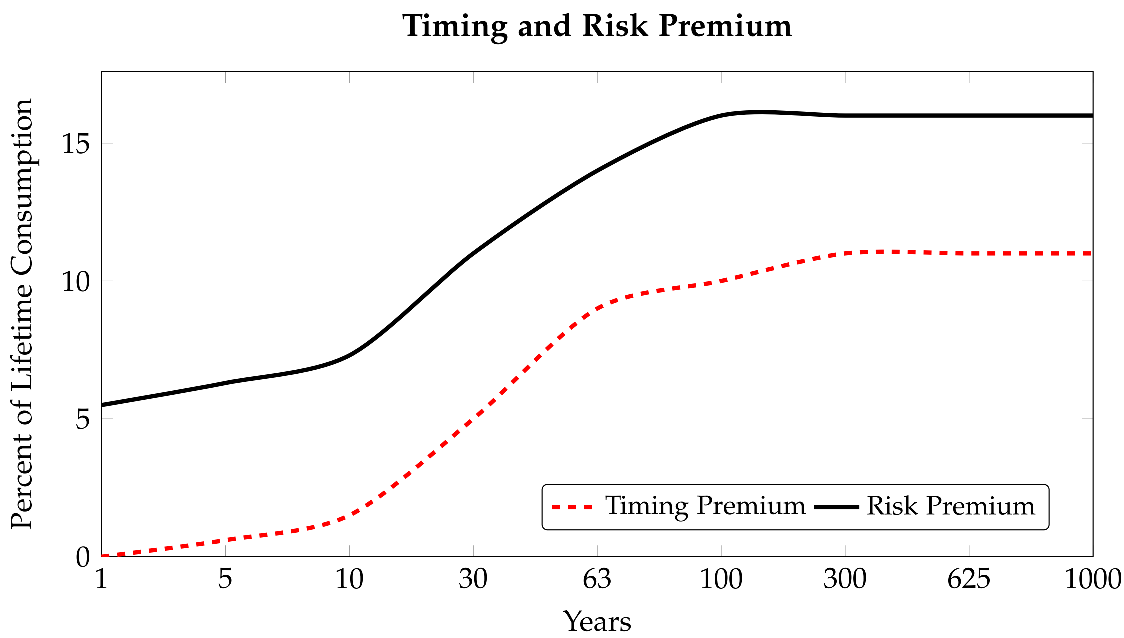
| **C.** Timing and Risk Premia | |
|---|---|
| Timing Premium | $\pi^* = 11\%$ |
| Risk Premium | $\bar{\pi} = 16\%$ |

Table 3.3.2 (Panel A, Full Model) reports the values of the estimated parameters of a full model. The estimate of risk aversion coefficient $\gamma$ is 1.86 and the estimate of the elasticity of intertemporal substitution $\psi$ is around 1.1865. We also find that the subjective discount factor is estimated at $\beta = 0.9914$ and the share of durable consumption in the intraperiod utility function $\alpha$ is about 15.5 percent. Table 3.3.2 (Panel B, Full Model) reports the model implied simulated moments. The simulated mean of the risk-free rate and of the risky return is 1.01 percent and 5.78 percent which close to the values observed in the data. The model implied volatility of risky return is about 17 percent (compared to 19 percent observed in the data) and the volatility of the risk-free rate is about 1.61 percent which is identical to that observed in the data.

The model also matches the mean and the volatility of the price-dividend ratio. All the data moments lie comfortably inside the corresponding model implied 95 percent confidence intervals.

Timing and risk premia for different time horizons in the simulated model are presented in Figure 3.3.3. $T = 30$ years corresponds to the duration of US Treasury bonds, $T = 63$ years corresponds to the sample size of the data used, and long time horizons are $T = 100, 300, 625$ and $1,000$ years. The timing premium increases from 5 percent for 30 years to 11 percent for 300 years and remains at that level for all longer time horizons. The risk premium increases from 11 percent for a 30 year time horizon to 16 percent for 100 years and then stays at that level. For $1,000$ years (4,000 periods) the model generates a timing premium of 11 percent and a risk premium of 16 percent (see also Table 3.3.2, Panel C).

FIGURE 3.3.3: **Timing Premium and Risk Premium.** The figure displays the timing premium (dashed line) and the risk premium (solid line) as functions of time horizon.




To further understand the contribution of durable consumption to these results, several scenarios are analyzed. First, we re-estimate both the linear model and the full model when the composition effect is absent (which is achieved by fixing $\rho = 1$ in the model). Table 3.3.3 contains the results. Comparing with the benchmark case in Table 3.3.2, one finds that without the composition effect the values of the estimated parameters as well as the estimated moments change substantially. Risk aversion and intertemporal elasticity of substitution (IES) are much higher, and, moreover, the model fails to match some of the asset pricing moments. There are also substantial differences between the linear and full model, e.g., the linear model fails to match the mean values of the risk-free and equity returns. While the model without composition effect generates a low value of the timing premium, it generates an unreasonably high value of the risk premium, about twice as high as in Bansal and Yaron (2004). The results in Table 3.3.3 suggest that the composition effect plays an important role for the estimated preference parameters and for matching the asset markets moments.

TABLE 3.3.3: No Composition Risk ($\rho = 1$)

| **A.** Preference Parameters | | |
|---|---|---|
| | **Linear Model** | **Full Model** |
| Risk aversion | $\gamma = 15.35$ | $\gamma = 4.83$ |
| IES | $\psi = 1.18$ | $\psi = 1.79$ |
| Subjective discount factor | $\beta = 0.997$ | $\beta = 0.998$ |
| Share of durable consumption | $\alpha = 0.42$ | $\alpha = 0.47$ |

| **B.** Unconditional Moments of Returns (in percent) | | | | | | | |
|---|---|---|---|---|---|---|---|
| | **Data** | **Linear Model** | | | **Full Model** | | |
| | | 5% | 50% | 95% | 5% | 50% | 95% |
| Mean$(r_f)$ | 0.95 | -0.87 | 0.34 | 1.39 | -0.99 | 0.26 | 1.31 |
| StdDev$(r_f)$ | 1.61 | 0.90 | 1.27 | 1.70 | 0.91 | 1.27 | 1.71 |
| Mean$(r_m)$ | 5.57 | -3.09 | 0.72 | 4.78 | 2.67 | 6.20 | 10.04 |
| StdDev$(r_m)$ | 18.94 | 14.61 | 18.88 | 23.41 | 13.46 | 17.43 | 21.61 |
| Mean$(p-d)$ | 4.93 | 8.43 | 8.49 | 8.55 | 4.22 | 4.28 | 4.33 |
| StdDev$(p-d)$ | 0.38 | 0.11 | 0.15 | 0.21 | 0.10 | 0.14 | 0.19 |

| **C.** Timing and Risk Premia | |
|---|---|
| Timing Premium | $\pi^* = 4\%$ |
| Risk Premium | $\bar{\pi} = 52\%$ |

We also analyze what happens to the asset pricing moments when the preference parameter values (risk aversion, IES and subjective discount factor) are those reported in Bansal and Yaron (2004). As we want to compare the original LRRM with extended model with durable consumption, we set the share of durable consumption to that obtained in our estimated linear model. The values of these preferences parameters are reported in Table 3.3.4 (Panel A).

Table 3.3.4 (Panels B and C) contains the results. Two main observations can be made. First, the model generates a very high risk-free rate of about 5 percent per year and an equity return in excess of 100 percent per year. Second, the model generates a timing premium of 69 percent and a risk premium of 80 percent. These numbers suggest that the presence of durable consumption lowers the values of the risk aversion and the EIS required to match key financial data moments and to generate reasonable values of timing and risk premia.

TABLE 3.3.4: Calibration of Bansal & Yaron (2004) Model with Durable Consumption Good

| **A.** Preference Parameters | | |
|---|---|---|
| Risk aversion | $\gamma = 7.5$ | |
| EIS | $\psi = 1.5$ | |
| Subjective discount factor | $\beta = 0.998$ | |
| Share of durable consumption | $\alpha = 0.30$ | |
| **B.** Unconditional Moments of Returns (in percent) | | |
| | Data | Model |
| Mean$(r_f)$ | 0.95 | 5.07 |
| StdDev$(r_f)$ | 1.61 | 1.66 |
| Mean$(r_m)$ | 5.57 | 103.75 |
| StdDev$(r_m)$ | 18.94 | 9.43 |
| Mean$(p-d)$ | 4.93 | 1.22 |
| StdDev$(p-d)$ | 0.38 | 0.07 |
| **C.** Timing and Risk Premia | | |
| Timing Premium | $\pi^* = 69\%$ | |
| Risk Premium | $\bar{\pi} = 80\%$ | |

## 3.4 Conclusion

We introduce a long-run risk model (LRRM) where durable and non-durable consumption goods are non-separable and gross complements, thus generating households' concern with short and long run composition risk, that is, fluctuations in the relative share of durables in their consumption basket. We show that our model matches the key stylized facts of financial markets and at the same time generates levels of timing and risk premia that are consistent with the conventional macroeconomic wisdom. In its benchmark calibration, our model matches financial data well with a risk aversion of 1.86, an elasticity of intertemporal substitution of 1.18 and an elasticity of substitution between durable and non-durable goods of 0.78. With this parametrization the timing premium is 11 percent and the risk premium is 16 percent. Compared to the its single consumption good counterpart, our model reduces the timing and risk premia by more than 50 percent. The paper holds two main lessons for financial economists: (a) the importance of durable consumption in obtaining reasonable timing and risk premia in LRRM and (b) the potential pitfalls in using linearization in an ill-specified LRRM.

# Appendix

## 3.A Solving the Non-Linear Model

In our estimation exercise, we use Euler equation to back out the asset returns as a function of model parameters and fully estimate these parameters. As the Euler equation does not admit analytical solution, we rely on numerical methods to fully estimate the parameters of the model. We proceed in several steps. First, we analytically derive the pricing kernel of the model, as well as price-dividend and wealth-consumption ratios. Second, we express returns in the model as functions of the price-dividend and wealth-consumption ratios, derived in the previous step, using a simple asset pricing identity. Thirdly, we approximate these ratios (and as a consequence the returns in the model) as a series of Chebyshev polynomials and apply projection methods to Euler equation to numerically derive the price-dividend and wealth-consumption ratio as a function of model parameters alone. This, in turn, allows us to estimate model parameters using the techniques described in main text.

### 3.A.1 Pricing Kernel

We first analytically derive the pricing kernel of the economy. Define the return on total consumption as

$$R_{W,t+1} = \frac{\widetilde{W}_{t+1}}{\widetilde{W}_t - C_t - Q_t D_t}$$

where total consumption $G_t$ is given by

$$G_t = C_t + Q_t D_t$$

and $Q_t$ denotes the user cost of the service flow for the durable good. Following Yogo (2006), $Q_t$ is given as a marginal rate of substitution between non-durable and durable consumption good

$$Q_t = \frac{\partial \mathcal{C}_t}{\partial D_t} \bigg/ \frac{\partial \mathcal{C}_t}{\partial C_t}.$$

Given the functional form for $\mathcal{C}_t$, we get

$$Q_t = \frac{\alpha}{1-\alpha} \left(\frac{D_t}{C_t}\right)^{-\frac{1}{\rho}}.$$

Define

$$F_t = \left(1 - \alpha + \alpha \left(\frac{D_t}{C_t}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}$$

then the intertemporal marginal rate of substitution can be written as

$$M_{t+1} = \beta^{\theta} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} \left(\frac{F_{t+1}}{F_t}\right)^{\theta\left(\frac{1}{\rho}-\frac{1}{\psi}\right)} R_{W,t+1}^{\theta-1}. \tag{3.A.1}$$

Furthermore,

$$R_{W,t+1} = \frac{\widetilde{W}_{t+1}}{\widetilde{W}_t - G_t} = \frac{\frac{\widetilde{W}_{t+1}}{G_{t+1}}}{\frac{\widetilde{W}_t}{G_t} - 1} \frac{G_{t+1}}{G_t}$$

where we can rewrite $G_t$ as

$$G_t = C_t + Q_t D_t = C_t + \frac{\alpha}{1-\alpha}\left(\frac{D_t}{C_t}\right)^{-\frac{1}{\rho}} D_t = C_t\left(1 + \frac{\alpha}{1-\alpha}\left(\frac{D_t}{C_t}\right)^{1-\frac{1}{\rho}}\right)$$

and $F_t$ as

$$F_t = (1-\alpha)^{\frac{1}{1-\frac{1}{\rho}}}\left(1 + \frac{\alpha}{1-\alpha}\left(\frac{D_t}{C_t}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}.$$

Substituting the terms in (3.A.1) using the above relations, we obtain

$$M_{t+1} = \beta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{\theta\left(1-\frac{1}{\psi}\right)-1}\left(\frac{A_{t+1}}{A_t}\right)^{\theta\left(1-\frac{1}{\psi}\right)-1}\left(\frac{\frac{\widetilde{W}_{t+1}}{G_{t+1}}}{\frac{\widetilde{W}_t}{G_t}-1}\right)^{\theta-1}$$

where

$$A_t = 1 + \frac{\alpha}{1-\alpha}\left(\frac{D_t}{C_t}\right)^{1-\frac{1}{\rho}}.$$

The evolution of $D_{t+1}/C_{t+1}$ (which enters $A_{t+1}$) can be written as

$$\frac{D_{t+1}}{C_{t+1}} = \frac{D_{t+1}/D_t \cdot D_t}{C_{t+1}/C_t \cdot C_t} = \frac{D_{t+1}}{D_t}\left(\frac{C_{t+1}}{C_t}\right)^{-1}\frac{D_t}{C_t}.$$

Setting $z_t = \log(D_t/C_t)$, we find

$$z_{t+1} = \Delta D_{t+1} - \Delta C_{t+1} + z_t = \mu_d + y_t + \sigma_d \varepsilon^d_{t+1} - \mu_c - x_t - \sigma_x \varepsilon^x_{t+1} + z_t.$$

## 3.A.2 Application of Projection Method

We now show how the projection method can be applied to wealth-Euler equation to numerically derive wealth-consumption ratio as a function of model parameters alone.

We start with Euler equation for wealth

$$\mathbb{E}_t \left[M_{t+1}R_{W,t+1}\right] = 1$$

where

$$M_{t+1} = \beta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\theta/\psi}\left(\frac{v(D_{t+1}/C_{t+1})}{v(D_t/C_t)}\right)^{\theta(1/\rho-1/\psi)} R_{W,t+1}^{\theta-1}$$

and

$$v\left(\frac{D_t}{C_t}\right) = F_t = \left[1-\alpha+\alpha\left(\frac{D_t}{C_t}\right)^{1-1/\rho}\right]^{1/(1-1/\rho)}.$$

Here, $R_{W,t+1}$ is the return on wealth. In logarithms, the Euler equation for wealth becomes

$$\mathbb{E}_t\left[\exp\left(\theta\log\beta - \frac{\theta}{\psi}\Delta c_{t+1} + \theta\left(\frac{1}{\rho}-\frac{1}{\psi}\right)\Delta f_{t+1} + (\theta-1)r_{w,t+1}\right)\right] = 1,$$

where lowercase variables denote the logs of the corresponding uppercase variables, and $\Delta c_{t+1} = c_{t+1} - c_t$ and $\Delta f_{t+1} = f_{t+1} - f_t$. Log-return on wealth $r_{w,t+1}$ can be further written as

$$r_{w,t+1} = \log\left(\frac{\widetilde{W}_{t+1}}{\widetilde{W}_t - C_t - Q_t D_t}\right) = \log\left(\frac{\frac{\widetilde{W}_{t+1}}{C_{t+1}}}{\frac{\widetilde{W}_t}{C_t} - 1 - Q_t\frac{D_t}{C_t}} \times \frac{C_{t+1}}{C_t}\right)$$

$$= wc_{t+1} - log\left(wc_t - 1 - Q_t\frac{D_t}{C_t}\right) + \Delta c_{t+1},$$

where $wc_t = \log(\widetilde{W}_t/C_t)$ is the log-wealth-consumption ratio.

We can then approximate today's and tomorrow's wealth-consumption ratio as a series of Chebyshev polynomials and substitute it back to the log-version of the wealth-Euler equation; we can then apply projection methods to numerically solve for wealth-consumption ratio.

