Aeromechanics Modelling of Tiltrotor Aircraft

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Nomenclature

**Greek Symbols**

\( \alpha \)  
Angle of attack [rad]

\( \alpha \)  
Damping factor

\( \alpha_s \)  
Shaft angle of attack [rad]

\( \beta \)  
Flapping angle of a blade [rad]

\( \beta' \)  
Resultant in-plane gimbal rate (dimensionless)

\( \beta_i \)  
Longitudinal gimbal tilt angle [rad]

\( \beta_p \)  
Precone angle of a blade [rad]

\( \beta_s \)  
Lateral gimbal tilt angle [rad]

\( \chi \)  
Rotor wake skew angle [rad]

\( \delta \)  
Pilot stick position

\( \Delta \)  
Disc-wind angle [rad]

\( \eta \)  
Propulsive efficiency

\( \varepsilon \)  
Wing downwash angle [rad]

\( \gamma \)  
Flight path angle [rad]

\( \gamma \)  
Generalised dihedral angle [rad]

\( \Gamma \)  
Dihedral angle of the starboard wing [rad]

\( \Gamma \)  
Resultant in-plane gimbal rate angle [rad]

\( \eta \)  
Elevator deflection [rad]

\( \kappa \)  
Induced power correction factor

\( \xi_f / \xi \)  
Flap/flaperon setting angles [rad]

\( \lambda \)  
Inflow ratio

\( \lambda_0 \)  
Mean induced velocity ratio

\( \lambda_c \)  
Longitudinal harmonic of induced velocity

\( \lambda_i \)  
Induced inflow ratio

\( \lambda_s \)  
Lateral harmonic of induced velocity
\( \lambda_{im} \)  Thrust-induced inflow ratio
\( \Lambda \)  Sweep angle \([\text{rad}]\)
\( \mu \)  Advance ratio
\( \mu_\infty \)  Airspeed ratio
\( \mu_z \)  Normal velocity ratio
\( \vec{\omega} \)  Angular velocity vector \([\text{rad s}^{-1}]\)
\( \Omega \)  Rotor shaft speed \([\text{rad s}^{-1}]\)
\( \phi \)  Inflow angle \([\text{rad}]\)
\( \psi \)  Blade azimuth angle \([\text{rad}]\)
\( \Psi \)  Span-axis rotation angle \([\text{rad}]\)
\( \rho \)  Air density \([\text{kg m}^{-3}]\)
\( \sigma \)  Rotor solidity
\( \tau \)  Rotor tilt angle \([\text{rad}]\)
\( \theta \)  Pitch angle \([\text{rad}]\)
\( \theta_0 \)  Collective pitch \([\text{rad}]\)
\( \theta_c \)  Lateral cyclic pitch \([\text{rad}]\)
\( \theta_s \)  Longitudinal cyclic pitch \([\text{rad}]\)
\( \theta_{tw} \)  Built-in blade twist \([\text{rad}]\)
\( \upsilon \)  Induced velocity \([\text{m s}^{-1}]\)
\( \nu_{RoE} \)  Correction factor to the induced velocity for the rotors-on-empennage interaction
\( \nu_{RoW} \)  Correction factor to the induced velocity for the rotors-on-wing interaction
\( [\xi] \)  Wake skew matrix
\( \zeta \)  Downstream distance from hub (dimensionless)

**Roman Symbols**

\( a \)  Lift-curve slope \([\text{rad}^{-1}]\)
\( A \)  Aspect ratio
\( b \)  Wing semi-span \([\text{m}]\)
\( b, i \)  Number of blades; refers to \( i \)th blade
\( B \)  Tip loss factor
\( c \)  Chord length \([\text{m}]\)
\( c_0 \)  Speed of sound \([\text{m s}^{-1}]\)
\( C_d \)  Drag coefficient
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<td>Angular momentum vector</td>
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\( \ell \) Shaft length [m]
\( l \) Station line ordinate [m]
\( [L] \) Inflow gain matrix
\( \mathcal{L} \) Lift force [N]
\( m_R \) Mass of the rotors [kg]
\( [M] \) Apparent mass matrix
\( M \) Section pitching moment [Nm]
\( \ddot{M} \) Moment vector [Nm]
\( M \) Mach number
\( M \) Pitching moment [Nm]
\( M_\beta \) Flapwise moment [Nm]
\( M_c \) Longitudinal harmonic of disc moment [Nm]
\( M_s \) Lateral harmonic of disc moment [Nm]
\( n \) Rotor direction specifier
\( \bar{p} \) Position vector [m]
\( P \) Rotor power [W]
\( q \) Dynamic pressure [Pa]
\( q \) Velocity about y-axis [rads\(^{-1}\)]
\( q_{RoE} \) Correction factor to the dynamic pressure for the rotors-on-empennage interaction
\( Q \) Rotor torque [Nm]
\( r \) Dimensionless radial position
\( \mathcal{R} \) Contracted wake radius [m]
\( R \) Rotor radius [m]
\( R_0 \) Blade root radius [m]
\( S \) Reference area \( [m^2] \)
\( t \) Time [s]
\( t/c \) Thickness-to-chord ratio
\( [T] \) Transformation matrix
\( T \) Thrust force of the rotor [N]
\( \bar{u} \) Velocity vector [ms]
\( \bar{u}_0 \) Velocity of rotating axis set
\( u \) Velocity along x-axis [ms\(^{-1}\)]
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</tr>
<tr>
<td>$V_P$</td>
<td>Perpendicular velocity of a blade section [ms$^{-1}$]</td>
</tr>
<tr>
<td>$V_R$</td>
<td>Radial velocity of a blade section [ms$^{-1}$]</td>
</tr>
<tr>
<td>$V_T$</td>
<td>Tangential velocity of a blade section [ms$^{-1}$]</td>
</tr>
<tr>
<td>$V_x$</td>
<td>Velocity along section $x$-axis [ms$^{-1}$]</td>
</tr>
<tr>
<td>$V_y$</td>
<td>Velocity along section $y$-axis [ms$^{-1}$]</td>
</tr>
<tr>
<td>$V_z$</td>
<td>Velocity along section $z$-axis [ms$^{-1}$]</td>
</tr>
<tr>
<td>$w$</td>
<td>Velocity along $z$-axis [ms$^{-1}$]</td>
</tr>
<tr>
<td>$\vec{H}$</td>
<td>Position vector of a rotor hub [m]</td>
</tr>
<tr>
<td>$\vec{W}$</td>
<td>Position vector of a wing section [m]</td>
</tr>
<tr>
<td>$\vec{x}$</td>
<td>State vector</td>
</tr>
<tr>
<td>$x$</td>
<td>Position along $x$-axis [m]</td>
</tr>
<tr>
<td>$x$</td>
<td>Radial position along a blade [m]</td>
</tr>
<tr>
<td>$X$</td>
<td>Body force along the $x$-axis [N]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Spanwise position along a fixed-wing [m]</td>
</tr>
<tr>
<td>$Y$</td>
<td>Side force of the rotor [N]</td>
</tr>
<tr>
<td>$z$</td>
<td>Position along $z$-axis [m]</td>
</tr>
<tr>
<td>$Z$</td>
<td>Body force along the $z$-axis [N]</td>
</tr>
</tbody>
</table>

**Superscripts**

- $'$ Azimuth derivative
- $\bar{}$ Averaged over a revolution (steady value)
- $\dot{}$ Time derivative

**Subscripts**

- $\alpha$ Denotes aerodynamic contribution
- $A$ Denotes the aircraft axes
- $\beta$ Denotes the gimbal states
- $B$ Denotes the body axes
- $BL$ Denotes the blade axes
- $C$ Denotes the chord axes
- $D$ Denotes the disc axes
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$E$</td>
<td>Denotes the empennage axes</td>
</tr>
<tr>
<td>$E$</td>
<td>Denotes the empennage</td>
</tr>
<tr>
<td>$CG$</td>
<td>Denotes the centre of gravity</td>
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<tr>
<td>$F$</td>
<td>Denotes the fuselage</td>
</tr>
<tr>
<td>$H$</td>
<td>Denotes the hub axes</td>
</tr>
<tr>
<td>$\infty$</td>
<td>Denotes freestream</td>
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<tr>
<td>$i$</td>
<td>Denotes interaction component</td>
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<tr>
<td>$\lambda$</td>
<td>Denotes the inflow states</td>
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<tr>
<td>$m$</td>
<td>Denotes inertial contribution</td>
</tr>
<tr>
<td>$N$</td>
<td>Denotes the nacelles</td>
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<tr>
<td>$P$</td>
<td>Denotes the pivot</td>
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<tr>
<td>$R$</td>
<td>Denotes the rotor</td>
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<tr>
<td>$S$</td>
<td>Denotes shaft axes</td>
</tr>
<tr>
<td>$SP$</td>
<td>Denotes the spinner</td>
</tr>
<tr>
<td>$W$</td>
<td>Denotes the wing axes</td>
</tr>
<tr>
<td>$W$</td>
<td>Denotes the wing</td>
</tr>
</tbody>
</table>
Abstract

Tiltrotor aircraft offer the hover and low-speed capabilities of helicopters coupled with the high-speed performance of fixed-wing aircraft. A unique operating regime of this configuration is the transition from rotor-borne to wing-borne flight. This flight envelope, expressed through the rotor tilt degree of freedom, is termed the conversion corridor. This thesis investigates the impact of the rotors/airframe interactions on the predicted conversion corridor, trim behaviour and aircraft performance. A generic aeromechanics model is developed that consists of a flight mechanics module and aerodynamic modules for each component. Three methods are rationalised to predict the conversion corridor with a simple trim sweep adopted due to the parallelisation capability for large domain investigations. A robust Newton-Raphson scheme that implements a variable damping factor is also presented to find the unknown trim quantities. A new formulation for the equations of motion of a gimballed rotor are presented that are applicable throughout the conversion corridor.

The rotors/airframe interaction can be classified as rotors-on-wing, rotors-on-empennage and wing-on-empennage. The overall effect of the interactions on the predicted corridor boundaries was fairly small but generally acted favourably to widen the corridor. The wing-on-empennage interaction was found to have the most impact on the predicted boundaries, decreasing the minimum-speed boundary towards aeroplane mode. The aerodynamic interactions were most pronounced on the trim characteristics, particularly the pitch attitude and stick position. The combined downwash and upwash at the empennage resulting from the rotors and wing was most important in helicopter mode to correlate the pitch and stick trim against literature data. As the rotors were tilted forwards, the increased pitch attitude at low-speeds increased the wing downwash and the wing-on-empennage interaction became dominant. The wing downwash at the tailplane had little effect on the pitch trim past a forward rotor tilt of 30° but significantly affected the stick trim. The rotors-on-wing interaction had a small effect on the trim behaviour for the configuration considered but did significantly increase the thrust and power required at low speeds due to the imposed download force on the airframe.
Declaration

No portion of the work referred to in this thesis has been submitted in support of an application for another degree of qualification of this or any other university or institute of learning.
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Publications

Papers in Conference Proceedings


Journal Papers in Preparation

Chapter 1

Introduction

This chapter introduces the tiltrotor configuration and the driving requirements of its inception. The unique flight regime associated with the tiltrotor configuration, the conversion corridor, is introduced and the bounding features of the corridor are explained. The latter part of this chapter introduces the focus of the thesis: the aerodynamic interaction between different components of the aircraft and its implication on trim and aircraft performance. The complex aerodynamic interactions are categorised and a qualitative explanation of their effects is given. The final section introduces the aims and objectives of the thesis.

1.1 Background

Rotary-wing and fixed-wing aircraft are used to perform a broad and diverse range of missions every day. The requirements of these missions determine both the design and configuration of the aircraft used to fulfil them. Rotary-wing aircraft have several unique capabilities that allow them to perform missions and manoeuvres that most fixed-wing aircraft are incapable of: vertical take-off and landing; hovering at altitude; omnidirectional flight; operating at low speed. These characteristics make rotorcraft invaluable for missions such as search and rescue, medical evacuations, offshore transport, troop deployment and many more. On the other hand, the speed, range and altitudes obtainable by fixed-wing aircraft are far superior to rotorcraft with both commercial and military sectors exploiting these attributes.

The technology of rotary-wing and fixed-wing aircraft has matured into aerodynamically
efficient and operationally safe designs, enabling aircraft to effectively perform their respective missions. However, mission requirements are continually changing that require newer, more advanced designs to fulfil them. Of particular interest is rotorcraft that can perform well in both hover and low-speed flight but also reach speeds greater than conventional helicopters. The requirement for high speed from conventional, single main rotor helicopters is difficult for several reasons [1]: the rotor thrust must simultaneously provide the lift, propulsive and control forces; the Mach numbers experienced by the blades towards the tip region on the advancing side may be high enough to induce shocks waves that incur severe power penalties; the reverse flow region on the retreating side of the rotor becomes larger with increasing forward speed which effectively decreasing the available lifting area of the disc; the blades on the retreating side begin to stall as a result of the blade dynamics and operating point; the control inputs required to augment the natural dynamic behaviour of the rotor may be excessive and beyond the control system capability; the aircraft trim state may become undesirable from a pilot perspective (e.g. excessive nose-down pitch attitude). For these reasons, advanced designs are required that deviate from the conventional main-rotor/tail-rotor designs. One example of high-speed rotorcraft is the compound helicopter, shown in Figure 1.1a, that offloads the main rotor in forward flight by deriving lift and propulsive thrust from a wing and propellers [2]. However, to realise the full potential of both rotary-wing and fixed-wing aircraft, a single aircraft capable of adapting to the required mission profile is desirable. This type of configuration was realised with the tiltrotor aircraft, shown in Figure 1.1b.

(a) Compound helicopter configuration (Airbus Racer) used to offload the main rotor in high-speed flight. Image from Airbus.

(b) Tiltrotor configuration operating (Bell-Boeing MV-22B Osprey) in aeroplane mode. Image from [3].

**Figure 1.1:** Examples of advanced rotorcraft configurations that can operate at higher forward speeds compared to conventional helicopters.

The realisation for an aircraft with the favourable traits of both rotary-wing and fixed-
wing aircraft was operationally realised around the 1950s [4]. Mission requirements from the U.S. Army and Air Force Convertiplane Program stipulated an aircraft with significant hover duration, low-speed manoeuvrability and both speed and range comparable to fixed-wing aircraft. This led to the development of the proof-of-concept Bell XV-3, pictured in Figure 1.2a, to be developed. This aircraft successfully demonstrated the potential of the tiltrotor configuration to deliver the hybrid mission requirements of rotary-wing and fixed-wing aircraft. However, the early development of the aircraft was plagued by aeroelastic instability due to the wing-tip mounted rotors which, at the time, was not well predicted from analytical models. Nevertheless, the XV-15 tiltrotor research aircraft, pictured in Figure 1.2b, was developed as the successor to the XV-3 to explore and develop the knowledge and technologies required to utilise this configuration [5, 6]. It was recognised early on in the development program that good modelling and simulation of such a complex aircraft was required to contribute to control concepts, cockpit configurations, pilot workload, handling qualities and flight performance [7]. The technology developed in the XV-15 program culminated in the Bell-Boeing V-22 Osprey, pictured in Figure 1.1b, that first flew in 1989. Flight testing of such a new and advanced aircraft in all flight modes was extensive and complemented by the continued development of modelling and simulation tools [8]. This highlights the necessity of these tools from the design stage right through to flight testing. To date, the V-22 Osprey is the only in-service tiltrotor worldwide and flight testing of its potential successor, the Bell V-280 Valor, is underway. Certification is also pending for the Leonardo AW609 civil tiltrotor, demonstrating the unique and diverse utility of these next-generation aircraft to the military and civil markets.

(a) Bell XV-3 operating in aeroplane mode. Image from [9].
(b) Bell XV-15 operating in helicopter mode. Image from [10].

