POWER SHARING, INCOME REDISTRIBUTION AND CORRUPTION

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Declaration

No portion of the work referred to in this thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

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Chapter 1

Introduction

Election works as the key mechanism of the policy making process in the democratic government. Since the seminal work of Downs (1957), spatial voting model has been used to provide the institutional framework of modern politics. The policy implementation process is usually modeled as in the winner-take-all system. However, in the real world politics even for the plurality electoral system there is power sharing in the policy making process such as separation of powers, veto power or agenda setting etc. The power sharing mechanism not only shape the policy but also determines the intensity of the electoral competition.

The research on the other extreme of institutional design as proportional representation in models of electoral competition starts with Ortuno-Ortin (1997), where the policy-making power of each party equals to their vote share. Coalition formation, policy polarization and strategic voting have been studied under the proportional representation framework. Later in the comparative politics literature pioneered by Persson and Tabellini (1999), the difference between majoritarian and proportional representation has been extensively studied both theoretically and empirically. Recently, Saporiti (2014), Herrera et al. (2016) and Matakos et al. (2016) studied policy polarization and voter turnout in the Downsian voting model with a reduced form of power sharing mechanism, which reflects power sharing not only in the electoral system but also in the political system in general. In this book, we aim to extend the analysis to multidimensional policy space, namely to introduce power sharing mechanism into the probabilistic voting model. The focuses of our study are the political redistribution policy and corruption.

In Chapter 2, we analyze a political competition model of redistributive policies. We introduce fairness concerned voters and politicians and power sharing mechanism into the canonical probabilistic voting model. We first prove that the pure strategy Nash equilibrium exists under some mild conditions on distribution function of the partisanship and power sharing rule. Next, we show that the net transfers to the income groups consist of two parts, called *altruistic* and *electoral* redistribution. More specifically, the net transfer is determined by a list of factors including: (i) the pre-tax income gap, (ii) the relative partisan independence of the poor, (iii) parties' and voters' fairness concern, and (iv) the power sharing disproportionality. In addition, we show that the income inequality as measured by Gini index rises with higher power sharing disproportionality. Finally, we extend the basic model to alternative income-sorting assumption, power sharing rule and asymmetric motivations.

In Chapter 3, we study the effect of power sharing on corruption through the electoral competition channel. The power sharing mechanism captures many institutional details in the political system. We show that a more disproportional system will perform better in terms of restricting corrupt behavior of the elected officials. Moreover, we confront the theoretical prediction with data from over 90 countries. The empirical results show a significant and robust linkage between power sharing and corruption which confirms our theoretical prediction.

Chapter 4 concludes this thesis and provides some future research directions. Overall, we find that in a more disproportional system we have narrower representation but better accountability.

Chapter 2

Power Sharing and Redistribution

2.1 Introduction

Since the mid twenty century an increasingly important activity of government in western democracies consists in redistributing income among different socio-economic groups. Quite often, this activity is not only motivated by the altruistic goal of reducing income disparities among citizens, but also by the tactical objectives of the political actors competing in the elections. In this chapter, we study several determinants of *altruistic* and *electoral* redistributive policies, emphasising in particular the role of fairness concern, partisanship, and political power sharing. The ultimate goal of this chapter is to build a theoretical framework that explains the empirical positive relationship between power sharing disproportionality and income inequality as shown on the following figure 2.1.[1]

Following recent research on fairness and redistribution, pioneered by Alesina and Angeletos (2005a,b),[2] we first modify the canonical model of redistributive politics due to Lindbeck and Weibull (1987) to allow voters to express a concern not just about their own well-being (e.g., disposable income), but also about the well-being of other members of society. This is consistent with data from laboratory experiments and neuro-imaging studies, which show that people are to some extent willing to sacrifice personal gains and share resources with others

[1]The data on Gini coefficient is the average of the Gini index on 2010 computed by The World Inequality database, while the data on electoral disproportionality is the Taagepera index computed by the author at the most recent elections before 2010.

[2]We discuss in greater detail the literature related to this chapter in Section 2.5.

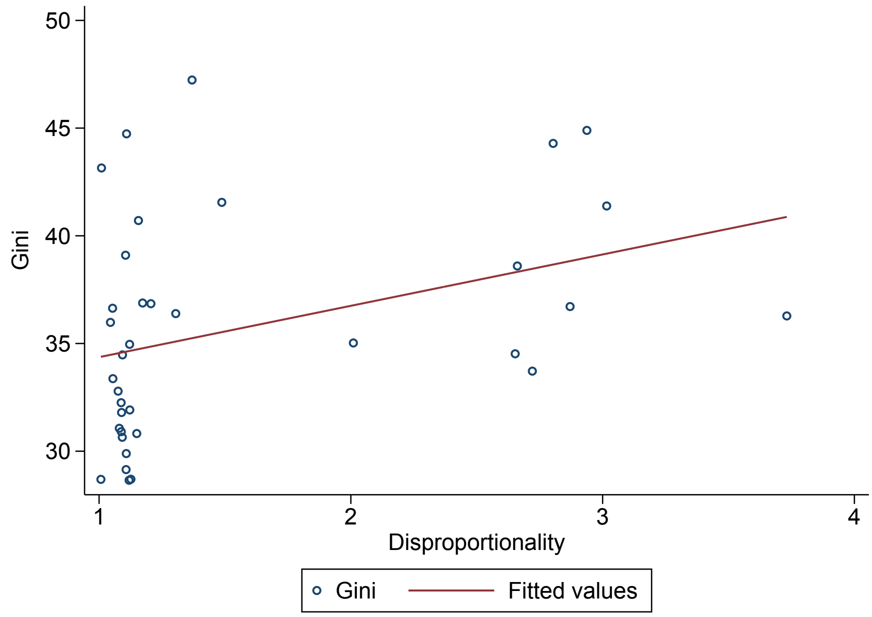

Figure 2.1: Disproportionality and Gini

to eliminate inequalities that they view as unfair.

In our model, the concern with fairness embedded into voters' and parties' preferences over redistributive policies is represented by a concern with egalitarianism, i.e., a dislike of unequal outcomes per se.[3] It is worth noting that, in contrast with the inequity aversion concept of Fehr and Schmidt (1999), which expresses envy and altruism and is self-centered,[4] our "public-value notion" of fairness, as is referred to by Corneo and Grüner (2002), is more in the vein of Arrow (1963), in the sense that individuals' attitude towards income redistribution reflects some ideal or principle of social justice about how resources ought to be distributed in society.

Besides introducing fairness into the utility functions, we also extend the redistribution model of Lindbeck and Weibull (1987) to accommodate a continuum of

[3]Experimental and neural evidence of egalitarian motives in humans strongly support the role of the anterior insula of the human brain (often associated with negative emotions such as pain and distress) in promoting egalitarian behavior (Dawes et al. 2007 and Dawes et al. 2012).

[4]By self-centerness we mean that fair-minded people in the inequity aversion sense is influenced by the comparison between their own payoffs and that of a reference group, but not by inequality per se, or by the differences among payoffs of other individuals. Interestingly, experimental evidence seems to indicate that the opposite might happen in simple distribution games, where people seem to consider also differences among others in their utility functions (Engelmann and Strobel 2004).

power sharing rules, ranging from purely proportional representation to winner-take-all. This is motivated by the fact that in modern democracies, politics is not "all or nothing", but most often it requires consensus and compromise among multiple policymakers (Powell 2000, Franzese 2010, Lijphart 2012). Indeed, majority and minority parties usually interact in government through a variety of channels and institutions; and the amount of policymaking power shared by these political actors shapes not only policy, but also the intensity of electoral competition.

Following the modeling strategy of Saporiti (2014), Matakos et al. (2016), and Herrera et al. (2016), we represent power sharing in our framework with the help of a *contest success function*. This function is meant to reflect in a reduced-form the institutions of representation, governance, and policy-making (such as, separation of powers, the electoral system, forms of government, agenda-setting and veto powers, amendment procedures, etc.) that shape the mechanism that transforms the votes of the parties, obtained in the election, into the decision-making power or "influence" over the implemented policy.[5] In our case, it specifically determines the post-election power of the political parties as a function of their *relative* electoral strengths, i.e., in relation to their *ratio* of votes. The implemented policy is then defined as a combination or *compromise* of the electoral proposals, each weighted by the party's corresponding share of policy-making power.[6]

The main results of this chapter are as follows. First we prove that under mild conditions the election game has a unique pure strategy Nash equilibrium. The first condition on the distribution function guarantees the vote share function to be concave, while the second condition guarantees that after the transformation the power share function is still concave. The first condition is a generalized version of Lindbeck and Weibull (1987) and it requires that the concavity of the utility function is greater than convexifying ability of the distribution function. The convexfying ability depends on the ratio of the second order derivative over

[5] Along this thesis, we employ the terms power sharing and electoral rules interchangeably and without making a distinction between them. However, as Herrera et al. (2016) point out, the former should be viewed as a much broader concept, representing not (like electoral rules) simply the mapping from votes shares into seat shares in the legislature, but the relationship between the electoral outcomes and parties' direct influence over the policy-making process.

[6] This assumption allows to abstract from the specificities of the (extensive-form) bargaining game that determines the implemented policy. Luckily, as Franzese (2010) points out, under complete information in any of these bargaining games the resulting equilibrium policy is a weighted-average combination of the policymakers' (or parties' in our case) most-preferred policies (platforms). Furthermore, this convex combination can be rationalized as the solution of a Nash bargaining process, where the optima of each of the players is weighted by its relative bargaining strength (Franzese 2010).

the first order derivative of the distribution function. This condition is satisfied by a group of well known distribution functions including the uniform distribution and doubly exponential distribution considered by Lindbeck and Weibull (1987). The second condition requires that the power sharing disproportionality to be in a reasonable range so that the power share is still a concave function over policy. Under these two conditions, we prove the existence of a unique pure strategy Nash equilibrium using the standard Debreu-Glicksberg-Fan's theorem.

Second, we show that the equilibrium redistributive policies of the modified probabilistic voting model can be divided into two parts. The first part coincides with the optimal policy of a purely altruistic party willing to achieve equality after redistribution, and is given by the difference or *gap* between the population and the group mean initial income. The second part represents the amount of tactical redistribution across income groups carried out for electoral purposes, and it depends on the interplay of three main factors: (i) the (relative) partisan independence of the poor, (ii) parties' and voters' fairness concern, and (iii) the electoral rule (dis)proportionality.

Finally, we derive from our equilibrium characterization a number of predictions. Among them, our analysis shows that the net transfers to all groups rise with the income gaps between the population and the group mean initial income. Likewise, the gap between the partisan independence of the poor and the average across all income groups increases the transfers to the poor and reduces income inequality. We also find that fairness concern curbs electoral redistribution and inequality, transferring resources from the middle class and the rich to the poorer segment of society. Interestingly, an effect in the opposite direction on the net group transfers is driven by power sharing disproportionality.

With regard to the after-tax income inequality, we prove that the Gini index after redistribution increases as policymaking power gets more concentrated in the majority winning party. The latter as well as the effect of power sharing over the net transfers take place if and only if parties are fair-minded, in which case the intensity of the electoral competition (determined by power sharing) affects parties' willingness to trade off equity for votes. By contrast, if parties maximize simply the expected vote shares, targeted spending to the swing voter groups is not affected by the power sharing regime.

The rest of this chapter is organized as follows. We set up the model in Section 2.2. The theoretical results are derived in Section 2.3. Section 2.4 presents several theoretical extensions of the model. Section 2.5 concludes this chapter summarizing the main findings and discussing the related literature. For convenience, proofs appear in Appendix.

2.2 The Model

2.2.1 Voters

Consider a continuum N of voters (large electorate) divided into three disjoined groups: the rich (R), the middle class (M), and the poor (P). Let $n_i \in (0,1)$ denote the size of group $i \in N$, with $\sum_{i\in N} n_i = 1$, and let $\sigma_i = n_i/(1-n_i)$ be group i's relative size in relation to the other groups. Suppose $e_i > 0$ denotes the initial income of every voter of group $i \in N$. Assume the income distribution is skewed to the right, with the mean income $e = \sum n_i\, e_i$ greater than the median $\bar{e}$, and $e_R > e > \bar{e} = e_M > e_P$. This assumption does not affect the main analysis, we assume this to make the model closer to the real world observations.

The initial allocation of incomes might not be seen as *fair* by the electorate. To represent voters' preferences for redistribution, let $\mathbf{z} = (z_i)_{i\in N} \in Z$ be an arbitrary income distribution, with $Z = \{\mathbf{z} \in \mathbb{R}_+^{|N|} \mid \sum_{i\in N} n_i\, z_i = \sum_{i\in N} n_i\, e_i\}$ denoting the set of all such allocations. The utility of a voter in group i over Z is given by

$$u_i(\mathbf{z}) = z_i - \alpha_i \sum_{j\in N} n_j\,(z_j - z)^2, \tag{2.1}$$

where for each $j \in N$, z_j denotes voter j's income under the distribution $\mathbf{z} \in Z$, $z = \sum_{j\in N} n_j\, z_j$ is the population mean income, and the parameter $\alpha_i \in \mathbb{R}_+$.

The preferences shown in equation (2.1) are additively separable in the voter's concern with his own well-being and his concern with the others', expressing a trade-off between self-interest and a pro-social motive. The first term of the right-hand side denotes voter i's *selfish utility* over his income z_i. The second term, i.e., the expression $-\alpha_i \sum_{i\in N} n_i\,(z_i - z)^2$, measures voter i's intrinsic concern with *fairness* (or inequality). To elaborate, taking the mean income under $\mathbf{z}$ as a reference point, voter i's concern with fairness is represented by the weighted sum of the distances between each group's average income and the reference point, with the weights given by the group sizes.

2.2.2 Political process

To remedy any social injustice created by the initial allocation of resources, there is a political process that redistributes income across groups through a tax-and-transfer policy. Let $x_i \in \mathbb{R}$ denote a net transfer imposed upon voters of group $i \in N$. A balanced-budget redistributive policy is a vector $\mathbf{x} = (x_i)_{i\in N} \in \mathbb{R}^{|N|}$ such that $\sum_{i\in N} n_i\, x_i = 0$ and $x_i \geq -e_i$ for all $i \in N$. We further restrict the set X of all such policies to guarantee *no income sorting*, in the sense that the ranking of disposable incomes $y_i = e_i + x_i$ after redistribution preserves the ordering of the initial incomes of the groups, that is, $y_R \geq y_M \geq y_P$.[7]

There are two political parties, indexed by $C \in \{A, B\}$, that compete in an election proposing simultaneously and independently a redistributive policy $\mathbf{x}^C \in X$.[8] Like in the Lindbeck-Weibull model, each voter has an partisan bias or preference toward the parties, which is unrelated to the current policy. This preference is fixed in the short-term, and may depend on prior political experience, attributes of the candidates, ideology, etc. Before the election, political parties are unsure about the partisan preferences of the electorate. More precisely, they view voter i's partisan bias θ_i as being drawn from a twice continuously differentiable distribution function $F_i(\cdot)$ over $\mathbb{R}$, with a density f_i that takes a value at zero (neutral bias) of $f_i(0) = \phi_i > 0$.

Following data about partisan independence and the income groups taken from the European Social Survey, we assume that $\phi_M > \phi > \phi_P > \phi_R$, where $\phi = \sum n_i\, \phi_i$. These conditions on the densities imply that the middle class is the "swing voter group" in our model, with the highest proportion of partisan independent voters, followed by the poor, and the rich.[9] In addition, the second inequality, that is, $\phi > \phi_P$, rules out the less compelling case where all voters have the same after-tax equilibrium income. Finally, to prevent any group to be fully expropriated and be left with a non-positive after-tax income, we assume that $\phi_P > \phi - 2\phi_\alpha\, e$, where $\phi_\alpha = \sum_{i\in N} n_i\, \phi_i\, \alpha_i$ is an average across groups reflecting

[7] Like in Lindbeck-Weibull model, under *income sorting* the ranking of the groups after redistribution changes in equilibrium in such a way that the rich becomes the lowest income group. This is not very realistic, since non-rich voters do not seem to possess in western democracies the political power to carry out such level of expropriation. Despite this, when income sorting is permitted, Section 2.4 shows that the main qualitative properties of the equilibrium transfers are similar.

[8] Hereafter, it is understood that the index $-C$ denotes B if $C = A$ and A if $C = B$.

[9] Persson and Tabellini (1999) also argue in favor of thinking of the group with the highest density of partisan independent voters as consisting of middle class voters.

independent voters' fairness concern.

At the election, each voter votes sincerely for the party's proposal that offers higher utility.[10] Specifically, a voter of group i votes for party A if $u_i(\mathbf{y}^A) \geq u_i(\mathbf{y}^B) + \theta_i$, where $\mathbf{y}^C = (y_i^C)_{i\in N}$, with $y_i^C = e_i + x_i^C$ representing group i's after-tax income under the policy of party C. Given that for every group $i \in N$, the initial income e_i is held fixed throughout the analysis, in the sequel we simply denote $u_i(\cdot)$ as a function of $\mathbf{x}^C$. Therefore, the probability that a voter in group i votes for party A given the platforms $\mathbf{x}^A$ and $\mathbf{x}^B$ is $\text{Prob}(\theta_i \leq u_i(\mathbf{x}^A) - u_i(\mathbf{x}^B)) = F_i(u_i(\mathbf{x}^A) - u_i(\mathbf{x}^B))$. As a result, the expected vote share of party A, denoted by v^A, is given by $v^A(\mathbf{x}^A, \mathbf{x}^B) = \sum_{i\in N} n_i\, F_i(u_i(\mathbf{x}^A) - u_i(\mathbf{x}^B))$. Assume there is no abstention, then party B's vote share is simply $v^B = 1 - v^A$.

After the election, the winning party and the opposition jointly determine the tax-and-transfer scheme $\mathbf{x} \in X$ in accord with their policy platforms $\mathbf{x}^C$ and their relative political strengths ρ^C. To be precise, we assume that $\mathbf{x} = \rho^A\, \mathbf{x}^A + \rho^B\, \mathbf{x}^B$, where $\rho^C = \Phi(v^C)$ denotes party C's power share ("influence") at the policymaking process as a nondecreasing function $\Phi : [0,1] \to [0,1]$ of party C's vote share v^C, with the usual requirement that $\rho^B = 1 - \rho^A$.[11]

Regarding the specific functional form of the power sharing function, we follow an string of the literature that sees party influence over policy as being determined by the relative electoral strengths of the parties, represented here by the ratio of votes. To be precise, we assume that the power sharing function ρ^C is given by

$$\rho^C = \frac{1}{1 + \left(\frac{1-v^C}{v^C}\right)^{\eta}}, \tag{2.2}$$

where $\eta \geq 1$ is a parameter interpreted below as the disproportionality of the electoral rule.[12]

Simple algebraic manipulation shows that (2.2) implies that ρ^C/ρ^{-C} =

[10] This entails no loss of generality because the probability of being pivotal at the election is zero given the continuum of voters.

[11] The influence over policy $\rho^C(\cdot)$ exerted by each party can be interpreted as its probability of determining alone policy $\mathbf{x} \in X$, which is expected to be nondecreasing in the party's vote share.