## 3.B Solving the Linear Model

An analytical solution to the linear model is obtained using a linear approximation to the conditional volatility process (3.2.7) and expressing volatility as a process that follows a Gaussian distribution:

$$\begin{aligned}\sigma_{t+1}^2 &\approx \bar{\sigma}^2(1-\rho_h) + \rho_h\sigma_t^2 + 2\bar{\sigma}^2\sigma_h\sqrt{1-\rho_h^2}w_{t+1} \\ &= \hat{\sigma} + \rho_h\sigma_t^2 + \sigma_w w_{t+1}.\end{aligned}$$

The endowment process for the economy is then given by

$$\begin{aligned}\Delta C_{t+1} &= \mu_c + x_t + \sigma_t\varepsilon_{t+1}^c \\ \Delta D_{t+1} &= \mu_d + y_t + \psi_d\sigma_t\varepsilon_{t+1}^d \\ \Delta S_{t+1} &= \mu_s + \phi_x x_t + \phi_y y_t + \pi_c\sigma_t\varepsilon_{t+1}^c + \pi_d\sigma_t\varepsilon_{t+1}^d + \psi_s\sigma_t\varepsilon_{t+1}^s \\ x_{t+1} &= \rho_x x_t + \psi_x\sigma_t\varepsilon_{t+1}^x \\ y_{t+1} &= \rho_y y_t + \psi_y\sigma_t\varepsilon_{t+1}^y \\ \sigma_{t+1}^2 &= \hat{\sigma} + \rho_h\sigma_t^2 + \sigma_w w_{t+1} \\ &\varepsilon_{t+1}^c, \varepsilon_{t+1}^d, \varepsilon_{t+1}^s, \varepsilon_{t+1}^x\varepsilon_{t+1}^y, w_{t+1} \sim \mathcal{N}(0,1).\end{aligned}$$

We derive the asset prices using the standard asset pricing condition

$$\mathbb{E}_t\left[e^{m_{t+1}+r_{i,t+1}}\right] = 1$$

for any asset $r_{i,t+1} = \log(R_{i,t+1})$, where the log-pricing kernel of the economy is

$$m_{t+1} = \theta\log\beta - \frac{\theta}{\psi}\Delta c_{t+1} + \theta\left(\frac{1}{\rho} - \frac{1}{\psi}\right)\Delta f_{t+1} + (\theta-1)\, r_{w,t+1}.$$

$r_{w,t+1}$ is the log return on the consumption claim, and $r_{m,t+1}$ is log market return. We use the approximation of Campbell and Shiller (1988*b*) for the returns:

$$r_{w,t+1} = z_{w,t+1} - \kappa_0 - \kappa_1 z_{w,t} - z_t + \Delta c_{t+1}$$

$$r_{m,t+1} = \kappa_0^m + \kappa_1^m z_{m,t+1} - z_{m,t} + \Delta s_{t+1}$$

where $z_t = \log(D_t/C_t)$, $z_{w,t}$ is the log-wealth-consumption ratio and $z_{m,t}$ is the log-price-dividend ratio. The approximating constants are given by

$$\kappa_0 = \log\left(e^{\bar{z}_w} - 1 - q(\bar{z})\right) + \frac{1}{e^{\bar{z}_w} - 1 - q(\bar{z})}\left[-e^{\bar{z}_w}\bar{z}_w - \frac{\alpha}{1-\alpha}\left(1 - \frac{1}{\rho}\right)e^{\left(1-\frac{1}{\rho}\right)\bar{z}}\bar{z}\right]$$

and

$$\kappa_0^m = \log\left(1 + e^{\bar{z}_m}\right) - \frac{e^{\bar{z}_m}\bar{z}_m}{1 + e^{\bar{z}_m}}, \quad \kappa_1^m = \frac{e^{\bar{z}_m}}{1 + e^{\bar{z}_m}}.$$

## 3.B.1 Consumption Claim

We conjecture that the log-wealth-consumption ratio $z_{w,t}$ is a linear function of state variables

$$z_{w,t} = A_0 + A_1 x_t + A_2 y_t + A_3 z_t + A_4 \sigma_t^2.$$

Then

$$\begin{aligned} r_{w,t+1} =& z_{w,t+1} - \kappa_0 - \kappa_1 z_{w,t} - \kappa_2 z_t + \Delta c_{t+1} \\ =& A_0 + A_1 x_{t+1} + A_2 y_{t+1} + A_3 z_{t+1} + A_4 \sigma^2_{t+1} \\ & - \kappa_0 - \kappa_1 A_0 - \kappa_1 A_1 x_t - \kappa_1 A_2 y_t - \kappa_1 A_3 z_t - \kappa_1 A_4 \sigma^2_t - \kappa_2 z_t + \Delta c_{t+1} \\ =& \{A_0(1-\kappa_1) - \kappa_0 + A_3(\mu_d - \mu_c) + A_4 \hat{\sigma} + \mu_c\} \\ & + \{A_1\rho_x - A_3 - \kappa_1 A_1 + 1\}\, x_t \\ & + \left\{A_2\rho_y + A_3 - \kappa_1 A_2\right\} y_t \\ & + \{A_3 - \kappa_1 A_3 - \kappa_2\}\, z_t \\ & + \{A_4\rho_h - \kappa_1 A_4\}\, \sigma^2_t \\ & + A_1\psi_x\sigma_t\varepsilon^c_{t+1} + A_2\psi_y\sigma_t\varepsilon^y_{t+1} + (1-A_3)\sigma_t\varepsilon^c_{t+1} + A_3\psi_d\sigma_t\varepsilon^d_{t+1} + A_4\sigma_w w_{t+1}. \end{aligned}$$

Using

$$\begin{aligned} \Delta f_{t+1} = & \underbrace{\frac{\rho}{\rho-1}\alpha \exp\left(\left(1-\frac{1}{\rho}\right)\bar{z}\right)\left(1-\frac{1}{\rho}\right)}_{K}(z_{t+1}-z_t) \\ & = K\left(\mu_d + y_t + \psi_d\sigma_t\varepsilon^d_{t+1} - \mu_c - x_t - \sigma_t\varepsilon^c_{t+1}\right) \end{aligned}$$

and

$$m_{t+1} = \theta\log\beta - \frac{\theta}{\psi}\Delta c_{t+1} + \theta\left(\frac{1}{\rho} - \frac{1}{\psi}\right)\Delta f_{t+1} + (\theta-1)\, r_{w,t+1}$$

we obtain

$$\begin{aligned} m_{t+1} = & \theta\log\beta - \frac{\theta}{\psi}\mu_c + \theta\left(\frac{1}{\rho}-\frac{1}{\psi}\right)K(\mu_c - \mu_d) \\ & + (\theta-1)(A_0(1-\kappa_1) - \kappa_0 + A_3(\mu_d-\mu_c) + A_4\hat{\sigma} + \mu_c) \\ & + \Big\{ -\frac{\theta}{\psi} - \theta\left(\frac{1}{\rho}-\frac{1}{\psi}\right)K + (\theta-1)\,(A_1\rho_x - A_3 - \kappa_1 A_1 + 1)\Big\} x_t \\ & + \Big\{\theta\left(\frac{1}{\rho}-\frac{1}{\psi}\right)K + (\theta-1)(A_2\rho_y + A_3 - \kappa_1 A_2)\Big\} y_t \end{aligned}$$

$$
\begin{aligned}
&+ \{(\theta-1)(A_3 - \kappa_1 A_3 - \kappa_2)\} z_t \\
&+ \{(\theta-1)(A_4 \rho_h - \kappa_1 A_4)\} \sigma_t^2 \\
&+ (\theta-1) A_1 \psi_x \sigma_t \varepsilon_{t+1}^x \\
&+ (\theta-1) A_2 \psi_y \sigma_t \varepsilon_{t+1}^y \\
&+ \left\{ (\theta-1)(1-A_3) - \frac{\theta}{\psi} - \theta \left( \frac{1}{\rho} - \frac{1}{\psi} \right) K \right\} \sigma_t \varepsilon_{t+1}^c \\
&+ \left\{ (\theta-1) A_3 + \theta \left( \frac{1}{\rho} - \frac{1}{\psi} \right) K \right\} \psi_d \sigma_t \varepsilon_{t+1}^d \\
&+ (\theta-1) A_4 \sigma_w w_{t+1}.
\end{aligned}
$$

Since both $m_{t+1}$ and $r_{w,t+1}$ are conditionally normal, the Euler equation for wealth can be written as

$$
\mathbb{E}_t \left[ m_{t+1} + r_{w,t+1} \right] + \frac{1}{2} \mathbb{V}\mathrm{ar}_t \left[ m_{t+1} + r_{w,t+1} \right] \approx 0.
$$

We use this equation to solve for the coefficients $A_0, ..., A_4$. These are

$$
\begin{aligned}
A_0 = & -\frac{1}{(\kappa_1 - 1)\ (\theta - 1)} \times \Bigg\{ (\theta-1)\ (\kappa_0 - \mu_c - A_4\, \hat{\sigma} + A_3\ (\mu_c - \mu_d)) - \theta \log \beta \\
& \qquad - \frac{A_4^2\, \sigma_w^2\, (\theta-1)^2}{2} + \frac{\mu_c\, \theta}{\psi} + K\, \theta \left( \frac{1}{\psi} - \frac{1}{\rho} \right) (\mu_c - \mu_d) \Bigg\} \\
A_1 = & \frac{1}{(\kappa_1 - \rho_x)\ (\theta - 1)} \times \left\{ \left( \frac{\kappa_2}{\kappa_1 - 1} + 1 \right) (\theta - 1) + K\, \theta \left( \frac{1}{\psi} - \frac{1}{\rho} \right) - \frac{\mu_c \theta}{\psi} \right\} \\
A_2 = & -\frac{1}{(\kappa_1 - \rho_y)\ (\theta - 1)} \times \left\{ K \theta \left( \frac{1}{\psi} - \frac{1}{\rho} \right) + \frac{\kappa_2\ (\theta - 1)}{\kappa_1 - 1} \right\} \\
A_3 = & -\kappa_2 / (\kappa_1 - 1) \\
A_4 = & \frac{1}{2\, (\kappa_1 - \rho_h)\ (\theta - 1)} \times \Bigg\{ {\psi_d}^2 \left( A_3\ (\theta - 1) - K\, \theta \left( \frac{1}{\Psi} - \frac{1}{\rho} \right) \right)^2 \\
& \qquad + \left( (A_3 - 1)\ (\theta - 1) - K\, \theta \left( \frac{1}{\Psi} - \frac{1}{\rho} \right) + \frac{\mu_c\, \theta}{\Psi} \right)^2 \\
& \qquad + {A_1}^2\, {\psi_x}^2\, (\theta - 1)^2 + {A_2}^2\, {\psi_y}^2\, (\theta - 1)^2 \Bigg\}.
\end{aligned}
$$

The innovation to $m_{t+1}$ is given by

$$m_{t+1} - \mathbb{E}_t[m_{t+1}] = \lambda_x \sigma_t \varepsilon^x_{t+1} + \lambda_y \sigma_t \varepsilon^y_{t+1} + \lambda_c \sigma_t \varepsilon^c_{t+1} + \lambda_d \sigma_t \varepsilon^d_{t+1} + \lambda_w \sigma_w w_{t+1},$$

where the coefficients $\lambda_\cdot$ represent the market price of risk for each source of risk:

$$\begin{aligned}\lambda_x =&(\theta-1)A_1\psi_x,\ \lambda_y = (\theta-1)A_2\psi_y,\ \lambda_c = (\theta-1)(1-A_3) - \frac{\theta}{\psi} - \theta\left(\frac{1}{\rho}-\frac{1}{\psi}\right)K,\\ \lambda_d =& \left((\theta-1)A_3 + \theta\left(\frac{1}{\rho}-\frac{1}{\psi}\right)K\right)\psi_d,\ \lambda_w = (\theta-1)A_4.\end{aligned}$$

Similarly, the innovation to $r_{w,t+1}$ is given by

$$r_{w,t+1} - \mathbb{E}_t[r_{w,t+1}] = -\beta_x \sigma_t \varepsilon^x_{t+1} - \beta_y \sigma_t \varepsilon^y_{t+1} - \beta_c \sigma_t \varepsilon^c_{t+1} - \beta_d \sigma_t \varepsilon^d_{t+1} - \beta_w \sigma_w w_{t+1},$$

where

$$\beta_x = -A_1\psi_x,\ \beta_y = -A_2\psi_y,\ \beta_c = -(1-A_3),\ \beta_d = -A_3\psi_d,\ \beta_w = -A_4.$$

The risk premium for the consumption claim is

$$\begin{aligned}\mathbb{E}_t[r_{w,t+1} - r_{f,t}] + \frac{1}{2}\mathbb{V}\mathrm{ar}_t[r_{w,t+1}] &= -\mathbb{C}\mathrm{ov}_t[m_{t+1}, r_{w,t+1}]\\ &= \left(\beta_x\lambda_x + \beta_y\lambda_y + \beta_c\lambda_c + \beta_d\lambda_d\right)\sigma_t^2 + \beta_w\lambda_w\sigma_w^2.\end{aligned}$$

### 3.B.2 Market Return

Conjecture that the log-price-dividend ratio for the claim on dividends is

$$z_{m,t} = B_0 + B_1 x_t + B_2 y_t + B_3 z_t + B_4 \sigma_t^2.$$

Then

$$\begin{aligned}
r_{m,t+1} &= \kappa_0^m + \kappa_1^m z_{m,t+1} - z_{m,t} + \Delta s_{t+1} \\
&= \kappa_0^m + \kappa_1^m \left(B_0 + B_1 x_{t+1} + B_2 y_{t+1} + B_3 z_{t+1} + B_4 \sigma_{t+1}^2\right) \\
&\quad - B_0 - B_1 x_t - B_2 y_t - B_3 z_t - B_4 \sigma_t^2 \\
&\quad + \mu_s + \phi_x x_t + \phi_y y_t + \pi_c \sigma_t \varepsilon_{t+1}^c + \pi_d \sigma_t \varepsilon_{t+1}^d + \psi_s \sigma_t \varepsilon_{t+1}^s \\
&= \{\kappa_0^m + B_0(\kappa_1^m - 1) + \kappa_1^m (B_3(\mu_d - \mu_c) + B_4 \hat{\sigma}) + \mu_s\} \\
&\quad + \{\kappa_1^m B_1 \rho_x - \kappa_1^m B_3 - B_1 + \phi_x\} x_t \\
&\quad + \left\{\kappa_1^m B_2 \rho_y + \kappa_1^m B_3 - B_2 + \phi_y\right\} y_t \\
&\quad + \{\kappa_1^m B_3 - B_3\} z_t \\
&\quad + \{\kappa_1^m B_4 \rho_h - B_4\} \sigma_t^2 \\
&\quad + \{\kappa_1^m B_1 \psi_x\} \sigma_t \varepsilon_{t+1}^x \\
&\quad + \left\{\kappa_1^m B_2 \psi_y\right\} \sigma_t \varepsilon_{t+1}^y \\
&\quad + \{-\kappa_1^m B_3 + \pi_c\} \sigma_t \varepsilon_{t+1}^c \\
&\quad + \{\kappa_1^m B_3 \psi_d + \pi_d\} \sigma_t \varepsilon_{t+1}^d \\
&\quad + \{\psi_s\} \sigma_t \varepsilon_{t+1}^s \\
&\quad + \{\kappa_1^m B_4\} \sigma_w w_{t+1}
\end{aligned}$$

Since both $m_{t+1}$ and $r_{m,t+1}$ are conditionally normal, the Euler equation can be written as

$$\mathbb{E}_t \left[m_{t+1} + r_{m,t+1}\right] + \frac{1}{2} \mathbb{V}\mathrm{ar}_t \left[m_{t+1} + r_{m,t+1}\right] \approx 0.$$