**Figure 1.2:** Examples of early tiltrotor aircraft used for proof-of-concept and technology development.
Tiltrotor aircraft operate as lateral-tandem rotorcraft with counter-rotating rotors mounted at the wing-tips. The counter-rotation of the rotors cancels the torque reaction from each rotor and eliminates the need for a tail rotor. The rotors are actuated to tilt through at least 90° about a lateral conversion axis; a small rearward tilt may also be permitted to improve rearward flight capability. The controllable tilt of the rotors allows hover and low-speed flight with the rotor shaft axes orientated towards vertical and high-speed flight towards horizontal. The capability of tilting the rotors while maintaining controllable flight expands the traditional flight envelopes of rotary-wing and fixed-wing aircraft to form a single, wider envelope. The operation of tiltrotor aircraft can be classified into three modes: helicopter, conversion and aeroplane. No universal definition of these flight modes exists and distinctions could be made depending on several factors: tilt angle alone; viability of hovering flight; share of lift between the rotors and wing; or trim behaviour exhibited. In this work, the first definition is used based on the tilt angle alone. The tilt angle, measured from rotors vertical, is denoted by \( \tau \). Helicopter mode is defined to be rotors vertical, \( \tau = 0° \), aeroplane mode rotors horizontal, \( \tau = 90° \), and conversion mode in between, \( 0° < \tau < 90° \). This definition was adopted as it is independent of specific design features.

A unique regime of tiltrotor aircraft is their conversion corridor. This can be interpreted as the aircraft flight envelope presented through the unique dimension of the rotor tilt angle. The conversion corridor bounds the permissible airspeeds as a function of the rotor tilt. A typical conversion corridor for the tiltrotor configuration is shown in Figure 1.3. The figure exemplifies the minimum and maximum boundaries as a function of airspeed through transitioning flight. In this work, the corresponding boundaries will be referred to as the min-speed and max-speed boundaries. The boundaries are determined from a set of performance constraints such as wing stall, control input limits, flapping limits, structural limits and installed power. Additionally, the boundaries could also be determined from a handling qualities perspective. The conversion corridor is generally broadest in helicopter mode with the airspeed ranging from hover through to the power-limited or control-limited max-speed boundary, whichever is first. As the rotors are tilted forwards, the corridor begins to narrow as the rotors cannot singularly provide the required lift and propulsive forces. Figure 1.3 also illustrates the adverse effect of increasing the aircraft weight, tending to narrow the permissible flight envelope by increasing the wing stall speed and required power.
Figure 1.3: Illustration of a typical conversion corridor highlighting the operating envelope as a function of the airspeed and rotor tilt. In this figure, the conversion angle is 90° for rotors vertical and 0° for rotors horizontal. Image from [6].

1.2 Overview of Tiltrotor Aerodynamic Interaction

Integrating a rotary-wing and fixed-wing aircraft into a single flight vehicle brings several design challenges: the mechanical transfer of shaft power to the rotors at any commanded tilt angle; the design of the rotor blades to operate efficiently throughout the operational domain; an aeroelastically stable wing-nacelle system that does not encounter whirl-flutter\(^1\); understanding the interactional aerodynamics between the rotors and airframe. The focus of this work is the latter, the influence of the interactional aerodynamics on flight behaviour and performance. The aerodynamic interactions for the case of tiltrotor aircraft can be categorised into three main interactions between the rotors, wing and empennage: rotors-on-wing, rotors-on-empennage and wing-on-empennage. This work investigates the individual and combined effects of these three interactions and the relative importance of including such interactions on the predictions of flight behaviour and performance. The flight regime studied is the conversion corridor and the influence on the predicted min-speed and max-speed boundaries is also investigated.

The rotors-on-wing interaction occurs as a result of the rotor wake interaction with the

\(^1\)Whirl-flutter is an aeroelastic phenomenon in which the elastic modes of the wing are excited by the rotor and can become unstable at high speed [11].
wing. From momentum theory of an actuator disc in hover, the induced velocity produced in the development of the rotor thrust is proportional to the square-root of the disc loading (thrust divided by the disc area). The power required to produce the slipstream is proportional to the induced velocity and, therefore, is smallest when the rotor thrust is produced over a large disc area. As a result, the radius of the rotors is mostly that of the wing semi-span and, therefore, a significant area of the wing is projected in the rotor wake; for the XV-15 rotor, approximately 78% of the wing area is projected under the rotor disc. The impingement of the rotor wakes on the wing creates a phenomenon known as download or vertical drag. The effect of the download is undesirable and increases the required thrust and power. Furthermore, having to overcome the download reduces the useful payload that can be lifted vertically [12]. When operating in helicopter mode, the download effect is exemplified by the plane of the wing being perpendicular to the rotor downwashes. This orientation results in significant drag forces arising from the flow separation around the wing. As the forward speed increases, the rotor wakes are convected with the freestream velocity rearwards off the wing and the download force is less significant. In this case, the rotor wake interaction is shifted to the empennage, detailed next.

The interaction from the rotors at the empennage occurs in forward flight when the wakes convect downstream. The lateral offset of the rotors from the fuselage centreline creates a different interaction from typical single main rotor helicopters. For the latter case, the tailplane usually operates in the downwash of the main rotor augmenting the tailplane lift depending on the local dynamic pressure and angle of attack [13, 14]. However, for the lateral offset in the tiltrotor case, the tailplane was found to experience an upwash. This upwash was induced by the vorticity in the rotor wake and was dependent on the thrust, airspeed and rotor incidence [15]. In addition to the rotor wash at the empennage, there is also downwash from the presence of the lifting wing. Due to the size of the main wing and the relative size and position of the empennage, this interaction should be accounted for in the aerodynamic model of the empennage.

### 1.3 Aim, Scope and Objectives

The primary aim of this thesis is to investigate the effects of interactional aerodynamics on the predicted trim and performance through the conversion corridor. The outcomes of this work will give insight into both the importance of modelling aerodynamic interactions and their impact, both singularly and collectively, on the predicted flight behaviour of tiltrotor
aircraft. The application of this work is directly applicable to the design and prediction of future tiltrotor aircraft as well as similar configurations such as tiltwings (where an outboard portion of the wing remains fixed to the rotors as they tilt). Furthermore, the outcomes may apply to the rapidly-expanding eVTOL sector (electric vertical take-off and landing) in which unconventional configurations are arising that could be susceptible to unforeseen behaviour during transitional flight. The primary aim of this thesis is undertaken through the development of an aeromechanics model that applies to generic tiltrotor aircraft. The term ‘generic’ used here is restricted to tiltrotors of conventional airframe designs with a pair of wing-tip mounted rotors. The aeromechanics model, in this context, refers to a flight mechanics model that is coupled with aerodynamic models for the rotor and airframe components. The flight mechanics model describes the aircraft kinematics and the aerodynamic models are used to calculate the forces and moment generated by each component. The interaction models between different aircraft components are detailed in their respective aerodynamic modelling chapters. Due to the large operating domain considered here, the aeromechanics model will focus on reduced-order models that can be readily reconfigured for preliminary design predictions.

The numerical model is written in MATLAB using an object-orientated approach. This easily facilitates the creation of generic designs through predefined component classes and streamlines the required input parameters and format. The numerical model consists of several modules: the aircraft model is first loaded followed by the list of operating points; this data is then passed to the flight mechanics module that attempts to find a trim solution; aerodynamic modules for each component are called from the flight mechanics module to calculate the aerodynamic loads on the airframe. The aircraft parameters and operating points are read from text file inputs that contain all the necessary data. The required inputs for a single component are primarily the configuration, geometry and aerodynamic data; control and interaction data can also be specified. The input data required for each component will be detailed in the relevant chapters.

The scope of this work is limited to a quasi-steady analysis of the aerodynamic interactions through the conversion corridor. The conversion corridor is defined by a series of trimmed operating points which removes the need to account for dynamic effects on the aerodynamic interactions. Additionally, the stability of the trim point is also not considered here. Instead, the focus is placed on the impact of the aerodynamic interactions on the trim quantities and aircraft performance as a whole. Only straight and wings-level flight conditions are simulated in this work, so-called longitudinal motion, due to the restrictive volume of available validation data. This also simplifies the modelling of the aircraft as it can be considered symmetric. As a
result, the aeromechanics model is only valid for flight behaviour at steady wings-level flight.

The objectives of this thesis are summarised as:

1. Construct a generic model for an aeromechanics analysis of tiltrotor aircraft.
2. Implement theoretical/empirical interaction models into the relevant component aerodynamic models to predict the effects of the rotors-on-wing, rotors-on-empennage and wing-on-empennage interactions on both the predicted conversion corridor and its associated trim behaviour and performance.
3. Determine a suitable and robust numerical method to solve the aircraft equations of motions for all operating conditions.
4. Rationalise and critically examine several approaches that could be used to determine the conversion corridor and decide on the most practical method.
5. Verify the predicted trim behaviour and performance of the aeromechanics model through simulated conversion corridors. In the first instance, the aerodynamic interaction is neglected to construct a baseline case.
6. Present an aeromechanics analysis of tiltrotor behaviour through the conversion corridor. Discuss the changes to the trim quantities, performance and conversion boundaries for each of the interaction cases and conclude the importance of each interaction to determine the conversion boundaries and trim behaviour.
Chapter 2

Literature Review

This chapter presents a review of literature pertaining to aeromechanics modelling of tiltrotor aircraft. The first part of this chapter gives a review of the research on interactional aerodynamics dominant for the tiltrotor configuration. The second part of this chapter reviews the aeromechanics models presented in literature and the effects of the interactional aerodynamics. Following this a review of rotor modelling for tiltrotor-type rotors due to their unconventional design compared to traditional helicopter rotors. Finally, the gaps in the literature are identified and the contributions of this thesis are detailed.

2.1 Aerodynamic Interaction Modelling

The aerodynamic interaction between the rotors and airframe can cause significant changes to local flow environments and has the potential to degrade the overall aircraft performance. The accurate prediction of the vehicle performance requires the incorporation of the rotor and airframe interactions even from a design stage [16]. For tiltrotor configurations, these interactions can be classified as rotors-on-wing, rotors-on-empennage and wing-on-empennage. Due to the lateral offset of the rotors from the fuselage centreline, the rotors-on-empennage interaction mechanism is different from that of conventional helicopters. Furthermore, a large portion of the wing is projected under the rotor disc in helicopter mode and the rotors-on-wing interaction is more significant than for conventional main rotors mounted above the fuselage. The interaction of the main wing with the empennage is well documented in fixed-wing literature, e.g. [17], and no research has specifically focussed on the wing-on-
empennage interaction for the tiltrotor configuration. Therefore, a review of this interaction will not be presented and could be investigated as future work of this project.

2.1.1 Rotors-on-Wing Interaction

The immersion of a portion of the airframe, such as the wing or fuselage, in the downwash of a hovering rotor exerts a vertical drag on the airframe termed a download. This effect is detrimental to the overall vehicle performance as this download must be overcome at the expense of additional power. The flowfield associated with the rotors-on-wing interaction is shown in Figure 2.1. Makofski and Menkick [18] experimentally investigated the download of a flat panel under the downwash of a hovering rotor. The estimated download from their empirical relation was relatively small but this could be attributed to the experimental rotor being unrepresentative of current tiltrotor designs. Several experimental studies have been conducted to estimate the rotors-on-wing download after the successful flight demonstration of the Bell XV-3 and development of the XV-15. A 1/5th scale model of the Bell Model 301 Tiltrotor Research Aircraft (the precursor of the XV-15 design) was tested in low-speed helicopter and conversion mode in [15]. The download in hover was found to be approximately 11.4% of the thrust, which was significantly greater than the predicted download of 7%. The experimental download was assumed to be too large due to the low Reynolds number of the test.
Figure 2.1: Flowfield associated with the download phenomenon for a tiltrotor aircraft in hover. The spanwise flow over the wing and formation of fountain flow at the fuselage centreline are illustrated. Image from [19].

Marr, Ford, and Ferguson [20] conducted tests on a 1/10th scale tiltrotor model and corroborated the previously observed download value. Additionally, the study also concluded that the interaction of the rotor wakes with the wing was small above a forward flight speed of 40 kn. Furthermore, flow visualisation at forward speeds as low as 20 kn showed the rotor wakes induced a strong upwash at the leading-edge of the wing nearer the fuselage. By a forward speed of 30 kn the rotor wakes were off the wing and alleviated any download. The convection of the rotor wakes past the wing at low speed was attributed to the nonuniform induced velocity of the wakes. Various flap settings were also tested to minimise the download on the wing but no significant reduction was found beyond a 50° deflection. Maisel, Laub, and McCroskey [21] went on to investigate aerofoil configurations that had the potential to alleviate the significant download force. They concluded that the download could be significantly reduced using a leading-edge umbrella flap with a traditional trailing-edge flap, as shown in Figure 2.2. However, the structural consequences of the umbrella flaps made them detrimental to the total weight; added structural weight was required to reinforce against whirl-flutter instability and lost fuel storage [12].
Accurately predicting the download is difficult due to the complexity of the three-dimensional, unsteady and separated flow [22]. A semi-empirical model to estimate the download was presented in [23], however, relied heavily on experimental flow visualisation and rotor downwash data. To estimate the download, the wing was discretised into both chordwise and spanwise panels. The download of each panel was found as a function of the panel area, dynamic pressure in the chordwise direction and a representative drag coefficient. The total download was found by summing the individual panel downloads. The dynamic pressure of the wing panels was based on experimental data. Flow within a specified radius from the shaft was assumed to be turned chordwise, whereas outside of this radius the flow followed along rays from the wing leading-edge. The predicted download showed good correlation to the experimental data for the V-22 but when applied to the XV-15 was found to consistently under-predict the download for all flap settings. This evidenced the high reliance on empirical data which, without an extensive experimental database, makes implementing such a method potentially unreliable. However, the computational cost was reasonable and could be easily be incorporated into design codes with a good empirical database to provide a first-order estimate of the download.

A simple analytical method to estimate the download was presented by McVeigh [19]. The model was based on a streamtube and used continuity of a streamline from the rotor disc to the wing to express the slipstream velocity at the wing as a function of the velocity at the rotor. The analysis found the download was not a function of the tip speed but instead the slipstream contraction and the spanwise induced velocity distribution. The induced velocity was expressed as a power series that accounted for the effects of thrust and blade twist. The coefficients of the power series and slipstream contraction were derived from experimental data and, when combined with a representative drag coefficient, gave good correlation to the measured download. The accuracy of the method is, however, reliant on the representative
drag coefficient that is calibrated to give the experimental download. The analytical method was only valid for hover and is, therefore, not readily applicable to forward flight.

Jordon, Patterson, and Sandlin [24] numerically investigated the rotors-on-wing interaction using a vortex ring representation of the rotor wake. The contraction of the slipstream under the disc was prescribed based on empirical data in [15]. The predicted download was found to correlate fairly well with experimental data but tended to under-predict its magnitude. However, the predicted velocity distribution along the wing was realistic with a download on the immersed wing sections and an upload on the inboard wing sections due to the vorticity in the wake. The complexity of this method makes it less suitable for aeromechanics investigations. Furthermore, the theory was derived for a hovering rotor and its validity into a forward flight was not considered.

As part of the Rotorcraft Handling, Interactions and Load Prediction (RHILP) project, Desopper et al. [25] presented an empirical model to estimate the download in hover and its evolution with forward flight. The download in hover was based on two parametric curves: the download as a function of thrust for zero flap setting; and the flap dependent download, normalised by the zero flap setting download. The download at a given operating point (thrust coefficient and flap setting) is then easily calculated from interpolation and can be readily implemented into an aeromechanics model. Two models were proposed for the download evolution in forward flight with both models dependent on the download in hover. The models showed a reasonable correlation to experimental data and were empirically adjusted to improve the results. Critically, the download models were based on total measurements and not on the local flow or geometry considerations like those in [19, 23].

CFD simulations of the complex download phenomenon, as shown in Figure 2.3, have been undertaken [26, 27]. However, due to their high computational cost have not been considered appropriate for the large domain investigation here. In reduced-order aeromechanics models, the induced velocity is typically modelled from momentum theory or uses a finite-state model owing to the low computational cost of these models. Therefore, the shape of the wakes as they convect downstream is not known and neither is the flowfield at the wing surface. In the real-time aeromechanics models of [28–30] the wing aerodynamics are correlated to the mean induced velocity at the rotors and employ empirical corrections to give the downwash velocity at the wing. The advantage of these models is that they are simple to implement and require very low computational cost, however, they rely heavily on being calibrated to experimental data. Given the reduced-order nature of this work, a similar model was appropriate here and one is derived that accommodates both hover and forward flight for all rotor tilt angles.
2.1.2 Rotors-on-Empennage

The rotors-on-empennage interaction for tiltrotor aircraft presents an interesting problem due to the lateral offset of the rotors from the fuselage. Early observations of the XV-3 aircraft showed a reversal of the pilot stick position with respect to forward speed around low-speed helicopter mode [15]. Experimental studies showed that in low-speed helicopter mode the rotor wakes quickly rolled-up downstream and the position of the wakes relative to the tailplane was found to induced an upwash velocity. Similar experimental studies in [20, 31] also corroborated this upwash effect. Modelling this interaction is challenging since it depends directly on the geometry of the rotor wakes which must be spatially resolved using, for example, a free wake method. In the early real-time aeromechanics model of the Bell 301 tiltrotor aircraft in [28], the rotors-on-empennage interaction was account for using a two-fold empirical model:

1. An upwash velocity at the empennage was calculated based on the mean induced velocity of the rotors. This was then superimposed with the freestream velocity and was used to calculate the angle of attack of the tailplane.