[12] An alternative to equation (2.2) would be to see parties' power shares as a function of the *margin of victory* (or electoral mandate), instead of the *ratio of votes*. The qualitative results of this chapter are robust to this alternative specification, since the equilibrium characterization under the "margin of victory" power sharing rule only suffers minor changes in comparison with that derived under (2.2). Details are omitted for the sake of brevity, but they are available in section 2.4.

$(v^C/v^{-C})^\eta$, which is Theil's (1969) well-known hypothesis about how vote shares translate into seat shares in a legislature. When $\eta = 1$, the expression above represents the purely proportional representation system, where the influence of each party coincides with its vote share. As the parameter η rises above 1, the electoral rule gets more disproportionate and biased in favour of the majority winning party.[13] In the limit, as η approaches infinity, (2.2) captures the winner-take-all system where the party holding more votes controls all branches of government and sets policy unilaterally.

To complete the model, we introduce the parties' payoff functions, $\Pi^C(\cdot)$, which are a combination of the interests of: (i) the politicians and party leaders, who seek power to influence policy, and (ii) other party members and supporters, who care to a certain extent about fairness in society. Formally, the payoff of party C is defined as $\Pi^C(\mathbf{x}^A, \mathbf{x}^B) = (1-\gamma)\cdot\rho^C - \gamma\,\frac{1}{2}\cdot\sum_{i\in N} n_i\,(y_i^C - y^C)^2$, where $y^C = \sum_{i\in N} n_i\,y_i^C$ is the mean after-tax income under party C's policy, and $\gamma \in [0,1]$ denotes party fairness concern.[14] When $\gamma = 0$, parties maximize their expected vote shares. At the other extreme when $\gamma = 1$, they are purely altruistic and seek to achieve an egalitarian distribution of income. In between these limit cases, parties compete motivated by *both* power and fairness.

2.2.3 Timing

Let $\mathcal{G} = (X, \Pi^C)_{C=A,B}$ denote the ***redistributive election game*** sketched above. The timing of this game is as follows. First, parties A and B propose simultaneously and non-cooperatively redistributive policies $\mathbf{x}^A$ and $\mathbf{x}^B$, respectively. At this stage, parties know the initial income of the groups, voters' preferences over the income distribution, and the group-specific cumulative distributions of the partisan bias, but not yet their realized values. Second, the actual values of θ_i are realized and all uncertainty is resolved. Third, voters cast their vote for one of the parties. Fourth, the vote and the power shares are determined and, together with the parties' proposals, they determine the implemented policy. Finally, fifth, parties and voters receive their respective payoffs.

[13]For instance, when $\eta = 3$, the seat allocation follows the "cube law", which is seen as approximating the distortions created in favour of the winner party in first-past-the-post elections.

[14]Alternatively, γ could be seen as the reputational cost for the party of campaigning on distributive policies perceived by the electorate as "socially insensible" (i.e., the cost of building the image of being a "nasty party" that only cares about the privileged few and not the many, as the British Conservative Prime Minister, Theresa May, put it in her 2002 party conference speech). The section 2.4 offers further results for the *asymmetric* case where $\gamma_A \neq \gamma_B$.

2.2.4 Discussion

The main objective of this chapter is to study the relationship between power sharing and income redistribution policy. The literature on electoral competition and income distribution has focused on the debate between swing voter or core voter arguments (i.e. Cox and McCubbins (1986), Cox (2009), Lindbeck and Weibull (1987), and Dixit and Londregan (1995, 1996, 1998)), while the differences between different political systems are neglected. For a detailed discussion about the debate please see Cox (2009). The overall message for the swing voter literature is that parties will target voters with less partisanship bias in order to get favorable position in the election. By introducing power sharing mechanisms, we can study whether this targeting strategy changes across political system.

Besides introducing merely power sharing rule, we also modify the preferences of politicians and voters by allowing them to have inequality aversion preferences. This is because without the inequality aversion preference the differences in power sharing disproportionality are not enough to change the redistributive policies. In any political system, the vote maximizing parties will promote the same policy regardless of the power sharing since power is increasing in vote share and the intensity of electoral competition cannot change the policy platform if both parties are win-motivated. Hence, we introduce the inequality aversion preferences in both voter's and parties' objectives. With the tradeoff between selfish utility and inequality aversion preference, the intensity of electoral competition do affect the equilibrium platform choices of the parties'. Moreover, the key contribution of this chapter is to see how power sharing rule affects the tradeoff between altruistic redistribution and electoral redistribution, which can only appear in a model with both power sharing rule and inequality aversion preference.

The main analysis is focused on two-party electoral competition. However, the power sharing function we employed can be easily extended to multi-party cases. Define the power share for each party as:

$$\rho^C = \frac{(v^C)^\eta}{\sum_C (v^C)^\eta}. \tag{2.3}$$

The effect of power sharing on electoral intensity would still be preserved, while there is difficulty in show equilibrium existence. A complete analysis including endogenous number of parties is left for future work.

2.3 Equilibrium

2.3.1 Existence

We begin the equilibrium analysis showing that under fairly general conditions, the redistributive election game has a unique equilibrium in pure strategies. To do that, for each group $i \in N$, let the index $\left(\sum_{j\in N} \xi_{ij}(\mathbf{x})\right)^{-1}$ be a measure of the overall concavity of the utility function $u_i(\cdot)$ at $\mathbf{x} \in X$, where $\xi_{ij}(\mathbf{x}) = -\frac{[\partial u_i(\mathbf{x})/\partial x_j]^2}{\partial^2 u_i(\mathbf{x})/\partial x_j \partial x_j}$. Fix C and $\mathbf{x}^{-C} \in X$, and define for each group $i \in N$ and for each party C's proposal $\mathbf{x}^C \in X$, the utility differential $t_i(\mathbf{x}^C) = u_i(\mathbf{x}^C) - u_i(\mathbf{x}^{-C})$. Let $\kappa_i(\mathbf{x}^C) = \frac{f_i'(t_i(\mathbf{x}^C))}{f_i(t_i(\mathbf{x}^C))}$ be the (logarithmic) rate at which the rate of change of party C's vote share $v^C(\,\cdot\,, \mathbf{x}^{-C})$ varies within group $i \in N$ in response to changes in $t_i(\mathbf{x}^C)$.[15] A sufficient condition for $v^C(\,\cdot\,, \mathbf{x}^{-C})$ to be concave on X is as follows:

Condition $\mathbb{C}_1$: For all $i \in N$, and all $\mathbf{x}^C \in X$, $\kappa_i(\mathbf{x}^C) \leq \left(\sum_{j\in N} \xi_{ij}(\mathbf{x}^C)\right)^{-1}$.

This condition is fulfilled in a number of meaningful cases, including the uniform distribution and the doubly exponential distribution (logit) case considered by Lindbeck and WeibullLindbeck and Weibull (1987), the latter when the overall concavity is greater than one. As a passing remark, notice that in the Lindbeck-Weibull model the right-hand side of Condition $\mathbb{C}_1$ reduces to $\left(\xi_{ii}(\mathbf{x}^C)\right)^{-1}$. The reason is the cross-derivatives of the vote share $v^C(\,\cdot\,, \mathbf{x}^{-C})$ with respect to party C's net transfers are all null, which simplifies greatly its Hessian matrix. Instead, in our case due to the inequality concern, the marginal increase in the percentage of votes that party C obtains by changing group i's transfers x_i^C varies with the transfers x_j^C to group $j \neq i$.

Although the condition above is enough to ensure that each party's vote share is concave given the policy of the other party, this is not sufficient for equilibrium existence. The reason is the parties' payoff functions involve the power shares ρ^C, which are an increasing transformation Φ of v^C. Therefore, in addition to $\mathbb{C}_1$, a restriction on Φ is needed to preserve the concavity of $\Phi(v^C(\,\cdot\,, \mathbf{x}^{-C}))$ on X. To do that, fix as before party C and the policy of the other party $\mathbf{x}^{-C} \in X$. For each group i, $j \in N$ and every policy $\mathbf{x}^C \in X$, denote $\delta_{ij}(\mathbf{x}^C) = \frac{\partial^2 v^C(\mathbf{x}^C, \mathbf{x}^{-C})/\partial x_i^C\, \partial x_j^C}{\partial v^C(\mathbf{x}^C, \mathbf{x}^{-C})/\partial x_i^C \,\cdot\, \partial v^C(\mathbf{x}^C, \mathbf{x}^{-C})/\partial x_j^C}$, and let $\delta(v^C) = \frac{\Phi''(v^C)}{\Phi'(v^C)}$ for each $v^C \in [0, 1]$. Next, define for each $\mathbf{x}^C \in X$ the 3×3 matrix $\Delta(\mathbf{x}^C) = \left(\delta_{ij}(\mathbf{x}^C)\right)_{i,j\in N}$,

[15] For instance, in the uniform case this ratio is equal to zero, meaning that changes in the utility differential affect the vote share of each political party at a constant rate.

where $\Delta^k(\mathbf{x}^C)$, $k = 1, 2, 3$, represents the determinant of the k-th leading principal minor of Δ at $\mathbf{x}^C$. Finally, given any square matrix $M = (m_{ij})$, denote by $M_{[i,\,u]}$ the matrix obtained from M by replacing in each column of the i-th row the entry scalar u.

Our next condition can be formally stated as follows:

Condition $\mathbb{C}_2$: For all $\mathbf{x}^C \in X$ and each k, $(-1)^{k+1}\,\Delta^k(\mathbf{x}^C) \leq (-1)^k \sum_{j\leq k} \Delta^k_{[j,\,\delta]}(\mathbf{x}^C)$.

In practice, depending on the different values adopted by the main parameters of the model, (particularly, on α_i and F_i), Condition $\mathbb{C}_2$ establishes an upper bound on the disproportionality parameter η below which the power sharing function exhibited in equation (2.2) satisfies concavity.[16] Therefore, with these two conditions $\mathbb{C}_1$ and $\mathbb{C}_2$ in place, we are now ready to state our result on equilibrium existence.

Proposition 2.1. *If conditions $\mathbb{C}_1$ and $\mathbb{C}_2$ hold for each party C, then the redistributive election game $\mathcal{G}$ has a unique Nash equilibrium in pure strategies.*

The proof of Proposition 2.1 is displayed for expositional convenience in section 2.6, as is the proof of any other result in the paper. The proof for existence relies on the standard Debrew-Glicksberg-Fan's theorem. Since our policy space is non-empty, convex and compact, we only need the payoff function to be quasi-concave for the existence. And Condition $\mathbb{C}_1$ and $\mathbb{C}_2$ exactly guarantee the concavity of the payoff function. Moreover, the uniqueness result relies on the strict quasi-concavity since this electoral competition game has the structure of the strictly competitive games.

Here it is worth pointing out that Proposition 2.1 guarantees the existence of Nash equilibrium in pure strategies in a broad family of tactical redistribution games $\mathcal{G}(\gamma^A, \gamma^B)$, including games with *symmetric* (i.e., $\gamma^A = \gamma^B$) and *asymmetric* (i.e., $\gamma^A \neq \gamma^B$) party fairness concerns. The latter family is particularly interesting because in reality political parties might care differently about fairness, due for example to different views about the driving forces behind income

[16] By being an increasing and concave transformation of the party's vote share, under condition $\mathbb{C}_1$ the power share function is always concave between half and one. The restriction on η is necessary to preserve concavity below half, that is, for the range of values of the vote share over which Φ is increasing but convex. Otherwise, if η were not constrained, it could be the case that the political parties increase their power sharing by reducing their percentage of votes in the election, which would eliminate the trade-off investigated in this paper between equity and votes.

inequality (e.g., luck vs effort); or because one party is "captured by" say the rich and the elite, and the other is heavily influenced by the unions and the working class.

2.3.2 Characterization

Despite the appeal of and the existence result under asymmetric party fairness, an equilibrium characterization for this case is typically hard to derive without severely restricting the model structure. Further, even when closed-form expressions can be calculated, there is not much one can say about how redistribution responds to changes in the main parameters of the model (i.e., power sharing, fairness, and ideological neutrality), because parties propose different transfers at the equilibrium, and that implies the expected vote shares depend on the specific distribution of the ideological bias.[17] To illustrate, we show in section 2.4 that in the simplest asymmetric scenario one could possibly imagine, where one party, say A, is not fair-minded and the other, say B, is purely altruistic, the transfers to group $i \in N$, namely, $x_i = e - e_i + \rho^A \cdot \widetilde{\beta}_i \cdot (\phi - \phi_P)$, with $\widetilde{\beta}_R = \widetilde{\beta}_M = \sigma_P \, (2\,\phi_\alpha)^{-1} = -\sigma_P \, \widetilde{\beta}_P$, depends on party A's equilibrium power share ρ^A, which is determined by equation (2.2), together with $v^A = \sum_{i \in N} n_i F_i \left(u_i \left(y^A\right) - u_i \left(y^B\right)\right)$, and $u_i \left(y^A\right) - u_i \left(y^B\right) = \widetilde{\beta}_i \cdot (\phi - \phi_P) - \alpha_i \cdot (\phi - \phi_P)^2 \cdot \sum_{i \in N} n_i \cdot \widetilde{\beta}_i^2$, where F_i represents the uniform distribution on $[-\frac{1}{2\phi_i}, \frac{1}{2\phi_i}]$. These are clearly complex expressions that do not allow to tell much about the comparative statics of the equilibrium of the game.

To circumvent this difficulty, we focus below on a much more tractable case where the two parties care equally about fairness, that is, $\gamma^A = \gamma^B = \gamma$. The resulting redistributive election game with symmetric fairness concerns, denoted by $\mathcal{G} = \mathcal{G}(\gamma, \gamma)$, is then used to investigate not only the equilibrium shape of the group transfers, but also the effects of targeted spending on inequality after redistribution. As we explained above, our excuse for focusing on symmetric party fairness is mostly pragmatic. However, we reckon that it is also partly justified by the fact that our modelling strategy deliberately abstract from considerations about the socio-economic structure of the political parties, and it does not distinguish either among alternative sources of individual income (e.g., effort, ability,

[17]By contrast, we show below that in the symmetric fairness case, regardless of the c.d.f. F_i, the expected vote shares are 1/2 at the equilibrium, because both parties campaign on the same policy.

luck, etc.), both of which might create different perceptions within the parties about the fairness of the income distribution. We come back to the asymmetric fairness case at Section 2.4, where we briefly discuss how the things we know about its equilibrium outcome compares with the symmetric equilibrium studied next.

We now characterize the equilibrium transfers emerging from the election, under the preferences for redistribution and the power sharing rule shown in (2.1) and (2.2), respectively, and assuming that $\gamma^A = \gamma^B = \gamma \in [0,1]$.

Proposition 2.2. *Let* $(\mathbf{x}^A, \mathbf{x}^B) \in X \times X$ *denote the pure-strategy equilibrium of the redistributive election game* $\mathcal{G}$. *For all* $i \in N$, $x_i^A = x_i^B$, *where*

$$x_i^C = \underbrace{(e - e_i)}_{AR} + \underbrace{\beta_i (\phi - \phi_P)}_{ER}, \;\; C = A, B, \tag{2.4}$$

with $\beta_P = -\frac{(1-\gamma)\,\eta}{(1-\gamma)\,2\,\eta\,\phi_\alpha + \gamma} < 0$ *and* $\beta_M = \beta_R = \frac{(1-\gamma)\,\eta\,\sigma_P}{(1-\gamma)\,2\,\eta\,\phi_\alpha + \gamma} > 0$.

The characterization given in Proposition 2.2 points out that despite the electoral system (that is, proportional representation, winner-take-all, or a system in between), the usual centripetal forces of electoral competition lead political parties to converge to a similar redistributive policy.[18] More importantly, it also shows that the tax-and-transfer policy to which parties converge consists of two parts:

- A first part, called ***altruistic redistribution*** (AR), which coincides with the policy chosen by an altruistic political party, and is equal to the gap between the population and the group mean initial income; and

- A second part that captures the amount of ***electoral redistribution*** (ER) carried out to increase voters' support, and that depends on three main factors: (i) the partisan independence gap of the poor, measured by the difference between the density of swing voters in that group and the average density in society, (ii) the proportionality of the electoral rule, and (iii) parties' and voters' inequality concern. The partisan independence gap of

[18] As a note of caution, it should be noticed that our equilibrium result doesn't produce the convergence to the median voter's most-preferred policy, as is the case in the Downsian framework. It does preserve however the principle of minimum differentiation of spatial competition, which is a common feature shared by other election games with two symmetric parties and full commitment to the campaign proposals.

the poor $\phi - \phi_P$ measures how the party cares more about the swing voter group at the cost of the poor group, while each β_i measures how each group benefit from this power seeking incentive of the parties.

The assumptions on the income distribution and on the group densities imply that the equilibrium transfers to the middle class are positive. For the other groups, the sign is indeterminate because AR and ER work in opposite directions. By playing with the magnitudes of these two, it could happen that either the middle class and the poor (resp., rich) benefit from income redistribution at the expense of the rich (resp., poor); or that the middle class is the only group benefiting from redistributive politics, a result known in the literature as Director's law.

From the utilitarian viewpoint, the equilibrium displayed in Proposition 2.2 is socially optimal, in the sense that it can be rationalized as the policy outcome obtained by maximizing a utilitarian social welfare function that weights voters' utilities according with the group sizes, the ex-ante distribution of partisan preferences, the fairness concern parameters, and the electoral rule disproportionality. To be more precise:

Corollary 2.1. *If* $\mathbf{x}^C \in X$ *denotes party* C*'s equilibrium policy at the election game* $\mathcal{G}$*, then* $\mathbf{x}^C = \arg\max_{\mathbf{x}\in X} \sum_{i\in\mathcal{N}} d_i\, u_i(\mathbf{x})$*, where* $d_i = (1-\gamma)\,\eta\, n_i\, f_i(0) + \gamma\,\frac{n_i}{2\sum_{i\in\mathcal{N}} n_i \alpha_i}$.

This result is common in the probabilistic voting model, where vote-maximizing parties propose the optimal policy as a utilitarian social planner. This is because the vote share function acts as a aggregating mechanism to aggregate voter's preference, hence the vote-maximizing policy is the same as utilitarian optimal. Moreover, in this chapter, because party also cares about the inequality of the society which is another aggregate preference of voters, we have the above result.

Besides revealing that altruistic redistribution only varies (rises) with the income gaps, Proposition 2.2 offers some insight as to how electoral redistribution is affected by the other parameters of the model. Corollaries 2.2-2.3 below collect these results.[19] To start, notice that an increase in ϕ_P raises the transfers to the poor, as is indicated by (2.2.A), because they become more responsive to policy

[19] In what follows, we assume that $\gamma \neq 1$, since otherwise group transfers consist only of altruistic redistribution and they are invariant to changes in the parameters investigated.

and their votes are easier to swing. Due to the non-income-sorting restrictions and the balanced-budget condition, both binding at the equilibrium, a greater ϕ_P decreases simultaneously (and in the same magnitude) the total transfers received by the non-poor.