We use this equation to solve for the coefficients $B_0, ..., B_4$. They are given by

$$B_0 = -\frac{1}{\kappa_1^m - 1} \times \left\{ \left( \frac{B_4^2 \kappa_1^{m\,2}}{2} + \frac{M_w^{2\;2}}{2} \right) \sigma_w^2 + M_0 + \kappa_0^m + \mu_s + \kappa_1^m \; (B_4\, \hat{\sigma} - B_3\; (\mu_c - \mu_d)) \right\}$$

$$B_1 = -\frac{M_x + \phi_x - B_3\, \kappa_1^m}{\kappa_1^m\, \rho_x - 1}$$

$$B_2 = -\frac{M_y + \phi_y + B_3\,\kappa_1^m}{\kappa_1^m\,\rho_y - 1}$$
$$B_3 = \frac{M_z}{1 - \kappa_1^m}$$
$$B_4 = -\frac{1}{2\,\kappa_1^m\,\rho_h - 2} \times \Big\{ 2M_\sigma + (\pi_c - B_3\kappa_1^m)^2 + \left(\pi_y + B_3\kappa_1^m\psi_d\right)^2 + M_{ec}^2 + M_{ed}^2 + M_{ex}^2$$
$$+ M_{ey}^2 + \psi_s^2 + B_1^2\kappa_1^{m2}\psi_x^2 + B_2^2\,\kappa_1^{m2}\,\psi_y^2 \Big\}$$

where

$$M_0 = \Big\{ \theta \log \beta - \frac{\theta}{\psi}\mu_c + \theta\left(\frac{1}{\rho} - \frac{1}{\psi}\right) K(\mu_c - \mu_d)$$
$$+ (\theta - 1)(A_0(1 - \kappa_1) - \kappa_0 + A_3(\mu_d - \mu_c) + A_4\hat{\sigma} + \mu_c) \Big\}$$
$$M_x = -\frac{\theta}{\psi} - \theta\left(\frac{1}{\rho} - \frac{1}{\psi}\right) K + (\theta - 1)\left(A_1\rho_x - A_3 - \kappa_1 A_1 + 1\right)$$
$$M_y = \theta\left(\frac{1}{\rho} - \frac{1}{\psi}\right) K + (\theta - 1)(A_2\rho_y + A_3 - \kappa_1 A_2)$$
$$M_z = (\theta - 1)(A_3 - \kappa_1 A_3 - \kappa_2)$$
$$M_\sigma = (\theta - 1)(A_4\rho_h - \kappa_1 A_4)$$
$$M_{ex} = (\theta - 1)A_1\psi_x$$
$$M_{ey} = (\theta - 1)A_2\psi_y$$
$$M_{ec} = (\theta - 1)(1 - A_3) - \frac{\theta}{\psi} - \theta\left(\frac{1}{\rho} - \frac{1}{\psi}\right) K$$
$$M_{ed} = \left\{ (\theta - 1)A_3 + \theta\left(\frac{1}{\rho} - \frac{1}{\psi}\right) K \right\} \psi_d$$
$$M_w = (\theta - 1)A_4.$$

The innovation to $r_{m,t+1}$ is given by

$$r_{m,t+1} - \mathbb{E}_t[r_{m,t+1}] =$$
$$- \beta_{m,x}\sigma_t\varepsilon_{t+1}^x - \beta_{m,y}\sigma_t\varepsilon_{t+1}^y - \beta_{m,c}\sigma_t\varepsilon_{t+1}^c - \beta_{m,d}\sigma_t\varepsilon_{t+1}^d - \beta_{m,s}\sigma_t\varepsilon_{t+1}^s - \beta_{m,w}\sigma_w w_{t+1}$$

where

$$\beta_{m,x} = -\kappa_1^m B_1 \psi_x,\ \beta_{m,y} = -\kappa_1^m B_2 \psi_y,\ \beta_{m,c} = \kappa_1^m B_3 - \pi_c,$$
$$\beta_{m,d} = -\kappa_1^m B_3 \psi_d - \pi_d,\ \beta_{m,s} = -\psi_s,\ \beta_{m,w} = -\kappa_1^m B_4.$$

The risk premium for the dividend claim is

$$\begin{aligned}\mathbb{E}_t[r_{m,t+1} - r_{f,t}] + \frac{1}{2}\mathbb{V}\mathrm{ar}_t[r_{m,t+1}] &= -\mathbb{C}\mathrm{ov}_t[m_{t+1}, r_{m,t+1}] \\ &= \left(\beta_{m,x}\lambda_x + \beta_{m,y}\lambda_y + \beta_{m,c}\lambda_c + \beta_{m,d}\lambda_d\right)\sigma_t^2 + \beta_{m,w}\lambda_w\sigma_w^2.\end{aligned}$$

### 3.B.3 Risk-Free Rate

Using the Euler equation the model-implied risk-free rate is given by

$$r_{f,t} = -\mathbb{E}_t\left[m_{t+1}\right] - \frac{1}{2}\mathbb{V}\mathrm{ar}_t\left[m_{t+1}\right].$$

Using the expression for $m_{t+1}$, the risk-free rate will be given by

$$r_{f,t} = C_0 + C_1 x_t + C_2 y_t + C_3 z_t + C_4 \sigma_t^2$$

where

$$
\begin{aligned}
C_0 &= -\left\{\theta \log \beta - \frac{\theta}{\psi}\mu_c + \theta\left(\frac{1}{\rho} - \frac{1}{\psi}\right) K(\mu_c - \mu_d)\right. \\
&\qquad \left. + (\theta - 1)(A_0(1-\kappa_1) - \kappa_0 + A_3(\mu_d - \mu_c) + A_4\hat{\sigma} + \mu_c) + \frac{\lambda_w^2 \sigma_w^2}{2}\right\} \\
C_1 &= -\left\{-\frac{\theta}{\psi} - \theta\left(\frac{1}{\rho} - \frac{1}{\psi}\right) K + (\theta - 1)\left(A_1\rho_x - A_3 - \kappa_1 A_1 + 1\right)\right\} \\
C_2 &= -\left\{\theta\left(\frac{1}{\rho} - \frac{1}{\psi}\right) K + (\theta - 1)(A_2\rho_y + A_3 - \kappa_1 A_2)\right\} \\
C_3 &= -\left\{(\theta - 1)(A_3 - \kappa_1 A_3 - \kappa_2)\right\} \\
C_4 &= -\left\{(\theta - 1)(A_4\rho_h - \kappa_1 A_4) + \frac{\lambda_x^2 + \lambda_y^2 + \lambda_c^2 + \lambda_d^2}{2}\right\}.
\end{aligned}
$$

# Chapter 4

# Consumer Sentiment, Durable Consumption, and Stock Returns

## 4.1 Introduction

We provide novel empirical evidence that consumers' beliefs about aggregate durable expenditure predict future movements in financial markets. Using the Survey of Consumers from the University of Michigan we show that the aforementioned beliefs predict future excess returns in both short and long horizons as well as the future price-dividend ratio. This chapter introduces in an otherwise of classic consumption-based asset pricing model with recursive preferences of Epstein and Zin (1989, 1991), consumption of durable goods, aggregate uncertainty about consumption growth and belief formation through Bayesian learning. These beliefs drive the price-dividend ratio and future expected returns through the intertemporal marginal rate of substitution. In order to discipline our asset-pricing model, we estimate the structural parameters of the model to match the levels and volatility of equity premium and the risk-free rate. The risk aversion coefficient and elasticity of intertemporal substitution required to match key financial variables is much lower then previously suggested and is consistent with the real business cycle literature. We therefore rationalize our empirical finding without compromising (and possibly improving) the model's ability to match standard macroeconomic and financial moments.

The intuition for our finding is the following. When making the decision about intertemporal consumption allocation, households form expectation about future consumption growth. These expectations enter the intertemporal marginal rate of substitution (and as a result the price-dividend ratio and the asset returns) in the form of beliefs. We find that in the model the price-dividend ratio (and hence the asset returns) is increasing function of beliefs. This implies that positive belief growth will predict positive future price-dividend ratio and positive expected returns.

In terms of the models ability to match standard macroeconomic and financial moments, we find that the presence of a durable component in the utility function is crucial for generating the value and the time properties of the risk-free rate and the equity premium. When utility is non separable between nondurable and durable consumption, and the elasticity of substitution between these two goods is higher than the elasticity of intertemporal substitution, the marginal utility of consumption is high when durable consumption is low. Since, empirically, the ratio of durable to nondurable consumption is highly pro-cyclical, this ratio magnifies the countercyclical property of marginal utility, and thus of the equity premium. We also show that uncertainty about the underlying state lowers the required risk aversion coefficient: the volatility of beliefs increases in bad times, and it makes bad times even worse for the consumer, thus lowering the risk aversion.

This study makes two important contributions to the consumption-based asset pricing literature. First, we provide novel empirical evidence that links consumers' beliefs about aggregate durable expenditure and future movements in financial markets.

Second, we extend an otherwise standard consumption-based asset-pricing model to incorporate consumption of durable goods and aggregate uncertainty. In the model, we are able to assess the relative contribution of each of the model's ingredients in explaining the key financial moments: the mean and the volatility of the equity premium and the risk-free rate.

**Related Literature** This paper relates to three strands of literature. The first of these strands analyses the ability of consumer surveys to predict the future movements in financial markets. Ang, Bekaert and Wei (2007); Coibion, Gorodnichenko and Kamdar (Forthcoming); Lahiri, Monokroussos and Zhao (2016); Madeira and Zafar (2015); Souleles (2004), for example, analyze the ability of consumers' expectations to predict the future expenditures and future inflation. In the context of financial markets, Huang et al. (2015); Jiang et al. (Forthcoming); Lemmon and Portniaguina (2006); Ludvigson (2004) study the role of consumer confidence in predicting stock returns. In line with this literature we provide empirical evidence that consumers' beliefs about durable expenditure do predict future movements in financial markets.

The second larger strand of literature analyzes the role of durable goods consumption and the nonseparability of durable and nondurable consumption in the recursive framework of Epstein and Zin (1989, 1991). Yogo (2006) proposes a consumption-based explanation of the cross-sectional variation in the expected stock returns and countercyclical variation in the equity premium. The model generalizes the previous durable consumption models (Dunn and Singleton, 1986; Eichenbaum and Hansen, 1990; Ogaki and Reinhart, 1998) by separating the risk-aversion coefficient from the elasticity of intertemporal substitution and is the first study to model durable consumption in a recursive framework. Yang (2011) models the economy in the spirit of a Bansal and Yaron (2004) type of long-run risk model and documents that there exists strong evidence of a highly persistent component in durable consumption growth. Eraker, Shaliastovich and Wang (2016) study the predictability of high expected inflation on low future real growth in the context of a two-good long-run risk economy. Unlike Yogo (2006), our paper studies the time-series rather than cross-sectional properties of asset returns. In contrast with Yang (2011) and Eraker, Shaliastovich and Wang (2016), who study asset prices in a long-run risk model with durable consumption, the focus of this paper is on the effect of learning about the hidden state on the prices of stocks and bonds in the presence of durable consumption and on predictability features of the beliefs, and not on the long-run risk with durable consumption.

The third and final strand of the literature that we relate to analyzes how financial time series change their behavior during periods of financial crises or rapid growth. Ang and Bekaert (2002); Cecchetti, Lam and Mark (1993); Dai, Singleton and Yang (2007) are the examples. To account for this feature of the data and to allow for aggregate uncertainty and belief formation in the economy, we model cash flows from non-durable consumption, durable consumption, and aggregate equity markets as subject to a regime shift that is unobservable for the agent in a similar vein as Veronesi (1999). This way of modelling the economy complements the existing recent literature on asset pricing (for example, Brandt, Zeng and Zhang, 2004; Ju and Miao, 2012, among others, model the growth rates of consumption and dividends subject to a hidden regime, and the agent learns about the hidden state of the economy). The unobservability of the underlying state induces endogenously time-varying uncertainty due to inference problems. Moreover, beliefs about the aggregate state enter the intertemporal marginal rate of substitution and therefore drive the formation of future excess returns.

## 4.2 Predictability of Returns and Price-Dividend Ratios

A rise in consumers' beliefs about the durable expenditure predicts a rise in future expected returns for both short and long horizons, as well as rise in future price-dividend ratio. We use the Survey of Consumers from the University of Michigan as a proxy for consumers' belief about the purchase of durable goods. We use 3 questions from the questionnaire, regarding the purchase of household durable goods, the purchase of vehicles and purchase of cars. The questions were: *"Generally speaking, do you think now is a good or a bad time for people to buy major household items?"*, *"Speaking of the automobile market – do you think the next 12 months or so will be a good time or a bad time to buy a car?"* and *"Generally speaking, do you think now is a good time or a bad time to buy a house?"*, respectively.

Table 4.2.1 reports the ability of constructed beliefs to predict excess returns (Panel A) and price-dividend ratios (Panel B). There is a positive relationship between the belief about durable purchase and the future excess returns. The values of the slope coefficients and corresponding $R^2$s rise with the return horizon. The car purchase and house purchase questions have less predictive power in the long horizon, with corresponding $R^2$s decreasing with horizon. There is also positive relationship between beliefs and price-dividend ratio. For all three questions the slope coefficient is positive and significant and $R^2$ is high.

In next section we develop a model that incorporates jointly the durable consumption and the belief system and test the model implications for financial markets.

TABLE 4.2.1: Predictive Power of Beliefs

| Panel A. Excess Returns[a,b] | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Durable Purchase | | | Car Purchase | | | House Purchase | | |
| Horizon | $b$ | $s.e.(b)$ | $R^2$ | $b$ | $s.e.(b)$ | $R^2$ | $b$ | $s.e.(b)$ | $R^2$ |
| 6 Months | 0.19 | (0.13) | 0.03 | 0.19 | (0.11) | 0.02 | 0.12 | (0.06) | 0.03 |
| 1 Year | 0.47 | (0.21) | 0.08 | 0.32 | (0.18) | 0.04 | 0.20 | (0.10) | 0.04 |
| 2 Years | 0.85 | (0.27) | 0.15 | 0.25 | (0.28) | 0.01 | 0.24 | (0.14) | 0.03 |
| 5 Years | 1.75 | (0.25) | 0.21 | 0.47 | (0.62) | 0.01 | 0.40 | (0.36) | 0.01 |
| Panel B. Price-Dividend Ratio[a,c] | | | | | | | | | |
| | Durable Purchase | | | Car Purchase | | | House Purchase | | |
| Horizon | $b$ | $s.e.(b)$ | $R^2$ | $b$ | $s.e.(b)$ | $R^2$ | $b$ | $s.e.(b)$ | $R^2$ |
| 1 Month | 2.18 | (0.46) | 0.23 | 2.90 | (0.34) | 0.39 | 1.87 | (0.19) | 0.45 |

[a] Excess return is the CRSP value-weighted return less the 3-month Treasury bill. Data is montly from 1952 - 2016, available from the CRSP. Price-Dividend Ratio is from Robert J. Shiller's website.
[b] The regression equation is $r^e_{t+1\to t+k} = a + b \times \pi_t + \varepsilon_{t+k}$, where $r^e_{t\to t+k}$ is the log excess return, continuously compounded over the horizon, and $\pi_t$ is the consumers belief about the purchase of durable goods. Return horizon is 0.5, 1, 2 and 5 years. Standard errors are Newey and West (1986), corrected for 10 lags.
[c] The regression equation is $pd_{t+1} = a + b \times \pi_t + \varepsilon_t$, where $pd_{t+1}$ is the log price-dividend ratio, and $\pi_t$ is the consumers belief about the purchase of durable goods. Standard errors are Newey and West (1986), corrected for 10 lags.

## 4.3 Model

### 4.3.1 Preferences and Endowments

Consider an infinitely lived representative household. In each period $t$, the household purchases $C_t$ units of non-durable consumption goods and $I_t$ units of durable consumption goods. The durable good provides a service flow for more than one period, while the non-durable consumption good is non-storable and is entirely consumed in period $t$. The household accumulates the stock of durable goods $K_t$ according to the law of motion

$$K_t = (1-\delta)K_{t-1} + I_t,$$

where $\delta \in (0,1)$ is the depreciation rate. The household's intertemporal utility is defined recursively as

$$U_t = \left\{ (1-\beta) V_t^{\frac{1-\gamma}{\theta}} + \beta \left( \mathbb{E}\left[ U_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{4.3.1}$$

where $V_t$ is given as the Cobb-Douglas function over $K_t$ and $C_t$

$$V_t = C_t^{1-\alpha} K_t^{\alpha}$$

with $0 < \alpha < 1$. The parameters of the household's utility function are the subjective discount factor $\beta \in (0,1)$, the relative risk aversion coefficient $\gamma > 0$, and the elasticity of intertemporal substitution $\psi \geq 0$ with $\theta = (1-\gamma)/(1-\frac{1}{\psi})$.

### 4.3.2 Assets and Dividends

The household has an initial $W_0$ units of wealth. In every period $t$, the household splits its current wealth $W_t$ between consumption and investment. The household invests $B_{i,t}$ units into one of the $N$ available tradable assets in the economy. Each asset realizes the gross rate of return $R_{i,t+1}$ in period $t+1$. The household's budget constraint in period $t$ is given by

$$W_t - C_t - P_t I_t = \sum_{i=1}^{N} B_{i,t},$$

where $P_t$ is the relative price of consumer durable goods in terms of non-durable goods. The $t+1$ period wealth of the household is given by

$$W_{t+1} = \sum_{i=1}^{N} B_{i,t} R_{i,t+1}.$$

I consider two types of assets: equity, that provides stochastic amount of dividends in each period, and risk-less bonds, that pay zero coupons and act as purely discount bonds. Consider a stochastic endowment economy, where each period non-durable consumption $C_t$ and durable consumption $I_t$ arrives. In equilibrium, agents purchase $C_t$ and $I_t$, such that markets clear and prices of these goods are determined endogenously. I model the growth rates of endowment $a$ (a non-durable good $C_t$, a durable good $I_t$, and equity) as a hidden Markov model in logs:

$$g^a_{t+1} = \mu^a_{S_t} + \sigma^a \varepsilon^a_{t+1}, \tag{4.3.2}$$

where $(\varepsilon^a_t)$'s are independent jointly standard normal error terms.