2. A nonlinear correction factor was applied to the empennage dynamic pressure to give the expected lift and pitching moment.

The upwash velocity was modelled as a nonlinear polynomial fit based on experimental data at trimmed operating points and, therefore, was not necessarily representative for untrimmed flight. In the later development of the Generic Tiltrotor Simulation (GTRS) model of the XV-15, a similar two-fold procedure was used for the rotors-on-empennage interaction based on the experimental data from [15, 31]. The nonlinear correction factors were contained in lookup tables as a function of airspeed, rotor tilt angle and fuselage angle of attack. Furthermore, the time delay of the rotor wakes to the empennage was accounted for and corrections for sideslip
Panel methods have been developed to analyse rotor-airframe interactions [32, 33]. These models used potential flow theory to determine the velocity and pressure fields around a geometry and permit the coupled flowfield effects between the rotor and airframe to be investigated. The analysis in [32] modelled the rotor using a steady-state actuator disc approximation and demonstrated the suitability of panel methods to the proposed problem. In [33], the VSAERO (Vortex Separation AEROdynamics) panel code was used to analyse the wake dynamics in transition from hover to forward flight. The wake geometry was calculated downstream and showed the wake roll-up and upwash at the tailplane. In the study, the tailplane was not modelled and, therefore, the interaction was not quantified. The suitability of panel methods to investigate the complex interaction between the rotors and airframe was, however, demonstrated. The disadvantage of panel methods is that they require a full geometrical description to accurately predict the rotor-airframe interactions and such geometries may not be known at initial design stages.

More recently, full CFD simulations of tiltrotor aircraft have been presented in literature [34–38]. These have focused on the validation of CFD solvers for the tiltrotor configuration and have been used to investigate the complex flowfields associated with different interactions throughout transitioning flight. Unfortunately, the results of these higher-order simulations have not been used to generate reduced-order models of the predicted loads. As a result, the rotors-on-empennage interaction model implemented in this work is derived from the empirical data used in the well-validated GTRS model.

2.2 Tiltrotor Aeromechanics Models

Modelling and simulation tools for tiltrotor aircraft were an integral part of the tiltrotor development programme [7]. The first simulation models of tiltrotor aircraft were presented by Bell [28] and Boeing [30] to complement both the design and training of pilots during the tiltrotor development phase in the 1970’s [4]. Both these aeromechanics models were real-time simulation models which, in order to meet this requirement, implemented reduced-order aerodynamic models. A suite of empirically derived lookup tables were used to calculate the aerodynamic forces and moments from each component throughout the operating domain. The rotors were modelled as actuator discs and used simplified models of the induced velocity and flapping dynamics. This simplification was required due to the implicit and time-
dependent relationship between the rotor loads, induced velocity field and rotor dynamics. Both aeromechanics models included a kinematic and dynamic description of the aircraft behaviour based on a point mass and rigid-body model [13, 39]. Due to the tilting degree of freedom of the rotors, the associated acceleration and velocity of the aircraft centre of gravity was also included in the dynamics model. The forces and moments acting on the airframe were determined by decomposing the aircraft into the major aerodynamic components: fuselage, rotors, wing and empennage. Aerodynamic models were then used to determine the loads produced by each component and summed to give the total airframe forces and moments.

The original aeromechanics model of the Bell 301 [28], the precursor to the design of the XV-15, has been used to develop several aeromechanics models [14, 40–43]. McVicar [41] improved the actuator disc model of [28] by modelling the rotor using an individual blade approach. Whilst this approach offers a better resolution of the blade loading, the aerodynamic data for the blade sections was restricted to a linear lift variation with angle of attack and a constant profile drag. Furthermore, the uniform induced velocity model was replaced by the three-state dynamic inflow model of [44]. The aeromechanics model included the rotors-on-wing, rotors-on-empennage and wing-on-empennage interactions collectively. The predicted trim behaviour of the aircraft agreed well with that presented in [28] and was used to investigate the flight dynamics of tiltrotor aircraft.

A flight dynamics investigation of a tiltrotor and tiltwing aircraft was also presented by Manimala and Bradley [42]. The tiltrotor model was based on that by McVicar for the XV-15 and, therefore, was restricted to linear aerodynamics. Due to a lack of available data for the tiltwing aircraft, the architecture was based on the XV-15. A trim map was constructed for the tiltrotor and tiltwing aircraft ranging from helicopter to aeroplane mode and from hover to a forward speed of 400 kn. No constraints were imposed on the trim state and, therefore, some of the presented trim solutions were unrealistic. Since no constraints were imposed, the trim map was not representative of the conversion corridor. Evidently, imposing realistic constraints are important to establish the conversion corridor, especially if the rotor implements linear aerodynamics that no not consider stall.

The most comprehensive aeromechanics model of tiltrotor aircraft is the Generic Tiltrotor Simulation model (GTRS) of the XV-15 presented by Ferguson [45]. This model was largely an extension of the Bell 301 model in [28] that implemented improved aerodynamic models to correlate with experimental and flight test data [29]. This model was again a real-time simulation model and implemented the same simplified representation of the rotor as [28]. Trim predictions of the aircraft as well as the trim controls and performance data were
presented in [29] and provide valuable comparison data for these type of aircraft. A significant improvement of the GTRS model was the interaction modelling between the rotors, wing and empennage; particularly the rotors-on-empennage interaction. In [28], the rotors-on-empennage interaction used a polynomial fit as a function of forward speed and tilt angle from experimental data at trim. In the GTRS model, this was extended as a function of airspeed, rotor tilt angle, fuselage angle of attack and sideslip, and also included a provision for the time-delay for the convection of the wakes from the rotors to the empennage. The rotor model was based on linearised aerodynamics with stall being imposed by conditional expressions. Transient changes in the rotor loads due to the inertia of the inflow and blade dynamics were accounted for using an equivalent ‘follow-on’ time.

Diaz, Mouterde, and Desopper [46] implemented an aeromechanics model to investigate the steady aircraft performance through the conversion corridor. The aeromechanics model was originally derived for single main rotor helicopters and was adapted for tiltrotor studies. Some aerodynamic interaction was considered: the wing download was modelled empirically from thrust and flap setting relations, and the wing downwash at the tailplane was modelled from lifting-line theory. The rotors-on-empennage interaction was not considered. A conversion corridor was presented for a generic tiltrotor aircraft but no validation of the code was shown. The effect of the wing flaps on the min-speed boundary was shown to have a considerable impact. Deflecting the flaps from 0° to 70° was found to decrease the min-speed boundary by approximately 40 kn in aeroplane mode. Based on this investigation, a flap schedule was produced to maximise performance, corridor width and passenger comfort.

A generic tiltrotor model, configured to the Bell-Boeing V-22 Osprey, was presented by Miller and Narkiewicz [47] to investigate performance, stability and control through arbitrary operating points. In their work, the trim equations were under-determined as the flap settings, elevator, cyclic and rotor tilt were all left as free parameters. Due to the under-determined system of trim equations, the trim solution was found using a Levenberg-Marquardt algorithm that allowed an arbitrary number of trim parameters. Performance results of several operating conditions were presented although no validation data for the V-22 Osprey was available for comparison. The under-determined system of trim equations and controls demonstrates the open control problem for tiltrotor aircraft. Additionally, only the rotors-on-wing interaction was considered and the omitted rotors-on-empennage and wing-on-empennage interactions may of influenced the trim behaviour and stability of the trim point.

Several aeromechanics models of tiltrotor aircraft have presented in literature both with and without validation data. The only validated aeromechanics model was provided by Ferguson
in the form of the GTRS model of the XV-15 [29, 45]. This model was validated against wind
tunnel and flight test data for the XV-15, however, the flight test data is not available in the
public domain. Due to a large volume of proprietary data for successive tiltrotor aircraft, the
GTRS model of the XV-15 remains the best source of publicly available data. Aeromechanics
models are important to investigate flight dynamics, aircraft stability and develop unique
control strategies for these type of aircraft. None of the aeromechanics models in literature have
investigated the singular and combined effects of the aerodynamic interactions on the predicted
conversion corridor or the associated trim behaviour and aircraft performance. This gap in the
literature is addressed in this thesis to give insight into the importance of the interactions and
their implication on aircraft performance and trim. Due to the large domain of the conversion
corridor, a reduced-order aeromechanics model is developed that is applicable to all operating
points. The GTRS model of the XV-15 remains the best source of publicly available data
with well-documented configuration parameters and aerodynamic data. Furthermore, trim and
performance data for the GTRS model is publicly available and, therefore, will be used as the
template aeromechanics model to investigate these aerodynamic interactions. The interaction
models for the rotors-on-empennage and wing-on-empennage interactions are derived from the
GTRS model. The rotors-on-wing interaction is modelled based on the cylindrical geometry of
the wake and is presented in Chapter 6.

2.3 Rotor Performance Modelling

The rotors are required to operate efficiently throughout the flight envelope; from hover
through to high-speed aeroplane mode - around 250 kn for the XV-15 [6]. The thrust required
by the rotors in different flight modes is drastically different and can represent almost an order
of magnitude difference. In helicopter mode, the rotors must support the aircraft weight
whereas in aeroplane mode, the rotors are required to provide only the propulsive forces.
The tilt degree of freedom gives a diverse operating environment and presents a difficult
optimisation problem to design an aerodynamically and structurally efficient blade for both
regimes, as well as intermediate flight between low-speed and high-speed [48, 49].

The performance modelling of a rotor system for tiltrotor aircraft is detached from the
performance models of typical helicopters and propellers. Compared to conventional helicopter
rotors, the freestream velocity component normal to the rotor plane experienced during high-
speed flight does not permit the usual assumption that the in-plane velocity of the blade is an
order of magnitude greater than the out-of-plane velocity. In a blade element analysis typically
employed in rotor performance modelling, this implies the inflow angle $\phi$ (the angle between the blade plane and resultant section velocity $V$ as shown in Figure 2.4) is not small and the small-angle approximation is not valid. On the other hand, high inflow angles are common in propeller analysis due to the large freestream inflow normal to the rotor disc. The rotor system is also required to provide control moments and, therefore, a dynamic analysis of the blade/rotor motion is also required in propeller mode since the rotor dynamics are not locked-out in aeroplane mode.

![Blade section angles](image)

**Figure 2.4:** Blade section angles: pitch angle $\theta$, angle of attack $\alpha$ and inflow angle $\phi$.

The hybrid rotor system for efficient helicopter and aeroplane mode flight results in highly twisted blades. This twist is largely necessitated for efficient propeller mode flight due to the large inflow angles arising from the high forward speed. The inflow angles at the blade root in aeroplane mode flight can be in the region of $70^\circ$. Blade element theory is commonly implemented to calculate the performance of a rotor system whereby the blade is discretised along the span into a finite number of sections. The aerodynamic analysis of a blade section was extended to accommodate high inflow angles in [50, 51] by assuming the section angle of attack $\alpha$ was small enough to be approximated by $\alpha \approx \sin \alpha$. The section angle of attack $\alpha = \theta - \phi$, as shown in Figure 2.4, was then expanded using the trigonometric identity for $\sin(\theta - \phi)$. The application of this method is best suited to when the lift and drag coefficients are analytic functions of the angle of attack (small angle of attack approximation). In both helicopter and aeroplane mode, the high degree of twist towards the root can cause this region to operate at stalled angles of attack [11, 52, 53] and, therefore, analytic expressions of the lift and drag are not representative.

The required twist of the tiltrotor blades also gives rise to unconventional loading in
helicopter mode forward flight. As noted by Johnson [54], the angle of attack of the advancing blade can give rise to negative lift and, therefore, negative blade loading. Due to this, the tip vortex becomes more difficult to model with some entrainment from the inboard vorticity. Johnson presented performance correlations using the comprehensive rotorcraft code CAMRAD (see [55] for an overview) against a 1/4 scaled V-22 rotor. A high-order free wake method was used to correlate good sectional aerodynamic loading against experimental data. He concluded that current wake models derived for helicopters are not representative of tiltrotor wakes and give inadequate performance predictions. Modelling differences included stall delay models, number of free-wake revolutions and identifying the location of the dominant tip vortices. An improved wake model was implemented into CAMRAD to try and better predict the vortex trajectories [56]. Mixed results were obtained for different blade loadings, highlighting the extremely complex wake pattern.

Gates [53] presented a CFD analysis of the XV-15 rotor in hover and propeller mode. In propeller mode, the spanwise thrust and torque distributions showed that a large portion of the blade was negatively loaded. This region was from the root towards the mid-span and, like the negatively loaded tip, implied the angles of attack produced negative lift and an accelerating torque. Comparisons between the predicted power and experimental power in [57] showed good agreement. The angle of attack distributions were not presented and, therefore, whether the blade sections were stalled could not be determined.

The earliest performance data for highly twisted blades was presented in [57]. The performance model employed reduced-order blade element momentum theory (BEMT). The induced velocity field was modelled as uniform in nonaxial flight and nonuniform in hover and axial flight. Overall, the predicted performance matched the experimental data well over the diverse operating conditions considered and demonstrated the viability of reduced-order models. The two-dimensional aerofoil data used to predict the performance was presented without reference angles of attack and, therefore, cannot readily be implemented into a blade element analysis. Furthermore, the lift and drag polars were not presented for negative lift which has been shown to be important in helicopter and propeller mode [53, 54].

Johnson [11] presented performance correlations of the XV-15 rotor using a reduced-order performance model. The induced velocity was modelled as uniform in axial flight and varied linearly over the disc in nonaxial flight. The results presented correlated reasonably well with experimental data for all flight modes considered: helicopter, conversion and aeroplane mode. However, the results relied upon empirical factors for improved correlations, particularly in hover and helicopter forward flight. In these flight modes, the induced power is significant
and under-predicted using a simplified induced velocity model. The diameter of the rotors are constrained by the wingspan and as a result, have a higher disc loading compared to helicopter rotors. Therefore, the induced power losses can be expected to be higher for this rotor system. The losses unaccounted for in simple BEMT are empirically corrected to match the experimental power by introducing the induced power factor $\kappa$. The induced power factors used by Johnson were $\kappa = 1.17$ in hover and $\kappa = 2.00$ in helicopter forward flight. However, the transition from hover to forward flight was not presented and it is difficult to generalise this transition. Johnson also concluded that the general performance correlations were not significantly improved using nonuniform induced velocity and that prescribed wake geometries developed for helicopter rotors are not adequate for tiltrotor wakes.

The inaccuracies of prescribed and semi-empirical wake models developed for helicopters to predict tiltrotor hover performance was corroborated by Bartie et al. [52]. Bartie et al. presented theoretical predictions of the XV-15 hover performance using baseline and ‘advanced technology’ blades against test data. The authors showed significant changes in the predicted performance when using different wake models. Deficiencies in the performance models were due not only to the inaccuracies of the wake model but also to aerofoil data and spanwise flow effects. The highly twisted blades experience high angles of attack at the root in hover and operate at, or beyond, the stall angles of attack. Provisions for high angle of attack aerodynamics are, therefore, required for accurate performance analysis. Furthermore, spanwise flow at the root, termed centrifugal pumping, accelerates the boundary layer and can significantly delay stall [58]. This can cause the inboard sections to produce more lift than predicted by two-dimensional blade element theory. The implementation of a BEMT model with stall delay corrections was shown to correlate well with experimental hover data in [59].