Corollary 2.2. *Let* $\mathbf{x}^C \in X$ *denote party* C*'s equilibrium policy at the redistributive election game* $\mathcal{G}$. *For all* $i \in N$, $\frac{\partial x_M^C}{\partial \phi_i} = \frac{\partial x_R^C}{\partial \phi_i} = -\sigma_P \frac{\partial x_P^C}{\partial \phi_i}$, *and*

(2.2.A) $\frac{\partial x_P^C}{\partial \phi_P} = \beta_P \cdot \frac{(n_P-1)\,\gamma + (1-\gamma)\,2\eta\,[(n_P-1)\,\phi_\alpha - n_P\,\alpha_P(\phi-\phi_P)]}{(1-\gamma)\,2\,\eta\,\phi_\alpha + \gamma} > 0,$

(2.2.B) $\frac{\partial x_i^C}{\partial \phi_i} = \beta_i \cdot \frac{n_i\,\gamma + (1-\gamma)\,2\eta\,n_i\,[\phi_\alpha - \alpha_i(\phi-\phi_P)]}{(1-\gamma)\,2\,\eta\,\phi_\alpha + \gamma},$ *with* $i = M, R.$

The first result of this corollary (2.2.A) is very intuitive and also common in the literature. It basically means that if the poor group becomes more important in terms of electoral competition they will get more transfer. As is shown in (2.2.B), the effect of a change in ϕ_M (resp., ϕ_R) over x_i^C is indeterminate, meaning that in contrast with Lindbeck and Weibull (1987), electorally motivated transfers do not necessarily rise in *all* groups with the percentage of swing voters.[20] On the one hand, a greater ϕ_M (resp., ϕ_R) raises the average density of swing voters across groups, reducing the electoral appeal of the poor vis-à-vis the middle class. Given that the non-sorting constraint of the middle class and the rich is binding at the equilibrium, this reduces also the appeal of the poor vis-à-vis the rich. Thus, the first effect (through the rise of the partisan independence gap of the poor) is positive for x_M^C and x_R^C, and negative for x_P^C. On the other hand, an increase in ϕ_M (resp., ϕ_R) also increases β_P and reduces the coefficients β_M and β_R. This works in the direction opposite to the first effect, capturing how fairness and power sharing interact with the partisan preferences. Therefore, the total effect of a change of ϕ_M (resp., ϕ_R) over x_i^C is ambiguous.

The second set of comparative statics results points out that the effect of (either citizens' or parties') inequality concern over ER-transfers is negative for the middle class and the rich, who benefit from this type of redistribution, and positive for the poor (see (2.3.A) and (2.3.B) below). This means that fairness preferences curb to some extent money transfers across income groups motivated by elections. As was pointed out before, AR-transfers are not directly affected by inequality concern.

[20] This result is, however, reestablished when income sorting is permitted. For more details, see section 2.4.

Corollary 2.3. *Let* $\mathbf{x}^C \in X$ *denote party* C*'s equilibrium policy at the redistributive election game* $\mathcal{G}$*. For all* $i \in N$*, and all* $t = \alpha_i,\ \gamma,\ \eta$*, sign* $\left(\frac{\partial x_i^C}{\partial t}\right) =$ *sign* $\left(\frac{\partial \beta_i}{\partial t}\right)$*, and*

(2.3.A) $\frac{\partial \beta_R}{\partial \alpha_i} = \frac{\partial \beta_M}{\partial \alpha_i} = -\sigma_P \, \frac{\partial \beta_P}{\partial \alpha_i} = -\frac{2\,(1-\gamma)^2\,\eta^2\,\sigma_P\,n_i\,\phi_i}{[(1-\gamma)\,2\,\eta\,\phi_\alpha+\gamma]^2} < 0,$

(2.3.B) $\frac{\partial \beta_R}{\partial \gamma} = \frac{\partial \beta_M}{\partial \gamma} = -\sigma_P \, \frac{\partial \beta_P}{\partial \gamma} = -\frac{\eta\,\sigma_P}{[(1-\gamma)\,2\,\eta\,\phi_\alpha+\gamma]^2} < 0,$

(2.3.C) $\frac{\partial \beta_R}{\partial \eta} = \frac{\partial \beta_M}{\partial \eta} = -\sigma_P \, \frac{\partial \beta_P}{\partial \eta} = \frac{(1-\gamma)\,\gamma\,\sigma_P}{[(1-\gamma)\,2\,\eta\,\phi_\alpha+\gamma]^2} > 0.$

Finally, the effect of the power sharing parameter on ER-transfers is positive for the high density group, that is, the middle class; and due to the non-income-sorting (resp., balanced-budget) constraint, it is also positive (resp., negative) for the rich (resp., poor). This captures that a political system that assigns policy influence more disproportionately among political parties rises the importance of winning a majority at the election, and thereby the stake of the parties in the swing voter group. This result is reminiscent of that derived in Persson and Tabellini (1999), according to which majoritarian elections make electoral competition stiffer, and that implies more targeted redistribution towards the politically influential middle class. In particular, (2.3.C) implies that electoral redistribution toward the middle class and the rich (resp., poor) is at the lowest (resp., highest) level under proportional representation, and increases (resp., decreases) smoothly as the power sharing system gets more disproportionate.

The closed-form expression of the tax-and-transfer policy allows also to investigate the effect of the parameters of the model on income inequality after redistribution. To do that, we follow a usual method of estimating the Gini coefficient when data is grouped into classes. This consists in approximating the Lorenz curve by a series of straight lines joining the known points, and then calculating the relevant area as a series of trapezia and triangles. The resulting estimation, denoted $\hat{G}$, can be written as

$$\hat{G} = 1 - \sum_{i \in N'} n_i\,(Y_i + Y_j), \quad j = i - 1, \tag{2.5}$$

where N' is a rearrangement of N in the order of increasing after-tax incomes, Y_ℓ denotes the percentage of cumulative income up until group ℓ (with $Y_0 = 0$), and $j = i - 1$ refers to the group immediate before group i in terms of its income share (Fuller 1979).

Corollary 2.4. *The groups' after-tax equilibrium incomes* $y_i = e+\beta_i(\phi-\phi_P)$, $i \in N$, *determine an estimate of the Gini coefficient equal to* $\hat{G} = n_P\,\beta_P\,(\phi_P-\phi)\,e^{-1}$. *Thus,*

(2.4.A) $\frac{\partial \hat{G}}{\partial \alpha_i} = n_P\,(\phi_P-\phi)\,e^{-1}\,\frac{\partial \beta_P}{\partial \alpha_i} < 0$, $i \in N$,

(2.4.B) $\frac{\partial \hat{G}}{\partial \gamma} = n_P\,(\phi_P-\phi)\,e^{-1}\,\frac{\partial \beta_P}{\partial \gamma} < 0$,

(2.4.C) $\frac{\partial \hat{G}}{\partial \eta} = n_P\,(\phi_P-\phi)\,e^{-1}\,\frac{\partial \beta_P}{\partial \eta} > 0$,

(2.4.D) $\frac{\partial \hat{G}}{\partial \phi_i} = n_P\,e^{-1}\,\beta_P\left[\frac{\partial \phi_P}{\partial \phi_i} - n_i + (\phi-\phi_P)\frac{(1-\gamma)\,2\,\eta\,n_i\,\alpha_i}{(1-\gamma)\,2\,\eta\,\phi_\alpha+\gamma}\right]$, $i \in N$.

The first two items of Corollary 2.4, that is, (2.4.A) and (2.4.B), confirm that income inequality decreases as society exhibits a greater fairness concern. More interestingly, (2.4.C) reveals that the Gini estimate is positively related with the disproportionately of the power sharing rule, which amount to say that income inequality rises as policymaking power gets more concentrated in the majority winning party. This is because in a more disproportional system winning the election becomes more important so that the middle class and the rich would receive more transfer, hence the inequality in the society would increase.

Finally, the swing voter effect over the Gini, given by (2.4.D), is negative for the poor, since the term in square brackets is positive and $\beta_P < 0$. This is pretty intuitive, since a larger density of independent voters within the poor induces more transfers to the group at the expense of both the rich and the middle class. For these two groups, the sign of (2.4.D) depends on the parameters of the model and it's therefore indeterminate.

2.4 Extensions

This section contains few theoretical extension of the basic model. Continuing with the numeration of section 2.3, first Proposition 2.3 restates the equilibrium transfers of the three income groups lifting the assumption of non-income-sorting. In the second place, Proposition 2.4 displays the equilibrium when the power sharing rule is given by the difference-form function (in the jargon of the contest literature), which implies that the influence of the parties at the policymaking process is determined by the margin of victory or electoral mandate, instead of by the ratio of votes. Finally, Lemma 2.1 and Lemma 2.2 deal with the equilibrium characterization of the redistributive policy when the two parties have different fairness concerns.

2.4.1 Income sorting

In the baseline model, we have assumed that the ranking of disposable incomes after redistribution preserves the ordering of the initial incomes of the groups, i.e., $y_R \geq y_M \geq y_P$, limiting consequently the amount of tactical redistribution among different socio-economic groups that the politicians can propose at the election. Let's suppose now that income sorting is possible, which might be the case for instance if social mobility occurs as result of targeted spending. The set of feasible policies is given by

$$X' = \left\{ \mathbf{x} \in \mathbb{R}^{|N|} : \sum_{i \in N} n_i \, x_i = 0, \; \& \; x_i \geq -e_i \; \forall i \in N \right\}.$$

Let's call $\mathcal{G}' = \left(X', \Pi^C\right)_{C=A,B}$ the redistributive election game determined by the model (with symmetric party fairness) of the paper and the policy set X'. Using the argument of Propositions 1 and 2, it is immediate to see that under Condition $\mathbb{C}$ and $\mathbb{D}$, this game has a unique equilibrium in pure strategies; and that parties announce the same transfers at the equilibrium. Specifically,

Proposition 2.3 (Income Sorting). *Let $\left(\mathbf{x}^A, \mathbf{x}^B\right) \in X' \times X'$ denote the pure-strategy equilibrium of the redistributive election game $\mathcal{G}'$. For all $i \in N$ and all $C = A, B$,*

$$x_i^C = \underbrace{(e - e_i)}_{AR} + \underbrace{\beta \cdot (\phi_i - \phi)}_{ER}, \quad \textit{where } \beta = \frac{(1-\gamma)\,\eta}{2\,\phi_\alpha\,\eta\,(1-\gamma)+\gamma}. \tag{2.6}$$

Notice in equation (2.6) above that the main feature of the transfer policy, namely, the "two-part structure", with the altruistic and electoral redistribution components, is the same under sorting and non-sorting. Actually, AR-transfers are the same in both cases. With regard to the ER-transfers, there are some minor differences, but essentially they are very similar. In particular, notice that now the β parameter is positive and the same for all groups; and that it is multiplied by the ideological neutrality gap of the group, instead of the gap of the poor. The AR- and ER-transfers of the middle class remains positive, which means that this group continues benefiting from targeted spending. On the contrary, for the rich both AR and ER are negative, meaning that the group pays for redistribution. The poor finally might benefit or not depending on whether AR is greater or smaller than ER, exactly like before.

As it happens in the standard Lindbeck-Weibull model without fairness and power sharing, notice that the ranking of the groups based on disposable incomes after redistribution changes under sorting in such a way that the rich people become the lowest income group, while the middle class becomes the richest and the poor the new middle class. This ranking is not very appealing, since income redistribution in the real world doesn't seem to produce such outcomes. To put it differently, though some social mobility occurs in practice, non-rich voters do not seem to possess the political power in a western democracy to carry out a level of expropriation of the rich that transforms the latter after taxes into the poorest group of society. That's why in section 2.3 we assume taxation and redistribution are limited by the "more natural" non-income-sorting condition.

Regarding the comparative statics effects associated with the equilibrium of Proposition 2.3, the results are as follows.

Corollary 2.5. *Let $\mathbf{x}^C \in X$ denote party C's equilibrium policy at the redistributive election game $\mathcal{G}'$. For all $i \in N$ and all $C = A, B$,*

$$\frac{\partial x_i^C}{\partial \phi_i} = \frac{(1-\gamma)^2\,\eta^2\,\left[(1-n_i)2\sum_{j\neq i} n_j\phi_j\alpha_j + 2n_i\alpha_i\sum_{j\neq i} n_j\phi_j\right] + (1-n_i)\,\gamma\,(1-\gamma)\eta}{(2\phi_\alpha\,(1-\gamma)\eta+\gamma)^2} > 0.$$

Corollary 2.2 displays the effect of a change in ϕ_i on x_i^C. As happens in the Lindbeck-Weibull model and in contrast with the result derived under non-income-sorting, equilibrium transfers rise in *all* groups with the density of swing voters.

Corollary 2.6. *Let $\mathbf{x}^C \in X$ denote party C's equilibrium policy at the redistributive election game $\mathcal{G}'$. For all $i \in N$,*

(2.6.A) $\frac{\partial x_i^C}{\partial \alpha_i} = -\frac{(\phi_i-\phi)2n_i\phi_i\,(1-\gamma)^2\eta^2}{(2\phi_\alpha\,(1-\gamma)\eta+\gamma)^2} \lesseqgtr 0 \;\Leftrightarrow\; \phi_i \gtreqless \phi,$

(2.6.B) $\frac{\partial x_i^C}{\partial \gamma} = -\frac{(\phi_i-\phi)\,\eta}{(2\phi_\alpha\,(1-\gamma)\eta+\gamma)^2} \lesseqgtr 0 \;\Leftrightarrow\; \phi_i \gtreqless \phi,$

(2.6.C) $\frac{\partial x_i^C}{\partial \eta} = \frac{(\phi_i-\phi)\,\gamma\,(1-\gamma)}{(2\phi_\alpha\,(1-\gamma)\eta+\gamma)^2} \gtreqless 0 \;\Leftrightarrow\; \phi_i \gtreqless \phi.$

Given our assumption that $\phi_M > \phi > \phi_P > \phi_R$, Corollaries (2.6.A) and (2.6.B) offer a similar conclusion than that derived under non-income-sorting, namely, fairness concern curbs electoral redistribution (ER-transfers) for those benefiting from targeting spending (here only the middle class). Further, altruistic redistribution isn't directly affected by fairness. With respect to (2.6.C), the power sharing effect on ER-transfers is positive for the high density group, that is, the middle class, and negative for the other two groups. The interpretation is similar to that given in the paper: as policymaking power gets more concentrated in the winning party, electoral spending flows from the less responsive to the more responsive groups of voters. The only difference is that under non-income-sorting the rich benefits even if they are the less responsive group because of the need to keep the ranking of disposable income unchanged after redistribution.

Corollary 2.7. *The groups' after-tax equilibrium incomes* $y_i = e + \beta \cdot (\phi_i - \phi)$, $i \in N$, *determines an estimate of the Gini coefficient equal to* $\hat{G} = \beta \cdot K$, *where* $K = e^{-1}\left[n_M(\phi_M - \phi) + n_R n_P(\phi_R - \phi_P)\right]$. *Thus,*

(2.7.A) $\frac{\partial \hat{G}}{\partial \alpha_i} = -K\,\beta^2\,2n_i\phi_i < 0,\ i \in N,$

(2.7.B) $\frac{\partial \hat{G}}{\partial \gamma} = -\frac{K\,\beta^2}{\eta\,(1-\gamma)^2} < 0,$

(2.7.C) $\frac{\partial \hat{G}}{\partial \eta} = \frac{\gamma\,K\beta^2}{(1-\gamma)\,\eta^2} > 0,$

(2.7.D) $\frac{\partial \hat{G}}{\partial \phi_i} = \beta\left(\frac{\partial K}{\partial \phi_i} - 2\,K\,\beta\,n_i\,\alpha_i\right),\ i \in N,$

where $\frac{\partial K}{\partial \phi_P} = -n_P\,(n_M + n_R)\,e^{-1} < 0$, $\frac{\partial K}{\partial \phi_M} = e^{-1}\,n_M(1 - n_M) > 0$, *and* $\frac{\partial K}{\partial \phi_R} = e^{-1}\,n_R\,(n_P - n_M) \gtreqless 0$ *depending on whether* $n_P \gtreqless n_M$.[21]

To conclude, the results shown in (2.7.A)-(2.7.D) indicates that the sign of the comparative statics effects of the main parameters of the model over the Gini are the same regardless of whether income-sorting is or isn't permitted.

2.4.2 Margin of victory

The equilibrium analysis carried out in the paper rests on the assumption that the influence of the parties at the policymaking process is determined by the ratio of vote shares, as is expressed by the rule

$$\rho^C = \frac{1}{1 + \left(\frac{1-v^C}{v^C}\right)^{\eta}}.$$

[21]We assume that $\frac{n_M}{n_P\,n_R} > \frac{\phi_P - \phi_R}{\phi_M - \phi}$, which ensures that $K > 0$ and the Gini index is well defined.

Although that seems to be the view adopted by other papers in the literature (e.g., Saporiti 2014, Matakos et al. 2016, and Herrera et al. 2016), an equally significant and intuitive hypothesis sees instead that influence to be determined by the absolute margin of victory, that is, by the difference of the vote shares, which in a democracy provides to the winning party the right according to law to carry out a particular political programme as approved by the electorate. In practice, however, a narrow margin of victory reduces the leeway of the winning party to implement policies aligned with its electoral platform. By contrast, the party that wins an election with a landslide victory receives from the public a clear mandate to govern and pursue its policy goals (Faravelli et al. 2015).

To formalize this argument, let party C's influence on policy be determined by the margin of victory or electoral mandate $v^{-C} - v^{C} = 1 - 2v^{C}$, so that

$$\hat{\rho}^{C} = \frac{1}{1 + \exp\left(\eta \left(1 - 2v^{C}\right)\right)}, \tag{2.7}$$

where the circumflex accent mark "hat" over the character ρ is used to distinguished this case from (2.2). In the theory of conflict, the expression in (2.7) is known as the difference-form contest success function, due to Hirshleifer (1989), whereas (2.2) is usually called the Tullock contest success function, after Tullock (2001).[22]

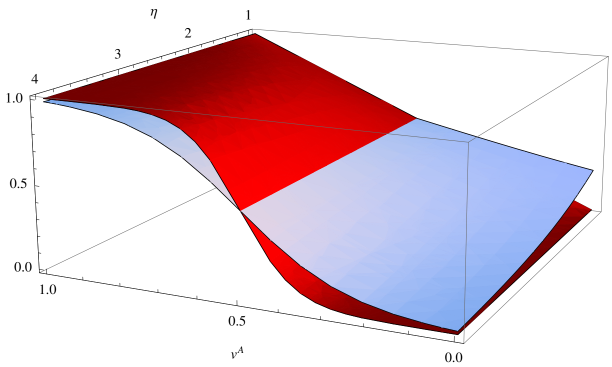

Figure 2.2: Party influence over policy: vote ratio vs margin of victory

The graph in Figure 2.2 illustrates party A's probability of determining the redistributive policy, that is, A's policy influence power, as a function of the ratio (in red) and the margin (in blue) of victory, as expressed in equations (2.2) and

[22]Skaperdas (1996) offers an axiomatic foundation of several popular contest success functions, including the two employed here, that is, the Tullock and the difference-form functions.

(2.7), respectively. The graph shows that both rules determine the same power distribution when parties' vote shares are equal. On the contrary, when parties have different vote shares, the ratio of victory determines a more disproportionate allocation of power, in the sense that the party with the higher vote share receives an even greater influence over policy. This discrepancy between the two expressions tends to narrow as the influence parameter η takes greater values.