The model is an extension of Hamilton (1989) and Cecchetti, Lam and Mark (1993). The predictable components $\mu^a_{S_t}$ are driven by the common Markov chain $S_t$ with the state space

$$\mathcal{S} = \{1 = \text{expansion}, 0 = \text{recession}\}.$$

The unobservability of the underlying state induces time-varying uncertainty due to inference problems. All dividend parameters are estimated using a Maximum Likelihood Estimation from postwar U.S. consumption and dividend data.

$S_t$ follows a two-state Markov chain with transition matrix $P = (p_{ij})$, where $p_{ij}$ is the conditional probability $\mathbb{P}(S_{t+1} = j|S_t = i)$ of the process being in state $j$ next period given it is in state $i$ this period. I further assume that $\mu^a_1 > \mu^a_0$, for every growth rate of endowment $a$, so that the growth rates during an expansion are higher than the growth rates during a recession .

Suppose we are in an economy with incomplete information, where the representative household knows the structure and the parameters of the model, but does not observe the state $S_t$. Let $\mathcal{F}_t$ be the information available to the household at time $t$, which consists of the observed growth rates of the endowment processes. We need to derive the evolution of the posterior state beliefs given $\mathcal{F}_t$. Let $\pi_t(i) = \mathbb{P}[S_t = i|\mathcal{F}_{t-1}]$ denote the posterior belief of state $t$ being $i$, and suppose $\pi_0$ (the stationary prior) is given. The agent uses Bayes' rule to update his belief about the hidden state:

$$\pi_{t+1}(i) \propto \sum_{j=0}^{1} \mathbb{P}(S_{t+1} = i | S_t = j) \cdot \mathbb{P}(S_t = j | \mathcal{F}_t),$$

where $\propto$ means that $\forall i,\ \pi_{t+1}(i)$ are multiplied by a constant term such that $\sum_i \pi_{t+1}(i) = 1$.

### 4.3.3 Consumption-Portfolio Choice

Define the total wealth of the household as

$$\widetilde{W}_t = W_t + (1 - \delta) P_t K_{t-1}$$

that is, it includes the value of the stock of durable goods expressed in terms of relative price $P_t$. Let

$$B_{N+1,t} = P_t K_t,$$

where $N$ is as before. Now, we will assume that the total number of assets available to the household is equal to three, and includes durable consumption, as well as equity and risk-less bonds. The return on the durable goods be equal to

$$R_{N+1,t+1} = (1 - \delta) \frac{P_{t+1}}{P_t}.$$

This way we can rewrite the budget constraint to include the return not only from the $N$ available assets but also the return on the durable goods:

$$\widetilde{W}_{t+1} = (\widetilde{W}_t - C_t) \sum_{i=1}^{N+1} \omega_{it} R_{i,t+1} \tag{4.3.3}$$

with the condition that

$$\sum_{i=1}^{N+1} \omega_{it} = 1. \tag{4.3.4}$$

Given the household's current wealth level $\widetilde{W}_t$, the household chooses the level of non-durable consumption $C_t$ and how much to invest into all of the available assets $\{\omega_{1,t}, \ldots, \omega_{N+1,t}\}$ to maximize utility (4.3.1) subject to (4.3.3) and (4.3.4). This leads to the Bellman equation for the problem of the form

$$J_t = \max_{C_t, \omega_{0,t}, \ldots, \omega_{N+1,t}} \left\{ (1-\beta) V_t^{\frac{1-\gamma}{\theta}} + \beta \left( \mathbb{E}_t \left[ J_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}.$$

I follow Yogo (2006) and conjecture that the value function $J_t$ is a function of wealth $J_t = J_t(\widetilde{W}_t) = \phi_t \widetilde{W}_t$. It can be shown that (see 4.A):

$$\phi_t = \left[ (1-\beta)(1-\alpha) v \left( \frac{C_t}{\widetilde{W}_t}, \omega_{N+1,t} \right)^{1-\frac{1}{\psi}} \right]^{\frac{1}{1-\frac{1}{\psi}}} \left( \frac{C_t}{\widetilde{W}_t} \right)^{\frac{1}{1-\psi}},$$

where

$$v \left( \frac{C_t}{\widetilde{W}_t}, \omega_{N+1,t} \right) = \left[ \frac{\omega_{N+1,t}}{P_t} \left( \frac{\widetilde{W}_t}{C_t} - 1 \right) \right]^{\alpha}.$$

### 4.3.4 Asset Pricing

Let $R_{m,t+1}$ denote the return on wealth from an optimal portfolio, defined as

$$\widetilde{W}_{t+1} = \left( \widetilde{W}_t - C_t \right) R_{m,t+1},$$

and let

$$M_{t+1} = \beta^{\theta} \left( \frac{C_{t+1}}{C_t} \right)^{-\theta} \left( \frac{V_{t+1}}{V_t} \right)^{\theta - \frac{\theta}{\psi}} R_{m,t+1}^{\theta-1} \tag{4.3.5}$$

be the intertemporal marginal rate of substitution (IMRS) of the economy, the pricing kernel. Epstein and Zin (1989, 1991) show that the first-order condition for the consumption and the portfolio choice implies that the return on every tradable asset $i$ in the economy satisfies the equation

$$\mathbb{E}_t[M_{t+1}R_{i,t+1}] = 1. \tag{4.3.6}$$

Equation (4.3.6) states that in equilibrium, the expected discounted value of any return is equal to one. There are few things to note about the IMRS. When there is no durable consumption ($V_t = C_t$) and the utility is time-separable (with $\theta = 1$), the IMRS becomes:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma},$$

meaning that IMRS depends only on the growth rate of nondurable consumption. When utility is Epstein and Zin's, the IMRS now depends on the return to market, which is in general non-observable and becomes:

$$M_{t+1} = \beta^{\theta} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{m,t+1}^{\theta-1}.$$

The presence of the durable consumption in the model generates two new features. First, the IMRS now includes the extra term:

$$M_{t+1} = \beta^{\theta} \left(\frac{C_{t+1}}{C_t}\right)^{-\theta} \left(\frac{V_{t+1}}{V_t}\right)^{\theta-\frac{\theta}{\psi}} R_{m,t+1}^{\theta-1},$$

which includes elasticity of substitution between nondurable and durable goods. Second, we now have an extra first-order condition due to durable goods.

This first-order condition with respect to the choice of durable goods is [1]

$$\mathbb{E}_t[M_{t+1}(R_{0,t+1} - R_{N+1,t+1})] = \frac{\frac{\partial V_t}{\partial K_t}}{P_t \frac{\partial V_t}{\partial C_t}},$$

where $V_t = C_t^{1-\alpha} K_t^{\alpha}$.

The effect of learning in the model could be seen by looking at the Euler equation (4.3.6). The conditional expectation operator in equation (4.3.6) is taken with respect to current state of the world:

$$\mathbb{E}_t[M_{t+1}R_{i,t+1}] = \pi_t \ \mathbb{E}[M_{t+1}R_{i,t+1}|S_t = 1] + (1 - \pi_t) \ \mathbb{E}[M_{t+1}R_{i,t+1}|S_t = 0], \quad (4.3.7)$$

where operator $\mathbb{E}$ is an expectation with respect to normally distributed variables. Equation (4.3.7) implies that IMRS as well as the returns in the model will depend on the beliefs of the consumer. As these beliefs are time-varying, it will imply time-variation in returns.

Equations (4.3.5) and (4.3.6) are used to derive the asset prices in the economy. In evaluating the model performance and estimating the preference parameters I follow the semiparametric procedure of Chen, Favilukis and Ludvigson (2013). Firstly, the consumption process is estimated directly from the data and the estimate of the hidden state is formed. Then, I treat the wealth-consumption ratio $\frac{\widetilde{W}_{t+1}}{C_{t+1}}$ (and therefore the price-dividend ratio $\frac{P_{a,t}}{D_{a,t}}$ on any asset $a$) as an unknown function that depends on a

[1]If we equate the marginal utility of non-durable consumption per unit spent to marginal utility of durable consumption per unit spent, and taking the non-durable good as a numèraire we get:

$$\frac{\frac{\partial V_t}{\partial C_t}}{1} = \frac{\frac{\partial V_t}{\partial K_t}}{Q_t},$$

where $Q_t$ can be thought of as a rental cost of service flow from the durable good. On the other hand, we can interpret $Q_t$ as

$$Q_t = P_t - (1 - \delta)\mathbb{E}_t[M_{t+1}P_{t+1}],$$

where we discount the future price $P_{t+1}$ by $M_{t+1}$. By definition, $R_{N+1,t+1} = \frac{(1-\delta)P_{t+1}}{P_t}$, which gives us the first-order condition above.

set of state variables $\boldsymbol{x}_t$. I take $\boldsymbol{x}_t$ to be a vector of two variables - the current belief $\pi_t$ and the ratio of investment into durable goods over the current stock of durable goods $\frac{I_t}{K_t}$. For any candidate set of preference parameters these unknown functions are estimated nonparametrically (see 4.B on how to apply the projection method of Judd (1992) to the model). Once the nonparametric estimate of the unknown function is obtained, the set of preference parameters are estimated using an suitable generalized method of moments procedure.

## 4.4 Data

### 4.4.1 Source and Construction

Personal consumption expenditure (PCE) data are retrieved from the U.S. National Income and Product Accounts as provided by the Bureau of Economic Analysis. The measure of non-durable consumption includes personal consumption expenditure on non-durable goods (food and beverages purchased for off-premises consumption, clothing and footwear, and gasoline and other energy goods) and personal consumption expenditure on services (housing, health care, transportation and other services). The corresponding seasonally adjusted quarterly quantity index for the sample period 1952:I–2016:IV is from lines 8 and line 13 of Table 2.3.3. (Real Personal Consumption Expenditures by Major Type of Product).

The measure of the stock of consumer durable goods includes motor vehicles, furnishings and durable household equipment, recreational goods and vehicles and other durable goods. The corresponding annual quantity index for the period 1952–2015 is from line 1 of Table 8.2 (Chain-Type Quantity Indexes for Net Stock of Consumer Durable Goods). The relative price of consumer durable goods is constructed as the ratio of the PCE price index for durable goods from line 3 over the PCE price index for non-durable goods from line 8 of Table 2.3.4 (Price Indexes for Personal Consumption

Expenditures by Major Type of Product). The BEA reports only the annual series of the net stock of consumer durable goods, quarterly series are interpolated by assuming that the depreciation rate is constant within the year and by finding its implied value, which is consistent both with the annual stocks of net consumer durables at the beginning as well as the end of the year, and with quarterly series of PCE expenditures on durable goods.[2] The U.S. population measure used to calculate per-capita quantities covers the period 1952–2016 and may be retrieved from the Federal Reserve Bank of St. Louis.

The quarterly and annual returns on the common stock market as well as the short-term nominal interest rate for the sample period 1952:I–2016:IV are from the CRSP and are provided by the University of Manchester. We deflate all asset returns with the PCE price index for non-durable goods to obtain real quantities because non-durable consumption is the numéraire in our analysis.

### 4.4.2 Basic Description and Business Cycle Properties of Consumption Data

Table 4.4.1 reports descriptive statistics for nondurable and durable goods consumption growth and durable stock growth. Nondurable consumption and services growth has a mean 0.49% and standard deviation of 0.46% per quarter. The growth of the expenditure to durable consumption has a mean 0.98% and standard deviation of 3.21% per quarter. Durable goods stock growth has mean a 0.83% and standard deviation of 1.01% per quarter. The first-order autocorrelations for the nondurable consumption growth, expenditure to durable goods growth and durable goods stock growth are equal to 0.45, -0.01, and 0.88, respectively.

[2]The law of motion of the consumer durable goods $K_{t+1} = (1-\delta_t) K_t + I_t$ yields after four iterations the equation $K_{t+4} = (1-\delta)^4 K_t + (1-\delta)^3 I_t + (1-\delta)^2 I_{t+1} + (1-\delta) I_{t+2} + I_{t+3}$ that implicitly defines the depreciation rate $\delta$ for the given year.

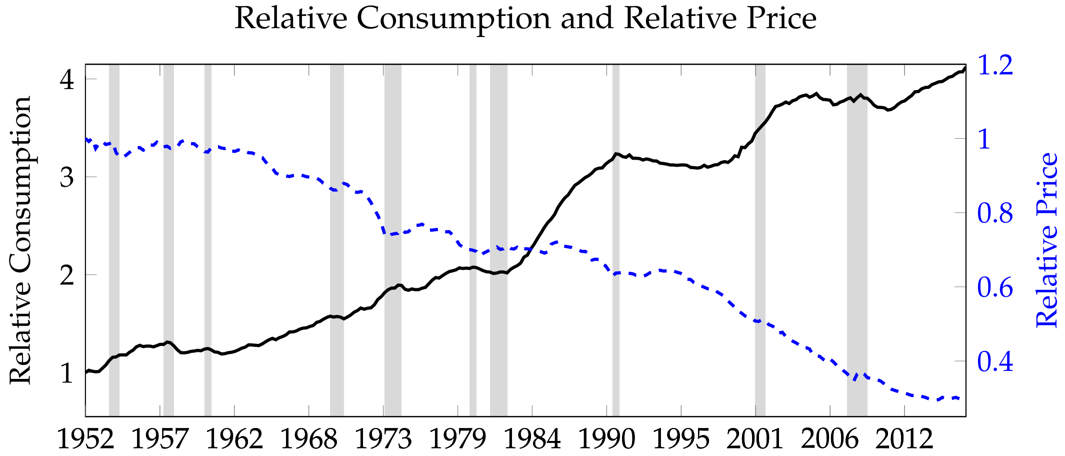
TABLE 4.4.1: Descriptive statistics[a]

| Time series | Mean[b] | SD[b] | Autocorrelation |
| --- | --- | --- | --- |
| | Est. (S.E.) | Est. (S.E.) | Est. (S.E.) |
| Nondurable Goods | 0.49 (0.06) | 0.46 (0.03) | 0.45 (0.08) |
| Durable Goods Expenditure | 0.98 (0.15) | 3.21 (0.25) | 0.01 (0.06) |
| Durable Goods Stock | 0.83 (0.15) | 1.01 (0.09) | 0.88 (0.04) |

[a] This table provides the descriptive statistics for the consumption data. Nondurable goods is the expenditure for nondurable consumption and services.
[b] All variables are in percentage. Standard errors obtained by performing a block bootstrap with each block having geometric distribution with length 32 quarters; 50,000 experiments performed. Sample period is 1952:I-2016:IV.

Figure 4.4.1 is a plot of the ratio of the stock of durable goods to nondurable consumption $\left(\frac{K_t}{C_t}\right)$ and the relative price of durables to nondurables. The upward trend in $\frac{K_t}{C_t}$ is consistent with a downward trend of the relative price. $\frac{K_t}{C_t}$ increased by factor of almost 2.5 over the data span while the relative price decreased by a factor of almost 3.5. The ratio $\frac{K_t}{C_t}$ is pro-cyclical, it rises during booms and falls during recessions. The shaded regions are the recessions as defined by the NBER. Figure 4.4.2 plots the time series of expenditures to durable goods (solid line) and nondurable goods consumption (dashed line). Both time series exhibit an upward trend in the sample period and are strongly pro-cyclical.

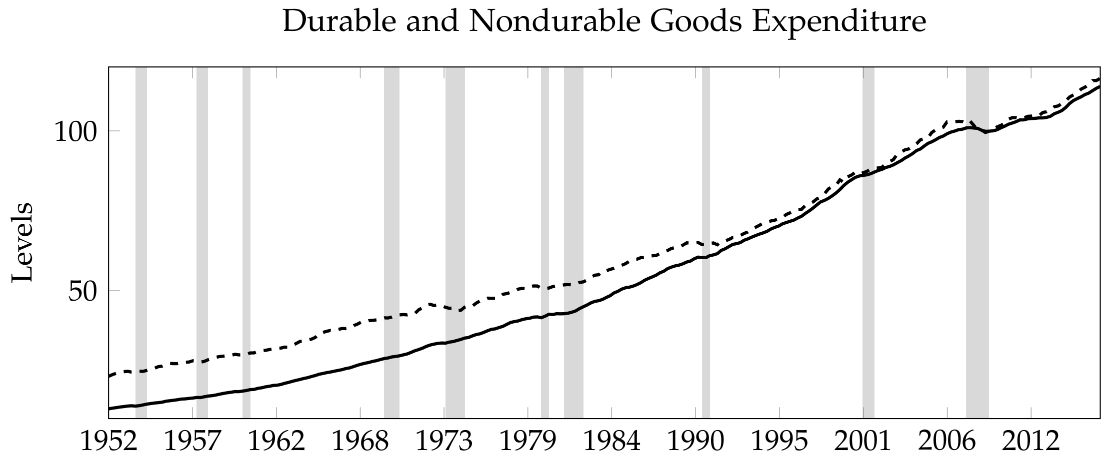
FIGURE 4.4.1: **Relative Consumption and Price.** Time series plot of durable consumption as a ratio of nondurable consumption (black solid line), and relative price of durable to nondurable consumption (blue dashed line). The sample period is 1952:I - 2016:IV, the shaded areas indicate NBER recessions.The 1952:I values are normalized to 1.