It is evident that accurately modelling the rotor wakes and predicting global, as well as local, performance requires higher-order models. This is due to the complex wake geometry arising from the highly twisted blades which were found to exhibit stall and unconventional spanwise loading. Higher-order models such as CFD and Vortex Particle Methods (VPM) solve the Navier-Stokes equations and can, therefore, more accurately predict the temporal and spatial aerodynamic loads. CFD studies have been undertaken in [37, 38, 53, 59] and have demonstrated good correlation with experimental data. These higher-order models are not practical for large domain investigations due to the computational time required and must be used selectively. The study here was focused on the conversion corridor regime of flight which consists of a large number of discrete airspeeds and rotor tilts. Therefore, reduced-order models were deemed more appropriate to afford less computation time. These models have
been shown to match experimental data sufficiently well for first-order accurate predictions of rotor performance [11, 57] and are robust tools commonly used in rotorcraft models. However, due to the reduced-order nature of the models, it is necessary to compare the predictions against experimental data.

2.3.1 Gimbaled Rotor Dynamics

A rotor is required to not only provide lift and propulsive forces, but also control moments. These are generated through the flapping degree of freedom that effectively tilts the rotor thrust vector to provide a control moment. A gimbaled rotor is the extension of a teetering rotor having three-or-more blades. These rotors do not have individual flap hinges and instead, the blades are connected to the rotorhead through a single joint [60, 61]. Since the blades are connected as a single structure, the dynamics of the gimbaled rotor depend on the combined aerodynamic and inertial moments from each blade.

In earlier works on gimbaled rotors, the equations of motion were obtained by considering the balance of inertial, aerodynamic and centrifugal moments on the rotor [62, 63]. The integrated loads from all the blades were resolved into the roll and pitch axes of the hub and summed over the blade count to give the gimbal tilt equations of motion. Johnson [61] concludes that the equations of motion for the gimbaled rotor are equivalent to the equations of motion of an articulated blade flapping about a hinge located on the rotor shaft. More recent work by Padfield [14] models the gimbaled rotor using a constant velocity joint on the rotorhead. This joint reorientates the angular momentum of the shaft parallel to the tip-path-plane axis and effectively tilts the plane of rotation. The blades are attached to the output shaft and, therefore, the centrifugal force does not act as a spring moment to restore the blades to the hub plane as found in classical blade dynamics. The gimbal equations of motion presented by Padfield were derived for helicopter mode flight with several simplifying assumptions. The rotor dynamics are present at all operating points through the corridor and, therefore, this work presents a derivation of the equations of motion valid for arbitrary operating points.

2.4 Literature Summary

This chapter has presented a review of literature relating to the aeromechanics modelling of tiltrotor aircraft through the conversion corridor. Previous investigations on the interactional
aerodynamics of tiltrotor aircraft have shown the significant effects on the performance and trim behaviour throughout the operational domain: rotors-on-empennage upwash resulted in a low-speed stick reversal and download forces in hover were as large as 11% of the rotor thrust. Several aeromechanics models have been developed for real-time simulations, performance investigations and flight dynamics studies. These models have inconsistently included the interactional aerodynamics of the rotors-on-wing, rotors-on-empennage and wing-on-empennage and have not been used to investigate the importance of these interactional aerodynamics on the predicted conversion corridor. This work will address this gap in the literature through the development of a reduced-order aeromechanics model to investigate the individual and combined effect of these interactions on the predicted conversion corridor, trim behaviour and aircraft performance.

A review of performance modelling for tiltrotor-type rotors was also presented that demonstrated the design compromises between efficient helicopter and propeller mode operation. Furthermore, the diverse loading exhibited by the rotor blades throughout transition was also highlighted that included positive and negative stall towards the blade root and negative tip loading. Established vortex models of helicopter wakes were found not to be directly transferable to tiltrotor wakes. Several higher-order models have been used to accurately predict the rotor performance throughout conversion but were not well suited here due to their expensive computational cost. Reduced-order models presented in literature were found to give reasonable first-order predictions of rotor performance and were more suitable to the large domain investigation undertaken here due to their lower computation time. A description of a gimballed rotor was also presented and a new formulation of the equations of motion for these kinds of rotors will be derived later in this thesis.

2.5 Thesis Structure

The structure and summary of the proceeding chapters and their relevant contributions are as follows:

• **Chapter 3:** This chapter describes the flight mechanics module of the aeromechanics model. The equations of motion are derived and the framework to compute the aerodynamic loads for generic aircraft is presented. The under-determined trim problem of tiltrotor aircraft is described and the closure of the trim problem detailed. This chapter contributes an adaptable trim routine that was found to be robust throughout the operating points considered in this
thesis.

- **Chapter 4:** This chapter describes how the aerodynamic forces and moments from a fuselage component are calculated.

- **Chapter 5:** This chapter describes how the aerodynamic forces and moments from a rotor component are calculated. The rotary-wing model is described and is applicable throughout the conversion corridor. The rotor dynamics and wake model are also described. This chapter makes a contribution to the dynamics of a gimballed rotor through a new derivation of their equations of motion for arbitrary operating conditions. The validation of the rotor model against experimental data for all flight regimes is also presented.

- **Chapter 6:** This chapter describes how the aerodynamic forces and moments from a wing component are calculated. This chapter contributes an interaction model for the rotors-on-wing interaction that is applicable throughout the conversion corridor.

- **Chapter 7:** This chapter describes how the aerodynamic forces and moments from an empennage component are calculated. This includes how the interaction of the rotor wakes and wing wake are accounted for in the aeromechanics model.

- **Chapter 8:** This chapter describes how the aerodynamic forces and moments from the rotor spinners and nacelle-wing interfaces are calculated.

- **Chapter 9:** This chapter presents a contribution to how the conversion corridor is approached. Three methods are rationalised and critically examined and the most practical method is selected. Additionally, a contribution to how a robust initial guess to the unknown trim controls is presented to aid the convergence of the trim solution.

- **Chapter 10:** This chapter presents the results of the simulated conversion corridors. The trim characteristics are investigated for each of the simulated interactions and comparisons to available data are made. This chapter contains a contribution to the relative importance of the interactions models on both the predicted conversion corridors, trim behaviour and aircraft performance.

- **Chapter 11:** This chapter draws conclusions from the results presented and gives suggestions for future work.
Chapter 3

Flights Mechanics Module

This chapter describes the flight mechanics module of the aeromechanics model. This chapter begins by detailing the generic framework used to establish the total loads on the airframe and details how the aircraft is discretised. Following this, the kinematic equations of motion relating the forces and moments on the airframe to the accelerations are derived. The resulting equations of longitudinal motion are established and the method used to solve these equations through the conversion corridor is presented. The under-determined trim problem that arises due to the amalgamation of the rotary-wing and fixed-wing control strategies is also covered and the closure of the system of equations is given. The final part of this chapter details the influence of the rotor tilt on the centre of gravity and aircraft inertia.

The general schematic of the flight mechanics module is illustrated in Figure 3.1. The required inputs to this module were the preloaded aircraft model and operating point. The role of the flight mechanics module was then to find a trim solution to the given operating point. The parameters that defined the operating point are detailed in Table 3.1. The centre of gravity location, moment of inertia in pitch, flap/flaperon setting and rotor speed are all specified in the operating point file since these parameters can be dependent on rotor tilt and airspeed. The range of operating points that can be specified in this manner is large, however, the analysis is limited to longitudinal motion only and all lateral/directional quantities are taken to be zero (i.e. sideslip and bank angle).
Figure 3.1: Overview of the flight mechanics module, illustrated by the red blocks, that is used to find the trim solution at the given operating point. The blue blocks indicate the aerodynamic modules.

Table 3.1: List of parameters contained in the operating point file.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{\infty}$</td>
<td>True airspeed of the centre of gravity (KTAS)</td>
<td>kn</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Rotor tilt angle measured from the body vertical</td>
<td>deg</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Flight path angle; positive for a climb</td>
<td>deg</td>
</tr>
<tr>
<td>$h$</td>
<td>Flight altitude above sea level used to determine the air density and speed of sound from an international standard atmosphere (ISA) model</td>
<td>m</td>
</tr>
<tr>
<td>$m$</td>
<td>Aircraft mass</td>
<td>kg</td>
</tr>
<tr>
<td>$I_{yy}$</td>
<td>Moment of inertia in pitch</td>
<td>kg m$^2$</td>
</tr>
<tr>
<td>$l_{CG}$</td>
<td>Station line ordinate of the centre of gravity; positive aft</td>
<td>m</td>
</tr>
<tr>
<td>$h_{CG}$</td>
<td>Water line ordinate of the centre of gravity; positive upwards</td>
<td>m</td>
</tr>
<tr>
<td>$\xi_f/\xi$</td>
<td>Integer value specify a predefined flap/flaperon setting</td>
<td>-</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Rotor speed</td>
<td>rpm</td>
</tr>
</tbody>
</table>

3.1 Aircraft Discretisation

The generic framework to compute the aerodynamic loads is to discretise the aircraft into the major aerodynamic components. For tiltrotor aircraft, these components are the fuselage, rotors, wing and empennage. This discretisation forms the basic framework for modelling
generic tiltrotor aircraft. The empennage is treated as a single component owing to the various configurations found on current tiltrotor aircraft: H-tail on the Bell XV-15 and Bell-Boeing V-22 Osprey; T-tail on the Leonardo AW609 civil tiltrotor; V-tail on the Bell V-280 Valor. The total generic force or moment, $X$, exerted on the airframe may then be written as

$$X = X_F + X_R + X_W + X_E$$

(3.1)

where the subscripts $F$, $R$, $W$ and $E$ denote the contributions from the fuselage, rotors, wing and empennage. The framework here is restricted to ‘conventional’ tiltrotor configurations with a single main wing and two counter-rotating, tip-mounted rotors. The similarities between the tiltrotor and tiltwing configurations allow the analysis to be directly extended to tiltwing configurations, however, some modification to the definition of the operating point would be required to account for the additional degree-of-freedom in the wing geometry.

To determine the moments exerted on the airframe, the spatial position of each component is required. It is convenient to write these positions relative to some fixed point, which is not necessarily the centre of gravity due to its dependence on rotor tilt, payload distribution and fuel burn. A Cartesian coordinate system, termed the aircraft frame and denoted by $O_A^{xyz}$, is introduced that defines the position of the aircraft components in this work. The $x$-axis points from nose to tail, the $y$-axis starboard and the $z$-axis upwards. The unit vectors of $O_A^{xyz}$ are denoted by $\hat{i}_A$, $\hat{j}_A$ and $\hat{k}_A$ and are shown in Figure 3.2. The position components along $\hat{i}_A$, $\hat{j}_A$ and $\hat{k}_A$ are termed the station line, buttock line and water line coordinates. The position of the aircraft frame origin $O_A$ can be arbitrary in space but is usually located at a convenient point, such as the aircraft nose.

![Figure 3.2: Aircraft coordinate system used to define the positions of the aircraft components. Image adapted from [64].](image-url)
The tiltrotor model was configured to the GTRS model of the XV-15 in [45]. This remains the only publicly available tiltrotor model that is validated against both experimental and flight test data. To be consistent with the GTRS model, the origin of the aircraft frame was placed 2.54 m (100 in) in front of the nose. The empennage for the XV-15 employs a H-tail configuration that consists of a tailplane and two tip-mounted vertical fins. The tilting degree of freedom of the rotors means their position is not fixed. Therefore, the position of the rotors is defined relative to the pivot locations on the conversion axis. The position of the rotor hubs can then be determined from the tilt angle and shaft length between the hub and pivot. The station, buttock and water line coordinates of the components used in the model are described in Table 3.2.

**Table 3.2**: Station, buttock and water line coordinates of the XV-15 components. Data from [29].

<table>
<thead>
<tr>
<th>Component</th>
<th>Station [m]</th>
<th>Buttock [m]</th>
<th>Water [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuselage¹</td>
<td>7.44</td>
<td>0.00</td>
<td>2.13</td>
</tr>
<tr>
<td>Wing²</td>
<td>7.95</td>
<td>0.00</td>
<td>2.37</td>
</tr>
<tr>
<td>Starboard pivot</td>
<td>7.62</td>
<td>4.90</td>
<td>2.54</td>
</tr>
<tr>
<td>Port pivot</td>
<td>7.62</td>
<td>-4.90</td>
<td>2.54</td>
</tr>
<tr>
<td>Tailplane³</td>
<td>14.22</td>
<td>0.00</td>
<td>2.62</td>
</tr>
<tr>
<td>Starboard fin⁴</td>
<td>14.22</td>
<td>1.96</td>
<td>2.62</td>
</tr>
<tr>
<td>Port fin⁴</td>
<td>14.22</td>
<td>-1.96</td>
<td>2.62</td>
</tr>
</tbody>
</table>

¹ Fuselage load location  
² Wing quarter-chord on fuselage centreline  
³ Tailplane quarter-chord on fuselage centreline  
⁴ Fin quarter-chord at tailplane tip quarter-chord

### 3.2 Equations of Motion

The equations of motion describe the temporal change in the kinematic states of the aircraft due to the applied forces and moments. The equations of motion are established in a body-fixed Cartesian coordinate system, denoted $O^B_{xyz}$, located at the instantaneous centre of gravity. The body-fixed frame of reference is shown in Figure 3.3. The location of the centre of gravity is strongly dependent on the tilt of the rotors due to their significant mass compared to the total
aircraft mass. In the dynamic case when the rotors are tilting, there is an associated velocity and acceleration of the centre of gravity. This greatly complicates the determination of the total moment since the masses of the components appear to be moving relative to the centre of gravity. To simplify this scenario, the rotors are only considered at fixed tilt angles, thereby neglecting the effects of the rotor tilting and the associated velocity and acceleration of the centre of the gravity. This permits the aircraft to be analysed at quasi-steady operating points but makes the equations of motion unrepresentative for dynamic analysis. Treating the aircraft as a rigid-body, the equations of motion can be derived from Newton’s second law of motion:

\[
\vec{F} = m \frac{d\vec{u}}{dt} \quad (3.2a) \\
\vec{M} = \frac{d\vec{h}}{dt} \quad (3.2b)
\]

where \( \vec{F} \) is the vector of external forces, \( m \) is the aircraft mass, \( \vec{u} \) is the velocity of the centre of gravity, \( t \) is time, \( \vec{M} \) is the vector of external moments and \( \vec{h} \) is the angular momentum about the centre of gravity. In Equation 3.2a, the mass has been taken as constant since the role of the flight mechanics module is to find the time-invariant trim solution at the specified operating point. In general, the aircraft is free to translate and rotate and, therefore, the body-fixed axes are a noninertial frame of reference. The equations of motion with respect to the body axes are obtained from Coriolis’ theorem that accounts for the rotation rate of the coordinate system:

\[
\vec{F} = m \left( \frac{\partial \vec{u}}{\partial t} + \vec{\omega} \times \vec{u} \right) \quad (3.3a) \\
\vec{M} = \frac{\partial \vec{h}}{\partial t} + \vec{\omega} \times \vec{h} \quad (3.3b)
\]

where the partial derivative indicates the time derivative taken in the rotating (body) frame and \( \vec{\omega} \) is the angular velocity vector of the body.
The inertia tensor \( [I] \) represents the distribution of mass from the rotational axes and transforms the angular velocity into the angular momentum vector:

\[
\vec{h} = [I] \vec{\omega}
\]  

(3.4)

Since the rotors have been considered at fixed rotor tilts, the angular momentum associated with the tilt rates of the rotors about the conversion axis can be neglected. An advantage of expressing the equations of motion in the body-fixed axes is that, when the rotor tilts are fixed, the inertia tensor is constant. Fixed-wing aircraft are usually symmetric about the plane \( O^Bxz \) and this property is used to simplify the inertia tensor by reducing the products of inertia \( I_{xy} \) and \( I_{xz} \) to zero. For rotorcraft, the assumption of symmetry about the plane \( O^Bxz \) is not generally true due to the asymmetry created by the tail rotor. For the case of tiltrotor aircraft, however, the opposing torques of the counter-rotating rotors make a tail rotor obsolete and, therefore, the aircraft can be assumed to be symmetric about the body plane \( O^Bxz \). The simplified inertia tensor is then given by

\[
[I] = \begin{bmatrix}
I_{xx} & 0 & -I_{xz} \\
0 & I_{yy} & 0 \\
-I_{xz} & 0 & I_{zz}
\end{bmatrix}
\]  

(3.5)

where \( I_{xx} \), \( I_{yy} \) and \( I_{xz} \) are the moments of inertia about the body \( x \), \( y \) and \( z \) axes and \( I_{xz} \) is the product of inertia.