For the purpose of the analysis conducted in this work, it is worth mentioning that the different power distribution emerging from (2.2) and (2.7) have minor implications on the equilibrium characterization. To see this, let's call $\hat{\mathcal{G}} = \left(X, \hat{\Pi}^C\right)_{C=A,B}$ the redistributive election game determined by the model (with symmetric party fairness) and the power sharing rule (2.7), where the payoffs $\hat{\Pi}^C$ have been appropriately redefined (specifically, $\hat{\Pi}^C\left(\mathbf{x}^A, \mathbf{x}^B\right) = (1-\gamma)\cdot\hat{\rho}^C - \gamma\,\frac{1}{2}\cdot\sum_{i\in N} n_i\left(y_i^C - y^C\right)^2$).

For the equilibrium existence, we need to modify the condition $\mathbb{D}$ to accomodate the changes in the power sharing rule $\hat{\Phi}(\cdot)$. Formally, define

Condition $\hat{\mathbb{D}}$: $\hat{\Phi}''(v) \leq \hat{\Phi}'(v)\min\{\Upsilon_1, \Upsilon_2, \Upsilon_3\}$, for all $i \in N$, $C = A$, B, and $\mathbf{x}^C \in X$.

Using the argument of Proposition 2.1, it is immediate to see that under Condition $\mathbb{C}$ and $\hat{\mathbb{D}}$, this game has a unique equilibrium in pure strategies; and that parties announce the same transfer policy at the equilibrium. To be more specific,

Proposition 2.4 (Margin of Victory)**.** *Let $\left(\mathbf{x}^A, \mathbf{x}^B\right) \in X\times X$ denote the pure-strategy equilibrium of the redistributive election game $\hat{\mathcal{G}}$. For all $i \in N$ and all $C = A, B$,*

$$x_i^C = e - e_i + \hat{\beta}_i\cdot(\phi - \phi_P)\,, \tag{2.8}$$

where $\hat{\beta}_R = \hat{\beta}_M = \frac{(1-\gamma)\,\eta\,\sigma_P}{2\,[(1-\gamma)\,\eta\,\phi_\alpha + \gamma]}$ and $\hat{\beta}_P = -\frac{(1-\gamma)\,\eta}{2\,[(1-\gamma)\,\eta\,\phi_\alpha + \gamma]}$.

The result stated in Proposition 2.4 shows that under the "margin of victory" power sharing rule, the pure-strategy equilibrium of the election game has the same structure and comparative statics effects than before. The only difference is that the coefficient in (2.8) that accompanies the ideological neutrality gap of the poor is smaller. Intuitively, this happens because a less disproportionate allocation of power under (2.7) diminishes the fierceness of political competition and the prominence of the swing voter group in the election, leading to less electoral redistribution and consequently to a more egalitarian distribution of income among the groups. Despite this, the qualitative results under the two power sharing regimes are similar.

2.4.3 Asymmetric fairness concern

So far, the analysis has focused on the symmetric motivation case where the two parties care equally about fairness, that is, $\gamma^A = \gamma^B = \gamma$. Obviously, it is possible to imagine an alternative scenario where parties, representing perhaps different socio-economic groups, express distinct concern with economic inequality. In particular, that might be the case if one party is "captured by" the rich and the elite, and the other is heavily influenced by the unions and the working class.

To fix ideas, let's consider a simple case of asymmetric motivation in which party A cares only about power, and party B is only concerned with fairness. Formally, let's assume $0 = \gamma^A \neq \gamma^B = 1$. The payoff functions of the parties in this case are

$$\widetilde{\Pi}^A \left(\mathbf{x}^A, \mathbf{x}^B\right) = \rho^A, \tag{2.9}$$

and

$$\widetilde{\Pi}^B \left(\mathbf{x}^A, \mathbf{x}^B\right) = -\frac{1}{2} \cdot \sum_{i \in N} n_i \left(y_i^B - y^B\right)^2. \tag{2.10}$$

Denote by $\widetilde{\mathcal{G}} = \left(X, \widetilde{\Pi}^C\right)_{C=A,B}$ the resulting redistributive election game, determined by the model and the payoffs (2.9) and (2.10). Using the argument of Proposition 2.1, it is immediate to see that under Condition $\mathbb{C}$ and $\mathbb{D}$, this game has a unique equilibrium in pure strategies. To be more specific,

Lemma 2.1 (Asymmetric Fairness Concern). *Let $\left(\mathbf{x}^A, \mathbf{x}^B\right) \in X \times X$ be the pure-strategy equilibrium of the election game $\widetilde{\mathcal{G}} = \left(X, \widetilde{\Pi}^C\right)_{C=A,B}$. Assume for all $i \in N$, θ_i is uniformly distributed over $[\frac{-1}{2\phi_i}, \frac{1}{2\phi_i}]$, with $\phi_M > \sum_{i \in N} n_i \phi_i > \phi_P > \phi_R$. Then,*

$$x_i^A = e - e_i + \widetilde{\beta}_i \cdot (\phi - \phi_P), \ i \in N \tag{2.11}$$

and

$$x_i^B = e - e_i, \ i \in N \tag{2.12}$$

where $\widetilde{\beta}_M = \widetilde{\beta}_R = \sigma_P (2\phi_\alpha)^{-1} = -\sigma_P \widetilde{\beta}_P$.

The result shown above offers several interesting insights. First, it shows that when parties have different fairness concerns, their redistributive policies diverge at the equilibrium. In particular, given that party B has been assumed to be purely altruistic, (2.12) dictates that B's equilibrium policy proposes a level of redistribution that equalizes the after-tax incomes of all socio-economic groups. For the policy of party A this is not the case obviously, since the middle class receives in addition an extra bit of positive tactical redistribution transfers.

Second, remember that the implemented policy is a compromise of the electoral proposals done by the parties, each weighted by the corresponding power share. For the equilibrium of lemma 2.1 it transpires therefore that for all $i \in N$,

$$x_i = e - e_i + \rho^A \cdot \widetilde{\beta}_i \cdot (\phi - \phi_P), \tag{2.13}$$

where ρ^A is given by (2.2), $v^A = 1/2 + \sum_{i \in N} n_i \, \phi_i \left(u_i \left(y^A \right) - u_i \left(y^B \right) \right)$, and $u_i \left(y^A \right) - u_i \left(y^B \right) = \widetilde{\beta}_i \cdot (\phi - \phi_P) - \alpha_i \cdot (\phi - \phi_P)^2 \cdot \sum_{i \in N} n_i \cdot \widetilde{\beta}_i^2$. These are obviously complex expressions that do not allow to say much about what happens with the transfer x_i of each group as the parameters of the model change. To be concrete, the problem is with the ER-transfers (AR-transfers are the same), which depend now on party A's power share, as shown in (2.13). How these shares respond to the parameters isn't easy to tell without imposing further restrictions on the model structure.

Third, it is interesting to see that (2.11) and (2.12) are particular instances of the redistributive policy characterized in the symmetric fairness case of the text, namely,

$$x_i^C = (e - e_i) + \beta_i \cdot (\phi - \phi_P)\,, \quad \text{with } \beta_M = \beta_R = \tfrac{(1-\gamma)\,\eta\,\sigma_P}{(1-\gamma)\,2\,\eta\,\phi_\alpha + \gamma} = -\sigma_P \, \beta_P, \tag{2.14}$$

when γ takes the values of 0 and 1, respectively. Having noted that, one might be tempted to think that perhaps the equilibrium of any other asymmetric case can be obtained in the same fashion by replacing the different levels of parties' fairness concern into the symmetric equilibrium shown in (2.14). We argue, however, that's correct in the limit case $0 = \gamma^A \neq \gamma^B = 1$ considered by Lemma 2.1, but not otherwise.

To elaborate, suppose party B remains altruistic (i.e., $\gamma^B = 1$), and let A care about power *and* fairness (i.e., $\gamma^A \in (0, 1)$). At the equilibrium, party B's redistributive policy continues to be the initial income gap $e - e_i$. By contrast, a closed-form expression for the policy of party A is hard to derive even under the assumption that voters' ideological bias is drawn from a uniform distribution.

The problem is parties do not converge to the same policy, and that transforms the first order partial derivative of the power share with respect to the expected vote share into a nontrivial expression (see equation (2.37) in the appendix). On the contrary, in the symmetric fairness case, regardless of the nature of the c.d.f. F_i, the expected vote shares are equal to 1/2 at the equilibrium, because parties propose the same redistributive policy. That implies that (2.37) is simply equal to η, and that simplifies enormously the calculation of the transfers.

Having said all that, it can be shown that party A's transfers (specifically, the TR-transfers) to the swing voter group (middle class) are now smaller than that given by (2.14). The reason is competition for votes in the asymmetric fairness case is less intense due to the fact that party B is by assumption less concerned with power sharing than under symmetry (in this example, B is not concerned at all with power). Other things equal, that reduces the level of tactical redistribution that a fair-minded party A is willing to implement and to trade against equity.[23]

Thus, although a closed-form solution for the previous asymmetric fairness case is hard to workout, compared with the symmetric case and provided that the relatively more opportunistic party is also fair-minded, the equilibrium transfers imply less targeted spending on the more responsive voter groups. This occurs by the fact that competition among political parties becomes less fierce, to which parties respond by curbing tactical redistribution. Below we state formally this observation and we generalize it for the case where none of the parties is purely altruistic.

Consider the redistributive election game $\tilde{\tilde{\mathcal{G}}}\left(\gamma^A, \gamma^B\right) = \left(X, \tilde{\tilde{\Pi}}^C\left(\gamma^C\right)\right)_{C=A,B}$, determined by the model and the payoff functions $\tilde{\tilde{\Pi}}^C$, $C = A, B$, where for each $\gamma^C \in [0,1]$, $\tilde{\tilde{\Pi}}^C\left(\gamma^C\right) = \left(1-\gamma^C\right)\cdot\rho^C - \gamma^C\frac{1}{2}\cdot\sum_{i\in N} n_i\left(y_i^C - y^C\right)^2$.

Lemma 2.2. *If Condition* $\mathbb{C}$ *and* $\mathbb{D}$ *hold, then the election game* $\tilde{\tilde{\mathcal{G}}}\left(\gamma^A, \gamma^B\right)$ *has a unique pure-strategy equilibrium* $\left(\mathbf{x}^A\left(\gamma^A, \gamma^B\right), \mathbf{x}^B\left(\gamma^A, \gamma^B\right)\right) \in X \times X$. *Denote the tactical redistribution as* $\left(\mathbf{x}^A_{TR}\left(\gamma^A, \gamma^B\right), \mathbf{x}^B_{TR}\left(\gamma^A, \gamma^B\right)\right)$, *where* $\mathbf{x}^A_{TRi}\left(\gamma^A, \gamma^B\right) = \mathbf{x}^A_i\left(\gamma^A, \gamma^B\right) - e + e_i$. *Moreover, if for all* $i \in N$, θ_i *is uniformly distributed over* $[\frac{-1}{2\phi_i}, \frac{1}{2\phi_i}]$, *with* $\phi_M > \sum_{i\in N} n_i\phi_i > \phi_P > \phi_R$, *then*

(2.2.A) $\left|x^C_{TRi}\left(\gamma^C, \gamma^{-C}\right)\right| < \left|x^C_{TRi}\left(\gamma^C, \gamma^C\right)\right|$ *for all* $0 < \gamma^C < \gamma^{-C} \leq 1$,

[23] What happens in the limit when party A is not fair-minded is that its willingness to trade votes for equity vanishes, and therefore it behaves independently of the intensity of electoral competition.

(2.2.B) $\lim_{\gamma^C \to 0} \; x_i^C\left(\gamma^C, \gamma^{-C}\right) = x_i^C\left(0, \gamma^{-C}\right)$ *for all* $\gamma^{-C} \in [0,1]$*,*

(2.2.C) $\left|x_{TRi}^{-C}\left(\gamma^C, \gamma^{-C}\right)\right| < \left|x_{TRi}^{-C}\left(\gamma^{-C}, \gamma^{-C}\right)\right|$ *for all* $0 \leq \gamma^C < \gamma^{-C} < 1$*, and*

(2.2.D) $\lim_{\gamma^{-C} \to 1} \; x_i^{-C}\left(\gamma^C, \gamma^{-C}\right) = x_i^{-C}\left(\gamma^C, 1\right)$ *for all* $\gamma^C \in [0,1]$*.*

The existence result stated above follows from the same argument used in the proof of Proposition 2.1. To be more concrete, regardless of the levels of party fairness concerns, Conditions $\mathbb{C}$ and $\mathbb{D}$ are sufficient to ensure the quasi-concavity of each party's conditional payoff function; and that's enough due to Debreu-Glicksberg-Fan's theorem to guarantee the existence of a pure-strategy equilibrium for the election game $\tilde{\tilde{\mathcal{G}}}\left(\gamma^A, \gamma^B\right)$ (where, remember, γ^A is not necessarily equal to γ^B).

With respect to the rest of the Proposition, (2.2.A) points out that so long as party C is fair-minded, the electoral redistribution is less than in the case where both parties have the same level of fairness concern because electoral competition is less intense under asymmetric fairness (differentiated parties). By contrast, (2.2.B) shows that in the limit, when party C is fully opportunistic, it behaves in the same way regardless of the intensity of competition. Likewise, (2.2.C) points out that so long as party $-C$ is not purely altruistic, it will also tactically redistribute less under asymmetric fairness because competition becomes more intense when the other party cares about inequality as much as party $-C$. Finally, (2.2.D) shows that in the limit, when $-C$ is purely altruistic, it chooses the same level of redistribution regardless of the intensity of competition.

We believe the main insights of Lemmas 2.1 and 2.2 hold under more general distribution functions of ideological preferences. However, the formal details of that claim remain an open question that goes beyond the scopes of this work and is left for future research.

2.5 Conclusion

In this chapter, we have reexamined the problem of income redistribution in a model of political competition with power sharing and fairness. We have characterized the equilibrium net transfers to the income groups and shown that they consist of two parts, called altruistic and electoral redistribution. We have also shown how these transfers vary with their main determinants, that is, with the gap between the population and the group average pre-tax incomes, the partisan independence gap of the poor, electoral rule disproportionality, and parties' and voters' concern with inequality. In particular, the theoretical and the empirical results suggest that the net transfers to the more responsive group of voters (i.e., the middle class) and the after-tax Gini index both rise as policymaking power gets more concentrated in the majority winning party.

These results add to the literature on "targeted spending" (Cox and McCubbins 1986, Lindbeck and Weibull 1987, Dixit and Londregan 1995 and Dixit and Londregan 1996), which has been used in the study of the size and the scope of public spending, social security, regional transfers, etc. (Persson and Tabellini 2002). Our work fills a gap in the current models by analysing electoral redistribution under a large variety of power sharing arrangements and in the presence of social preferences.

Our research also contributes to the literature on redistribution and other-regarding preferences. Within the Meltzer and Richard's (1981) model of redistributive politics, preferences for redistribution that goes beyond those motivated by the agents' own economic benefit have been studied in Galasso (2003), Alesina and Angeletos (2005a,b), Tyran and Sausgruber (2006), Dhami and al Nowaihi (2010a,b), Luttens and Valfort (2012), and Flamand (2012). In the context of the probabilistic voting model, to our knowledge the only article that incorporates preferences for fairness is Alesina et al. (2012). The latter analyzes a dynamic extension of the Lindbeck-Weibull model with a winner-take-all election at the end of each period. The aim of the paper is to show how different perceptions of fairness of the market outcomes can lead to different steady states of redistribution and growth. Our paper complements Alesina et al. (2012) by analyzing the consequences of different distributions of policymaking power over the redistributive policies and income inequality. By being dynamic, Alesina et al.'s (2012) framework isn't appropriate for that goal due to the lack of an accepted theory in political economy about how political power sharing evolves over time.

Finally, our work also adds to the literature on redistribution and inequality under different electoral rules (namely, firs-past-the-post (FPTP) and proportional representation (PR)). A central prediction is that PR favors spending on goods that benefit broad social groups, whereas FPTP favors spending on goods provided to specific subsets of voters (Persson and Tabellini 1999; Lizzeri and Persico 2001; Milesi-Ferretti et al. 2002; and Funk and Gathmann 2013). To our knowledge, this paper constitutes the first attempt to bring this insight into a framework with a rich variety of *mixed* electoral systems, which not only reflects better the reality of many democracies, but it also allows to quantify the effects of small changes in these rules over both redistribution and the Gini.[24]

2.6 Appendix

Proof of Proporsition 2.1. To show that $\mathcal{G} = (X, \Pi^C)_{C=A,B}$ has a unique pure-strategy equilibrium, we employ Debreu-Glicksberg-Fan's existence result. First, note that the strategy space X is non-empty, compact, and convex. Second, each function $\Pi^C(\mathbf{x}^A, \mathbf{x}^B)$ is continuous on $(\mathbf{x}^A, \mathbf{x}^B) \in X^2$. Thus, it remains to prove that, under conditions $\mathbb{C}_1$ and $\mathbb{C}_2$, each conditional payoff function $\Pi^C(\,\cdot\,, \mathbf{x}^{-C})$ is strictly quasi-concave on X. The uniqueness result follows from the structure of the game and strictly quasi-concavity of the payoff function.

Fix any policy $\overline{\mathbf{x}}^B \in X$, and consider the resulting conditional payoff function $\Pi^A(\,\cdot\,, \overline{\mathbf{x}}^B)$ of party A. The proof for party B is similar. Note that the second term of party A's conditional payoff, namely, $-\,\gamma\,\frac{1}{2}\cdot\sum_{i\in N} n_i\,(y_i^A - e)^2$, is strictly concave in the party's own strategy. Thus, to prove that $\Pi^A(\,\cdot\,, \overline{\mathbf{x}}^B)$ is strictly quasi-concave on X, it suffices to show that the power share function $\rho^A(\mathbf{x}^A, \overline{\mathbf{x}}^B)$ is concave in $\mathbf{x}^A$.