The upward trend in nondurable and durable consumption expenditure as well as a downward trend in the relative price imply that the series might be co-integrated. 4.C provides a complete co-integration analysis. While we find that there is a long-term relationship between nondurable goods, the expenditure to durable goods, and their relative price, we do not model the relative price explicitly and rather assume that the relative price adjusts such that this long-term relationship holds. [3]

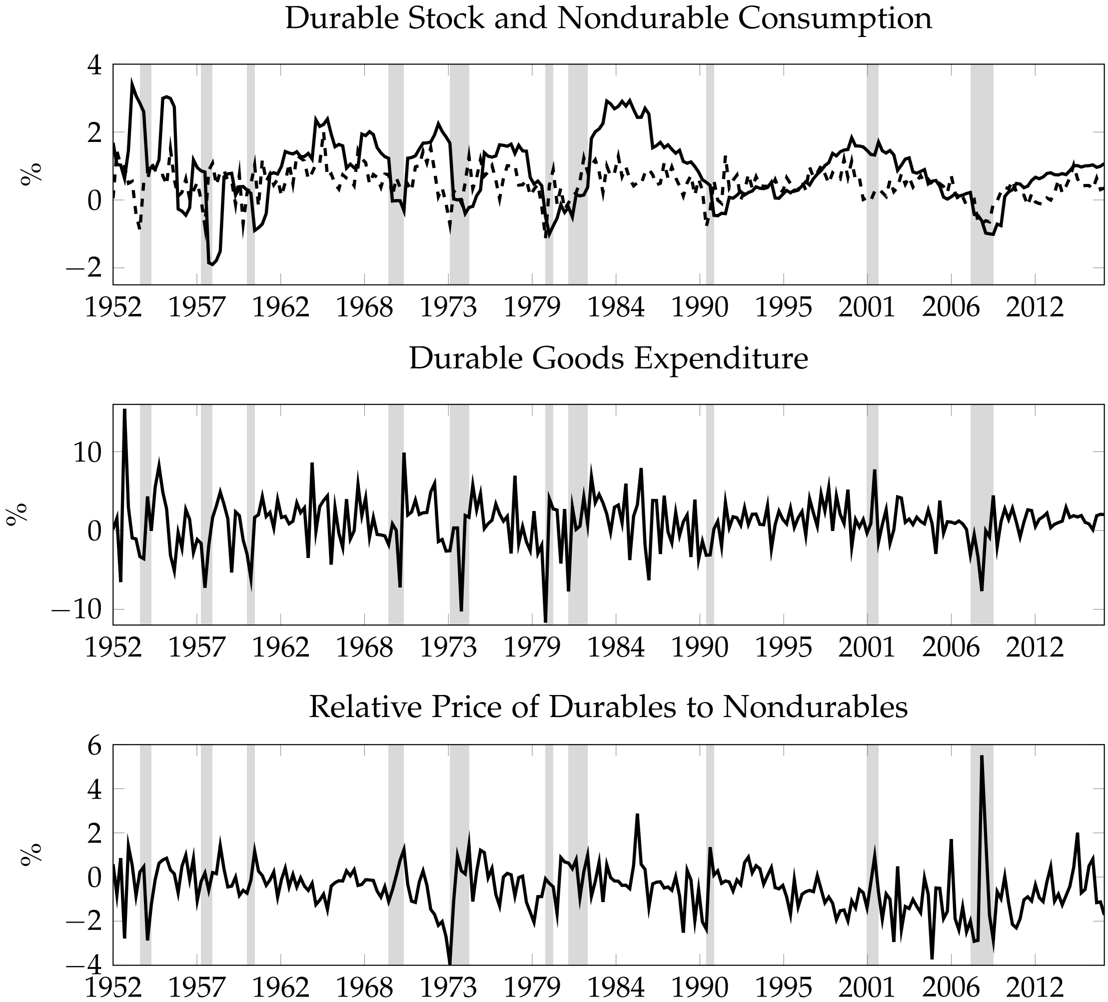
[3]There are two important points to note here. Firstly, we model the endowment economy in terms of growth rates, thus removing any trend in the consumption and dividends. Secondly, we abstract from the upward trend in ratio of durable to nondurable composition by using Cobb-Douglas interperiod utility function. We could model the interperiod utility as a CES aggregator to allow for the upward trend; this, however, would generate a composition risk as in Monika Piazzesi, Martin Schneider and Selale Tuzel (2007), which would move the focus of the paper.

FIGURE 4.4.2: **Durable and Nondurable Goods Expenditure.** Time-series plot of the real durable goods expenditure (solid line) and the real nondurable goods consumption (dashed line), in levels. The sample period is 1952:I - 2016:IV; the shaded regions indicate NBER recessions.




The top panel of Figure 4.4.3 plots the corresponding growth rates of the stock of durable goods and of nondurable consumption, respectively. The middle panel plots the growth rate of expenditure to durable goods, while the bottom panel plots the growth rate of relative price of durables. The growth rates of durable stock, expenditure to durable goods and nondurable consumption are strongly pro-cyclical, whereas the growth rate of the relative price is strongly countercyclical. The growth rate of expenditure to durable goods is more pro-cyclical than nondurable consumption, and thus is a good business cycle indicator.

FIGURE 4.4.3: **Growth Rates.** The top panel is a time-series plot of the real growth rates of the stock of durables (thick solid line) and nondurable consumption (thin solid line), the middle panel plots the real growth rate of durable goods expenditure, and the bottom panel plots the growth rate of relative price of durables to nondurables. The sample period is 1952:I - 2016:IV; the shaded regions indicate NBER recessions.




## 4.5 Model Estimation

In this section we describe the estimation procedure of the model. We first estimate the regime-switching endowment process using the consumption and dividend data.

We then estimate the preference parameters of the model to match the unconditional moments of asset returns.

### 4.5.1 Estimation of the Endowment Process

We fully estimate the model at quarterly frequency. The endowments coefficients are estimated using the US consumption and dividends data, and the preference parameters are estimated semiparametrically in the spirit of Chen, Favilukis and Ludvigson (2013). Since the model does not admit an analytical solution, we solve the model numerically and run simulations to compute the model implied moments. We then use these simulated moments to estimate the preference parameters using the data on returns. We also study the shape of the wealth-consumption and the price-dividend ratios, and the time-varying properties of the asset prices.

Coefficients of equations (4.3.2) are estimated using the Maximum Likelihood Estimation procedure from the post-war US data on consumption and dividends. Table 4.5.1 reports the estimation results. Non-durable consumption does not grow in the recession state and grows at the rate of more than 0.6% in the boom state. The volatility of non-durable consumption is estimated to be almost 0.38%. The growth rate of the expenditure to durable consumption declines at the rate of 1.95% in the recession state and grows at the rate of almost 1.04% in the boom state. Durable consumption is more volatile than non-durable consumption with an estimated volatility of approximately 3%. Finally, the growth rate of dividends in the recession state is estimated at -1.83% whereas in a boom state at 1.60%. The dividends are very volatile, with the estimated volatility of greater than 5%. Transition probabilities for the regime-switching process are estimated as 0.95 and 0.75 for the boom and recession states, respectively, therefore implying that the expected duration of a recession is equal to four quarters.

Figure 4.5.1 displays the plot of filtered (solid line) and smoothed (dashed line) probabilities of the recession state from the postwar US data over the NBER recessions. We

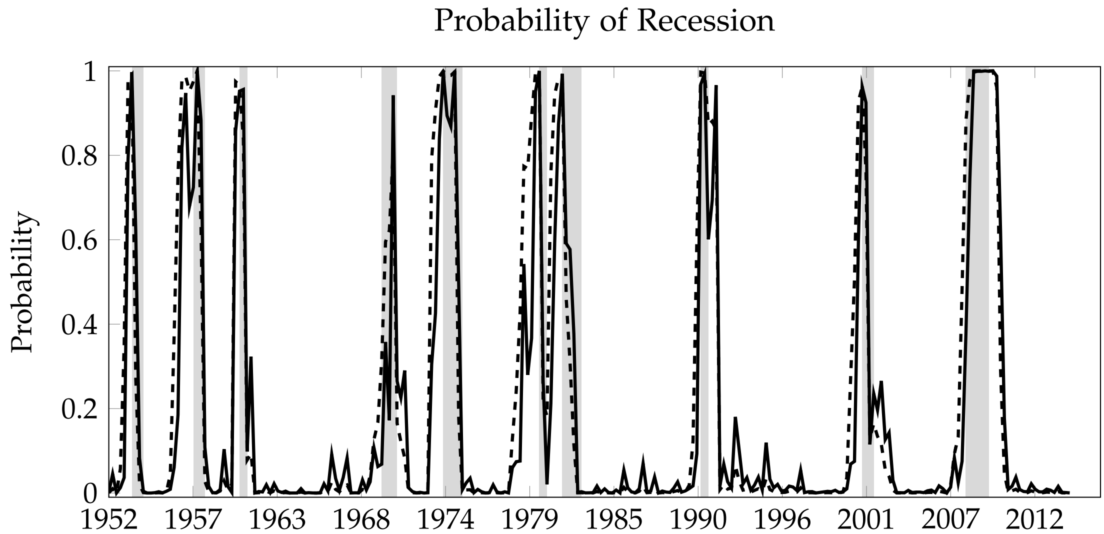
Table 4.5.1: Maximum Likelihood Estimation of a Two-State Model [a]

| Parameter | Estimate | | Parameter | Estimate | |
|---|---|---|---|---|---|
| | Value[b] | SE[b] | | Value[b] | SE[b] |
| $\mu_0^c$ | 0.00 | 0.07 | $p$ | 0.95 | NA |
| $\mu_0^d$ | -1.83 | 1.10 | $q$ | 0.75 | NA |
| $\mu_0^e$ | -1.95 | 0.54 | | | |
| $\mu_1^c$ | 0.63 | 0.03 | $\sigma^c$ | 0.37 | 0.01 |
| $\mu_1^d$ | 1.04 | 0.41 | $\sigma^d$ | 5.07 | 2.01 |
| $\mu_1^e$ | 1.60 | 0.23 | $\sigma^e$ | 2.97 | 0.59 |

[a] This table provides the maximum likelihood estimation of the endowment process for nondurable and durable consumption expenditure and dividend series. $p$ and $q$ denotes the probability of expansion and recession, respectively. $\mu_i^c, \mu_i^d$ and $\mu_i^e$ denote the estimated growth rates for nondurable consumption, dividends, and expenditure to durable consumption, respectively, in the state $i$, where $i = 0$ is recession and $i = 1$ is expansion. $\sigma^c, \sigma^d$, and $\sigma^e$ denote the estimated standard deviations of growth rates of nondurable consumption, dividends, and expenditure to durable consumption.
[b] All variables are in percentage. Data is quarterly. Sample period is 1952:I-2016:IV.

can see that the estimated probability tracks NBER recessions well.

Figure 4.5.1: **Probability of recession.** Figure displays the filtered (solid line) and smoothed (dashed line) probabilities of recession. The sample period is 1952:I - 2014:IV; the shaded regions indicate NBER recessions.



### 4.5.2 Estimation of the Preference Parameters

We then estimate three model parameters: the risk aversion coefficient $\gamma$, the elasticity of intertemporal substitution $\psi$ and the share of durable consumption $\alpha$. We do this numerically in a following manner. We treat wealth-consumption and price-dividend ratio as unknown functions of state variables and approximate them nonparametrically using the projection method of Judd (1992) for a candidate set of preference parameters. Given the nonparametric estimate of these unknown functions, we estimate the set of preference parameters using a generalized method of moments procedure. For that we simulate the model-implied moments and take identity matrix as a weighting matrix in GMM. We use three moments - the mean and the variance of equity return and the variance of risk-free rate to estimate three preference parameters mentioned above. The estimated value for the risk aversion coefficient is 2.1, which lies well between the most commonly accepted values of 1 and 3 (some studies even suggesting values as low as 0.2 and as high as 10). The implied value of the elasticity of the intertemporal substitution is 1.09, which also lies comfortably between the reported values of 0.2 and 2 (see, for example, Havránek, 2015, for recent survey). The value of the elasticity of the intertemporal substitution bigger than one is required to generate a preference for early resolution of uncertainty as emphasized by Bansal and Yaron (2004). The share of durable consumption is estimated to be equal to 0.3 and is consistent with the data (durables make up of approximately 30% of the average consumers consumption basket). The other parameters of the model are set in the following way. The subjective discount factor $\beta$ is set to match exactly the mean value of risk-free rate in the model. The implied value for $\beta$ is 0.985. The depreciation rate of durables $\delta$ is set to 6%, which is the average sample observed value. Table 4.5.2 reports the median estimated values of the parameters.

TABLE 4.5.2: Benchmark values of the preference parameters

| Parameter | | Value |
|---|---|---|
| EIS | $\psi =$ | 1.09 |
| Risk aversion | $\gamma =$ | 2.1 |
| Subjective discount factor | $\beta =$ | 0.985 |
| Share of durable consumption | $\alpha =$ | 0.3 |
| Depreciation rate | $\delta =$ | 0.06 |

This tables lists the benchmark values for the preference parameters. The value of the subjective discount factor is set to match the level of risk-free interest rate implied by the model. The value for the depreciation rate is set to match the average quarterly depreciation rate from the data for the sample period 1952:Q1-2016:Q4.

### 4.5.3 Unconditional Moments of Returns

For the aforementioned set of parameters we also analyze two benchmark models in order to emphasize the importance of each of the ingredients of the model. Benchmark Model I analyses the endowment economy in which the agent has a recursive utility over consumption. The endowments are subject to the regime switch, but the state of the world is observable for the agent. Moreover, the agent has utility from nondurable consumption only. Benchmark Model II extends the previous benchmark model and analyses the case of recursive utility with durable consumption; again the underlying state of the economy is observed by the agent.

TABLE 4.5.3: Unconditional Moments of Returns [a]

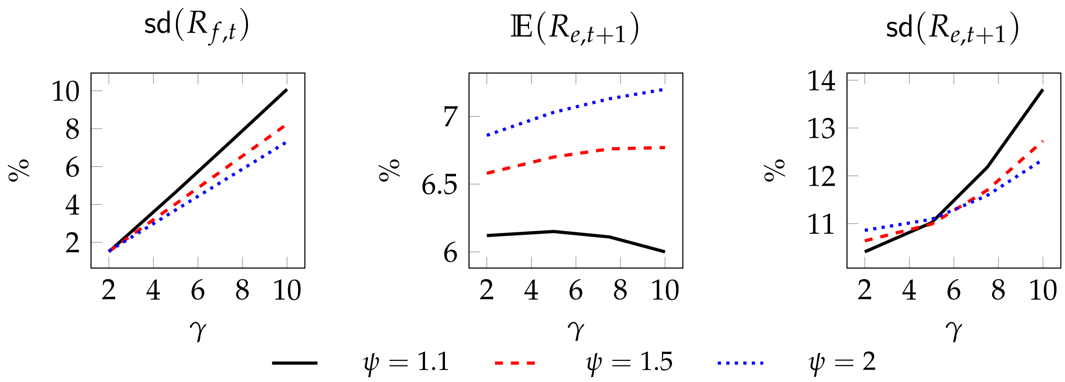
| | $\mathbb{E}(R_{f,t})$ | $\mathsf{sd}(R_{f,t})$ | $\mathbb{E}(R_{e,t+1})$ | $\mathsf{sd}(R_{e,t+1})$ |
|---|---|---|---|---|
| US Data [b] | 1.20 | 0.99 | 5.57 | 18.94 |
| Full Model [c] | 1.20 | 0.99 | 6.60 | 10.38 |
| Only Nondurable Consumption No Uncertainty [c] | 1.20 | 0.30 | 1.88 | 11.14 |
| Durable and Nondurable Consumption No uncertainty [c] | 1.20 | 2.90 | 5.15 | 10.52 |

[a] This table reports the unconditional moments of returns from the data, the Full Model (with uncertainty and durables) and for Benchmark Model I (no uncertainty and no durables) and Benchmark Model II (no uncertainty, durables).
[b] Risk-free rate is the 3-month Treasury bill. Equity premium is defined as CRSP value-weighted return less the 3-month Treasury bill. All data is quarterly for the period 1952:Q1 - 2016:Q4, reported as annualized percentage values, and is available from CRSP.
[c] Model implied moments are obtained as a sample values from simulated 100,000 data points and annualized to match the corresponding data moments.