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Taking the inertia tensor to constant in the body-axes, the translational and rotational accelerations of the airframe may then be calculated from:

\[
\frac{\partial \ddot{u}}{\partial t} = \frac{\vec{F}}{m} - \vec{\omega} \times \ddot{u} \\
\frac{\partial \ddot{\omega}}{\partial t} = [I]^{-1} \left( \vec{M} - \vec{\omega} \times [I]\vec{\omega} \right)
\]

These governing equations form the principal system of nonlinear equations that are solved for the trim solution throughout the operational domain. They are also used to assess several important aspects of flight such as the stability around the trim point. In this work, only the wings-level longitudinal behaviour was of interest in the construction of the conversion corridor. In this case, the freestream velocity is considered only in the plane \( O_{xz} \) and the angle of sideslip is zero. At the trim point, the translational and rotational accelerations are zero and there is no aircraft angular velocity (\( \vec{\omega} = \vec{0} \)). The equations of motion are then reduced to the translation along \( \hat{i}_B \) and \( \hat{k}_B \), and rotation about \( \hat{j}_B \). Expanding out Equation 3.6 gives the longitudinal equations of motion describing the time derivatives of the kinematic quantities \( u, \dot{w}, \) and \( \dot{q} \) as:

\[
\dot{u} = \frac{X}{m} \\
\dot{w} = \frac{Z}{m} \\
\dot{q} = \frac{M}{I_{yy}}
\]

where the dot denotes the time derivative.

It is common practice to decompose the body forces into the aerodynamic and gravitational contributions. The aircraft motion is described in the body-fixed axes and, therefore, the aircraft weight must also be expressed in these axes. The pitch of the fuselage \( \theta \) is measured from the horizon to the \( \hat{i}_B \) direction and defined positive in the nose-up sense. The pitch angle is used to express the aircraft weight \( mg \) along the body-fixed axes as

\[
-mg \sin \theta \hat{i}_B + mg \cos \theta \hat{k}_B
\]

where \( g \) is the acceleration due to gravity. The translational acceleration equations of motion
can then be written as

\[ \ddot{u} = \frac{X}{m} - g \sin \theta \]  
(3.9a)

\[ \ddot{w} = \frac{Z}{m} + g \cos \theta \]  
(3.9b)

where \( X \) and \( Z \) are now the \textit{aerodynamic} forces on the aircraft. The solution to the trim problem is then to find the aircraft orientation and control inputs required to hold the aircraft at the given operating point with zero acceleration \( \ddot{u} = \ddot{w} = \ddot{q} = 0 \).

### 3.3 Under-Determined Trim Problem

The solution of the general trim problem for both fixed-wing and rotary-wing aircraft involves the solution of the equations of motion. Several methods have been developed to solve these equations such as force-moment balance, harmonic balance and periodic shooting [65]. In all cases, the trim solution is defined when the calculated translational and rotational accelerations are sufficiently small to within some prescribed criteria. The solution of the trim problem is formulated as an inverse problem: given an operating point, find the controls inputs and airframe orientation required to hold steady flight.

The translational accelerations are driven by the forward and vertical forces, \( X \) and \( Z \), on the body-fixed equations of motion. These forces are controlled by the collective pitch of the rotors \( \theta_0 \) and the airframe pitch angle \( \theta \). For longitudinal trim, fixed-wing aircraft typically employ a tailplane with a moveable elevator that is deflected to augment the tailplane lift and provide a control moment. Rotorcraft employ a similar strategy whereby a cyclic pitch change is applied to the blades to effectively tilt the rotor thrust to provide a control moment. Tiltrotor aircraft possess both kinds of control strategies to provide sufficient control moments over a wide range of rotor tilts and airspeeds. In hover and at low speeds, the dynamic pressure over the tailplane is relatively small and control authority is dominated by the rotor cyclic. On the other hand, at higher airspeeds the elevator authority is dominant (assuming a sufficiently long moment arm). The existence of two control strategies to generate the trim moment in the equations of motion leads to an under-determined system of equations. This problem is not limited to just longitudinal control, as illustrated in Figure 3.4. The existence of an under-determined system of trim equations presents an interesting problem to engineer the scheduling of the rotary-wing and fixed-wing controls throughout the operational domain. This, however, was not the focus of this research.
In order to create a determined set of trim equations, the longitudinal controls are parametrically expressed in terms of a single variable, the longitudinal stick position \( \delta \). The limits of the longitudinal stick are normalised such that \( \delta = 1 \) represents the stick at the full-forward position and \( \delta = -1 \) represents the stick at the full-aft position. A neutral stick position is then \( \delta = 0 \). The trim solution, if it exists, is then uniquely dependent on the parametric functions relating the stick position to the longitudinal cyclic pitch input \( \theta_s \) and elevator input \( \eta \):

\[
\theta_s = f(\delta) \quad (3.10a)
\]

\[
\eta = g(\delta) \quad (3.10b)
\]

where \( f \) and \( g \) are arbitrary function. The determination of \( f \) and \( g \) should be chosen appropriately to offer good handling and control qualities and need not be functions of stick position only. In the flight simulation models of Harendra et al. [28] and Ferguson [45], the longitudinal cyclic pitch input is washed out with rotor tilt towards aeroplane mode whereas the elevator input is kept constant throughout the flight envelope. Both control inputs are linear functions of the stick displacement. The control strategies implemented here are made.

Figure 3.4: Control strategies in helicopter mode (left) and aeroplane mode (right). Image from [14].
consistent with the GTRS model in [45]:

\[ \theta_s(\delta, \tau) = A \delta \cos \tau \]  \hspace{0.5cm} (3.11a)

\[ \eta(\delta) = B \delta \]  \hspace{0.5cm} (3.11b)

where \( A = -10^\circ \) and \( B = 20^\circ \) are, respectively, the maximum cyclic pitch inputs and maximum elevator input. The control inputs are, therefore, symmetrical about the neutral stick position. In the GTRS model, there is also a static longitudinal cyclic pitch input that is independent of the stick position such that the cyclic control is given by

\[ \theta_s(\delta, \tau) = A \delta \cos \tau + A_1 (1 - \cos \tau) \]  \hspace{0.5cm} (3.12)

where \( A_1 = -1.5^\circ \). This applies a constant negative cyclic pitch acting to flap the rotor disc forward. The exact source of this term is not given by Ferguson but it is assumed this helps to reduce the rearward flapping of the rotor at positive fuselage angles of attack in aeroplane mode. At these conditions, there is a small in-plane component of the forward speed that tends to cause the rotor to flap backwards and since no cyclic control is applied through the swashplate in aeroplane mode, this static rigging term may be used to compensate against this effect.

### 3.4 Solving the Equations of Motion

The trim solution requires finding the orientation and control inputs required to hold a given operating point. The problem is written as

\[ \bar{f}(\bar{x}) = \bar{0} \]  \hspace{0.5cm} (3.13)

where \( \bar{f} \) is the vector function containing the translational and rotational accelerations of the airframe and \( \bar{x} \) is the vector of unknown trim quantities to be solved for. The unknown trim quantities are the collective pitch, aircraft pitch attitude and longitudinal stick position:

\[ \bar{x} = \{\theta_0, \theta, \delta\}^T \]  \hspace{0.5cm} (3.14)
For zero climb, the pitch attitude is equivalent to the fuselage angle of attack $\alpha$ and is used to give the translational velocity components of the body axes:

\[ u = V_\infty \cos \alpha \quad (3.15a) \]
\[ w = V_\infty \sin \alpha \quad (3.15b) \]

The airframe acceleration functions form the following system of equations:

\[ f_1(\theta_0, \theta, \delta) = \frac{X(\theta_0, \theta, \delta)}{m} - g \sin \theta \quad (3.16a) \]
\[ f_2(\theta_0, \theta, \delta) = \frac{Z(\theta_0, \theta, \delta)}{m} + g \cos \theta \quad (3.16b) \]
\[ f_3(\theta_0, \theta, \delta) = \frac{M(\theta_0, \theta, \delta)}{l_{yy}} \quad (3.16c) \]

Several methods exist to solve systems of nonlinear equations. The in-built solver originally used in MATLAB\(^1\) performed well overall but was found to be too persistent when no viable trim solution existed i.e. the solver did not terminate the search for the trim solution. An algorithm that had good convergence criteria throughout the operating domain and terminated iterations quickly when a solution did not exist was, therefore, required. In this work, the newton-Raphson method was adopted:

\[ \vec{x}_{n+1} = \vec{x}_n + \vec{\Delta} \quad (3.17) \]

where $n$ is the iteration counter and $\vec{\Delta}$ is the predicted step vector. The step vector at each iteration is determined from

\[ \vec{\Delta} = -[J(\vec{x}_n)]^{-1}\vec{f}(\vec{x}_n) \quad (3.18) \]

where $[J]$ is the Jacobian matrix containing the first-order partial derivatives and $\vec{f}$ is the trim function.

The convergence of nonlinear systems of equations can be difficult and are susceptible to divergence and sporadic oscillations. To improve the convergence, a good initial guess is usually required. The convergence of nonlinear equations can be improved by implementing under-relaxation (effectively damping the step size) or limiting the step size to some predefined value/s. Improving the convergence of the system of equations usually increases the number function evaluations (iterations) and, thereby, the computational cost. This is unfavourable.

---

\(^1\)See documentation for fsolve: https://uk.mathworks.com/help/optim/ug/fsolve.html.
when the function evaluation is computationally expensive. One strategy that balances the convergence rate against the computational cost is a variable damping factor that is dependent on the step size at the current iteration. This strategy was implemented here due to the large operating domain to be simulated and was found to be robust for all attempted operating points. The Newton-Raphson scheme in Equation 3.17 is recast as

$$\vec{x}_{n+1} = \vec{x}_n + \alpha \Delta$$

(3.19)

where $\alpha$ is the damping factor computed from the predicted step size:

$$\alpha = \frac{1}{1 + \|\Delta\|}$$

(3.20)

whereby $\alpha \to 1$ as $\|\Delta\| \to 0$ (indicating a converged solution). A convergence criteria of $\|\vec{x}_{n+1} - \vec{x}_n\| \leq 10^{-6}$ was used throughout. The determination of the initial guess is detailed in Section 9.2.

Each iteration of the Newton-Raphson scheme requires the determination of the Jacobian matrix $[J]$. Analytic expressions of the first-order partial derivatives are unrealistic due to the nonlinearity of the equations of motion. Therefore, the elements of the Jacobian matrix were approximated numerically at each iteration using a forward difference scheme:

$$\frac{\partial f_i}{\partial x_j} \approx \frac{f_i(x_j + h) - f_i(x_j)}{h},$$

(3.21)

where $i$ and $j$ are row-column specifiers and $h$ is a small step size compared to $x_j$. A step size of $h = 10^{-8}$ was used in the present work. For computational efficiency, the function value and Jacobian were computed in a single function call that avoided the necessity of a loop to perturb each element of $\vec{x}$ individually. A higher-order central difference scheme was also implemented to compare the predicted trim solutions but showed negligible difference compared to the forward difference scheme. Therefore, the forward difference scheme was retained.

### 3.5 Variable Inertia and Centre of Gravity

The centre of gravity and inertia tensor are determined by the distribution of mass about the body. The rotors and nacelles account for a significant portion of the total aircraft mass and, when tilted, change the inertia tensor and location of the centre of gravity. The dynamic
tilting of the rotors also causes a velocity and acceleration associated with the centre of gravity. In this thesis, the rotors are considered only at fixed tilt angles and, therefore, this effect is not considered. Consequently, the flight mechanics model is not suitable to analyse the dynamic behaviour associated with the tilting of the rotors.

The displacement of the centre of gravity and the tilt-dependent inertia parameters of the aircraft are modelled using the expressions described in the GTRS model [45], detailed hereafter. The position vector of the centre of gravity $\vec{p}_{CG}$ is written as

$$\vec{p}_{CG}(\tau) = l_{CG}(\tau) \hat{i}_A + d_{CG}(\tau) \hat{j}_A + h_{CG}(\tau) \hat{k}_A \quad (3.22)$$

where $l$, $d$ and $h$ denote the station, buttock and water line coordinates and $\tau$ is the rotor tilt angle. It is assumed that the lateral offset of the centre of gravity is zero and, therefore, the centre of gravity is described by the station and water line coordinates only. These position components $l_{CG}$ and $h_{CG}$ are expressed as

$$l_{CG}(\tau) = l_{CG}^{\tau=0} + \Delta l_{CG}(\tau), \quad (3.23a)$$
$$h_{CG}(\tau) = h_{CG}^{\tau=0} + \Delta h_{CG}(\tau), \quad (3.23b)$$

where the superscript $\tau = 0$ denotes the centre of gravity at zero rotor tilt (helicopter mode) and $\Delta$ is the displacement due to the rotor tilt. These displacements are expressed as functions of the rotor tilt angle:

$$\Delta l_{CG}(\tau) = k_1 (1 - \cos \tau) + k_2 \sin \tau \quad (3.24a)$$
$$\Delta h_{CG}(\tau) = k_2 (1 - \cos \tau) - k_1 \sin \tau \quad (3.24b)$$

where the coefficients $k_1$ and $k_2$ are determined from

$$k_1 = \frac{m_R}{m} (l_N - l_P) \quad (3.25a)$$
$$k_2 = \frac{m_R}{m} (h_N - h_P) \quad (3.25b)$$

with $m_R/m$ being the ratio of the total rotor mass to the aircraft mass and the subscripts $N$ and $P$ refer to the positions of the nacelle and pivot. Values of these respective quantities can be found in Table 3.3.

The change in the distribution of the aircraft mass due to the rotor tilt also changes the inertia tensor. The forward tilt of the rotors causes the moments of inertia in roll and pitch, $I_{xx}$
and $I_{yy}$, and also the product of inertia $I_{xz}$ to decrease. On the other hand, the moment of inertia in yaw, $I_{zz}$, increases. The change in the moments of inertia are modelled similarly to the centre of gravity. For the case of longitudinal flight, only the inertia in pitch is required which is modelled as a linear function of the rotor tilt from helicopter mode to aeroplane mode:

$$I_{yy}(\tau) = I_{yy}^{\tau=0} - \frac{dI_{yy}}{d\tau} \tau,$$

(3.26)

where $dI_{yy}/d\tau$ is the inertia gradient with respect to the rotor tilt. Values of these respective quantities are found in Table 3.3.

Table 3.3: Parameters for the XV-15 used in the flight mechanics model. Data from [45].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{CG}^{\tau=0}$</td>
<td>7.65 m</td>
<td>At the aft centre of gravity limit</td>
</tr>
<tr>
<td>$h_{CG}^{\tau=0}$</td>
<td>2.07 m</td>
<td></td>
</tr>
<tr>
<td>$m_R$</td>
<td>1900 kg</td>
<td></td>
</tr>
<tr>
<td>$m$</td>
<td>5900 kg</td>
<td>Aircraft design weight</td>
</tr>
<tr>
<td>$l_N$</td>
<td>7.41 m</td>
<td></td>
</tr>
<tr>
<td>$h_N$</td>
<td>3.00 m</td>
<td></td>
</tr>
<tr>
<td>$I_{yy}^{\tau=0}$</td>
<td>29000 kg m$^2$</td>
<td>At the design weight and aft centre of gravity</td>
</tr>
<tr>
<td>$dI_{yy}/d\tau$</td>
<td>15.2 kg m$^2$ deg$^{-1}$</td>
<td></td>
</tr>
</tbody>
</table>

3.6 Summary

This chapter has presented the flight mechanics module of the aeromechanics model. The generic discretisation framework of a tiltrotor aircraft was presented that decomposes the aircraft into the major aerodynamic components. The under-determined trim problem was presented and closed by parametrically relating the rotary-wing and fixed-wing controls to the pilot stick position. A damped Newton-Raphson scheme was presented to solve for the trim quantities and was found to be stable and robust throughout the operating domain considered in this work. Finally, models for the variable centre of gravity location and moment of inertia in pitch were presented as functions of the rotor tilt angle.
Chapter 4

Aerodynamic Module: Fuselage

This chapter details the aerodynamic module of the fuselage. The module is used to determine the force and moment contributions that are substituted into the equations of motion from Chapter 3. In this work, the fuselage is considered to be an isolated component that neglects any interaction with the rotors and/or wing. A first-order model for estimating the parasitic drag is presented first, followed by a more general approach to determine the aerodynamic forces and moments.