Starting with party A's vote share, recall that for all $\mathbf{x}^A \in X$, $v^A(\mathbf{x}^A, \overline{\mathbf{x}}^B) = \sum_{i\in N} n_i\, v_i^A(\mathbf{x}^A, \overline{\mathbf{x}}^B)$, with $v_i^A(\mathbf{x}^A, \overline{\mathbf{x}}^B) = F_i(u_i(\mathbf{x}^A) - u_i(\overline{\mathbf{x}}^B))$. Consider the Hessian matrix associated to each $v_i(\mathbf{x}^A,\, \overline{\mathbf{x}}^B)$, i.e., $H_i(\mathbf{x}^A) = \left[\frac{\partial^2 v_i(\mathbf{x}^A, \overline{\mathbf{x}}^B)}{\partial x_i^A \partial x_j^A}\right]_{i,j\in N}$. Since for all $i \neq j$, $i, j \in N$, the second-order partial derivatives $\frac{\partial^2 u_i(\mathbf{x}^A)}{\partial x_i^A \partial x_j^A} = 0$, it is easy to show that the matrix $H_i(\mathbf{x}^A)$ is negative semi-definite on X if for all $\mathbf{x}^A \in X$,

$$\frac{\partial^2 v_i(\mathbf{x}^A, \overline{\mathbf{x}}^B)}{\partial x_i^A \partial x_i^A} \leq 0 \iff \kappa_i(\mathbf{x}^A) \leq \Big(\xi_{ii}(\mathbf{x}^A)\Big)^{-1}$$

$$\begin{vmatrix} \frac{\partial^2 v_i(\mathbf{x}^A, \overline{\mathbf{x}}^B)}{\partial x_i^A \partial x_i^A} & \frac{\partial^2 v_i(\mathbf{x}^A, \overline{\mathbf{x}}^B)}{\partial x_i^A \partial x_j^A} \\ \frac{\partial^2 v_i(\mathbf{x}^A, \overline{\mathbf{x}}^B)}{\partial x_j^A \partial x_i^A} & \frac{\partial^2 v_i(\mathbf{x}^A, \overline{\mathbf{x}}^B)}{\partial x_j^A \partial x_j^A} \end{vmatrix} \geq 0 \iff \kappa_i(\mathbf{x}^A) \leq \Big(\xi_{ii}(\mathbf{x}^A) + \xi_{ij}(\mathbf{x}^A)\Big)^{-1},\ i \neq j$$

$$|H_i(\mathbf{x}^A)| \leq 0 \iff \kappa_i(\mathbf{x}^A) \leq \left(\sum_{j\in N} \xi_{ij}(\mathbf{x}^A)\right)^{-1}.$$

[24]For a comparative analysis of mixed electoral systems, see Moser and Scheiner (2004).

Thus, since $\Big(\sum_{j\in N}\xi_{ij}(\mathbf{x}^A)\Big)^{-1} < \Big(\xi_{ii}(\mathbf{x}^A)+\xi_{ij}(\mathbf{x}^A)\Big)^{-1} < \Big(\xi_{ii}(\mathbf{x}^A)\Big)^{-1}$, condition $\mathbb{C}_1$ guarantees that each $v_i^A(\,\cdot\,,\overline{\mathbf{x}}^B)$ is concave on X.

Finally, we use condition $\mathbb{C}_2$, and we show that the concavity of $v^A(\,\cdot\,,\overline{\mathbf{x}}^B)$ proved above is preserved under the increasing transformation Φ, establishing the desired result that the power share function $\rho^A(\,\cdot\,,\overline{\mathbf{x}}^B)$ is also concave on X. To do that, recall that by definition, for each $\mathbf{x}^A \in X$, $\frac{\partial^2\rho^A(\mathbf{x}^A,\overline{\mathbf{x}}^B)}{\partial x_i\partial x_j} = \Phi''(v^A(\mathbf{x}^A,\,\overline{\mathbf{x}}^B))\frac{\partial v^A(\mathbf{x}^A,\overline{\mathbf{x}}^B)}{\partial x_i}\frac{\partial v^A(\mathbf{x}^A,\overline{\mathbf{x}}^B)}{\partial x_j} + \Phi'(v^A(\mathbf{x}^A,\,\overline{\mathbf{x}}^B))\frac{\partial^2 v^A(\mathbf{x}^A,\overline{\mathbf{x}}^B)}{\partial x_i\partial x_j}$, which can be rewritten using the notation of Section 2.3 as $\frac{\partial^2\rho^A(\mathbf{x}^A,\overline{\mathbf{x}}^B)}{\partial x_i\partial x_j} = \frac{\partial v^A(\mathbf{x}^A,\overline{\mathbf{x}}^B)}{\partial x_i}\frac{\partial v^A(\mathbf{x}^A,\overline{\mathbf{x}}^B)}{\partial x_j}\Phi'(v^A(\mathbf{x}^A,\,\overline{\mathbf{x}}^B))\Big(\delta(v^A(\mathbf{x}^A,\,\overline{\mathbf{x}}^B)) + \delta_{ij}(\mathbf{x}^A,\,\overline{\mathbf{x}}^B)\Big)$. Therefore, given that $\Phi' > 0$, it follows that

$$\frac{\partial^2\rho^A(\mathbf{x}^A,\overline{\mathbf{x}}^B)}{\partial x_i^A\partial x_i^A} \le 0 \iff \delta(\mathbf{x}^A) \le -\Delta^1(\mathbf{x}^A)$$

$$\begin{vmatrix} \frac{\partial^2\rho^A(\mathbf{x}^A,\overline{\mathbf{x}}^B)}{\partial x_i^A\partial x_i^A} & \frac{\partial^2\rho^A(\mathbf{x}^A,\overline{\mathbf{x}}^B)}{\partial x_i^A\partial x_j^A} \\ \frac{\partial^2\rho^A(\mathbf{x}^A,\overline{\mathbf{x}}^B)}{\partial x_j^A\partial x_i^A} & \frac{\partial^2\rho^A(\mathbf{x}^A,\overline{\mathbf{x}}^B)}{\partial x_j^A\partial x_j^A} \end{vmatrix} \ge 0 \iff \Delta^2_{[1,\,\delta(\mathbf{x}^A)]} + \Delta^2_{[2,\,\delta(\mathbf{x}^A)]} \ge -\Delta^2(\mathbf{x}^A)$$

$$|H_{\rho^A}(\mathbf{x}^A)| \le 0 \iff \sum_{j\le 3}\Delta^3_{[j,\,\delta(\mathbf{x}^A)]}(\mathbf{x}^A) \le -\Delta^3(\mathbf{x}^A),$$

which results by applying condition $\mathbb{C}_2$ in the Hessian matrix of $\rho^A(\,\cdot\,,\overline{\mathbf{x}}^B)$ to be negative semi-definite on X. □

Proof of Proporsition 2.2. We first show that the equilibrium is symmetric. Given the policy of the other party, party A and B face the same optimazaiton problem, namely,

$$\max_{\mathbf{x}^C}\ \Pi^C(\mathbf{x}^A,\mathbf{x}^B)$$

$$\text{s.t.}\quad \sum_{i\in N} n_i x_i^C = 0, \tag{2.15}$$

$$x_i^C + e_i \ge 0 \text{ for all } i \in N, \tag{2.16}$$

$$e_R + x_R^C \ge e_M + x_M^C, \tag{2.17}$$

$$e_M + x_M^C \ge e_P + x_P^C, \tag{2.18}$$

$$\mathbf{x}^{-C} \text{ given.}$$

Without loss of generality, consider next party A's problem. The Lagrange function is $\mathcal{L} = \Pi^A(\mathbf{x}^A,\mathbf{x}^B) + \lambda[0 - \sum_{i\in N} n_i x_i^A] + \sum_{i\in N}\mu_i(x_i^A + e_i) + \delta_1(e_R + x_R^A -$

$e_M - x_M^A) + \delta_2(e_M + x_M^A - e_P - x_P^A)$, where λ, μ_i, δ_1 and δ_2 are the multipliers associated with the constraints listed in (2.15)-(2.18). Because of the symmetry of the maximisation problem, party A and B have the same first order constraints. Consider the case where $\lambda > 0$, $\mu_i = 0$ for all $i \in N$, $\delta_1 > 0$, and $\delta_2 = 0$, we show that the equilibrium policies are symmetric, i.e. $\mathbf{x}^A = \mathbf{x}^B$. The proof for other cases are using the same argument. Under this configuration of values of the Lagrange multipliers, the system of first-order conditions reduces to (2.16) and (2.18) together with the following equations:

$$\frac{\partial \Pi^A}{\partial x_R^A} - \lambda^A n_R + \delta_1^A = 0, \tag{2.19}$$

$$\frac{\partial \Pi^A}{\partial x_M^A} - \lambda^A n_M - \delta_1^A = 0, \tag{2.20}$$

$$\frac{\partial \Pi^A}{\partial x_P^A} - \lambda^A n_P = 0, \tag{2.21}$$

$$\sum_{i \in N} n_i \, x_i^A = 0, \tag{2.22}$$

$$e_R + x_R^A - e_M - x_M^A = 0. \tag{2.23}$$

Rearranging (2.19) and (2.20), we have that $(\frac{\partial \Pi^A}{\partial x_R^A} + \frac{\partial \Pi^A}{\partial x_M^A})\frac{1}{n_R + n_M} = \lambda^A$. And from (2.21), we have that $\frac{\partial \Pi^A}{\partial x_P^A}\frac{1}{n_P} = \lambda^A$. With the same first order conditions on party B, we can derive the following:

$$\frac{\frac{\partial \Pi^A}{\partial x_R^A} + \frac{\partial \Pi^A}{\partial x_M^A}}{\frac{\partial \Pi^B}{\partial x_R^B} + \frac{\partial \Pi^B}{\partial x_M^B}} = \frac{\lambda^A}{\lambda^B}, \qquad \frac{\frac{\partial \Pi^A}{\partial x_P^A}}{\frac{\partial \Pi^B}{\partial x_P^B}} = \frac{\lambda^A}{\lambda^B}, \tag{2.24}$$

where $\frac{\partial \Pi^A}{\partial x_i^A} = \frac{\partial \rho^A}{\partial v^A}\Big(n_i f_i(t_i) - 2n_i(\tilde{e}_i + x_i^A)\sum_{i \in N} n_i f_i(t_i)\alpha_i\Big)$, and $\frac{\partial \Pi^B}{\partial x_i^B} = \frac{\partial \rho^B}{\partial v^B}\Big(n_i f_i(t_i) - 2n_i(\tilde{e}_i + x_i^B)\sum_{i \in N} n_i f_i(t_i)\alpha_i\Big)$. Notice that the $\frac{\partial \rho^A}{\partial v^A} = \frac{\partial \rho^B}{\partial v^B}$ because of no abstention. Suppose $\mathbf{x}^A \neq \mathbf{x}^B$, we have that $x_P^A > x_P^B$ or $x_P^A < x_P^B$, since $e_M + x_M^C = e_R + x_R^C$. We show that there is a contradiction in the first case, the proof for the second case is similar. If $x_P^A > x_P^B$, then from the first part of equation (2.24), we have that $\lambda^A > \lambda^B$. However, from the second part, we have that $\lambda^A < \lambda^B$. Therefore, at equilibrium, $\mathbf{x}^A = \mathbf{x}^B$.

Now, we characterize the equilibrium. First we consider the case where $\lambda > 0$, $\mu_i = 0$ for all $i \in N$, $\delta_1 > 0$, and $\delta_2 = 0$. Since $\mathbf{x}^A = \mathbf{x}^B$, the vote share of party

A is 1/2, implying that

$$\frac{\partial \Pi^A(\mathbf{x}^A, \mathbf{x}^B)}{\partial x_i^A} = (1-\gamma)\,\eta\,\frac{\partial v^A(\mathbf{x}^A, \mathbf{x}^B)}{\partial x_i^A} - \gamma\, n_i\,(\tilde{e}_i + x_i^A), \tag{2.25}$$

where $\tilde{e}_i = e_i - e$ and $\frac{\partial v^A(\mathbf{x}^A,\mathbf{x}^B)}{\partial x_i^A} = n_i\phi_i - 2n_i(\tilde{e}_i + x_i^A)\phi_\alpha$. Adding (2.19) and (2.20), we have that $\frac{\partial \Pi^A}{\partial x_R^A} + \frac{\partial \Pi^A}{\partial x_M^A} - \lambda n_R - \lambda n_M = 0$, which implies using (2.25) that

$$\lambda = \frac{n_M}{n_M + n_R}[(1-\gamma)\eta\phi_M - (\tilde{e}_M + x_M^A)D] + \frac{n_R}{n_M + n_R}[(1-\gamma)\eta\phi_R - (\tilde{e}_R + x_R^A)D], \tag{2.26}$$

where $D = (1-\gamma)\eta 2\phi_\alpha + \gamma$. Notice that from (2.21) and (2.25), it also follows that

$$\lambda = (1-\gamma)\eta\phi_P - (\tilde{e}_P + x_P^A)D. \tag{2.27}$$

Combining (2.26) and (2.27) together with (2.23),

$$x_P^A = \frac{(1-\gamma)\eta}{D}\frac{\phi_P - \phi}{n_R + n_M} + e_M + x_M^A - e_P. \tag{2.28}$$

Substituting (2.23) and (2.28) into (2.22), we get the transfer to the middle class, namely, $x_M^A = e - e_M + \beta_M(\phi - \phi_P)$, where $\beta_M = \frac{(1-\gamma)\,\eta\,\sigma_P}{(1-\gamma)\,2\,\eta\,\phi_\alpha + \gamma}$ and $\sigma_P = \frac{n_P}{1-n_P}$. The transfer to the rich and the poor are obtained by replacing x_M^A into (2.23) and (2.28), respectively. Moreover, using (2.27), we have that $\lambda = (1-\gamma)\eta\phi$, which is strictly positive as required. Finally, it's easy to verify that these critical values of x_i^A satisfy (2.16) and (2.18). Therefore, they constitute the solution of party's A constrained optimization problem. It is left for the reader to check that any other configuration of values of the Lagrange multipliers violates one or more of the first-order conditions. □

Proof of Corollary 2.1. Without loss of generality, we show the result for party A. Recall that A maximizes the payoff function $\Pi^A = (1-\gamma)\rho^A - \gamma\frac{1}{2}\sum_{i\in\mathcal{N}} n_i(y_i^A - y^A)^2$ with respect to $\mathbf{x}^A \in X$ subject to the constraints listed in (2.15)-(2.18). With regard to the second part of party A's payoff function, notice first that sum of voters' utility functions $\sum_{i\in\mathcal{N}} n_i\, u_i(\mathbf{x}^A)$ is

$$\sum_{i\in\mathcal{N}} n_i\, u_i(\mathbf{x}^A) = y - \hat{\alpha}\left[\sum_{i\in\mathcal{N}} n_i\,(y_i^A - y^A)^2\right],$$

where $\hat{\alpha} = \sum_{i\in\mathcal{N}} n_i\,\alpha_i$. Thus, $-\gamma\frac{1}{2}\sum_{i\in\mathcal{N}} n_i(y_i^A - y^A)^2$ is equivalent to

$\gamma\frac{1}{2\hat{\alpha}}\sum_{i\in\mathcal{N}} n_i u_i(\mathbf{x}^A) - \gamma\frac{y}{2\hat{\alpha}}$. Next the first order conditions for maximising Π^A is equivalent to the first order conditions of maximising $\sum_{i\in\mathcal{N}} d_i\, u_i(\mathbf{x}^A)$, with $d_i = (1-\gamma)\,\eta\, n_i\, f_i(0) + \gamma\,\frac{n_i}{2\sum_{i\in\mathcal{N}} n_i\alpha_i}$, which proves the desired result. s □

Proof of Proposition 2.3. Like in the proof to Proposition 2, we consider only the problem of party A, which is (given the policy $\mathbf{x}^B \in X'$ of the other party)

$$\max_{\mathbf{x}^A} \; \Pi^A\left(\mathbf{x}^A, \mathbf{x}^B\right)$$

$$\text{s.t.} \quad \sum_{i\in N} n_i x_i^A = 0, \tag{2.29}$$

$$x_i^A + e_i \geq 0 \text{ for all } i \in N. \tag{2.30}$$

The main difference between this optimization problem and party A's problem under the non-income-sorting constraints is the restrictions (8) and (9) of Appendix A in the paper, which are now lifted. The Lagrange function is $\mathcal{L} = \Pi^A\left(\mathbf{x}^A, \mathbf{x}^B\right) + \lambda[0 - \sum_{i\in N} n_i x_i^A] + \sum_{i\in N}\mu_i\left(x_i^A + e_i\right)$, where λ and μ_i stand for the Lagrange multipliers associated with (2.29) and (2.30), respectively. Consider the case where $\lambda \geq 0$ and $\mu_i = 0$ for all $i \in N$. The first order conditions reduce to equation (2.29) and:

$$\frac{\partial \Pi^A}{\partial x_R^A} - n_R\lambda = 0, \tag{2.31}$$

$$\frac{\partial \Pi^A}{\partial x_M^A} - n_M\lambda = 0, \tag{2.32}$$

$$\frac{\partial \Pi^A}{\partial x_P^A} - n_P\lambda = 0. \tag{2.33}$$

The first order derivative of the payoff function is: $\frac{\partial \Pi^A}{\partial x_i^A} = (1-\gamma)\,\eta\cdot\frac{\partial v^A}{\partial x_i^A} - \gamma n_i\left(\tilde{e}_i + x_i^A\right)$, where $\tilde{e}_i = e_i - e$ and $\frac{\partial v^A}{\partial x_i^A} = n_i\phi_i - 2n_i\left(\tilde{e}_i + x_i^A\right)\phi_\alpha$. Combining (2.31) and (2.32) and following the steps of proof for proposition 2.2 , we have that

$$x_R^A = e_M - e_R + x_M^A - \frac{(1-\gamma)\,\eta\,(\phi_M - \phi_R)}{(1-\gamma)\,2\phi_\alpha + \gamma}. \tag{2.34}$$

By the same token, using (2.32) and (2.33), it follows that

$$x_P^A = e_M - e_P + x_M^A - \frac{(1-\gamma)\,\eta\,(\phi_M - \phi_P)}{(1-\gamma)\,2\phi_\alpha + \gamma}. \tag{2.35}$$

Finally, substituting (2.34) and (2.35) into (2.29), we get the transfer to the middle class:

$$x_M^A = e - e_M + \frac{(1-\gamma)\,\eta\,(\phi_M - \phi)}{(1-\gamma)\,2\phi_\alpha + \gamma}. \tag{2.36}$$

The transfers to the rich and the poor are obtained by replacing (2.36) back into (2.34) and (2.35), respectively. □

Proof of Proposition 2.4. The proof is identical to the proof of Proposition 2.2. The only difference is the value of the first order partial derivative of the power sharing function with respect to the vote share. In the ratio of victory case, this derivative is

$$\frac{\partial \rho^A}{\partial v^A} = \frac{1}{\left(1+\left(\frac{1-v^A}{v^A}\right)^{\eta}\right)^2} \cdot \eta \left(\frac{1-v^A}{v^A}\right)^{\eta-1} \cdot \frac{1}{\left(v^A\right)^2}, \tag{2.37}$$

whereas in the margin of victory case is

$$\frac{\partial \hat{\rho}^A}{\partial v^A} = \frac{1}{\left(1+e^{\eta(1-2v^A)}\right)^2} \cdot e^{\eta\left(1-2v^A\right)} \cdot 2\eta.$$

Since at the equilibrium $\mathbf{x}^A = \mathbf{x}^B$ and $v^A = \frac{1}{2}$, it follows that $\frac{\partial \rho^A}{\partial v^A} = \eta$, and $\frac{\partial \hat{\rho}^A}{\partial v^A} = \frac{1}{2}\eta$. The rest of the proof proceeds in the same manner as the proof of Proposition 2.2. □

Proof of Lemma 2.1. First of all, it is immediate to verify that the policy of party B that maximizes its objective function subject to the usual constraints is $x_i^B = e - e_i$, $i \in N$.