Table 4.5.3 summarizes the results. The first row of table 4.5.3 reports the annualized data moments. The average annual risk-free rate is equal to 1.2%, with a volatility of 0.99%. The mean equity premium is marginally higher than 5.5% with a large volatility of almost 19%. The second row of table 4.5.3 reports the results for the full model with recursive preferences over nondurable and durable consumption, and uncertainty about the endowments. We see, that both the mean and the volatility of the risk-free rate are matched perfectly. The model implied mean equity premium is slightly above the data value and is equal to just over 6%. On the other hand, the model implied volatility, is slightly lower than 60% of the value observed in the data.

We next analyze how sensitive are the values reported in the second row of 4.5.3 to the change in preference parameters. Rather than focusing on the standard error of the estimates, we solve the model for different values of risk aversion coefficient and elasticity of intertemporal substitution and report the model implied moments. This allows to assess all possible non-linearities in the model. As before, we fix the value of subjective discount factor to match the mean value of risk-free rate. We also fix the value of $\alpha = 0.3$.

FIGURE 4.5.2: **Unconditional Moments of Returns.** This figure displays model implied volatility of the risk-free rate $\text{sd}(R_{f,t})$, mean and volatility of equity premium, $\mathbb{E}(R_{e,t+1})$ and $\text{sd}(R_{e,t+1})$, as a function of risk aversion $\gamma$. The black solid line is for $\psi = 1.1$, the dashed red line is for $\psi = 1.5$, and the blue dotted line is for $\psi = 2$.




As we see on Figure 4.5.2, the volatility of the risk-free rate is increasing function of risk aversion coefficient $\gamma$, and the effect is stronger for lower values of EIS $\psi$.The same effect holds for the volatility of the equity premium, the higher the risk aversion coefficient $\gamma$ is, the higher is the value of implied volatility of the equity premium. The mean equity premium, on the other hand, is more stable with respect to risk aversion $\gamma$ but is highly dependent on EIS $\psi$. In this case the average value of mean equity premium increases by about 0.5% when we increase $\psi$ from 1.1 to 1.5. We report other sensitivity analysis in Appendices 4.D and 4.E.

We next turn to analyzing which channel of the model contributes most to explaining the model implied moments of returns. As the first exercise we fix the values of the preference parameters at the previously estimated values and re-solve the model under different scenarios. We then used the model to simulate the risk-free and equity returns and compare those to the data and to the original model.

First, we consider Benchmark Model I, where we shut down both channels of the model. In this scenario, the agent has a recursive utility over nondurable consumption only. While the endowments are still subject to the regime switch, the state of the world is now observable for the agent. The third row of table 4.5.3 reports the unconditional moments of returns for this scenario. As before, we fix the value of $\beta$ to match the mean value of risk-free rate. Compared to the full model, this model generates very low volatility of the risk-free rate (only 0.3% compared to about 1% in the data) and very low value of equity premium (only 1.88% compared to about 5.5% in the data and a bit more than 6% in the full model). This model generates roughly the same volatility of the equity premium as the full model.

Next we look at what is the effect of durable consumption in the model. We call this scenario Benchmark Model II. In this model, the agent now has the utility over both nondurable and durable consumption. The state of the world is still observable for the agent, as in case of Benchmark Model I. The effect of durable consumption can be seen in the last row of table 4.5.3. On one hand, durable consumption generates substantial equity premium (of more than 5%) and about the same volatility of equity premium as other models (about 10%). On the other hand, durable consumption also generates very high volatility of risk-free rate (about 3 times as large as in the data). The effect of durable consumption is clearly visible through equation (4.3.5). High cyclicality of durable consumption generates high market price of expected growth risk. In combination with cyclical dividends this generates high equity premium. At the same time, however, this excess movement leads to high volatility of risk-free rate. With respect to volatility of equity premium - most of the movement is due to dividends, that outweighs the movement in durable consumption (see table 4.5.1).

The effect of learning in the model thus can be seen by comparing full model with Benchmark Model II. Even with low values of risk aversion learning (in combination with durables) generates a sizable equity premium (of more than 6%). It also helps bring the volatility of the risk-free rate down. As the risk-free rate is the inverse of the IMRS, equation (4.3.7) suggests that movements in risk-free rate will be driven by movements in beliefs. With constant volatility of durable goods, the volatility of beliefs (which increases in bad times and decreases in good times) will now overweight bad times, but will also underweight good times (which are more likely), and thus bringing the overall volatility down.

## 4.6 Predictive Power of Beliefs

In this section we asses whether our model can reconcile empirical observations reported in Section 4.2. We start with analyzing the properties of price-dividend ratio and its dependence on beliefs. We find that in the model, price-dividend ratio is an increasing function of beliefs. We then simulate the model and run the same regressions as in Section 4.2 for simulated data.

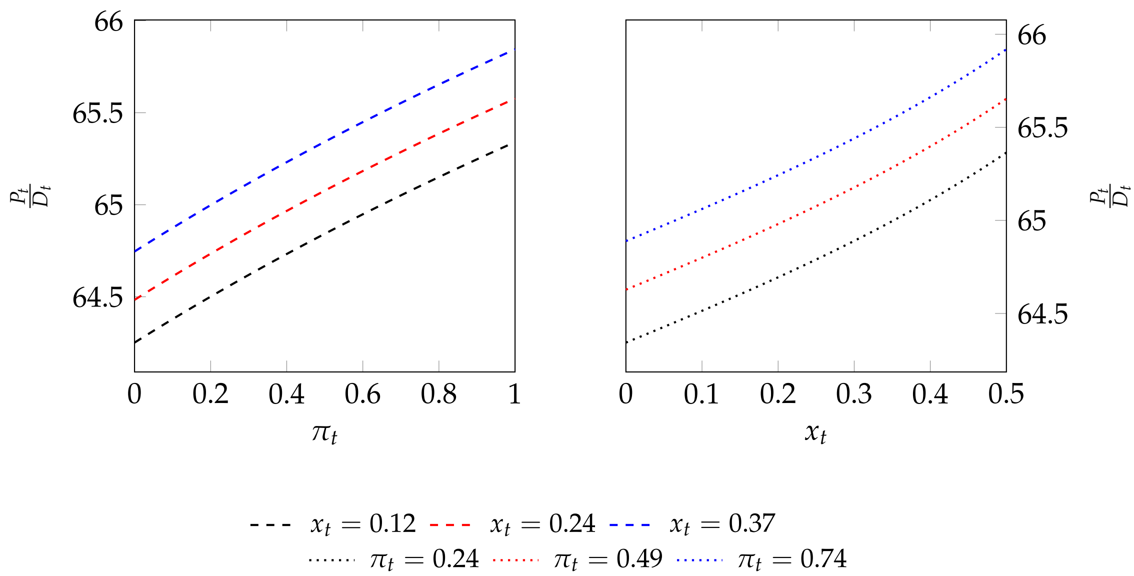
### 4.6.1 Properties of Price-Dividend Ratio

To understand what explains the value and the dynamics of asset returns in the model it is worth looking at the properties of price-dividend ratios. Figure 4.6.1 presents the price-dividend ratio as a function of two state variables in the model: the posterior probabilities $\pi_t$ (blue solid line) and the ratio of durable goods expenditure to stock of durables $x_t$ (blue dashed line) for the baseline values of parameters from section 4.5.

As we see from left panel figure 4.6.1 , the price-dividend ratio is an increasing and convex function of $\pi_t$. The intuition for this fact is similar to Veronesi (1999) in the case of time-additive expected exponential utility. When $\pi_t$ is close to 1 (meaning that the times are good), bad news decreases $\pi_t$ and hence decreases future consumption growth. At the same time, the agent's uncertainty about the dividend growth is increased, since $\pi_t$ is now closer to 0.5. The agents want to be compensated for being exposed to more risk, and thus they will require an extra discount on their dividend claim. Therefore, this reduction of the price because of bad news is higher than the reduction in expected future dividend. On the contrary, if $\pi_t$ is close to zero, and times are bad, the good piece of news increases the expected future consumption growth, but also increases the agent's uncertainty ($\pi_t$ is closer to 0.5). Thus, the price-dividend ratio is increasing and convex.

Right panel in figure 4.6.1 also depicts the price-dividend ratio as a function of $x_t$. Notice, that the price-dividend ratio is concave and increasing functions of the state variable $x_t$. Moreover, the variation of the price-dividend ratio with respect to $x_t$ are much higher than with respect to $\pi_t$, giving further evidence in favor of using durable consumption in the asset-pricing models. The fact that the ratio is increasing in $x_t$ is straightforward due to our ordering of states. Intuitively, the concavity property means that the price-dividend ratio dampens the low realizations of $x_t$ (when the state of the world is bad) but exaggerate the high realizations (when the state of the world is good).

FIGURE 4.6.1: **Price-Dividend Ratio.** Figure displays price-dividend ratio as a function of state variables, keeping the other state variable fixed.



### 4.6.2 Predictability of Price-Dividend Ratio and Excess Returns

In this section we report the predictability of model-simulated beliefs for simulated price-dividend ratio and future excess return. To compare the model with the empirical data we run the regressions of the same form as reported in Section 4.2. We start with assessing the predictability of beliefs for excess returns. For that, we run the regression of the form

$$r^e_{t+1\to t+k} = a + b \times \pi_t + \varepsilon_{t+k},$$

where $r^e_{t\to t+k}$ is the log excess return, continuously compounded over the horizon, and $\pi_t$ is the consumers belief about the purchase of durable goods. Return horizon is 0.5, 1, 2 and 5 years. The results are reported in Panel A of Table 4.6.1.

Similarly, we test the beliefs ability to predict future price-dividend ratio. The regression equation is

$$pd_{t+1} = a + b \times \pi_t + \varepsilon_t,$$

where $pd_{t+1}$ is the log price-dividend ratio, and $\pi_t$ is the consumers belief about the purchase of durable goods. The results are reported in Panel B of Table 4.6.1.

As in the data, the model-generated returns is significant predictor of future excess returns and future price-dividend ratio. The coefficient for the future excess returns is increasing in horizon. $R^2$ is increasing for short horizons and eventually decreases. Similarly, model-generated beliefs significantly predict future price-dividend error.

In Appendix 4.F we also look at the direct link between price-dividend ratio and future excess return.

TABLE 4.6.1: Predictive Power of Beliefs (Model)

| Panel A. Excess Returns[a] | | | |
|---|---|---|---|
| Horizon | $b$ | $s.e.(b)$ | $R^2$ |
| 6 Months | 0.07 | (0.001) | 0.02 |
| 1 Year | 0.11 | (0.001) | 0.05 |
| 2 Years | 0.20 | (0.002) | 0.01 |
| 5 Years | 0.44 | (0.004) | – |
| **Panel B. Price-Dividend Ratio[b]** | | | |
| Horizon | $b$ | $s.e.(b)$ | $R^2$ |
| 1 Month | 4.35 | (0.02) | – |

[a] The regression equation is $r^e_{t+1\to t+k} = a + b \times \pi_t + \varepsilon_{t+k}$, where $r^e_{t\to t+k}$ is the log excess return, continuously compounded over the horizon, and $\pi_t$ is the consumers belief about the purchase of durable goods. Return horizon is 0.5, 1, 2 and 5 years.
[b] The regression equation is $pd_{t+1} = a + b \times \pi_t + \varepsilon_t$, where $pd_{t+1}$ is the log price-dividend ratio, and $\pi_t$ is the consumers belief about the purchase of durable goods.

## 4.7 Conclusion

We provide novel empirical evidence that consumers' beliefs about aggregate durable expenditure predicts future movements in financial markets. Using the Survey of Consumers from the University of Michigan we show that the aforementioned beliefs predict future excess returns in both short and long horizons as well as future price-dividend ratio. This paper introduces in an otherwise of classic consumption-based asset pricing model with recursive preferences of Epstein and Zin (1989, 1991), consumption of durable goods, aggregate uncertainty about consumption growth and belief formation through Bayesian learning. These beliefs drive the price-dividend ratio and future expected returns through the intertemporal marginal rate of substitution. As in data, we show that model-generated beliefs predict future excess return for both short and long horizons and future price-dividend ratio. We discipline our asset-pricing model and estimate the structural parameters of the model to match the levels and volatility of equity premium and the risk-free rate. The model generates high equity premium, low and stable risk-free rate, and explains up to 60 percent of the volatility of equity premium with a level of risk aversion of 2.1 and elasticity of intertemporal substitution of 1.09.

# Appendix

## 4.A Numerical solution

In this subsection, we present the numerical solution to the model. I start by guessing a solution of the form $U_t = \phi_t \widetilde{W}_t$, where $\widetilde{W}_t$ denotes the total wealth

$$\widetilde{W}_{t+1} = W_{t+1} + (1-\delta)P_{t+1}K_t = \sum_{i=1}^{N} \omega_{i,t} R_{i,t+1} + (1-\delta)P_{t+1}K_t = \sum_{i=1}^{N+1} \omega_{i,t} R_{i,t+1},$$

meaning I treat the durable good as an asset. We can further rewrite the budget constraint as

$$\widetilde{W}_{t+1} = (\widetilde{W}_t - C_t) \sum_{i=1}^{N+1} \omega_{i,t} R_{i,t+1} = (\widetilde{W}_t - C_t) \cdot \omega_t' R_{t+1},$$

where $\omega_t = (\omega_{1,t}, \ldots, \omega_{N+1,t})'$ is the vector of weights and $R_{t+1} = (R_{1,t+1}, \ldots, R_{N+1,t+1})'$ is the vector of returns. For some $x$ and $y$ lets define function

$$v(x,y) = \left[ 1 - \alpha + \alpha \left( \frac{y}{P_t} \left( \frac{1}{x} - 1 \right) \right)^{1-\frac{1}{\rho}} \right]^{\frac{1}{1-\frac{1}{\rho}}}, \quad \text{when } \rho \neq 1,$$

and

$$v(x,y) = \left[ \frac{y}{P_t} \left( \frac{1}{x} - 1 \right) \right]^{\alpha}, \quad \text{when } \rho = 1,$$

Note that

$$V_t = C_t \left[ 1 - \alpha + \alpha \left( \frac{\omega_{N+1,t}}{P_t} \left( \frac{\widetilde{W}_t}{C_t} - 1 \right) \right)^{1-\frac{1}{\rho}} \right]^{\frac{1}{1-\frac{1}{\rho}}} = C_t\, v \left( \frac{C_t}{\widetilde{W}_t}, \omega_{N+1,t} \right).$$

Using the equations above, Bellman equation

$$J_t = \left\{ (1-\beta) V_t^{\frac{1-\gamma}{\theta}} + \beta \left( \mathbb{E}_t \left[ J_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{4.A.1}$$

simplifies to

$$\left( \phi_t \widetilde{W}_t \right)^{1-\frac{1}{\psi}} = (1-\beta) \left( C_t \cdot v \left( \frac{C_t}{\widetilde{W}_t}, \omega_{N+1,t} \right) \right)^{1-\frac{1}{\psi}} + \beta \left( \widetilde{W}_t - C_t \right)^{1-\frac{1}{\psi}} y_t \tag{4.A.2}$$

with

$$y_t = \mathbb{E}_t \left[ (\phi_{t+1} R_{m,t+1})^{1-\gamma} \right]^{\frac{1}{\theta}}.$$

We are interested in deriving the expression for $\phi_t$ as a function of $\frac{C_t}{\widetilde{W}_t}$ and $\omega_{N+1,t}$. Using a similar approach as Epstein and Zin (1989, 1991), the FOC of the equation (4.A.2) with respect to $C_t$ is

$$0 = (1-\beta) \frac{\partial \left( \left( C_t \cdot v \left( \frac{C_t}{\widetilde{W}_t}, \omega_{N+1,t} \right) \right)^{1-\frac{1}{\psi}} \right)}{\partial C_t} - \left( 1 - \frac{1}{\psi} \right) \beta \left( \widetilde{W}_t - C_t \right)^{-\frac{1}{\psi}} y_t,$$

$$\frac{\partial \left( (C_t \cdot v(\cdot,\cdot))^{1-\frac{1}{\psi}} \right)}{\partial C_t} = \left( 1 - \frac{1}{\psi} \right) (C_t \cdot v(\cdot,\cdot))^{-\frac{1}{\psi}} \left[ v(\cdot,\cdot) + C_t \frac{\partial v(\cdot,\cdot)}{\partial C_t} \right] =$$
$$\left( 1 - \frac{1}{\psi} \right) (C_t \cdot v(\cdot,\cdot))^{-\frac{1}{\psi}} \left[ v(\cdot,\cdot) + \left( v(\cdot,\cdot) + (\alpha - 1) v(\cdot,\cdot)^{\frac{1}{\rho}} \right) \frac{-\widetilde{W}_t}{\widetilde{W}_t - C_t} \right],$$

where

$$v(\cdot,\cdot) = v \left( \frac{C_t}{\widetilde{W}_t}, \omega_{N+1,t} \right).$$