4.1 Aerodynamic Forces and Moments

In a first-order analysis, the aerodynamic forces and moments produced by the fuselage are generally unknown. The contribution of the fuselage to the parasitic power at high airspeeds is significant and should, therefore, be accounted for in an aeromechanics model. In helicopter literature, a first-order approximation of the fuselage drag force $D$ is found by expressing the drag force in the form [13, 60, 61]

$$D = \frac{1}{2} \rho V_\infty^2 f$$

(4.1)

where $\rho$ is the air density, $V_\infty$ is the freestream velocity and $f$ represents a flat-plate area. This area is determined from

$$f = SC_d$$

(4.2)

where $S$ is a reference area and $C_d$ is the drag coefficient. The equivalence of a flat-plate area removes any ambiguity in the reference area [60]. The flat-plate area can then be approximated.
from historical aircraft of a similar type, weight and design.

Once the fuselage geometry for an aircraft is known, higher-order models such as panels methods or CFD can be used to determine the aerodynamic loads. However, the increased computational cost associated with these higher-order models can inhibit their use in flight mechanics models over large domain investigations. Alternatively, the fuselage loads can be derived experimentally through wind-tunnel tests. To determine the aerodynamic forces and moments for generic tiltrotor aircraft, a more universal method is required. In order to facilitate various configurations, the aerodynamic loads are determined from supplied lookup tables. These tables contain the variations of the fuselage forces and moments with respect to the angles of attack. In aeromechanics models, it common practise to express the fuselage forces and forces relative to the freestream dynamic pressure [28, 45, 66]:

\[ q_\infty = \frac{1}{2} \rho V_\infty^2 \]  

Furthermore, the loads are expressed in a wind frame of reference, i.e. relative to a vector parallel to the freestream velocity \( \vec{u} \). The fuselage forces and moments are then applied as point loads at a predefined location. Since only longitudinal motion was considered here, models for the lateral and directional loads are not presented. The aerodynamic lookup tables and fuselage centre pressure are the required inputs for the fuselage component.

The fuselage lift and drag forces, expressed relative to the freestream dynamic pressure, \( \mathcal{L}/q_\infty \) and \( \mathcal{D}/q_\infty \), are given by:

\[ \frac{\mathcal{L}}{q_\infty} = \mathcal{L}_\alpha(\alpha) \]  

\[ \frac{\mathcal{D}}{q_\infty} = \mathcal{D}_\alpha(\alpha) \]  

where \( \mathcal{L}_\alpha \) and \( \mathcal{D}_\alpha \) are supplied lookup tables as a function of the fuselage angle of attack. The lookup tables are linearly interpolated at the required angle of attack. The lift and drag forces are defined perpendicular and parallel to the freestream velocity \( V_\infty \) and must be rotated into the body-fixed axes for substitution into the equations of motion. From Figure 4.1, the fuselage lift and drag contributions to the body-fixed \( X \) and \( Z \) forces are:

\[ X = q_\infty (\mathcal{L}_\alpha \sin \alpha - \mathcal{D}_\alpha \cos \alpha) \]  

\[ Z = -q_\infty (\mathcal{L}_\alpha \cos \alpha + \mathcal{D}_\alpha \sin \alpha) \]
Figure 4.1: Aerodynamic forces of the fuselage applied at the fuselage centre of pressure and resolved into the body-fixed axes.

The pressure distribution around the surface of the fuselage also produces an aerodynamic pitching moment in longitudinal flight and, similarly, roll and yaw moments in sideslip. The pitching moment $\mathcal{M}$ is expressed in the same manner as the lift and drag forces:

$$\frac{\mathcal{M}}{q_\infty} = \mathcal{M}_\alpha(\alpha)$$

(4.6)

where $\mathcal{M}_\alpha$ is the supplied lookup table that is again linearly interpolated with respect to the fuselage angle of attack. The pitching moment of the fuselage is expressed in the wind frame of reference, which, in the case of longitudinal flight is equivalent to the body-fixed pitch axis $\hat{j}_B$.

The total moment produced by the fuselage about the centre of gravity includes the contribution from the fuselage forces that can, in general, be offset from the centre of gravity. Denoting the position vector of the fuselage centre of pressure as $\vec{p}$, the total moment about the centre of gravity is found from

$$\vec{M} = \vec{p} \times \vec{F} + \vec{M}$$

(4.7)

where $\vec{M} = \{0, M, 0\}^T$ is the resultant moment vector, $\vec{F} = \{X, 0, Z\}^T$ is the force vector and $\vec{M} = \{0, M, 0\}^T$ is the fuselage moment vector. The total pitching moment about the centre of gravity in the body-fixed axes is then found to be

$$M = zX - xZ + \mathcal{M}$$

(4.8)
where $x$ and $z$ are the components of $\vec{p}$ along the $\hat{i}_B$ and $\hat{k}_B$ directions. These components are found from the known positions of the centre of gravity and fuselage centre of pressure:

$$x = -(l_F - l_{CG})$$

(4.9a)

$$z = -(h_F - h_{CG})$$

(4.9b)

where $l$ and $h$ are the components of the position vector along the $\hat{i}_A$ and $\hat{k}_A$ directions and the subscripts $F$ and $CG$ refer to the fuselage and centre of gravity.

The same methodology is also used in the GTRS model of the XV-15 [45] and, therefore, is readily implemented here as the reference aircraft for investigation. The variations of $L_\alpha$, $D_\alpha$ and $M_\alpha$ with respect to the angle of attack are shown in Figure 4.2. The aerodynamic loads are applied at the fuselage location detailed in Table 3.2. In Figure 4.2, the data in the angle of attack domain $[-20^\circ, 20^\circ]$ is derived from static wind-tunnel tests and was empirically extended to the domain $[-180^\circ, 180^\circ]$. However, in the context of longitudinal trim considered here, the fuselage angle of attack was not found to exceed $|\alpha| \approx 25^\circ$ as will be shown in Chapter 10. To simulate other generic tiltrotor aircraft, and in the absence of experimental/reference data, it is reasonable to assume that the GTRS XV-15 data provides a good first-order approximation of the fuselage loads.

![Figure 4.2: Variation of fuselage lift, drag and pitching moment with respect to angle of attack. Data from [45].](image)

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4.2 Summary

This chapter has presented the calculation of the aerodynamic forces and moments of the fuselage module for substitution into the flight mechanics module. A simple and universal approach was adopted whereby the aerodynamic forces and moments are supplied through lookup tables containing the lift, drag and pitching moment variations with respect to the angle of attack. This easily facilitates the implementation of new, similar or updated aerodynamic data for aeromechanics modelling.
Chapter 5

Aerodynamic Module: Rotor

This chapter describes the aerodynamic module for the rotor. Due to the implicit relationship between the rotor loads, the induced flowfield and the rotor dynamics, the aerodynamic module of the rotor is the most computationally expensive component. The chapter starts with an overview of the rotor wake models used to familiarise the reader with the different methods that could have been used in the aerodynamics modules. After this, a description of the coordinate systems used in the analysis is presented followed by the kinematic description of the blades. The calculation of the rotor loads on both the hub and airframe are presented next. Following this, the selected wake model of the rotor inflow is described and the equations of motion for a gimballed rotorhead are derived from first principals. The final sections of this chapter detail the validation efforts of the rotor model against experimental data.

5.1 Overview of Rotor Wake Modelling

The aerodynamic and inertial forces and moments generated by a rotor are implicitly related to the induced velocity field and the dynamic motion of the blades. The flowfield associated with these loads is unsteady and three-dimensional constituting a complex problem to accurately capture the spatial and temporal load variations. These flowfields are best captured using physics-rich methods such as CFD or VPM that solve the Navier-Stokes equations. However, the computational cost of these tools generally restricts their implementation until the latter stages of rotor/blade design. Methods that offer similar accuracy
with a more affordable computational cost are vortex methods that discretise the blade into either a single bound vortex (lifting line formulation) or a finite number of panels along the chord and span (lifting surface formulation). The unknown bound circulations are found at each time step through the application of the flow tangency condition applied to each panel (see Katz and Plotkin [67] for a thorough description of these potential flow methods). The lifting line method is commonly adopted in rotary-wing literature as compressible and viscous aerofoil data can be introduced through lookup tables in the calculation of the bound circulation [54, 68]. The lifting-line and lifting-surface methods require the wake-induced velocity at the blades to compute the aerodynamic loads and, therefore, the geometry of the wake is required. Accurately predicting the geometry of the wake is, therefore, paramount for accurate predictions from vortex methods. Free-wake methods allow the wake geometry to deform in time and space by convecting each wake node with the local convection velocity calculated the freestream and wake-induced velocities. Since every point in the wake is updated and new wake nodes are generated at each time step, this method can quickly become computationally prohibitive. Alternatively, experimental or empirical data for the shape of the wake can also be used, the prescribed wake model, but the accuracy of rotor performance is then reliant on the experimental/empirical data.

For large domain and first-order approximations of rotor performance, momentum and finite-state models of the rotor wake are used due to their robustness and computational cost [69]. The classical momentum models relate the rotor thrust to the change in the slipstream velocity; the induced velocity at the rotor plane is half the far-downstream value. The classical momentum models are based on steady-state operating conditions and do not, in their first-order form, consider the temporal degree of freedom. The time-dependency of the rotor-induced flowfield is introduced in the finite-state models that capture the time dependency between the aerodynamic loading and transient induced inflow that started with the work of Carpenter and Fridovich [70]. The finite-state models are based on a set of first-order differential equations that relate the time-dependent aerodynamic loading to the wake degrees of freedom. Such a model will be implemented in this work as described in Section 5.4.

5.2 Kinematics

The aerodynamic loads produced by a blade element are a function of its velocity that determines its dynamic pressure and angle of attack. To determine the velocity of a blade element, several Cartesian coordinates systems are used here and are described now. The first
coordinate system is the nonrotating hub system $O_H^{xyz}$ that is located on the shaft axis and centred on the hub as shown in Figure 5.1. The unit vectors along the hub axes are denoted by $\hat{i}_H$, $\hat{j}_H$ and $\hat{k}_H$. The orientation of these axes are parallel to the body-fixed axes in helicopter mode. As the rotors are tilted, the orientation of these axes relative to the body-fixed axes are described by the transformation matrix

$$\begin{pmatrix}
\cos \tau & 0 & \sin \tau \\
0 & 1 & 0 \\
-\sin \tau & 0 & \cos \tau
\end{pmatrix}$$

(5.1)

Figure 5.1: Nonrotating hub coordinate system located on the rotor shaft.

The second coordinate system is typically one fixed to the shaft that describes the angular displacement of the blades. However, for the case of gimbaled rotors, the plane of rotation may tilt relative to the hub plane due to a gimbal joint on the rotor head [14]. Therefore, the second coordinate system used in this work is the Cartesian $O_D^{xyz}$ disc system that describes the plane of rotation of the rotor disc due to the gimbal joint. The unit vectors along the disc axes are denoted by $\hat{i}_D$, $\hat{j}_D$ and $\hat{k}_D$. The origin of these axes are co-located with the hub axes and their orientation relative to the nonrotating hub axes is described through the lateral and longitudinal gimbal tilt angles, $\beta_s$ and $\beta_c$. The orientation of the disc axes is initially parallel to the hub axes when the lateral and longitudinal gimbal tilt angles are zero. The rotations from the hub axes into the disc axes are performed such that the steady-state first-harmonic flapping of a blade may be expressed in a Fourier series expanded in the azimuth angle as

$$\beta = \beta_p + \beta_s \cos \psi + \beta_c \sin \psi$$

(5.2)
where $\beta$ is the flapping angle from the hub plane, $\beta_p$ is the blade precone angle and $\psi$ is the azimuth angle. The azimuth angle describes the angular position of the blades around the shaft axis and in this work, is taken as positive in the direction of rotation measured from the rear centreline of the hub $(-\hat{i}_H)$. Since tiltrotor aircraft operate as lateral-tandem aircraft with counter-rotating rotors, the lateral flapping angle (equivalently the lateral gimbal tilt angle) of the right and left rotors, $\beta_s$, is of opposite sign when viewed in the nonrotating hub system as shown in Figure 5.2. However, it is convenient to describe the rotor aerodynamics by a single set of equations. In order to do so, the rotation specifier $n$ is introduced which has the conditional value

$$n = \begin{cases} 
1 & \text{for positive rotation} \\
-1 & \text{for negative rotation} 
\end{cases}$$

(5.3)

where the sense of rotation is defined from the right-hand convention about the shaft axis along the $-\hat{k}_H$ direction. The orientation of the disc axes relative to the hub axes is defined first by a rotation through the gimbal roll angle about $\hat{i}_H$ followed by a rotation through the gimbal pitch angle about $\hat{j}_D$ as shown in Figure 5.3. This transformation is described through the transformation matrix

$$\begin{pmatrix}
\hat{i}_D \\
\hat{j}_D \\
\hat{k}_D 
\end{pmatrix} = \begin{bmatrix}
1 & 0 & \beta_c \\
0 & 1 & -n\beta_s \\
-\beta_c & n\beta_s & 1 
\end{bmatrix} \begin{pmatrix}
\hat{i}_H \\
\hat{j}_H \\
\hat{k}_H 
\end{pmatrix}$$

(5.4)

where the small-angle approximation has been made to simplify the rotation matrix. The product of the gimbal tilt angles $n\beta_s\beta_c$ has been assumed to be negligibly small and neglected in the transformation matrix. The lateral and longitudinal gimbal tilt angles used in the transformation matrix are not strictly the first-harmonic coefficients used to describe the blade flapping as in Equation 5.2. These coefficients define the lateral and longitudinal tilts relative to the nonrotating frame, whereas the definitions used in the rotation matrix are Euler angles. However, these angles are nearly identical when the small-angle approximation is invoked.

![Figure 5.2: Lateral flapping as viewed from the rear of the nonrotating hub axes. The $\hat{i}_H$ direction points into the page.](image)
The third coordinate system is the shaft system $O_{xyz}^s$ that describes the angular position of the shaft. The unit vectors along the shaft axes are denoted by $\hat{i}_S$, $\hat{j}_S$ and $\hat{k}_S$. To give consistent unit vectors in the chordwise and chord-normal directions of a blade section, the orientation of the shaft axes relative to the blade span are dependent on the direction of rotation, as shown in Figure 5.4. The origin of the shaft axes is co-located with the disc axes at the hub centre since the gimbal joint reorientates the plane of rotation. The orientation of the shaft axes relative to the disc axes is defined by a rotation through the azimuth angle about $\hat{k}_D$ and is described by the rotation matrix

$$
\begin{pmatrix}
\hat{i}_S \\
\hat{j}_S \\
\hat{k}_S
\end{pmatrix} =
\begin{pmatrix}
-n \cos \psi & \sin \psi & 0 \\
\sin \psi & n \cos \psi & 0 \\
0 & 0 & -1
\end{pmatrix}
\begin{pmatrix}
\hat{i}_D \\
\hat{j}_D \\
\hat{k}_D
\end{pmatrix}.
$$

Figure 5.3: Axis transformation from the hub axes into the disc axes through the lateral gimbal tilt angle (left; $\hat{i}_H$ into the page) followed by the longitudinal tilt angle (right; $\hat{j}_D$ into the page).
Figure 5.4: Azimuth angle transformation for the left and right rotors. The $\hat{k}_D$ direction points into the page and $\hat{k}_S$ direction points out the page.

The final coordinate system is the Cartesian $O^{BL}$-XYZ blade-fixed system. The unit vectors along the blade-fixed axes are denoted by $\hat{i}_{BL}$, $\hat{j}_{BL}$ and $\hat{k}_{BL}$ and termed the radial, tangential and perpendicular directions. The radial axis spans along the aerodynamic centre of the blade. The orientation of the blade-fixed axes relative to the shaft axes are shown in Figure 5.5 and are described by a rotation through the precone angle about $\hat{j}_S$:

$$
\begin{bmatrix}
\hat{i}_{BL} \\
\hat{j}_{BL} \\
\hat{k}_{BL}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & n\beta_p \\
0 & 1 & 0 \\
-n\beta_p & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\hat{i}_S \\
\hat{j}_S \\
\hat{k}_S
\end{bmatrix}
$$

where the small angle approximation has been made since the precone angle is typically a few degrees. The precone angle is measured positive for the blade upwards and, therefore, is dependent on the direction of shaft rotation. The rotation matrices in Equation 5.5 and 5.6 are combined to resolve the disc axes directly into the blade-fixed axes:

$$
\begin{bmatrix}
\hat{i}_{BL} \\
\hat{j}_{BL} \\
\hat{k}_{BL}
\end{bmatrix} =
\begin{bmatrix}
-n\cos\psi & \sin\psi & -n\beta_p \\
\sin\psi & n\cos\psi & 0 \\
\beta_p \cos\psi & -n\beta_p \sin\psi & -1
\end{bmatrix}
\begin{bmatrix}
\hat{i}_D \\
\hat{j}_D \\
\hat{k}_D
\end{bmatrix}
$$

This completes the description of the coordinate systems used in the aerodynamic analysis herein.