Second, party A's optimization problem consists in maximizing with respect to $\mathbf{x}^A$ the power sharing function $\rho^A\left(\mathbf{x}^A, \mathbf{x}^B\right)$, given that $x_i^B = e - e_i$ $\forall i \in N$, and subject to the following set of restrictions:

$$\sum_{i \in N} n_i\, x_i^A = 0, \tag{2.38}$$

$$x_i^A + e_i \geq 0 \text{ for all } i \in N, \tag{2.39}$$

$$e_R + x_R^A \geq e_M + x_M^A, \tag{2.40}$$

$$e_M + x_M^A \geq e_P + x_P^A. \tag{2.41}$$

Suppose $\lambda \geq 0$, $\mu_i = 0$ for all $i \in N$, $\delta_1 > 0$ and $\delta_2 = 0$, where λ, μ_i, δ_1, and δ_2 are the Lagrange multipliers associated with (2.38)–(2.41). The first order conditions are (2.38), (2.39), (2.41), together with

$$\frac{\partial \rho^A}{\partial x_R^A} - \lambda n_R + \delta_1 = 0, \tag{2.42}$$

$$\frac{\partial \rho^A}{\partial x_M^A} - \lambda n_M - \delta_1 = 0, \tag{2.43}$$

$$\frac{\partial \rho^A}{\partial x_P^A} - \lambda n_P = 0, \tag{2.44}$$

$$e_R + x_R^A - e_M - x_M^A = 0. \tag{2.45}$$

Combining (2.42) and (2.43), we get

$$\frac{\partial \rho^A}{\partial v^A}\left(\frac{\partial v^A}{\partial x_M^A} + \frac{\partial v^A}{\partial x_R^A}\right) = (n_M + n_R)\,\lambda. \tag{2.46}$$

Meanwhile, note that (2.44) can be rewritten as

$$\frac{\partial \rho^A}{\partial v^A}\frac{\partial v^A}{\partial x_P^A} = n_P \lambda, \tag{2.47}$$

where $\frac{\partial v^A}{\partial x_i^A} = n_i \phi_i - 2n_i\left(\tilde{e}_i + x_i^A\right)\phi_\alpha$. Combining (2.46) and (2.47) and after some algebraic manipulation, we have that

$$x_P^A + e_P - e_M + \frac{\phi - \phi_P}{n_M + n_R}\frac{1}{2\phi_\alpha} = x_M^A. \tag{2.48}$$

Thus, substituting (2.48) and (2.45) into (2.38), we get the transfer to the middle class, namely,

$$x_M^A = e - e_M + \sigma_P \frac{1}{2\phi_\alpha}\left(\phi - \phi_P\right),$$

from which we also obtain the transfer to the poor and the rich. □

Proof of Lemma 2.2. Following the reasoning of the proof to Proposition 2.2, we can derive an (implicit) expression for the equilibrium transfers of the asymmetric case, namely,

$$x_i^A(\gamma^A, \gamma^B) = e - e_i + \beta_i(\phi - \phi_P), \text{ for } i \in \mathcal{N}$$

where $\beta_R = \beta_M = \frac{(1-\gamma)\sigma_P\, \partial\rho^A/\partial v^A}{\partial\rho^A/\partial v^A\,(1-\gamma)\,2\,\phi_\alpha+\gamma} = -\sigma_P\beta_P$. Note that this is not a closed-form solution for $x_i^A(\gamma^A,\gamma^B)$, since $\frac{\partial\rho^A}{\partial v^A}$ depends on x_i^A. However, we prove below that this partial derivative is bounded, specifically, $0 < \frac{\partial\rho^A}{\partial v^A} \leq \eta$. Hence, $\lim_{\gamma^A\to 0} x_i^A\left(\gamma^A,\,\gamma^B\right) = e - e_i + \lim_{\gamma^A\to 0}\beta_i\left(\phi - \phi_P\right) = x_i^A\left(0,\,\gamma^B\right)$, with $\beta_R = \beta_M = \sigma_P\left(2\,\phi_\alpha\right)^{-1} = -\sigma_P\beta_P$, which proves (2.2.B). The proof for (2.2.D) is conducted in a similar fashion.

To prove (2.2.A), let's differentiate first ρ^A, that is, equation (2.2), with respect to v^A,

$$
\begin{aligned}
\frac{\partial\rho^A}{\partial v^A} &= \frac{1}{\left(1+\left(\frac{1-v^A}{v^A}\right)^{\eta}\right)^2}\eta\left(\frac{1-v^A}{v^A}\right)^{\eta-1}\cdot\frac{1}{\left(v^A\right)^2}\\
&= \eta\cdot\frac{1}{v^Av^B}\cdot\frac{1}{2+\left(\frac{v^B}{v^A}\right)^{\eta}+\left(\frac{v^A}{v^B}\right)^{\eta}}.
\end{aligned}
$$

Let $F(\eta) = \frac{1}{v^Av^B}\frac{1}{2+\left(\frac{v^B}{v^A}\right)^{\eta}+\left(\frac{v^A}{v^B}\right)^{\eta}}$. By definition, $\eta \geq 1$. Moreover, it is easy to see that $F(1) = 1$, and that

$$
\frac{\partial F}{\partial\eta} = -\frac{\left(\frac{v^B}{v^A}\right)^{\eta}\ln\left(\frac{v^B}{v^A}\right)+\left(\frac{v^A}{v^B}\right)^{\eta}\ln\left(\frac{v^A}{v^B}\right)}{v^Bv^A\left[2+\left(\frac{v^B}{v^A}\right)^{\eta}+\left(\frac{v^A}{v^B}\right)^{\eta}\right]^2} < 0. \tag{2.49}
$$

Therefore, for $\eta > 1$ the expression in (2.49) implies that $F(\eta) < 1$, and consequently $\frac{\partial\rho^A}{\partial v^A} = \eta\cdot F(\eta) < \eta$. Finally, the latter implies that $\left|x_{TRi}^A\left(\gamma^A,\,\gamma^B\right)\right| < \left|x_{TRi}^A\left(\gamma^A,\,\gamma^A\right)\right|$. The remaining item, that is (2.2.C), is proved following a similar reasoning. □

Chapter 3

Power Sharing and Corruption

3.1 Introduction

Corruption is a common phenomenon that attracts attention from scholars in both political science and economics. The literature have found many economic, cultural, historical and political factors explaining the causes of corruption. In this chapter, we emphasize the role of political power sharing. We find that a more disproportional system restricts corruption better. In addition, the cross country empirical analysis also provide evidence confirming a significant relationship between power sharing and corruption.

To formalize our argument, we adopt Persson and Tabellini's (1999) model and extend it with a continuum of power sharing rules, ranging from purely proportional to winner-take-all. We model the power sharing rule with the help of the Tullock's contest success function. This reduced form of power sharing function captures many institutional details in the policy making and governance process including the forms of government, agenda-setting, separation of powers, electoral rules and veto power etc. This is motivated by the observation that in the modern politics party's influence over the policy is not 'all or nothing'. It is usually a compromise between the majority and minority parties. Moreover, the interaction and negotiation between parties not only shape the policy but also determine the intensity of electoral competition.

As for the result, we show that a more disproportional system leads to lower corruption due to more intensive electoral competition. The intuition is that in a more disproportional system winning is more important, so politicians' incentive to take rents has been mitigated more severely by the incentive to win. Hence, there are less rents in a more disproportional system.

In addition, we evaluate this result with an empirical assessment using data from over 90 countries in the period 2000-2015. We employ the well known perceived corruption index from the Transparency International and the World Bank. On the other hand, to measure power sharing we rely on the electoral disproportionality index proposed by Taagepera (1986), which fits well with our theoretical framework. To construct the index we obtain data on the total number of votes and seats, and the average electoral district magnitude from various sources, including the International Institute for Democracy and Electoral Assistance and the Database of Political Institutions of Cruz et al. (2015).

More specifically, we conduct both cross sectional and panel analysis. The control variables are adopted from the literature including the standard economic, geographical and political variables. We put a special emphasis on the electoral characteristics. We also use the starting year of the current electoral system and the persistence of the majoritarian system as instruments to address the endogenity problem. The persistence of majoritarian system is the difference of numbers majoritarian system and PR system in the electoral reform history. The first instrument was proposed by Persson, Tabellini, and Trebbi (2003), while we propose the second. Our identification assumption is that controlling for other determinants of the policy, the starting period of the electoral system and the persistence of the majoritarian system are not directly related to corruption. In the panel fixed effect analysis, we also present the results from some sub-sample regressions, where the democratic systems are more mature.

The empirical results are strongly supportive to our theoretical prediction that power sharing disproportionality negatively correlates with corruption. In the cross sectional analysis, we found that electoral disproportionality negatively correlates with perceived corruption at 1% significant level. This effect is weaker if we include legal origins and other electoral characteristics in the control set. In the IV regressions, the results are significant at 5% even when other electoral rules are controlled for and they are generally larger in magnitude. Moreover, we did some robustness check by using alternative measures of corruption and electoral disproportionality. The significant relationship holds when we use World Bank's corruption index, while the Gallagher index gives little support. Finally, in the panel analysis we find that this negative relationship between disproportionality and corruption is stronger when we restrict the attention to mature democracies.

This chapter is related to three strands of literature. The first strand focuses on the effects of electoral rules on corruption. The closest work related to this chapter is Persson and Tabellini (1999), which builds on the model of Polo (1998). They show that the majoritarian system has lower political rents than the proportional system. Another influential paper in this topic is Myerson (1993). He compares different electoral rules in their ability to deter corrupt politicians. The plurality rule allows both non-corrupt equilibrium and corrupt equilibrium, whereas the proportional representation rule is fully effective in the sense that it only permits non-corrupt equilibrium. The driving forces of this result are voters' coordination and their tradeoff between ideology and corruption. However, here we concentrate the incentives of parties to compete in the election so that the policy with less corruption is implemented. For the relationship between electoral rules and other economic policies please see Persson and Tabellini (2004). A comparison between the proportional and the majoritarian system is a common feature of these papers. However, in this chapter, the aim is to focus on the power sharing disproportionality which is a reduced form of all the power sharing in the political system, capturing not only the electoral rule but also other determinant factors like separation of powers and allocation of veto powers etc.

The second strand of literature we are related to is the literature on the consequence of different power sharing rules. Among those studies that use the Tullock contest function to model power sharing, we are closely related with Saporiti (2014), Matakos, Troumpounis, and Xefteris (2016), Herrera, Morelli, and Palfrey (2014) and Debowicz, Saporiti, and Wang (2016). The first two papers focus on equilibrium existence and platform polarization in the spatial model of electoral competition, while the latter two center on voters' turnout and redistributive politics. This chapter shares the same technique in modeling the power sharing disproportionality, while our aim is to bring accountability into the discussion.

Finally, our work also contributes to the understanding of the causes of corruption from an empirical view. Treisman (2007, 2000) give a summary of the empirical studies on the causes of corruption. Among many socioeconomic factors, Protestant tradition, British legal origin, economic development and openness to trade are associated with lower corruption. Moreover, a long time of liberal democratic systems and freedom of press also curbs corruption. Among these studies that explain the cause of corruption, causal inference is difficult to build because of the possible endogeneity issue. Persson, Tabellini, and Trebbi (2003) specifically focuses on the aspects of electoral system. They find that the majoritarian system (measured as a binary dummy) tends to generate lower corruption than the proportional system when other electoral characteristics have also been controlled. Different to their work, we use a continuous measure of disproportionality. Angeles and Neanidis (2015) use geographic and historically determined factors to instrument European settlement and they find robustly that European settlement leads to higher corruption. In this chapter, we use the starting time and persistence of the majoritarian system as instruments to deal with the endogeneity problem.

The rest of this Chapter is organized as follows. In the next section we outline our theoretical model. The theoretical results are derived in Section 3.3. Section 3.4 describes the data and methodology employed in the empirical analysis. The empirical results are displayed in Section 3.5. Section 3.6 concludes this Chapter. For expositional convenience, the proof, data sources and original tables are relegated in Appendix 3.7.1, 3.7.2 and 3.7.3 respectively.

3.2 The Model

Consider a finite electorate N of voters. Each voter $i \in N$ has a gross income $y_i > 0$, with $y = \sum_{i \in N} y_i$. Government policies are income tax $t \in [0,1]$ and provision of a pure public good $g \in \mathbb{R}_+$. Voter i's preference over policies is presented by the following utility function:

$$u_i(g,t) = (1-t)y_i + a(g), \tag{3.1}$$

where function $a : \mathbb{R}_+ \mapsto \mathbb{R}$ represents the preference over public good and it is the same for all voters. We assume $a(g)$ is a concave and monotonically increasing function. Government policies are assumed to always satisfy the following budget balance constraint:

$$t\,y = g + r, \tag{3.2}$$

where $r \geq 0$ represents the misuse of tax revenues for private purposes, or in other words, political rents.

There are two political parties $C \in \{A, B\}$ proposing simultaneously public policies $x_C \in X = [0,1] \times \mathbb{R}_+$. Each voter $i \in N$ has an ideological bias σ_i

towards party B. For all $i \in N$, σ_i follows a twice continuously differentiable distribution function $F_i(\cdot)$ over $\mathbb{R}$, with positive density $f_i(\cdot)$ that takes value at zero $f_i(0) = \phi_i$. Voters vote for the party which offers higher utility. To be more specific, voter i votes sincerely for party A if and only if $u_i(x_A) - u_i(x_B) \geq \sigma_i$. As a result, the expected vote share party A get is:

$$v_A(x_A, x_B) = \frac{1}{N} \sum_{i \in N} F_i(u_i(x_A) - u_i(x_B)). \tag{3.3}$$

We assume there is no abstention. Therefore $v_B = 1 - v_A$.

After the election, the winning party and the opposition jointly determine the implemented public policy $x = \sum_C \rho_C \, x_C$, where ρ_C denotes party C's power share, which is defined as following using the Tullock function.

$$\rho_C = \frac{v_C^\eta}{v_C^\eta + (1 - v_C)^\eta}, \tag{3.4}$$

where $\eta \in [1, \infty)$ and $\rho_A + \rho_B = 1$. The parameter η measures the disproportionality of the political system. When it takes the value 1, it represents the purely proportional system, while when it is ∞, it becomes the winner-take-all system.

This way of modeling power sharing is a reduced form which captures many other power sharing mechanisms in the political system. The advantage of doing this is to use an abstract concept of power sharing to capture many aspects of the political system. Although it has no correspondence in the real political system, it capture the overall power sharing of the political system. Moreover, in the empirical part of this chapter we will see exactly the power sharing indices that measures this power sharing concept. In other word, with the help of this reduced form power sharing rule we achieved a theoretical concise way of modeling power sharing without losing it empirical applicability.

Parties' objectives are to maximize the expected rents appropriated according to their power share. More specifically, party's payoff function is defined as following:

$$\Pi_C(x_A, x_B) = \rho_C \cdot (\gamma \, r_C + R) \tag{3.5}$$

where $R > 0$ represents ego rents and $\gamma \in [0, 1]$ measures the transaction cost of taking rents. Notice that the ego rents would not affect the main mechanism of the model, however to be consistent with the literature we still keep it.

Let $\mathcal{G} = (X, \Pi_C)_{C=A,B}$ denote the election game described above. The timing of this game is as follows. First, both parties announce their campaign pledges simultaneously; Second, each ideological bias σ_i is realized and is observed by voter i; Third, each voter casts her vote for one party; Fourth, the implemented policy is decided through the power sharing rule; Finally, fifth, voters and political parties receive their respective payoffs.

3.3 Theoretical Results

We now show the characterization result for the election game, under parties' payoff functions and the power sharing rule in (3.5) and (3.4).

Proposition 3.1 (Characterization). *Let (x_A^*, x_B^*) be the pure strategy equilibrium of the election game $\mathcal{G}$. It is symmetric and unique and has the following property, for $C = A, B$:*

$$\begin{aligned} g_C^* &= (a')^{-1}(\frac{\phi_\alpha}{\phi}); \\ r_C^* &= \max[0, \frac{N}{2\eta\phi_\alpha} - \frac{R}{\gamma}]; \\ t_C^* &= \frac{r_C^* + g_C^*}{y}; \end{aligned} \tag{3.6}$$

where $\phi = \sum_i \phi_i$, and $\phi_\alpha = \sum_i \phi_i \frac{y_i}{y}$.

The equilibrium is symmetric because the symmetric position of the two parties and the concavity of the utility function, while the uniqueness follows from the monotonicity of the first order condition. There are several interesting findings emerging from this proposition. Firstly, the provision of public good is at the socially optimal level since it is maximizing the weighted sum of voters' utility. Secondly, the level of political rents is not at the socially optimal level 0. Only for the extreme case of the winner-take-all system(i.e. when η goes to infnity), the electoral competition drives the political rents to the minimum level. The intuition behind this result is that the competition between parties are the fiercest in the winner-take-all system. Therefore, the competition over policy platform will drive the level of political rents to the lowest possible. Different to Persson and Tabellini's (2002) result, where due to electoral uncertainty competition cannot keep the political rents at an efficient level, we show that if power sharing is considered then winner-take-all system can achieve the efficient level of political rents.[1] Another interesting finding is about the relationship between the power sharing disproportionality and political rents. Formally, we have the following result.

Corollary 3.1. *Let r^* denote the equilibrium rent level, when it is positive we have that:*

$$\frac{\partial r^*}{\partial \eta} = -\frac{N}{2\eta^2 \phi_\alpha}. \tag{3.7}$$

The level of political rents is decreasing in the power sharing disproportionality. In other words, a more disproportional power sharing system leads to less corruption. This is because in a more disproportional system parties' incentive to takes rents are better constrained by the incentive to win. Hence less power sharing makes the voter better off.

3.4 Data and Methodology

We now embark on the description of the key variables used in the empirical analysis. Our sample consists of over 90 democracies in the period 2000-2015. The Data Appendix provides a detailed description of the data source and the constructing method.

3.4.1 Corruption

There are two types of data available on corruption: perception based data and experience based data. Two commonly used perception indices on corruption are the Corruption Perceptions Index (CPI) constructed by Transparency International (TI) and a rating of control of corruption published by Daniel Kaufmann at the World Bank (WB). In our data set, CPIrev is the reversed CPI index with higher value referring to higher corruption. And WBrev is the reversed World Bank index with lower value indicates better control over corruption.[2]

As for the second type of corruption data, TI's "Global Corruption Barometer" (GCB) survey and the World Bank's World Business Environment Survey (WBES) are two well known experience-based corruption data. However, these

[1] Parties are not perfect substitutes from voters' perspective with the presence of electoral uncertainty.

[2] Note that the reversing method is a linear transformation. For CPI, we use 100 minus the original CPI and for WB we use the following formula: $WBrev = 100 * (\max(WBcc) - WBcc)/(\max(WBcc) - \min(WBcc))$.

experience-based corruption data are not measuring the grand corruption as in our theoretical model, where the elected official can appropriate the residual of budget. The measures of experience-based corruption are not adequate for our analysis since they are based on surveys where people are asked whether they have experienced bribery from police officers or traffic controllers etc., which measures petty corruption. Therefore, our main empirical results are relying on the perception-based data.

3.4.2 Electoral Disproportionality

As for the first measure of disproportionality we adopt the Taagepera's (1986) index (Disprop_Tag), which is generated by dividing the logarithm of the total number of votes by the logarithm of the total number of parliamentary seats, and powering the result to the inverse of the mean electoral district magnitude.[3] This index goes from 1 (purely proportional representation) to infinity (winner-take-all) coinciding with what the theory indicates. We use this index because it is a generalized version that suits both plurality system and proportional representation system and also accommodates the multi-district multi-party system. To construct the index, data on the total number of votes for each election and country is collected from the International Institute for Democracy and Electoral Assistance (IDEA) Database.[4] On the other hand, the total number of seats and the electoral district magnitudes are gathered from the Database of Political Institutions of Cruz et al. (2015). Although the Taagepera index precisely measures the disproportionality of the seat and vote relationship, it is a good proxy for the overall power sharing disproportionality since parliamentary seats are the major source of political power.