The FOC can be rewritten as

$$\beta\left(\widetilde{W}_t - C_t\right)^{1-\frac{1}{\psi}} y_t = (1-\beta)(1-\alpha)v^{\frac{1}{\rho}}\widetilde{W}_t\left(C_t \cdot v\right)^{-\frac{1}{\psi}} - (1-\beta)\left(C_t \cdot v\right)^{1-\frac{1}{\psi}}.$$

Substituting expression above to the Bellman equation (4.A.2) we get:

$$\left(\phi_t\widetilde{W}_t\right)^{1-\frac{1}{\psi}} = (1-\beta)(1-\alpha)v^{\frac{1}{\rho}}\widetilde{W}_t\left(C_t \cdot v(\cdot,\cdot)\right)^{-\frac{1}{\psi}}.$$

Hence,

$$\phi_t = \left[(1-\beta)(1-\alpha)v\left(\frac{C_t}{\widetilde{W}_t}, \omega_{N+1,t}\right)^{\frac{1}{\rho}-\frac{1}{\psi}}\right]^{\frac{1}{1-\frac{1}{\psi}}}\left(\frac{C_t}{\widetilde{W}_t}\right)^{\frac{1}{1-\psi}}.$$

Following Yogo (2006), the Euler equation is of the form:

$$\mathbb{E}_t\left[\beta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\rho}}\left(\frac{V_{t+1}}{V_t}\right)^{\frac{\theta}{\rho}-\frac{\theta}{\psi}} R^{\theta}_{m,t+1}\right] = 1, \tag{4.A.3}$$

where $R_{m,t+1}$ denotes the return on wealth from an optimal portfolio, defined as

$$\widetilde{W}_{t+1} = \left(\widetilde{W}_t - C_t\right) R_{m,t+1}.$$

Using the functional form for $V_t$, we can further eliminate $\frac{V_{t+1}}{V_t}$ from the Euler equation (4.A.3). We have

$$\frac{V_{t+1}}{V_t} = \left\{\frac{(1-\alpha)\left(\frac{C_{t+1}}{K_{t+1}}\right)^{1-\frac{1}{\rho}}\left(\frac{K_{t+1}}{K_t}\right)^{1-\frac{1}{\rho}} + \alpha\left(\frac{K_{t+1}}{K_t}\right)^{1-\frac{1}{\rho}}}{(1-\alpha)\left(\frac{C_t}{K_t}\right)^{1-\frac{1}{\rho}} + \alpha}\right\}^{\frac{1}{1-\frac{1}{\rho}}}.$$

Consider the case when $\rho = 1$. Since $\rho \neq 1$ introduces a new state variable into the model, we will consider the Cobb-Douglas specification of the intratemporal utility

$$V_t = C_t^{1-\alpha} K_t^{\alpha}.$$

Following Yang (2011), it can be shown that

$$\frac{V_{t+1}}{V_t} = \left(\frac{K_{t+1}/C_{t+1}}{K_t/C_t}\right)^{\alpha} \frac{C_{t+1}}{C_t}.$$

From the budget constraint we get

$$\widetilde{W}_{t+1} = \left(\widetilde{W}_t - C_t\right) R_{m,t+1},$$

which can be rewritten as

$$R_{m,t+1} = \frac{\frac{\widetilde{W}_{t+1}}{C_{t+1}}}{\frac{\widetilde{W}_t}{C_t} - 1} \frac{C_{t+1}}{C_t}.$$

Let $\xi_t = \frac{\widetilde{W}_t}{C_t}$ denote the wealth-consumption ratio. Then,

$$R_{m,t+1} = \frac{\xi_{t+1}}{\xi_t - 1} \frac{C_{t+1}}{C_n}.$$

The Euler equation then becomes

$$\mathbb{E}_t \left[ \beta^{\theta} \left(\frac{C_{t+1}}{C_t}\right)^{\theta\left(1-\frac{1}{\psi}\right)} \left(\frac{K_{t+1}/C_{t+1}}{K_t/C_t}\right)^{\alpha\theta\left(1-\frac{1}{\psi}\right)} \xi_{t+1}^{\theta} \right] = (\xi_t - 1)^{\theta}.$$

4.B explains how to solve for $\xi_t$. Taking the FOC of the Bellman equation with respect to $\omega_{i,t}$ results in another Euler equation of the form:

$$\mathbb{E}_t \left[ \beta^{\theta} \left(\frac{C_{t+1}}{C_t}\right)^{1-\theta} \left(\frac{K_{t+1}/C_{t+1}}{K_t/C_t}\right)^{\alpha\theta\left(1-\frac{1}{\psi}\right)} R_{m,t+1}^{\theta-1} R_{i,t+1} \right] = 1.$$

The return $R_{i,t+1}$ can be written as

$$R_{i,t+1} = \frac{P_{R,t+1} + D_{R,t+1}}{P_{R,t}} = \frac{\frac{P_{R,t+1}}{D_{R,t+1}} + 1}{\frac{P_{R,t}}{D_{R,t}}} \frac{D_{R,t+1}}{D_{R,t}} = \frac{\lambda_{t+1} + 1}{\lambda_t} \frac{D_{R,t+1}}{D_{R,t}}.$$

Therefore, the Euler equation can be written as

$$\mathbb{E}_t\left[\beta^\theta\left(\frac{C_{t+1}}{C_t}\right)^{\theta\left(1-\frac{1}{\psi}\right)-1}\left(\frac{K_{t+1}/C_{t+1}}{K_t/C_t}\right)^{\alpha\theta\left(1-\frac{1}{\psi}\right)}\left(\frac{\xi_{t+1}}{\xi_t-1}\right)^{\theta-1}\frac{D_{R,t+1}}{D_{R,t}}\ (\lambda_{t+1}+1)\right]=\lambda_t.$$

4.B explains how to solve for $\lambda_t$.

## 4.B Application of the projection method

This section describes the application of the projection method of Judd (1992) to our model. We start with equilibrium wealth-consumption ratio. Using the budget constraint, we express $R_{m,t+1}$ in terms of a wealth-consumption ratio as:

$$R_{m,t+1} = \frac{\frac{\widetilde{W}_{t+1}}{C_{t+1}}}{\frac{\widetilde{W}_t}{C_t} - 1} \frac{C_{t+1}}{C_t}.$$

We make a conjecture that the wealth-consumption ratio is a function $\xi_t$ of state variables $\boldsymbol{x}_t = (\pi_t, \frac{I_t}{K_t})$ and thus

$$R_{m,t+1} = \frac{\xi_{t+1}(\boldsymbol{x}_{t+1})}{\xi_t(\boldsymbol{x}_t) - 1} \frac{C_{t+1}}{C_t},$$

and we use the Euler equation and apply the projection method to obtain the functional form of $\xi_t$. By the Euler equation

$$\mathbb{E}_t \left[ \beta^\theta \left( \frac{C_{t+1}}{C_t} \right)^{\theta\left(1-\frac{1}{\psi}\right)} \left( \frac{K_{t+1}/C_{t+1}}{K_t/C_t} \right)^{\alpha\theta\left(1-\frac{1}{\psi}\right)} \xi_{t+1}^\theta \right] - (\xi_t - 1)^\theta = 0.$$

We can further rewrite the Euler equation as:

$$\mathbb{E}_t \left[ \beta^\theta \left( \frac{C_{t+1}}{C_t} \right)^{\theta\alpha\left(\frac{1}{\psi}-1\right)} \left( \frac{K_{t+1}}{K_t} \right)^{\theta\alpha\left(1-\frac{1}{\psi}\right)} \xi_{t+1}^\theta \right] = (\xi_t - 1)^\theta.$$

Let $x_t$ denote $\frac{I_t}{K_t}$. Then, using $K_{t+1} = (1-\delta)K_t + I_t$ we get

$$\beta^\theta \mathbb{E}_t \left[ e^{\theta\alpha\left(\frac{1}{\psi}-1\right)\Delta c_{t+1}} (1-\delta+x_t)^{\theta\alpha\left(1-\frac{1}{\psi}\right)} \xi_{t+1}^\theta \right] - (\xi_t - 1)^\theta = 0.$$

where $\Delta c_{t+1} = \log\left(\frac{C_{t+1}}{C_t}\right)$. Let $\kappa = \theta\alpha\left(\frac{1}{\psi} - 1\right)$ and $\zeta = -\kappa = \theta\alpha\left(1 - \frac{1}{\psi}\right)$. We conjecture that $\xi_t$ is a function of $x_t$ and $\pi_t$, where $x_t = \frac{I_t}{K_t}$ and $\pi_t$ is a posterior state belief.

Then, equation becomes:

$$\begin{aligned}\beta^{\theta}\mathbb{E}_t\left[e^{\kappa\Delta c_{t+1}}\cdot(1-\delta+x_t)^{\zeta}\cdot\xi^{\theta}_{t+1}(x_{t+1},\pi_{t+1})\right]&\\ -(\xi_t(x_t,\pi_t)-1)^{\theta} &= 0.\end{aligned}$$

We approximate function $\xi_t$ as

$$\hat{\xi}_t(x_t,\pi_t)=\sum_{i,j=0}^{n}\phi_{i,j}\psi_i(x_t)\psi_j(\pi_t),$$

where $\{\psi_i(\cdot)\}_{i=1}^{n}$, $\{\psi_j(\cdot)\}_{j=1}^{n}$ is a basis of complete Chebyshev polymonials of order $n$, and $\phi_{i,j}$ are the coefficients of the polynomials. We next define the residual equation

$$\begin{aligned}R(x_t,\pi_t;\phi) &= \beta^{\theta}\mathbb{E}_t\left[e^{\kappa\Delta c_{t+1}}\cdot(1-\delta+x_t)^{\zeta}\cdot\xi^{\theta}_{t+1}(x_{t+1},\pi_{t+1})\right]\\ &- (\xi_t(x_t,\pi_t)-1)^{\theta}.\end{aligned}$$

Substituting for $\hat{\xi}_t$ we get

$$\begin{aligned}\hat{R}(x_t,\pi_t;\phi) &= \beta^{\theta}\mathbb{E}_t\left[e^{\kappa\Delta c_{t+1}}\cdot(1-\delta+x_t)^{\zeta}\cdot\hat{\xi}^{\theta}_{t+1}(x_{t+1},\pi_{t+1})\right]\\ &- (\hat{\xi}_t(x_t,\pi_t)-1)^{\theta}.\end{aligned}$$

Denote by $f(S_t,\varepsilon_{t+1})$ the integrand in the equation above. Then,

$$\mathbb{E}_t\left[f(S_t,\varepsilon_{t+1})\right]=\int\left[\pi_t f(1,\varepsilon_{t+1})+(1-\pi_t)f(0,\varepsilon_{t+1})\right]d\Phi(\varepsilon),$$

where $\Phi(\varepsilon)$ is a cdf of the standard normal distribution, and

$$f(S_t, \varepsilon_{t+1}) = e^{\kappa(\mu^c_{S_t} + \sigma^c \varepsilon_{t+1})}(1 - \delta + x_t)^{\zeta}\, \hat{\xi}^{\theta}_{t+1}(x_{t+1}(S_t), \pi_{t+1}(S_t)), \quad S_t \in \{0, 1\}$$

The residual function becomes

$$\hat{R}(x_t, \pi_t; \phi) = -(\hat{\xi}_t(x_t, \pi_t) - 1)^{\theta} + \beta^{\theta} \int [\pi_t f(1, \varepsilon_{t+1}) + (1 - \pi_t) f(0, \varepsilon_{t+1})]\, d\Phi(\varepsilon).$$

We choose $\phi$ so that $\hat{R}$ is exactly zero at $n$ collocation points. These points are chosen as a roots of $n$th order Chebyshev polynomial. The integral

$$\int [\pi_t f(1, \varepsilon_{t+1}) + (1 - \pi_t) f(0, \varepsilon_{t+1})]\, d\Phi(\varepsilon)$$

is evaluated using the Gauss-Hermite quadrature.

Further, let $P_{a,t}$ denote the date $t$ price of dividend claim $a$. We conjecture that the price-dividend ratio $\frac{P_{a,t}}{D_{a,t}}$ is a function $\lambda_t$ of state variables $\boldsymbol{x}_t$, which are as above. By definition,

$$R_{a,t+1} = \frac{P_{a,t+1} + D_{a,t+1}}{P_{a,t}} = \frac{D_{a,t+1}}{D_{a,t}} \frac{1 + \lambda(\pi_{t+1}, x_{t+1})}{\lambda(\pi_t, x_t)}.$$

Using the same procedure as before, we can solve for the price-dividend ratio.

## 4.C Cointegration Analysis

One can easily show that the intratemporal first-order condition states that the marginal utility per last dollar spent must be the same across all consumption goods:

$$\frac{V_C(C_t, K_t)}{1} = \frac{V_K(C_t, K_t)}{rc_t},$$

where $rc_t$ is the rental cost for durable goods (taking nondurable goods as a numèraire). The above condition can be rewritten as

$$\frac{V_C(C_t, K_t)}{V_K(C_t, K_t)} = rc_t.$$

On the other hand, the no-arbitrage argument implies the connection between rental cost of durable goods and their relative price, namely

$$rc_t = q_t - (1 - \delta)\mathbb{E}_t[M_{t+1}q_{t+1}],$$

where $M_{t+1}$ denotes the stochastic discount factor.

The right hand side of the equation states that one can buy a unit of durable good for $q_t$ and sell it next period for $(1 - \delta)q_{t+1}$ (after depreciation). Following a similar argument to Pakoš (2011) one can show that the growth rate of nondurable consumption, the growth rate of stock of durable goods, and their relative price share a single common stochastic trend. As the growth rate of stock of durables is directly related to expenditure on durable goods, this section explores the nature of the long-term relationship between nondurable goods, expenditure on durable goods, and their relative price (we assume that the long-run relationship is of the form $\Delta c_t - \lambda \Delta q_t - \eta \Delta e_t \sim I(0)$).

I first test for the presence of unit roots in time series, then use the Johansen Likelihood Ratio test to test for the number of cointegrating vectors, and report the estimated vector error correction model and corresponding estimated cointegrating vector.

I test the null hypothesis that the growth rates of nondurable consumption, expenditures to durable goods, and relative price of durables are difference stationary against the alternative hypothesis of trend stationarity, using the tests of Elliott, Rothenberg and Stock (1996) and Ng and Perron (2001). In all cases we are unable to reject the hypothesis about difference stationarity (see Table 4.C.1).

TABLE 4.C.1: Testing for unit roots [a]

| Time Series | ERS | DF-GLS | MPP |
|---|---|---|---|
| In logs | | | |
| Nondurable Consumption | 9.01 | -2.24 | -2.24 |
| Durable Good Stock | 5.30 | -3.12 | -3.15 |
| Expenditure to Durable Goods | 7.40 | -2.55 | -2.48 |
| Relative Price of Durable Goods | 88.2 | 0.32 | 0.33 |

[a] ERS denotes the test of Elliott, Rothenberg and Stock (1996) and DF-GLS and MPP denote the modified Phillips-Perron tests of Ng and Perron (2001). Sample period is 1952:I–2015:IV.

Because the time series is trending, I compute the Johansen Likelihood Ratio test assuming an unrestricted constant. Let $H_0(r) : r = r_0$ be the null hypothesis of exactly $r_0$ cointegrating vectors, and $H_1(r) : r > r_0$ denote the alternative hypothesis of more than $r_0$ cointegrating vectors.

Panel A in Table 4.C.2 reports the value of the trace statistics and maximum eigenvalue statistics for the vector of nondurable goods, durable goods expenditure and relative price. Based on the values of trace statistics reported, we cannot accept $H_0(0)$ at 1%, 5% or 10% significance level, but we accept $H_0(1)$. Similarly, the value of maximum eigenvalue statistics suggests that we cannot accept $H_0(0)$ at 5% and 10% significance level, but we accept $H_0(1)$ at 5%. Thus, both test statistics suggest there is exactly one cointegrating vector.

Panel B in Table 4.C.2 reports the value of the trace statistics and maximum eigenvalue statistics for the vector of nondurable goods, durable goods expenditure and relative price when we impose VAR(2) in levels. Based on the values of trace statistics reported, we cannot accept $H_0(0)$ at 1%, 5% and 10% significance level, but we accept $H_0(1)$ at 5% significance level. Similarly, the value of maximum eigenvalue statistics suggests that we cannot accept $H_0(0)$ at 10% significance level, but we accept $H_0(1)$.