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Figure 5.5: Axis transformation from the shaft axes into the blade-fixed axes through the precone angle. The $\hat{j}_S$ direction points into the page.

The blade aerodynamics are calculated in the blade-fixed axes and, therefore, the absolute velocity of a blade element in these axes is required. The absolute velocity of a fixed point in a set of rotating axes is calculated from the generic kinematic equation

$$\vec{u} = \vec{u}_0 + \vec{\omega} \times \vec{p}$$

where $\vec{u}$ is the absolute velocity, $\vec{u}_0$ is the linear velocity of the rotating axes, $\vec{\omega}$ is the angular velocity of the rotating axes and $\vec{p}$ is the position of a point in the rotating axes. Since the hub, disc and blade-fixed axes have the same origin on the hub centre, the velocity of this point is first described. In the work undertaken here, the rotors are considered only at static tilt angles and the aircraft trim behaviour is of interest. In the trim condition, there is no angular velocity of the airframe and, therefore, the linear velocity of the hub origin is due simply to the freestream velocity. The freestream velocity in the body axes is given by

$$\vec{u}_0 = u\hat{i}_B + w\hat{k}_B$$

which is expressed equivalently in the hub axes by transforming the body components through the rotor tilt angle:

$$\vec{u}_0 = u_H\hat{i}_H + w_H\hat{k}_H$$

(5.10a)
where the components \( \{u_H, 0, w_H\}^T \) are given by

\[
\begin{align*}
    u_H &= u \cos \tau + w \sin \tau \\
    w_H &= -u \sin \tau + w \cos \tau
\end{align*}
\]  

(5.10b)  

(5.10c)

The freestream velocity is now rotated into the disc axes through the tilt angles of the gimbal:

\[
\tilde{u}_0 = u_D \hat{i}_D + v_D \hat{j}_D + w_D \hat{k}_D
\]  

(5.11a)

where the components \( \{u_D, v_D, w_D\}^T \) are given by

\[
\begin{align*}
    u_D &= u_H + w_H \beta_c \\
    v_D &= -w_H n \beta_s \\
    w_D &= w_H - u_H \beta_c
\end{align*}
\]  

(5.11b)  

(5.11c)  

(5.11d)

The in-plane velocity components of the disc are used to define the advance ratio of the disc \( \mu_D \) and the disc-wind angle \( \Delta_D \):

\[
\mu = \sqrt{\frac{u_D^2 + v_D^2}{\Omega R}}
\]  

(5.12a)

\[
\Delta = \arctan \frac{v_D}{u_D}
\]  

(5.12b)

where \( \Omega R \) is the tip speed of the blade due the shaft rotation. The out-of-plane velocity component of the disc is similarly normalised by the tip speed to define the normal velocity ratio:

\[
\mu_z = \frac{w_D}{\Omega R}
\]  

(5.13)

The advance ratio, normal velocity ratio and disc wind angle are used to simplify the kinematic expressions by reducing the velocity representation into two components; the resultant in-plane component and the out-of-plane component. The linear velocity of the blade-fixed axes can then be written in the disc axes in terms of the these components:

\[
\tilde{u}_0 = \Omega R \left[ \mu \cos \Delta \hat{i}_D + \mu \sin \Delta \hat{j}_D + \mu_z \hat{k}_D \right]
\]  

(5.14)

Finally, these velocity components are rotated into the blade-fixed axes through Equation 5.7.
to give the linear velocity of the blade-fixed axes:

$$\vec{u}_0 = \Omega R \begin{Bmatrix} -\mu_n \beta_p - n \mu \cos(\psi + n\Delta) \\ \mu \sin(\psi + n\Delta) \\ \mu \beta_p \cos(\psi + n\Delta) - \mu_c \end{Bmatrix}$$  \hspace{1cm} (5.15)

The angular velocity contribution $\vec{\omega} \times \vec{p}$ to the absolute velocity of the blade is now derived. The angular velocity of the the airframe and rotor tilt rate in this work are zero and, therefore, the angular velocity of a blade element contains two contributions. The first contribution is from the angular velocity of the gimbal tilt rates, $\dot{\beta}_s$ and $\dot{\beta}_c$, and the second contribution is from the shaft speed $\Omega$. The angular velocity vector of a blade element is then given by

$$\vec{\omega} = -n \dot{\beta}_s \hat{i}_H - \dot{\beta}_c \hat{j}_D - n \Omega \hat{k}_D$$  \hspace{1cm} (5.16)

When dealing with the rotor motion it is more convenient to describe the blades with respect to the azimuth angle rather than time and, therefore, the tilt rates of the gimbal are recast as:

$$\dot{\beta}_s = \Omega \beta'_s$$  \hspace{1cm} (5.17a)

$$\dot{\beta}_c = \Omega \beta'_c$$  \hspace{1cm} (5.17b)

where the dash represents differentiation with respect to the azimuth angle (dimensionless time). The variables $\beta'_s$ and $\beta'_c$ are then treated as states of the gimbal motion. The angular velocity vector is then written equivalently as

$$\vec{\omega} = \Omega \begin{bmatrix} -n \beta'_s \hat{i}_H - \beta'_c \hat{j}_D - n \Omega \hat{k}_D \end{bmatrix}$$  \hspace{1cm} (5.18)

Rotating the lateral flapping term $-n \beta'_s$ from the hub axes introduces a small angular velocity component about $\hat{k}_D$. This term is neglected since its contribution is an order of magnitude smaller than the angular velocity of the shaft. The angular velocity of a blade element in disc axes is then

$$\vec{\omega} = \Omega \begin{bmatrix} -n \beta'_s \hat{i}_D - \beta'_c \hat{j}_D - n \hat{k}_D \end{bmatrix}$$  \hspace{1cm} (5.19)

In a similar fashion to the linear velocities, it is more convenient to write the kinematics in two-dimensions and, therefore, the resultant in-plane angular velocity and tilt-rate angle are
defined as:

\[ \beta' = \sqrt{(\beta'_s)^2 + (\beta'_c)^2} \]  

(5.20a)

\[ \Gamma = \arctan \frac{-\beta'_c}{-n\beta'_s} \]  

(5.20b)

The angular velocity of the blade axes is then written equivalently in the disc axes as

\[ \bar{\omega} = \Omega \left[ \beta' \cos \Gamma \hat{i}_D + \beta' \sin \Gamma \hat{j}_D - n \hat{k}_D \right] \]  

(5.21)

and rotating into the blade-fixed axes gives

\[ \bar{\omega} = \Omega \begin{pmatrix} \beta_p - n \beta' \cos(\psi + n \Gamma) \\ \beta' \sin(\psi + n \Gamma) \\ n \end{pmatrix} \]  

(5.22)

where the contribution \( \Omega \beta_p \beta' \cos(\psi + n \Gamma) \hat{k}_{BL} \) is much smaller than the shaft speed and has been neglected. The position of a blade element in the blade-axes is simply

\[ \bar{p} = nr \hat{r}_{BL} \]  

(5.23)

where \( r \) is the dimensionless radial position and the rotation specifier is required due to the orientation of the blade span axis. Performing the cross product gives the angular contribution to the absolute velocity of a blade element as

\[ \bar{\omega} \times \bar{p} = \Omega R \left[ r \hat{j}_{BL} - nr \beta' \sin(\psi + n \Gamma) \hat{k}_{BL} \right] \]  

(5.24)

The absolute velocity of a blade element is now found by summing the linear velocity of the axis set in Equation 5.15 with the angular contribution in Equation 5.24 to give

\[ \bar{u} = \Omega R \begin{pmatrix} -\mu \beta_p - n \mu \cos(\psi + n \Delta) \\ r \mu \cos(\psi + n \Delta) \\ \mu \beta_p \cos(\psi + n \Delta) - \mu \beta - nr \beta' \sin(\psi + n \Gamma) \end{pmatrix} \]  

(5.25)

In addition to the absolute velocity of the blade from the freestream and rotation rates, the blade also experiences an additional velocity component due to the flow induced by the rotor.
loading. The induced velocity is defined in the disc axes and is denoted dimensionally by

\[ \vec{v} = -v \hat{k}_D. \] (5.26)

In general, the induced velocity is spatially and temporally dependent on the loading of the rotor and is a three-dimensional vector field. In this work, only the induced velocity normal to the rotor disc is considered since its effect on the rotor loads is significantly greater than the in-plane components. The induced inflow ratio is defined as the ratio of the induced velocity to the tip speed:

\[ \lambda_i = \frac{v}{\Omega R} \] (5.27)

and is used in the dimensionless representation of the blade kinematics. The components of the aerodynamic velocity of the blade in the blade-fixed axes are denoted by \( \{V_R, V_T, V_P\}^T \) representing the radial, tangential and perpendicular components. Summing the absolute velocity of the blade from Equation 5.25 with the induced inflow ratio gives the velocity components:

\[ V_R = \Omega R [\lambda n \beta_p - n \mu \cos(\psi + k \Delta)] , \] (5.28a)
\[ V_T = \Omega R [r + \mu \sin(\psi + n \Delta)] , \] (5.28b)
\[ V_P = \Omega R [\lambda + \mu \beta_p \cos(\psi + n \Delta) - nr \beta' \sin(\psi + n \Gamma)] , \] (5.28c)

where \( \lambda \) is the inflow ratio representing the total inflow normal to the rotor disc due to the freestream and induced velocity:

\[ \lambda = \lambda_i - \mu_c \] (5.29)

This completes the kinematic description of a blade element to find its velocity components as required for the calculation of the aerodynamic forces and moments.

### 5.3 Aerodynamics Forces and Moments

The calculation of the aerodynamic forces generated by the rotor is now described. The aerodynamic loads relative to the hub axes are first established in order to compare the performance predictions of the rotor against experimental data. The hub loads are then rotated into the body-fixed axes for substitution into the equations of motion.

The aerodynamic forces and moments were modelled using quasi-steady blade element
theory where the blade is discretised along the span axis into a finite number of two-
dimensional sections. First-order blade element theory assumes the aerodynamic forces of
each section are independent of the adjacent sections and generated by the two-dimensional
flow over the aerofoil section. The radial velocity component as calculated in Equation 5.28a
is, therefore, neglected on the assumption it has a negligible effect on the section aerodynamics.
This is a valid assumption for high aspect ratio blades typical of rotors. The effect of radial flow
yaws the resultant velocity over the blade section and can increase the skin friction drag and
delay the onset of stall [61, 71], however, these effects are second-order and can be omitted in
a first-order analysis of the rotor performance. The blades of tiltrotors are designed to be stiff
for efficient performance in propeller mode and, therefore, the blades are assumed to be rigid
and undergo no aeroelastic deformation when loaded. Furthermore, the blade is assumed to be
straight so that the quarter-chord of every blade element lies on the span axis. This assumption
neglects blade sweep and complex geometries such as dihedral/anhedral tips, however, these
geometries require improved wake models to capture both the local loading and wake effects
(methods such as lifting line/surface, VPM or CFD).

The lift and drag forces produced by a blade element of elementary span $dx$ are proportional
to the dynamic pressure, chord length and respective coefficient:

$$dL = C_l \frac{1}{2} \rho V^2 c \, dx$$  \hspace{1cm} (5.30a)

$$dD = C_d \frac{1}{2} \rho V^2 c \, dx$$  \hspace{1cm} (5.30b)

where $\rho$ is the air density, $V$ is the resultant velocity, $\rho V^2/2$ is the dynamic pressure, $c$ is the
chord length, $C_l$ is the lift coefficient and $C_d$ is the drag coefficient. A pitching moment may
also be developed about the aerodynamic centre but this is neglected here as its contribution
to the roll and pitch moments on the rotor are small compared to the lift contribution.
Furthermore, the pitching moment on the hub is only transferred to the airframe if a hub spring
exists. The drag force contains two contributions: the profile drag at zero lift $dD_0$; and the
induced drag due to the production of lift $dD_i$. The total drag produced by the blade section is
then written as

$$dD = dD_0 + dD_i$$  \hspace{1cm} (5.31a)

$$= (C_{d_0} + C_{d_i}) \frac{1}{2} \rho V^2 c \, dx$$  \hspace{1cm} (5.31b)

where $C_{d_0}$ and $C_{d_i}$ are the profile and induced drag coefficients. The resultant velocity and
determination of the aerodynamic coefficients is described next.
The resultant velocity of the blade element is defined by the two-dimensional flow over the aerofoil in the tangential and perpendicular directions. The resultant velocity is then

\[ V = \sqrt{V_T^2 + V_P^2} \]  

(5.32)

and the Mach number of the blade element follows directly:

\[ M = \frac{V}{c_0} \]  

(5.33)

where \( c_0 \) is the speed of sound calculated from an ISA model. The aerodynamic coefficients are linearly interpolated from incompressible user-supplied lookup tables as a function of angle of attack and spanwise position. The tabulated angle of attack data is required in the domain \([-180^\circ, 180^\circ]\) to facilitate section stall and reverse flow conditions on the retreating side of the disc. The span dimension in the lookup table also facilitates different aerofoil sections to be implemented. The angle of attack \( \alpha \) of the blade element is defined as the difference between the pitch angle \( \theta \) and the inflow angle \( \phi \), as shown in Figure 5.6:

\[ \alpha = \theta - \phi \]  

(5.34)

The pitch angle of the blade element is measured from the span plane to the section chord line. The inflow angle is defined as the angle between the resultant velocity and the span plane. Both the pitch and inflow angles are measured positive above the span plane. The inflow angle is calculated from the tangential and perpendicular velocity components of the blade section:

\[ \phi = \tan^{-1} \frac{V_P}{V_T} \]  

(5.35)

To facilitate the correct orientation of the resultant velocity vector in reversed flow, the four-quadrant tangent operator is used.
Figure 5.6: Schematic of an aerofoil section showing the velocity components, flow angles and section forces.

Current tiltrotors operate in helicopter and conversion modes at a tip Mach number somewhere roughly between 0.65 and 0.70 and, therefore, compressibility effects on the section aerodynamics are important. In this work, the first-order Prandtl-Glauert correction was used to correct the interpolated incompressible lift coefficient:

\[ C_l = \frac{[C_l]_{M=0}}{\sqrt{1-M^2}} \]  

(5.36)

where \( M \) is the local Mach number. The Prandtl-Glauert correction factor is derived from small-disturbance theory and is only valid for small angles of attack and thin aerofoils. Once transonic flow becomes present the predicted loads start to deviate from the simple Prandtl-Glauert correction and it is no longer applicable [72]. The onset of transonic flow is determined by the critical Mach number which can be approximated from the Korn equation [73] or similar methods [74] and is predominantly a function of the lift coefficient and thickness-to-chord ratio. The Prandtl-Glauert correction is applied up to a Mach number of 0.70 where the local flow over the aerofoil was assumed to be subsonic. This limit is not an accurate reflection of the thicker aerofoil sections towards the root where the critical Mach number is smaller. However, this was accepted since the dynamic pressure is much larger towards the tip where the aerofoil sections are thinner and the imposed limit is more realistic. No corrections were made to the section drag coefficient for compressibility effects as the drag is typically small compared to the lift. More detailed compressible aerodynamics are not currently modelled but can be easily facilitated as a Mach number dimension in the aerodynamic lookup tables.