Another very commonly used disproportionality index is Gallagher's least square index. We also use this index to check the robustness of our result. However, this index measures the discrepancy between votes and seats, it does not measure the way that votes are translated to seats. Therefore, we should put more value on the Taagepera's index. Moreover, since the sample size for Taagepera index is larger we use Taagepera for most of our studies, while we use Gallagher index for robustness check.

[3]Please see section 3.7.2 for more details about the Taagepera index.

[4]International Institute for Democracy and Electoral Assistance (IDEA) Database, (multiple countries; 1945-2014). Stockholm: IDEA. Web address: http://www.idea.int/db.

3.4.3 Other Explanatory Variables

There are many empirical studies looking at the causes of corruption in the last decades. They have identified a number of economic, social, cultural and political variables that correlate with the level of perceived corruption. We follow these studies in formulating our basic estimation specification. Among those studies, the control variables we use followed very closely with Persson et al. (2003).

To control for the age and quality of the democracy we use several political variables. DemocracyAge measures the years since the beginning of the last democratic regime. FH_poli and FH_civil are respectively the political right index and the civil right index published by the Freedom House. Also there are some electoral rule characteristics like the elected representatives on the party list (Plist) and the average district magnitude (Dismag), which are controlled for IV regressions but not for the basic OLS regressions, since they severely restrict the sample size. However we always make it clear whether these are in the control panel or not.

Previous work on corruption has found that economic development is a very significant cause for lower corruption (see Treisman, 2007). Therefore we use the logarithm of per capita GDP as a control variable. We also consider a measure of openness (Trade), defined as the percentage of the sum of import and export over GDP.

Some variables on country characteristics are also included. Several studies have found that higher linguistic and ethnic fractionalization causes higher corruption in the government. We adopt Alesina et al.'s (2003) ethnic fractionalization index. We measure country's level of education by the secondary school gross enrollment ratio. Country's religious belief are measured as percentage of Protestants of the total population. We also control for the media transparency of each country measured by the "freedom of the press" index conducted by freedom house (see Brunetti and Weder, 2003).

Moreover, the geographical location and legal origin are also in our control panel. Especially as the British legal origin tends to have majoritarian systems, it is important to control for this variable. We also try to include other legal origins like German and Scandinavian, but they never show significant results. To keep our specification parsimonious, we only include British and French legal origin in our main studies.

3.4.4 Methodology

To test our theoretical prediction, we estimate two sets of regressions: cross sectional and panel. Specifically our empirical specification in the cross sectional analysis will be:

$$corruption = \alpha + \beta\, disproportionality + \sum_k \gamma_k\, x_k + \mu, \tag{3.8}$$

where the disproportionality index is taken from the most recent election year before the year of the corruption data. x_k stands for the control variables, which are borrowed from the literature on corruption. It is clearly presented in the previous section. From the theoretical framework we should expect a negative β, as corruption decreases with disproportionality.

However, since previous corruption affect both disproportionality and current corruption the disproportionality index would be endogenous. Previous corruption (if observed) would make voters dissatisfied with the incumbent so that the disproportionality would be larger (more concentrated to the challenger) for the next election. If not observed, previous corruption would increase the incumbent's advantage so that next election the power is more concentrated in the incumbent. In both ways, previous corruption, which is not controlled for, are correlated with the endogenous variable disproportionality index. Hence, without getting into the uncommonly used dynamic panel regressions in the literature, we use two instrumental variables to isolate the effect of disproportionality. They are the starting time of the current electoral system and the number of majoritarian system in the previous electoral history.

In the panel analysis, we have the following general specification.

$$\begin{aligned} corruption_{it} = \alpha + \beta\, disproportionality_{it\_e} + \sum_k \gamma_k\, x_{kit} \\ + \gamma_t\, year_t + \delta_i\, D_i + \mu_{it}, \end{aligned} \tag{3.9}$$

where all control variables are taken from the same year as corruption and D_i stands for country fixed effect. The subscript of the disproportionality index t_e means the year of election before t, this specification identifies the lagged effect of the electoral competition. This is consistent with our cross sectional analysis. As for the control variables, since some of them are time-invariant in the period we are studying such as religion and ethnic fraction we exclude them from the control panel. The country fixed effect and time dummies are included in all of the regressions.

3.5 Empirical Results

3.5.1 Cross Section Analysis

We first present in Table 3.1 the result of the cross sectional analysis in equation (3.8). The original tables are relegated in Appendix 3.7.3. The dependent variables are the average of the CPI corruption index over 2010 to 2015 except for column (2), where we use the corruption index for 2010. We choose these years because there are more available data in these years. The regression method for column (1), (3), (4) and (5) are weighted least square, in which the weights are the inverse of the square of the standard deviations over the 5 years. Moreover, the disproportionality indices are taken from the most recent election year before 2010. The control variables in column (1) and (2) are our full set of controls including legal origin and geographical locations, whereas column (3) excludes legal origins and column (4) and (5) include separately the percentage of representatives elected on the party list and district magnitude.

Table 3.1: Cross Section Analysis

	(1)	(2)	(3)	(4)	(5)
	CPIrev10_15	CPIrev10	CPIrev10_15	CPIrev10_15	CPIrev10_15
Disprop_Tag	-3.138**	-2.950*	-4.037***	-4.447	-6.436**
	(1.461)	(1.523)	(1.077)	(3.062)	(3.136)
Method	WLS	OLS	WLS	WLS	WLS
Legal Origins	Yes	Yes	No	Yes	Yes
Other controls	No	No	NO	Plist	dismag
Countries	96	94	100	65	65
R^2	0.886	0.845	0.878	0.899	0.898

Standard errors in parentheses.
Continental dummies are included.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Our main interest is the effect of disproportionality on corruption, as displayed in Table 3.1. The effect of disproportionality is negative and significant in the regressions using Taagepera's index except column (4) when Plist is controlled. In the contrast, as shown in figure 3.1 in the appendix, the countries with higher disproportionality have larger corruption. We show that after controlling for other important causes of corruption, this positive correlation breaks down. The insignificant result in column (4) might be driven by the high correlation between Plist and disproportionality. This is because usually in a highly disproportioanal system the percentage of elected representatives on the party list is very low, while for a more proportional system the percentage of elected representatives would be very high. In other words, by controlling Plist, we are restricting the variation of the disproportionality too much. Therefore, when Plist is controlled, both Plist and disproportionality are insignificant. Notice that when legal origins are excluded, the effect of disproportionality are slightly larger and more significant. This is because the electoral disproportionality is capturing some effect of the legal origin. To be more specific, countries with British legal origin are more likely to have majoritarian system hence more disproportional elections. Also British legal origin has been found to cause a lower corruption level in the literature. So when we omit the legal origins, the effect of the disproportionality is larger. However, since we want to separate the effect of disproportionality from the effect of legal origin in the following analysis we always include legal origin in the control set. Overall, the OLS and WLS regression results suggest that higher disproportionality leads to lower corruption.

To deal with the possible endogeneity problem, we use instrumental variable to isolate the effect of disproportionality on corruption. The results for these IV regressions are in Table 3.2. However, since the instruments we use does not vary over time in the panel analysis we cannot conduct the same IV regression. The overall IV regression results are strongly supporting our prediction and it has larger effect than the OLS and WLS regressions. We conducted a Limited Information Maximum Likelihood (LIML) estimator for the model specification in column (1) to see whether the estimate are biased or not. It turns out that the 2SLS and LIML are very similar so that our estimates are not too biased even though the sample size is very small.

The instrumental variables we use are the starting year of the current electoral system: A (elec_origin)[5] and the difference between the numbers of the majoritarian system and the proportional system in the electoral reform history: B (Net_maj). All of the first stage regressions are included in the Appendix 3.7.3. To argue for the relevance of the instrument, it is well known that the

[5] This instrument is first proposed by Persson, Tabellini, and Trebbi (2003).

Table 3.2: IV Regressions

	(1)	(2)	(3)	(4)	(5)
	CPIrev10_15	CPIrev10_15	CPIrev10	CPIrev10_15	CPIrev10_15
Disprop_Tag	-9.755**	-9.965**	-8.816**	-12.481*	-12.687**
	(3.925)	(4.047)	(4.101)	(6.758)	(6.186)
Instruments	A B	A B	A B	A	A B
Other Controls	Plist	Plist	Plist	Plist	dismag
Method	2SLS	LIML	2SLS	2SLS	2SLS
F Statistic	7.00	7.00	7.00	8.81	4.47
Hansen J test	0.42	0.43	0.29	No	0.69
Countries	58	58	58	58	58
R^2	0.866	0.865	0.862	0.848	0.851

Standard errors in parentheses.
Continental dummies are included.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

majoritarian system tends to exaggerate the winning party's share of seats, see Norris (1997). Also we can see from the correlation table in the appendix that the correlations between the majoritarian index (Net_maj) and our disproportionality indices are both positive and very high. Moreover, the electoral reforms taken more recently is more inclined to have proportional systems. So countries with most recent electoral reforms are more likely to have lower disproportionality. To argue for the exogeneity, corruption can hardly determine which year was the current electoral system being formed. Neither does the starting time of the electoral system affects corruption directly. As for the persistence of the electoral system, after controlling for economic development and the electoral disproportionality and other democratic variables, the system characteristics can be seen as exogenous to corruption. The reason is that for the electoral system to affect corruption it would mainly go through the election results, because the election is where the party gets power from. Moreover, it is not straightforward which electoral system can lead to higher corruption, as Persson, Tabellini, and Trebbi (2003) show that a switch from majoritarian to proportional system only have a slight increase in corruption. Therefore, the mechanism that corrupted officials manipulate the system to be more favorable to corruption is very feeble. In other words, corruption can hardly determine the persistence of the majoritarian system.

Table 3.3: Robustness Check

	(1)	(2)	(3)	(4)
	WBrev10_15	WBrev10	CPIrev10_15	CPIrev10_15
Disprop_Tag	-11.781**	-9.194**		-3.784
	(4.912)	(4.303)		(2.451)
Disprop_Gal			-0.579	
			(0.812)	
Instruments	A B	A B	A B	No
Other controls	Plist	Plist	Plist	Plist
F Statistic	7.00	7.00	2.01	No
Hansen J test	0.482	0.613	0.324	No
Method	IV	IV	IV	OLS
Countries	58	58	49	58
R^2	0.867	0.887	0.931	0.880

Standard errors in parentheses.
Continental dummies are included.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

In Table 3.3, we present the results of some robustness check for the IV analysis, where we use alternative measures for corruption and disproportionality in column (1) to (3).[6] Notice that the Gallagher index has no significant relationship with the corruption index, but Taagepera index continues to be significantly correlated with the WB corruption index. Column (4) is an OLS analysis using the same sample for the IV regression to check whether the significant result comes from sample selection bias. Apparently, the driving force of the strong IV result is not from the sample selection bias since the OLS regression result is not significant. Therefore the instruments we use indeed isolate the effect of disproportionality and the significance of the IV result strongly support our theoretical result.

Overall, the cross sectional analysis suggests that electoral disproportionality is negatively affecting the grand corruption. This strongly supports our theoretical result that through an electoral competition channel disproportionality affects corruption negatively.

[6]We have also tested our thesis with experience-based corruption data as mentioned in section 3.4. None of the experience-based corruption measures reflects a relationship between disproportionality and corruption. Results are available upon request.

3.5.2 Panel Analysis

Table 3.4: Panel Analysis

	(1) CPIrev	(2) CPIrev	(3) CPIrev	(4) CPIrev	(5) WBrev
Disprop_Tag	-2.509*	-4.854***	-4.610***		-1.822
	(1.375)	(1.677)	(1.733)		(1.357)
Disprop_Gal				0.032	
				(0.155)	
Countries	94	70	67	58	73
Sample	Full	fhpoli < 4	fhcivil < 4	fhpoli < 4	fhpoli < 4
N	289	212	202	181	231
R^2	0.226	0.275	0.270	0.215	0.162

Standard errors in parentheses.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

In Table 3.4, we present the panel analysis results for the lagged effect model. The corruption and control variables are taken from years of 2000, 2005, 2010 and 2015, while the disproportionality indices are collected from the most recent election year before the time when corruption is measured. Column (1) displays the result for the full sample, while column (2) and (3) put different restrictions on the sample, which mainly requires the country to be more developed in terms of its democratic system by having more freedom in political rights and civil rights. Column (4) and (5) represent the same restricted sample but with different measures of corruption and disproportionality. As for the result, Gallagher index cannot give any solid inference, while the Taagepera's index shows a negative relationship confirming our theoretical prediction. Moreover, when we restrict the sample to developed and free democratic countries the effect becomes stronger.

To sum up, the panel method also supports our theoretical hypothesis that higher disproportionality leads to lower corruption. This electoral competition effects work better with more mature and developed democracies, since in these countries the election can regulate the politicians better.

3.6 Conclusion

In this chapter, we have studied the effect of power sharing over political rents in a probabilistic voting model with endogenous political rents. The characterization result shows that corruption is better constrained in a more disproportional system. We also found supporting empirical evidence for this prediction using both cross sectional and panel data. In other words, less power sharing creates more incentive for politicians to compete so that it regulates them better.

3.7 Appendix

3.7.1 Proofs

Proof of Proposition 3.1. First we show that the equilibrium is symmetric, i.e. $x_A = x_B$. Conditional on the other party's policy, party A and B face the same optimization problem. Without loss of generality, we consider party A's problem.

$$\begin{aligned} &\max_{x_A} \quad \Pi_A(x_A, x_B) \\ \text{s.t.} \quad & r_A + g_A = t_A y && (3.10) \\ & t_A \in [0,1] && (3.11) \\ & r_A \geq 0 && (3.12) \\ & g_A \geq 0 && (3.13) \end{aligned}$$

The Lagrange function is $L = \rho_A(\gamma r_A + R) + \lambda(t_A y - r_A - g_A) + \mu_1 t_A + \mu_2(1 - t_A) + \mu_3 r_A + \mu_4 g_A$. Notice because of the symmetry of the problem, party A and B will have the same first order conditions. We consider the case with interior solutions where $\lambda > 0, \mu_1 = \mu_2 = \mu_3 = \mu_4 = 0$. We show that in this case the equilibrium is symmetric, and the same argument applies to other cases. Under this configuration of Lagrange multipliers, the first order conditions reduce to the following:

$$\begin{aligned} \rho_A \gamma - \lambda_A &= 0 \\ (\gamma r_A + R)\rho'_A(s_A) \sum_{i \in N} F'_i u'_i(g_A) - \lambda_A &= 0 \\ (\gamma r_A + R)\rho'_A(s_A) \sum_{i \in N} F'_i u'_i(t_A) - \lambda_A y &= 0 \end{aligned}$$

Similarly we have these equations for party B. Hence, we can deduce that:

$$\frac{\rho_A \gamma}{\rho_B \gamma} = \frac{\lambda_A}{\lambda_B} \tag{3.14}$$

$$\frac{(\gamma r_A + R)\rho'_A(v_A)\sum_{i\in N} F'_i(t_i)u'_i(g_A)}{(\gamma r_B + R)\rho'_B(v_B)\sum_{i\in N} F'_i(t_i)u'_i(g_B)} = \frac{\lambda_A}{\lambda_B} \tag{3.15}$$

$$\frac{(\gamma r_A + R)\rho'_A(v_A)\sum_{i\in N} F'_i(t_i)u'_i(t_A)}{(\gamma r_B + R)\rho'_B(v_B)\sum_{i\in N} F'_i(t_i)u'_i(t_B)} = \frac{\lambda_A}{\lambda_B}, \tag{3.16}$$

where $t_i = u_i(x_A) - u_i(x_B)$. Since $u'_i(t_B) = u'_i(t_A)$, for all $t_A \neq t_B$, and $\rho'_A(v_A) = \rho'_B(v_B)$, from (3.15) and (3.16), we can deduce that $g_A = g_B$. Now suppose that $r_A > r_B$, then from (3.14) we know that $\frac{\lambda_A}{\lambda_B} < 1$, since ρ_C is decreasing in r_C. From (3.15) we know that $\frac{\lambda_A}{\lambda_B} > 1$, which contradicts each other. Hence, the equilibrium policy is symmetric.

Next, we characterize the equilibrium. Substitute $t_A = \frac{r_A+g_A}{y}$ into the payoff function, we have $v_A = \frac{1}{N}\sum_i F_i[(1-\frac{r_A+g_A}{y})y_i + a(g_A) - (1-\frac{r_B+g_B}{y})y_i - a(g_B)]$. Given that $v_A = v_B = \frac{1}{2}$, it is true that $\frac{\partial \rho_A}{\partial v_A} = \eta$. We assume that the income is higher enough so that a fully tax appropriation would never happens.

Therefore, the first order condition for g_A is:

$$(\gamma r_A + R)\eta \frac{1}{N}\sum_i \phi_i[-\frac{y_i}{y} + a'(g_A)] = 0. \tag{3.17}$$

This implies that the public good supply at equilibrium satisfies: $a'(g_A^*) = \frac{\phi_\alpha}{\phi}$.

The first order condition for the political rent r is:

$$\gamma \rho_A + (\gamma r_A + R)\eta \frac{1}{N}\sum_i \phi_i(-)\frac{y_i}{y} = 0 \tag{3.18}$$

Since $\rho_A = \frac{1}{2}$, this implies $r_A^* = \max[0, \frac{N}{2\eta\phi_\alpha} - \frac{R}{\gamma}]$. Finally, t_A is determined by the budget constraint: $t_A = \frac{r_A^* + g_A^*}{y}$.

Moreover, this equilibrium is the unique equilibrium of $\mathcal{G}$, since from equation (3.17) and (3.18) we can see that the first order derivatives are both strictly monotonic decreasing in g_A and r_A. Hence there can only be one solution satisfying the first order conditions. □

3.7.2 Data Appendix

CPI10 = Transparency International's CPI index in 2010. Source: https://www.transparency.org

CPIrev10= 100- CPI10. The reversed Transparency International's CPI index in 2010.

DemocracyAge = Years since the beginning of last democratic regime adopted from Treisman (2007).

Dismag = Average district magnitude that ranges from 0 to 1, taking a value of 0 for a system with only single-member districts, and close to 1 for a system with a single electoral district. Source: Persson, Tabellini, and Trebbi (2003).

Disprop_Gal= Gallagher's least square index for electoral disproportionality of the nearest election before 2000. (Source: https://www.tcd.ie/Political_Science/people/michael_gallagher/ElSystems/index.php)

Disprop_Tag=Taagepera index of the nearest election before 2000. It is generated using the equation (5) of Taagepera (1986) in the following way:

$$Disprop\_Tag = (\frac{\log V}{\log S})^{1/M}$$

where V and S are the total votes and total seats respectively and M is the mean electoral district magnitude. To construct the index, data on the total number of votes for each election and country is collected from the International Institute for Democracy and Electoral Assistance (IDEA) Database. (Web address: http://www.idea.int/data-tools.) On the other hand, the total number of seats and the electoral district magnitudes are gathered from the Database of Political Institutions of Cruz et al. (2015). (Web address: https://publications.iadb.org/handle/11319/7408)

Edu= Secondary school enrollment ratio taken from the WDI database. Source: same as lnGDP.