We also compute the Johansen Likelihood Ratio test for nondurable goods, durable goods stock and relative price. The values of both trace statistics and maximum eigenvalue statistics (Panel C in Table 4.C.2) suggest that we accept $H_0(0)$ at 10%, 5% and 1% significance level. Finally, Panel D in Table 4.C.2 reports the values of both trace statistics and maximum eigenvalue statistics of the Johansen Likelihood Ratio test for nondurable goods minus expenditure to durable goods and relative price. In both cases we accept $H_0(0)$ at any convenient significance level.

As the results in Table 4.C.2 indicate, the vector of time series $[\Delta c_t, \Delta e_t, \Delta q_t]'$ follows a cointegrated VAR(2), and hence the lag length for the vector error correction model (VECM) is 1.

Table 4.C.3 reports the estimated VECM model for the time series. Table 4.C.4 reports the estimated cointegrating vector for nondurable goods, expenditure to durable goods, and relative price (Panels A and B) and nondurable goods minus expenditure to durable goods and relative price (Panel C). We can conclude that there is a long-term relationship between nondurable goods, the expenditure to durable goods, and their relative price. At the current stage of research I do not include this relationship in the estimation procedure so as not to fall for the "curse of dimensionality" and over-complicate the dynamics of the model. As a possible extension for future research it is possible to add the fourth time series (the relative price of durable and nondurable goods) into the model.

TABLE 4.C.2: Testing for cointegration

| | Eigenvalue | Trace Stats | 90% CV | 95% CV | Max Stats | 90% CV | 95% CV |
|---|---|---|---|---|---|---|---|
| **Panel A. Nondurable Goods, Durable Good Expenditures and Relative Price [a]** | | | | | | | |
| H(0)++** | 0.09 | 37.57 | 28.71 | 31.52 | 22.47 | 18.90 | 21.07 |
| H(1)* | 0.06 | 15.09 | 15.66 | 17.95 | 13.76 | 12.91 | 14.90 |
| H(2) | 0.01 | 1.33 | 6.50 | 8.18 | 1.33 | 8.18 | 8.18 |
| **Panel B. Nondurable Goods, Durable Good Expenditures and Relative Price [b]** | | | | | | | |
| | Eigenvalue | Trace Stats | 90% CV | 95% CV | Max Stats | 90% CV | 95% CV |
| H(0)++* | 0.08 | 36.63 | 28.71 | 31.52 | 20.27 | 18.90 | 21.07 |
| H(1)+** | 0.06 | 16.36 | 15.66 | 17.85 | 15.71 | 12.91 | 14.90 |
| H(2) | 0.00 | 0.64 | 6.50 | 8.18 | 0.64 | 8.18 | 8.18 |
| **Panel C. Nondurable Goods, Durable Good Stock and Relative Price** | | | | | | | |
| | Eigenvalue | Trace Stats | 90% CV | 95% CV | Max Stats | 90% CV | 95% CV |
| H(0) | 0.08 | 25.87 | 28.71 | 31.52 | 18.63 | 18.90 | 21.07 |
| H(1) | 0.03 | 7.24 | 15.66 | 17.85 | 6.51 | 12.91 | 14.90 |
| H(2) | 0.00 | 0.73 | 6.50 | 8.18 | 0.73 | 8.18 | 8.18 |
| **Panel D. Nondurable Goods Minus Durable Good Expenditures and Relative Price** | | | | | | | |
| | Eigenvalue | Trace Stats | 90% CV | 95% CV | Max Stats | 90% CV | 95% CV |
| H(0) | 0.04 | 10.65 | 15.66 | 17.95 | 10.65 | 12.91 | 14.90 |
| H(1) | 0.00 | 0.00 | 6.50 | 8.18 | 0.00 | 8.18 | 8.18 |

[a] Akaike, Bayesian and Hannan-Quinn criteria all suggest VAR(1) in levels.
[b] Imposed VAR(2) in levels.

TABLE 4.C.3: Vector error correction model [a]

| | Intercept | $c_t - \eta q_t - \lambda e_t$ | $\Delta c_t$ | $\Delta e_t$ | $\Delta q_t$ | $R^2$ |
|---|---|---|---|---|---|---|
| $\Delta c_{t+1}$ | 0.024 | -0.015 | 0.127 | 0.035 | 0.074 | 44.04 |
| | (0.015) | (0.012) | (0.070) | (0.016) | (0.046) | |
| | [1.558] | [-1.224] | [1.810] | [2.202] | [1.619] | |
| $\Delta e_{t+1}$ | -0.097 | 0.082 | 1.196 | -0.126 | 0.253 | 18.59 |
| | (0.067) | (0.052) | (0.308) | (0.070) | (0.201) | |
| | [-1.436] | [1.565] | [3.881] | [-1.799] | [1.259] | |
| $\Delta q_{t+1}$ | -0.075 | 0.056 | -0.112 | 0.023 | 0.258 | 26.66 |
| | (0.022) | (0.017) | (0.099) | (0.023) | (0.065) | |
| | [-3.447] | [3.333] | [-1.130] | [1.028] | [3.969] | |

[a] Asymptotic standard errors in parentheses whereas the $t$-statistics are in square brackets. Sample period is quarterly 1952:I–2015:IV.

TABLE 4.C.4: Estimated cointegrating vector

| Panel A. Nondurable Goods, Durable Good Expenditures and Relative Price [a] | | | |
|---|---|---|---|
| Coint. Parameter | Estimates | S.E. | t-stats |
| $\lambda$ | 0.59 | (0.03) | [-18.89] |
| $\eta$ | 0.26 | (0.09) | [-2.86] |
| **Panel B. Nondurable Goods, Durable Good Expenditures and Relative Price of** | | | |
| Coint. Parameter | Estimates | S.E. | t-stats |
| $\lambda$ | 0.60 | (0.03) | [-17.95] |
| $\eta$ | 0.26 | (0.10) | [-2.67] |
| **Panel C. Nondurable Goods Minus Durable Good Expenditures and Relative Price** | | | |
| Coint. Parameter | Estimates | S.E. | t-stats |
| $\eta$ | 0.90 | (0.32) | [2.87] |

[a] Akaike, Bayesian and Hannan-Quinn criteria all suggest VAR(1) in levels.

# 4.D Sensitivity Analysis

As the second exercise we perturbate the preference parameters and re-solve both the full model and two benchmark scenarios. Again, we use the models to simulate risk-free and equity returns and compare those between models to further understand the effects of each of the components in the model.

We start by fixing the preference parameters on a arbitrary level (we take the estimated parameters $\gamma = 2.1$, $\psi = 1.09$, $\alpha = 0.3$ as a benchmark). We then change all the parameters in turn, keeping the other parameters fixed, we change two of the parameters, keeping the other one fixed, and, finally, we change all three parameters at the same time. The values reported in table 4.D.1 are relative to those reported table 4.5.3.

The first part of table 4.D.1 reports the analysis for Benchmark Model I (no uncertainty and no durable consumption). The volatility of risk-free rate as well as the volatility of the equity premium are not very sensitive to the parameter change: the impact of change in EIS, risk aversion, or both is less then 1%. The mean equity premium, however, is more sensitive to change in preference parameters. Increase in EIS $\psi$ decreases equity premium by about 11% whereas increase in risk aversion $\gamma$ increases the equity premium by about 10%. Increase in both parameters simultaneously leads to decrease in equity premium, however it is very small.

The second part of table 4.D.1 reports the analysis for Benchmark Model II (no uncertainty but durable consumption). There is a substantial difference between both benchmark scenarios. The presence of durable consumption inflates the effect of change in risk aversion coefficient, confirming the previous intuition that durable consumption inflates equity premium, thus requiring lower risk aversion to match it. Durable consumption also reverses the impact of EIS in the model - increase in EIS leads to increase in equity premium. The change in durable share itself (keeping other parameters fixed) increases all of the moments, with the highest impact being on equity premium. The cross effects of increase in preference parameters are further inflated in the presence of durable consumption (see table 4.D.1).

Table 4.D.1: Sensitivity Analysis [a]

| | No Uncertainty, No Durables | | | No Uncertainty, Durables | | | Full Model | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\mathrm{sd}(R_{f,t})$ | $\mathbb{E}(R_{e,t+1})$ | $\mathrm{sd}(R_{e,t+1})$ | $\mathrm{sd}(R_{f,t})$ | $\mathbb{E}(R_{e,t+1})$ | $\mathrm{sd}(R_{e,t+1})$ | $\mathrm{sd}(R_{f,t})$ | $\mathbb{E}(R_{e,t+1})$ | $\mathrm{sd}(R_{e,t+1})$ |
| $\frac{\partial}{\partial\psi}$ | 0.5 | (11.1) | (0.6) | (0.1) | 44.1 | 2.0 | 1.4 | 11.4 | (1.2) |
| $\frac{\partial}{\partial\gamma}$ | 0.9 | 9.5 | 0.0 | 14.1 | 42.5 | 0.7 | 6.4 | 14.9 | 0.9 |
| $\frac{\partial}{\partial\alpha}$ | – | – | – | 28.1 | 43.7 | 2.0 | 15.0 | 1.2 | (0.3) |
| $\frac{\partial^2}{\partial\psi\partial\gamma}$ | (0.6) | (1.6) | (0.2) | 15.41 | 39.2 | 2.9 | 8.5 | 3.9 | 0.9 |
| $\frac{\partial^2}{\partial\psi\partial\alpha}$ | – | – | – | 30.0 | 46.4 | 2.3 | 16.8 | 12.8 | (1.4) |
| $\frac{\partial^2}{\partial\gamma\partial\alpha}$ | – | – | – | 48.3 | 44.2 | 5.0 | 23.2 | 16.2 | 0.6 |
| $\frac{\partial^3}{\partial\gamma\partial\psi\partial\alpha}$ | – | – | – | 51.3 | 40.7 | 3.5 | 25.8 | 5.3 | 0.7 |

[a] This table displays percentage increase of model implied moments with respect to change in preference parameters. Model implied moments are evaluated numerically. Negative numbers are in parentheses. We change the value of EIS $\psi$ by 0.02, the value of risk aversion $\gamma$ by 0.2 and the value of the share of durables $\alpha$ by 0.1.

Finally, we analyze what is the effect of change in preference parameters in the full model (last part of table 4.D.1). The effect of learning is clearly visible if we compare the previous case to the full model. The increase in risk aversion $\gamma$, EIS $\psi$ and share of durables $\alpha$ increases the equity premium, now with durable consumption itself having small effect. Durable consumption, however still has substantial effect on volatility of risk-free rate, but effect is much smaller compared to "no uncertainty" scenario. The same holds for the cross effects of change in preference parameters. It is also worth noting that the volatility of the equity premium decreases slightly with increase in durable consumption, and the effect is stronger when we also increase EIS $\psi$.

We see that with only three free parameters (risk aversion, the elasticity of the intertemporal substitution and the share of durable consumption), the full model with recursive preferences over nondurable and durable consumption, and uncertainty about the endowments matches the three moments of the returns observed in the data (the mean and volatility of the risk-free rate, and the mean of the equity premium), while also generating more than 50% of the equity premium volatility observed in the data. We conclude the section by further studying the properties of asset prices, such as the shape of the wealth-consumption and price-dividend ratios, and the predictability of the price-dividend ratio.

## 4.E Further Sensitivity Analysis

TABLE 4.E.1: Unconditional Moments of Returns for Different Values of Risk Aversion and EIS[a]

| Risk Aversion ($\gamma$) | $\mathbb{E}(R_{f,t})$ | $\mathsf{sd}(R_{f,t})$ | $\mathbb{E}(R_{e,t+1})$ | $\mathsf{sd}(R_{e,t+1})$ |
|---|---|---|---|---|
| | | Panel A: $\psi = 1.1$ | | |
| 2 | 1.20 | **1.50** | **6.12** | 10.41 |
| 5 | 1.20 | 4.65 | 6.15 | 11.02 |
| 7.5 | 1.20 | 7.35 | 6.11 | 12.18 |
| 10 | 1.20 | 10.08 | 6.00 | **13.81** |
| | | Panel B: $\psi = 1.5$ | | |
| 2 | 1.20 | **1.52** | **6.58** | 10.64 |
| 5 | 1.20 | 4.03 | 6.70 | 10.99 |
| 7.5 | 1.20 | 6.14 | 6.76 | 11.70 |
| 10 | 1.20 | 8.24 | 6.77 | **12.73** |
| | | Panel C: $\psi = 2$ | | |
| 2 | 1.20 | **1.54** | **6.86** | 10.86 |
| 5 | 1.20 | 3.71 | 7.03 | 11.09 |
| 7.5 | 1.20 | 5.51 | 7.13 | 11.59 |
| 10 | 1.20 | 7.31 | 7.20 | **12.34** |

[a] This table reports the annualised unconditional moments of model implied returns calculated as a sample values from simulated 100,000 data points. The values are reported for different values of risk aversion and EIS. Values in bold highlight the moments that are the closest to the data.

## 4.F Predictive Regressions

One way to capture variation in the conditional equity risk is to run predictive regressions. The most popular regressor in the literature is the dividend-price ratio (as in Campbell and Shiller, 1988*a*; Fama and French, 1988, etc.). To see how the model captures return predictability by the dividend-price ratio, we compute predictive regressions for the simulated return series and compare those to the data. Table 4.F.1 reports the estimated coefficients, t-values and $R^2$ for regressions of (continuously compounded) one-, three-, five-, and ten-year excess returns on the dividend-price ratio. The $R^2$ is high for short-horizons (and close to data counterpart), and then decreases at the long-horizons. The slope coefficients also increase with the horizon (we can see the same effect when we run the regression for the data). Judging by the t-statistics, the slope coefficients are also statistically significant.

TABLE 4.F.1: Long-horizon Predictive Regressions [a]

| | 1 Year | | | 3 Years | | | 5 Years | | | 10 Years | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $b$ | $t(b)$ | $R^2$ | $b$ | $t(b)$ | $R^2$ | $b$ | $t(b)$ | $R^2$ | $b$ | $t(b)$ | $R^2$ |
| Data[b] | 0.07 | (2.69) | 0.97% | 0.21 | (4.84) | 3.16% | 0.30 | (5.72) | 4.51% | 0.55 | 7.64 | 8.41% |
| Model | 0.32 | (8.23) | 0.67% | 0.32 | (4.71) | 0.22% | 0.34 | (3.93) | 0.15% | 0.34 | (2.80) | 0.08% |

[a] The regression equation is $r^e_{t\to t+k} = a + b \times dp_t + \varepsilon_{t+k}$, where $r^e_{t\to t+k}$ is the log excess return, continuously compounded over the horizon, and $dp_t$ is the log dividend-price ratio. Return horizon is 1, 3, 5 and 10 years.
[b] Excess return is the CRSP value-weighted return less the 3-month Treasury bill. Data is quarterly from 1952:Q1 - 2016:Q4, available from the CRSP.

# Chapter 5

# Conclusions

This thesis is a collection of three essays that analyze the interplay between financial and mortgage markets, and what are the spillover effects from the activity on those markets on households' consumption.

In chapter one, we studied the spillovers from government intervention in the mortgage market on households' consumption. In particular, we showed that after an expansionary mortgage market operation, the consumption response of homeowners with mortgage debt was large and significant, while the consumption response of homeowners without the mortgage debt is small and insignificant. Non-homeowners also increased their consumption but less than mortgagors. We also found that expansionary policy significantly increased consumption inequality of mortgagors. We explained these facts through the lens of a life-cycle model with incomplete markets and endogenous housing choice in the spirit of Huggett (1996). The intuition for this result was the following. Reduction in credit rates creates extra wealth for the mortgagors while the reduction in interest rates shifts this wealth towards consumption. Increase in wealth is bigger for those with a larger mortgage – this exacerbates consumption inequality.

In chapter two, we studied the role of durable consumption in the context of long-run risk models. The long-run risk models of Bansal and Yaron (2004) became a cornerstone in the macro-finance literature for their ability to capture key asset price phenomena. They are, however, known to entail implausibly high levels of timing and risk premia (see, for example, Epstein, Farhi and Strzalecki, 2014). In this chapter, we addressed this puzzle by considering the consumption of durable goods in addition to that of non-durable goods. In our estimated model, the timing premium reduced to 11 percent and the risk premium to 16 percent of lifetime consumption. These values are about a third of the previously implied premia and are more consistent with empirical and experimental evidence.

In chapter three we used the Michigan Survey of Consumers to provide novel evidence that a rise in consumers' beliefs about current and future aggregate durable expenditure predicts a rise in expected returns. We rationalized this finding through a lens of consumption-based asset pricing model with recursive preferences over non-durable and durable goods and uncertainty about the underlying endowments. The model generated high equity premium, low and stable risk-free rate, and explained up to 60% of the volatility of equity premium, with calibrated parameters consistent with the macroeconomic literature (risk aversion of 2.1 and elasticity of intertemporal substitution of 1.09).

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