Elec_origin = The originating year when the current electoral system was formed. Source: Colomer (2016). The handbook of electoral system choice. Springer.

Ethnic_frac= Ethnic fractionalization data from Alesina et. al., with higher value indicating more fractionalized population. Alesina et al. (2003). Source: http://www.anderson.ucla.edu/faculty_pages/romain.wacziarg

FH_civil= Freedom house civil liberty index in 2010 with 1 representing most free and 7 represents least free. Source: see FH_poli.

FH_poli= Freedom house political right index in 2010 with the same ranking as FH_civil. Source: https://freedomhouse.org/content/freedom-world-data-and-resources

FH_press = The freedom house press transparency index. Source: see FH_poli.

FH_pressrev =100 - fhpress. This is the reverse of the freedom house press transparency index. Source: see FH_poli.

GCB_bri= Transparency International's Global corruption barometer total bribery rate in 2017. Source: https://www.transparency.org

lnGDP= Logarithm of per capita GDP in 2010 collected from World Bank's World Development Indicators (WDI) database. Source: https://datacatalog.worldbank.org/dataset/world-development-indicators.

Net_maj = Number of majoritarian system − number of other systems in the year from 1800 to 2010. Source: see Elec_origin.

Plist = The percentage of representatives elected on a party list. Plist ranges between 0, under plurality rule in every district, and 1, in a system with full proportioality. Source: Persson, Tabellini, and Trebbi (2003).

Protestant=Percentage of protestant in the total population. Source:http://www.pewforum.org/2011/12/19/table-christian-population-as-percentages-of-total-population-by-country/

Trade=Total import and export as a percentage of GDP provided by WDI database. Source: see lnGDP.

WBcc2010= World Banks's control of corruption measure in 2010. Source: http://info.worldbank.org/governance/wgi/#home.

WBrev10 = The reversed WBcc2010 and rescaled to 0 to 100 range.

WBES= The latest World Bank's World Business Environment Survey's corruption experience measure. Source: Enterprise Surveys (http://www.enterprisesurveys.org), The World Bank.

3.7.3 Original Tables

Table 3.5: Summary Statistics Panel

	count	mean	sd	min	max
CPIrev	336	52.5	23.1	0	89
WBrev	384	53.1	26.7	-1.10e-14	98.9
Disprop_Tag	360	1.69	.868	1	4.15
Disprop_Gal	244	7.38	6.52	.26	32.5
lnGDP	382	24.9	2.15	20	30.4
Edu	338	82.3	30.9	6.04	167
FH_poli	384	2.78	1.94	1	7
FH_civil	384	2.86	1.65	1	7
FH_press	384	40.9	23	5	96
Trade	378	84.6	48.3	19.8	410

Table 3.6: Summary Statistics Cross Section

	count	mean	sd	min	max
CPIrev10_15	182	58	20	8.17	91.6
CPIrev10	177	60	20.9	7	89
WBrev10_15	196	59.9	24.9	0	100
WBrev10	196	58.7	24.6	1.08e-14	100
lnGDP	193	8.52	1.51	5.44	11.9
Protestant	193	.211	.267	.001	.954
Ethnic_frac	183	.435	.258	0	.93
FH_poli	190	3.37	2.16	1	7
FH_civil	190	3.19	1.83	1	7
FH_pressrev	190	55.2	25.2	4	92
DemocracyAge	170	8.99	21.9	0	169
Trade	181	89.4	46.9	.175	373
Edu	142	81.9	26	13.5	132
Leg_British	174	.333	.473	0	1
Leg_French	174	.443	.498	0	1
OECD	189	.159	.366	0	1
Elec_origin	94	1983	18.6	1918	2002
Net_maj	94	-.383	2.44	-6	8
Disprop_Tag	138	1.85	.967	1	4.99
Disprop_Gal	110	7.97	6.15	.3	32.3
N	201				

Table 3.7: Correlations

	CPIrev10	Disprop_Tag	Disprop_Gal	lnGDP	Protestant	Ethnic_frac	FH_poli	FH_pressrev	Elec_origin	Net_maj
CPIrev10	1.000									
Disprop_Tag	0.137	1.000								
Disprop_Gal	0.140	0.462	1.000							
lnGDP	-0.783	-0.217	-0.144	1.000						
Protestant	-0.224	0.271	-0.075	-0.001	1.000					
Ethnic_frac	0.420	0.107	0.127	-0.397	-0.112	1.000				
FH_poli	0.595	0.134	0.168	-0.476	-0.272	0.323	1.000			
FH_pressrev	-0.625	-0.133	-0.189	0.514	0.330	-0.304	-0.865	1.000		
Elec_origin	0.530	-0.103	0.054	-0.408	-0.351	0.196	0.215	-0.288	1.000	
Net_maj	0.082	0.448	0.268	-0.148	0.021	0.057	0.125	-0.093	-0.138	1.000

Figure 3.1: Corruption and disproportionality

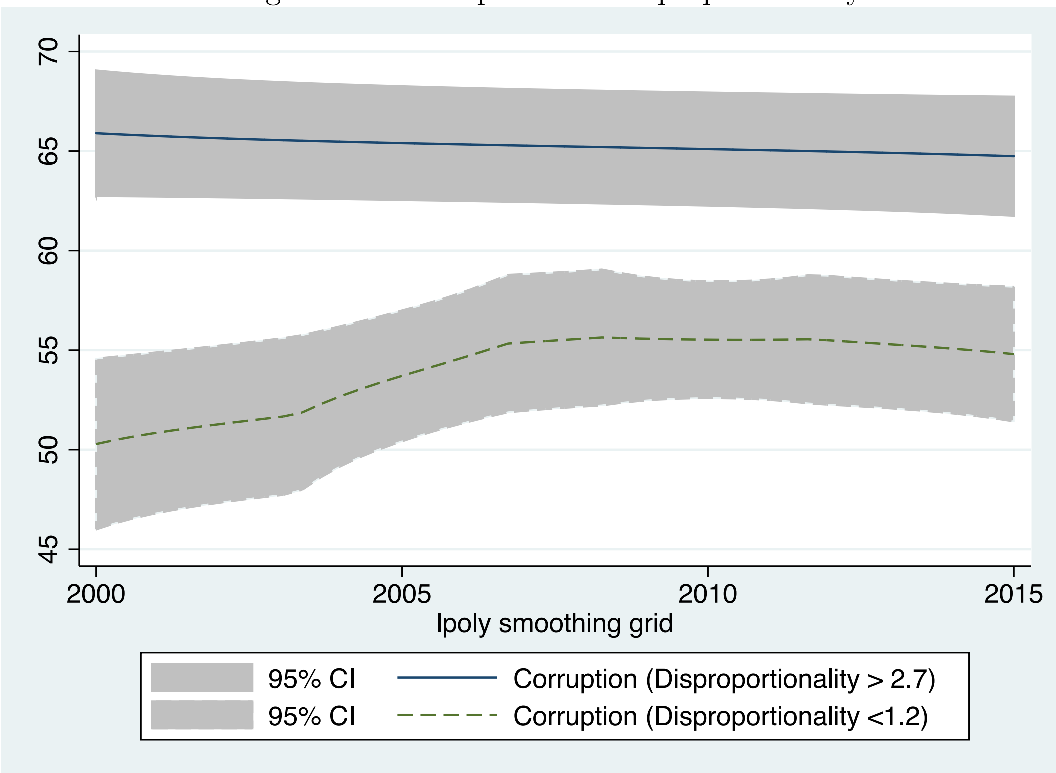

Table 3.8: Cross Section Analysis

	(1)	(2)	(3)	(4)	(5)
	CPIrev10_15	CPIrev10	CPIrev10_15	CPIrev10_15	CPIrev10_15
Disprop_Tag	-3.138**	-2.950*	-4.037***	-4.447	-6.436**
	(1.461)	(1.523)	(1.077)	(3.062)	(3.136)
lnGDP	-3.338	-5.365***	-4.167*	-2.968	-2.457
	(2.269)	(1.862)	(2.207)	(3.191)	(3.228)
Protestant	6.544	4.068	3.190	-10.816	-10.238
	(4.491)	(6.753)	(4.578)	(10.200)	(10.126)
Ethnic_frac	-3.602	-2.547	-7.138	-7.439	-7.304
	(6.142)	(5.395)	(7.438)	(6.707)	(6.708)
FH_poli	1.160	-1.391	1.817	-1.050	-1.568
	(1.934)	(1.898)	(2.012)	(2.471)	(2.231)
FH_civil	4.636	7.006**	4.074	8.399*	9.378**
	(3.105)	(3.066)	(2.848)	(4.373)	(4.074)
DemocracyAge	0.063	0.034	0.073	-0.018	-0.012
	(0.048)	(0.049)	(0.044)	(0.063)	(0.064)
Trade	0.002	-0.045*	0.011	-0.049	-0.052
	(0.026)	(0.026)	(0.022)	(0.031)	(0.031)
Edu	-0.127	-0.098	-0.122	-0.080	-0.097
	(0.110)	(0.085)	(0.117)	(0.148)	(0.148)
FH_pressrev	-0.104	0.028	-0.103	-0.283	-0.267
	(0.149)	(0.136)	(0.130)	(0.180)	(0.174)
Leg_British	1.141	-1.436		4.619	4.762
	(6.428)	(6.783)		(9.093)	(9.124)
Leg_French	5.504	6.316		1.460	0.787
	(6.123)	(5.997)		(8.253)	(8.260)
OECD	-7.226	-10.578**	-5.983	4.313	4.305
	(5.496)	(4.627)	(4.138)	(6.753)	(6.859)
Plist				3.002	
				(4.611)	
Dismag					-2.073
					(5.594)
N	96	94	100	65	65
R^2	0.886	0.845	0.878	0.899	0.898

Standard errors in parentheses

Continental dummies are included.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3.9: IV Regressions

	(1)	(2)	(3)	(4)	(5)
	CPIrev10_15	CPIrev10_15	CPIrev10	CPIrev10_15	CPIrev10_15
Disprop_Tag	-9.755**	-9.965**	-8.816**	-12.481*	-12.687**
	(3.925)	(4.047)	(4.101)	(6.758)	(6.186)
lnGDP	-4.890**	-4.853**	-4.876**	-4.406*	-5.063**
	(2.305)	(2.319)	(2.339)	(2.562)	(2.552)
Protestant	-0.528	-0.425	-1.629	0.817	-4.623
	(9.274)	(9.333)	(10.513)	(10.218)	(8.499)
Ethnic_frac	-5.917	-6.025	-4.681	-7.325	-4.913
	(6.506)	(6.542)	(7.155)	(6.921)	(6.702)
FH_poli	-2.093	-2.120	-2.217	-2.455	-1.486
	(2.222)	(2.226)	(2.432)	(2.460)	(2.241)
FH_civil	9.506***	9.561***	10.405***	10.230***	9.668***
	(2.966)	(2.971)	(3.323)	(3.399)	(3.116)
DemocracyAge	-0.007	-0.007	-0.009	-0.012	-0.017
	(0.047)	(0.048)	(0.047)	(0.051)	(0.048)
Trade	-0.102***	-0.103***	-0.106***	-0.116***	-0.098***
	(0.035)	(0.036)	(0.040)	(0.044)	(0.037)
Edu	-0.006	-0.007	-0.069	-0.015	0.013
	(0.106)	(0.107)	(0.110)	(0.113)	(0.115)
FH_pressrev	-0.192*	-0.192*	-0.113	-0.197*	-0.105
	(0.108)	(0.109)	(0.118)	(0.117)	(0.135)
Leg_British	3.676	3.760	5.486	4.768	4.024
	(8.670)	(8.735)	(9.573)	(9.606)	(8.914)
Leg_French	2.327	2.303	7.152	2.015	0.735
	(6.230)	(6.264)	(7.147)	(6.813)	(6.913)
OECD	-4.236	-4.275	-4.411	-4.742	-5.230
	(6.021)	(6.046)	(6.544)	(6.248)	(6.142)
Plist	-6.682	-6.868	-5.217	-9.105	
	(4.586)	(4.663)	(5.048)	(6.625)	
Dismag					-13.902
					(9.915)
N	58	58	58	58	58
R^2	0.866	0.865	0.862	0.848	0.851

Standard errors in parentheses

Continental dummies are included.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3.10: IV Regressions First Stage

	(1)	(2)	(3)
	Disprop_Tag	Disprop_Tag	Disprop_Tag
Elec_origin	-0.016***	-0.016***	-0.014**
	(0.006)	(0.005)	(0.006)
Net_maj	0.092**		0.051
	(0.037)		(0.042)
lnGDP	0.334	0.253	0.226
	(0.210)	(0.222)	(0.268)
Protestant	-0.029	0.081	-0.443
	(0.613)	(0.670)	(0.431)
Ethnic_frac	-0.289	-0.460	-0.128
	(0.451)	(0.431)	(0.498)
FH_poli	-0.283*	-0.176	-0.139
	(0.161)	(0.173)	(0.173)
FH_civil	0.506*	0.402	0.387
	(0.253)	(0.261)	(0.306)
DemocracyAge	-0.003	-0.004*	-0.004*
	(0.002)	(0.002)	(0.002)
Trade	-0.006**	-0.008**	-0.005**
	(0.003)	(0.003)	(0.002)
Edu	-0.009	-0.007	-0.005
	(0.009)	(0.009)	(0.010)
FH_pressrev	-0.000	0.000	0.009
	(0.011)	(0.012)	(0.012)
Leg_British	0.237	0.325	0.182
	(0.477)	(0.466)	(0.448)
Leg_French	0.006	0.014	-0.123
	(0.455)	(0.414)	(0.470)
OECD	-0.147	-0.292	-0.256
	(0.330)	(0.319)	(0.332)
Plist	-0.788***	-0.889***	
	(0.284)	(0.308)	
Dismag			-1.301***
			(0.432)
N	58	58	58
R^2	0.753	0.709	0.782

Standard errors in parentheses

Continental dummies are included.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3.11: Robustness Check

	(1)	(2)	(3)	(4)
	WBrev10_15	WBrev10	CPIrev10_15	CPIrev10_15
Disprop_Tag	-11.781**	-9.194**		-3.784
	(4.912)	(4.303)		(2.451)
lnGDP	-6.691**	-5.533**	-4.329**	-5.973**
	(2.948)	(2.630)	(2.028)	(2.859)
Protestant	-1.038	-1.514	-1.294	-3.911
	(11.330)	(10.267)	(6.963)	(10.022)
Ethnic_frac	-1.936	-2.596	2.847	-3.085
	(7.762)	(7.262)	(6.125)	(7.138)
FH_poli	-2.264	-1.997	-7.686***	-1.273
	(2.766)	(2.538)	(2.628)	(2.890)
FH_civil	10.155***	9.946***	18.171***	7.921*
	(3.546)	(3.343)	(2.434)	(3.972)
DemocracyAge	-0.026	-0.013	0.112***	0.001
	(0.054)	(0.045)	(0.030)	(0.053)
Trade	-0.132***	-0.118***	-0.085*	-0.070
	(0.043)	(0.038)	(0.046)	(0.042)
Edu	-0.042	-0.111	0.097	0.010
	(0.136)	(0.119)	(0.137)	(0.134)
FH_pressrev	-0.315**	-0.260**	-0.397***	-0.185
	(0.130)	(0.118)	(0.132)	(0.151)
Leg_British	4.928	3.790	-0.066	2.213
	(10.953)	(10.057)	(5.246)	(9.166)
Leg_French	0.798	2.359	4.049	3.050
	(7.817)	(7.418)	(4.293)	(7.320)
OECD	-0.978	-3.684	6.210	-2.612
	(7.447)	(6.523)	(6.338)	(6.790)
Plist	-6.894	-4.393	-4.885	
	(5.785)	(5.216)	(4.005)	
Disprop_Gal			-0.579	
			(0.812)	
N	58	58	49	58
R^2	0.867	0.887	0.931	0.880

Standard errors in parentheses
Continental dummies are included.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3.12: Panel Analysis

	(1) CPIrev	(2) CPIrev	(3) CPIrev	(4) CPIrev	(5) WBrev
Disprop_Tag	-2.509*	-4.854***	-4.610***		-1.822
	(1.375)	(1.677)	(1.733)		(1.357)
lnGDP	-14.912***	-17.009***	-17.144***	-19.407***	-8.251**
	(2.997)	(3.636)	(3.805)	(4.190)	(3.578)
Edu	0.027	0.024	0.023	0.023	-0.046
	(0.029)	(0.030)	(0.029)	(0.033)	(0.030)
FH_poli	-0.524	0.985	0.885	0.878	1.070
	(0.682)	(1.215)	(1.295)	(1.463)	(1.424)
FH_civil	0.552	1.596	0.691	0.941	-0.480
	(0.976)	(1.174)	(1.301)	(1.336)	(0.934)
FH_press	-0.042	0.026	0.018	0.072	0.160*
	(0.066)	(0.093)	(0.098)	(0.117)	(0.093)
Trade	-0.008	-0.023	-0.028	-0.030	-0.024
	(0.032)	(0.043)	(0.045)	(0.044)	(0.022)
2005.year	2.654**	3.293***	2.804**	2.582*	-0.455
	(1.029)	(1.157)	(1.127)	(1.296)	(0.998)
2010.year	5.264***	5.453***	5.194***	5.160***	0.490
	(1.385)	(1.528)	(1.512)	(1.764)	(1.458)
2015.year	4.296**	5.049**	4.661**	5.114**	0.961
	(1.910)	(2.009)	(2.055)	(2.183)	(1.960)
Disprop_Gal				0.032	
				(0.155)	
N	289	212	202	181	231
R^2	0.226	0.275	0.270	0.215	0.162

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Chapter 4

Conclusion

The aim of this thesis is to extend the analysis of power sharing in electoral competition to multidimensional policy space. In chapter 2 we study the effect of power sharing on income redistribution policies, and in chapter 3 we analyze the effect of power sharing on corruption. With the help of Tullock contest function, we are able to show that income inequality increases but corruption decreases with power sharing disproportionality.

The central story here about power sharing is that in a more disproportional system we have narrower representation but better accountability. As shown in chapter 2, the reason that more disproportional system has higher income inequality is the swing voter group (middle class) become more important and better represented than the poor group. In addition, we find in chapter 3 that a more disproportional system punishes politicians' rent seeking behavior more severely so that there is better accountability.

We will finish this book with a list of future research directions. First, the election games considered in both chapters are static. We have not studied the power transition in a dynamic setting. It is a promising topic to study the transition between parties when voting is adaptive. Moreover, the equilibrium policy choice also depends on parties' electoral advantage. Second, in chapter 3, the effect of strategic voting and multiparty competition are both unclear. These are both very interesting questions in political economy. Finally, the effect of power sharing on politicians' signaling behavior is still an open question. Introducing power sharing to signaling games of electoral competition would be a non-trivial problem.

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