The lift and drag forces produced by a blade element act perpendicular and parallel to the resultant velocity. Therefore, the inflow angle is used to resolve the section lift and drag forces
into the blade-fixed axes as shown in Figure 5.6. The blade element forces in the tangential and perpendicular directions are then found to be:

\[
dF_T = dL \sin \phi + dD \cos \phi \tag{5.37a}
\]
\[
dF_P = dL \cos \phi - dD \sin \phi \tag{5.37b}
\]

The tangential force is defined positive against the rotation of the shaft along \(-\hat{j}_{BL}\). In conventional helicopter analysis, the inflow angle \(\phi\) is assumed to be small \((V_P \ll V_T)\) such that the small angle approximation is used to linearise the sine and cosine terms. For the airspeeds and rotor incidence angles of conventional helicopters, this is a reasonable approximation due to the lower normal velocity ratios. However, the larger normal velocity ratios experienced by tiltrotor aircraft does not permit this assumption and, therefore, the inflow angle is not linearised. The section tangential and perpendicular blade forces are now resolved into the nonrotating hub axes to give the in-plane, side and thrust force contributions from a blade section. The tangential and perpendicular section forces are first rotated into the disc axes through Equation 5.7. The section in-plane, side and thrust forces generated by the rotor are, in general, time dependent quantities when the rotor operates in nonaxial flight. In the disc axes, these section forces are denoted by \((-dH_D, dY_D, -dT_D)^T\) and are found to be:

\[
dH_D = dF_T \sin \psi - \beta_p dF_P \cos \psi \tag{5.38a}
\]
\[
dY_D = -n dF_T \cos \psi - n\beta_p dF_P \sin \psi \tag{5.38b}
\]
\[
dT_D = dF_P \tag{5.38c}
\]

The total force produced by a blade is then found by integration of the section forces along the span. This integration is dependent on the shaft rotation direction since the blade spans along or away from the positive \(\hat{j}_{BL}\) axis. The generalised spanwise integration can therefore be written as

\[
n \int_{r_0}^{R} \ldots dx \tag{5.39}
\]

where \(R_0\) is the root radius, \(R\) is the tip radius and \(x\) is the spanwise ordinate. The tip and root radii are defined as positive quantities and the root radius is determined from \(R_0 = eR\), where \(e\) is the root cut-off fraction. The total force produced by the rotor is then found by summing the individual blade contributions over the number of blades. The in-plane, side and thrust forces...
in the disc axes are then found to be:

\[ H_D = \sum_{i=1}^{b} \left( n \int_{nR_0}^{nR} \frac{dH_D}{dx} \, dx \right)_i \]  
(5.40a)

\[ Y_D = \sum_{i=1}^{b} \left( n \int_{nR_0}^{nR} \frac{dY_D}{dx} \, dx \right)_i \]  
(5.40b)

\[ T_D = \sum_{i=1}^{b} \left( n \int_{nR_0}^{nR} \frac{dT_D}{dx} \, dx \right)_i \]  
(5.40c)

where the integration is performed numerically using trapezoidal integration. The disc forces are now rotated into the hub axes through the gimbal tilt angle rotations described in Equation 5.4. The rotor forces in the hub axes are denoted by \(-H_H, Y_H, -T_H\)^T and are found to be:

\[ H_H = H_D - T_D \beta_c \]  
(5.41a)

\[ Y_H = Y_D - nT_D \beta_s \]  
(5.41b)

\[ T_H = T_D + H_D \beta_c + nY_D \beta_s \]  
(5.41c)

The aerodynamic forces of the blade element give rise to aerodynamic moments about the blade hinge. These moments give rise to the tilting of the plane of rotation on the gimballed rotorhead. The aerodynamic moment is taken positive in the conventional sense of blade flapping upwards out of the blade plane. The integrated aerodynamic flap moment \(M_\beta\) is given by

\[ M_\beta = n \int_{nR_0}^{nR} x \frac{dF_P}{dx} \, dx \]  
(5.42)

where \(x\) is the radial ordinate. The in-plane aerodynamic force generates an aerodynamic torque about the shaft axis that acts to decelerate the shaft speed. In order to maintain constant rotor speed, an equivalent torque must be applied to the shaft. The torque required to drive a single blade about the shaft axis, \(Q_i\), is found from

\[ Q_i = n \int_{nR_0}^{nR} x \frac{dF_T}{dx} \, dx \]  
(5.43)

and is taken positive against the shaft motion again. The total torque on the rotor \(Q\) is found by summing the torque contributions from all \(b\) blades attached to the rotor:

\[ Q = \sum_{i=1}^{b} Q_i \]  
(5.44)
The torque is defined in the disc axes since the gimballed rotorhead transforms the angular momentum normal to the tip-path-plane. It is assumed that the gimbal joint is ideal and has no frictional losses. The power required to drive the rotor shaft at an angular speed \( \Omega \) is then \( P = \Omega Q \).

The aerodynamic forces in Equations 5.38 and 5.41 are time dependent quantities. The outputs of the rotor aerodynamic module are the steady forces that are substituted into the equations of motion. The steady forces, denoted with an overbar, are found by averaging the time-dependent quantities over a rotor revolution:

\[
\bar{f} = \frac{1}{2\pi} \int_{0}^{2\pi} f(\psi) \, d\psi
\]

where \( f \) is an arbitrary function. In steady flight, the inertial loads that arise from the acceleration of the blade equate to zero when averaged around the rotor azimuth due to the periodicity of the forcing [13] and hence, have been neglected in the analysis here. In this work, only longitudinal flight is considered and, therefore, the lateral forces generated by the counter-rotating rotors cancel out. The contribution of the steady rotor forces to the body \( X \) and \( Z \) forces are then found from Equations 5.1 to be:

\[
X = \bar{T}_H \sin \tau - \bar{H}_H \cos \tau
\]

\[
Z = -(\bar{H}_H \sin \tau + \bar{T}_H \cos \tau)
\]

The balance of aerodynamic and inertial out-of-plane moments determines the orientation of the tip-path-plane and, therefore, no out-of-plane moment is transferred to the hub. However, the reaction moment of a hub spring can be used to improve the control power. The steady pitch moment on the hub \( \bar{M}_H \) is proportional to the steady longitudinal tilt angle of the gimbal:

\[
\bar{M}_H = -K \tilde{\beta}_c
\]

where \( K \) is the hub spring stiffness. A roll moment is also generated by the steady lateral tilt angle of the gimbal, however, in steady longitudinal flight the contributions of both rotors give zero net moment. The steady pitching moment on the airframe is then found from the contributions from the rotor forces and hub pitching moment:

\[
M = -(h_P + \ell \cos \tau - h_{CG})X + (l_P - \ell \sin \tau - l_{CG})Z + \bar{M}_H
\]

where \( l \) and \( h \) are the station and waterline ordinates, \( \ell \) is the shaft length and the subscripts \( P \)
and $CG$ denote the pivot and centre of gravity. This completes the calculation of the aerodynamic forces and moment outputted from the rotor aerodynamic module.

When comparing rotors of different sizes and designs, it is convenient to assess their performance in terms of dimensionless quantities. For rotor performance, the parameters of interest are thrust, torque and power. The definitions of thrust, torque and power coefficients used in this work are

\[ C_T = \frac{T}{\rho \pi R^2 (\Omega R)^2} \]
\[ C_Q = \frac{Q}{\rho \pi R^2 (\Omega R)^2 R} \]
\[ C_P = \frac{P}{\rho \pi R^2 (\Omega R)^3} \]

where $\rho$ is the air density, $\pi R^2$ is the rotor disc area, $\Omega R$ is the tip speed and $R$ is the rotor radius. The power and torque coefficients are identical since $P = \Omega Q$. The thrust value is defined relative to some convenient axis, e.g. shaft axis or tip-path-plane axis. Experimental data for rotors is usually presented with the steady flapping trimmed to zero ($\bar{\beta}_s = \bar{\beta}_c = 0$), in which case the shaft axis and tip-path-plane axis are aligned. An alternative dimensionless parameter can be defined that normalises the thrust, torque and power on the total blade area rather than the disc area and this emphasizes the loading on the blades [60]. Normalising with respect to the total blade area is achieved through the rotor solidity $\sigma$ since this represents the ratio of the blade area to disc area:

\[ \sigma = \frac{bc}{\pi R} \]

where $c$ is the mean blade chord. The parameter $C_T/\sigma$ is referred to as the blade loading coefficient.

### 5.4 Inflow Dynamics

This section presents the mathematical model for the calculation of the induced velocity generated by the rotor. The induced velocity developed in the production of the rotor forces and moments is extremely complex. Generally, only the induced velocity axially through the disc is modelled since it is significantly larger than the azimuthal and radial induced velocity components. Given the large design space and operating conditions of tiltrotor aircraft, the
induced velocity models used in this research are reduced-order and derived from actuator disc
theory owing to their lower computational cost compared to higher-order vortex wake models. These models are advantageous due to their finite-state representation and are easily integrated into aeromechanics codes [41]. The models are derived by treating the rotor as an infinitely thin disc of equivalent area, capable of sustaining a pressure discontinuity. Actuator disc theory considers the fluid flow from a global perspective rather than the intricate details of the vortex wake which is able to deform under self and mutual induced velocities.

Dynamic inflow is an unsteady actuator disc model that relates the induced inflow to the aerodynamic loading. Pitt [75] developed a perturbation model in disc-wind axes that was viable for both hover and forward flight and correlated well with experimental data [76]. Peters and HaQuang [44] present the theory of dynamic inflow, formulated in disc axes, using total loads rather than perturbation loads. Since the rotor loads in this work are total quantities, the model of Peters and HaQuang was used to model the inflow dynamics. This model is nonlinear in thrust dynamics but linear in roll and pitch dynamics.

The induced inflow ratio is expressed as a linear, first-harmonic Fourier series over the disc:

\[ \lambda_i(r, \psi) = \lambda_0 + r \lambda_s \sin \psi + r \lambda_c \cos \psi, \]  
(5.51)

where \( \lambda_i \) is the total induced inflow, \( r \) is the dimensionless radial ordinate, \( \psi \) is the azimuth angle, \( \lambda_0 \) is the mean induced inflow component and \( \lambda_s \) and \( \lambda_c \) are the lateral and longitudinal induced inflow components. The induced inflow representation is extended for the counter-rotating rotors by rewriting the lateral induced inflow component dependent on the shaft rotation direction

\[ \lambda_i(r, \psi) = \lambda_0 + r \lambda_s \sin n \psi + r \lambda_c \cos \psi \]  
(5.52)

where \( n \) is the rotor specifier and \( \psi \) is positive definite. Note, \( n \) is omitted in the cosine term since the cosine function is even. The quantities \( \lambda_0 \), \( \lambda_s \) and \( \lambda_c \) are the states of the induced velocity field and are dynamically related to the thrust, roll and pitch moments produced by the rotor. The dynamic inflow model is written in state-space form as

\[ [M] \ddot{\vec{x}}_\lambda + [L]^{-1} \vec{x}_\lambda = \vec{f}_\alpha, \]  
(5.53)

where \([M]\) is the apparent mass matrix, \( \vec{x}_\lambda \) is the inflow state vector, \([L]\) is the inflow gain matrix and \( \vec{f}_\alpha \) is the forcing vector. The subscript \( \alpha \) implies only the aerodynamic contributions are
included (omitting the inertial and spring loads). The inflow state vector is expressed as
\[
\vec{x}_\lambda = \{\lambda_0, \lambda_s, \lambda_c\}^T
\] (5.54)
and the forcing vector as
\[
\vec{f}_\alpha = \{C_T, C_L, -C_M\}^T
\] (5.55)
where \(C_T, C_L\) and \(C_M\) are the thrust, roll moment and pitch moment coefficients:
\[
C_T = \frac{T_D}{\pi \rho \Omega^2 R^4}
\] (5.56a)
\[
C_L = \frac{L_D}{\pi \rho \Omega^2 R^5}
\] (5.56b)
\[
C_M = \frac{M_D}{\pi \rho \Omega^2 R^5}
\] (5.56c)
The aerodynamic roll and pitch moments on the disc are found by resolving the blade flap moments into the relevant disc axes and summing over the blade count.

The mass matrix contains the apparent mass and inertia of an impermeable disc and models the time delay in the rotor response due to the unsteady wake. It is given by
\[
[M] = \text{diag}\{M_1, M_2, M_2\}
\] (5.57a)
where the mass matrix elements \(M_1\) and \(M_2\) are given by
\[
M_1 = \frac{128}{75 \pi}
\] (5.57b)
\[
M_2 = \frac{16}{45 \pi}
\] (5.57c)
The induced inflow gain matrix \([L]\) is written in terms of the mass flow matrix \([V]\) and wake-skew gain matrix \([\xi]\) as
\[
[L]^{-1} = [V][\xi]^{-1}
\] (5.58)
where the elements of \([\xi]\) depend only on the skew angle of the rotor wake from the shaft \(\chi\). In the original theory of Pitt [75], the wake-skew matrix was presented in a disc-wind coordinate system and is denoted by \([\xi_{WD}]\). The method of Peters and HaQuang is written in the disc
coordinate system and transforms the wake-skew gain matrix as

\[ [\xi] = [T][\xi_{WD}][T]^{-1} \]  

(5.59a)

where the transformation matrix \([T]\) is given by

\[
[T] = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \Delta & \sin \Delta \\
0 & -\sin \Delta & \cos \Delta
\end{bmatrix},
\]  

(5.59b)

and \(\Delta\) is the disc-wind angle from Section 5.2.

The matrices \([V]\) and \([\xi_{WD}]\) are defined in terms of the advance ratio ratio \(\mu\), inflow ratio \(\lambda\), thrust-induced inflow ratio \(\lambda_{iw}\) and wake skew angle \(\chi\). The wake skew angle relative to the tip-path-plane axis is given by

\[ \chi = \text{atan} \left( \frac{\mu}{|\lambda|} \right) \]  

(5.60)

where the absolute value of the inflow ratio is used because the mass flow must be positive in actuator disc theory [44]. The mass-flow matrix is given by

\[ [V] = \text{diag}\{V_1, V_2, V_2\} \]  

(5.61a)

where the elements \(V_1\) and \(V_2\) are given by

\[ V_1 = \sqrt{\mu^2 + \lambda^2}, \]  

(5.61b)

\[ V_2 = \frac{\mu + \lambda (\lambda + \lambda_{iw})}{V_1} \]  

(5.61c)

The term \(V_1\) represents the resultant velocity at the disc and \(V_2\) is the weighted downstream velocity. The thrust-induced inflow is given by

\[ \lambda_{iw} = \frac{1}{2} \{1, 0, 0\}^T [\xi]^{-1} \bar{x}_\lambda \]  

(5.62)
Finally, the wind wake skew matrix is given by

\[
\begin{bmatrix}
\frac{1}{2} & 0 & -\frac{15\pi \kappa}{64} \\
0 & 2(1 + \kappa^2) & 0 \\
\frac{15\pi \kappa}{64} & 0 & 2(1 - \kappa^2)
\end{bmatrix}
\]  

(5.63)

where $\kappa = \tan(\chi/2)$. The thrust-induced inflow ratio and wake skew angle are implicitly related and, therefore, are solved iteratively at each time step. This completes the nonlinear inflow model of Peters and HaQuang.

### 5.5 Gimbaled Rotor Dynamics

This section presents one of the contributions of this thesis and derives the equations of motion for a gimbaled rotor. Padfield [14] presented the first analysis of a gimbaled rotor in which the plane of rotation is free to tilt in space, however, the arrival at the equations of motion was based on several assumptions that permitted an analytical treatment of the motion and were valid only for helicopter flight. The equations of motion derived here are for arbitrary operating conditions that satisfy the assumptions invoked and are applicable to the entire conversion domain.

The blades on gimbaled rotors are attached to the rotorhead without individual flap or lead/lag hinges [61]. Therefore, the motion of the gimbaled rotorhead is coupled to the inertial and aerodynamic loads from all $b$ blades, as well as any hub spring loads present. The blades are very stiff and can be well-approximated as rigid. The derivation of the equations of motion follows similarly from Padfield [14], however, is extended to arbitrary operating conditions through transitioning flight. The difference in gimbaled rotor dynamics is the absence of the centrifugal moment restoring the blade to the hub plane. The gimbal is attached through a constant-velocity joint that transfers the input drive shaft speed to the output shaft. The rotor blades are attached to the output shaft and, therefore, the centrifugal force always acts radially outwards. In gimbal dynamics, the centrifugal moment is then only apparent if the blades are preconed. The gimbaled configuration reduces the Coriolis-induced loads since the individual blades do no flap towards or away from the shaft axis. Furthermore, without mechanical hinges the mass of the rotorhead and maintenance requirements are reduced compared to, for example, an articulated rotorhead.
The equations of motion for the gimbaled rotor are derived using a blade element approach by equating the aerodynamic, inertial and spring loads at the gimbal joint. First, consider a blade element at spanwise location $x_n$ from the centre of rotation. The out-of-plane moment exerted about the gimbal joint by a blade element on the $i^{th}$ blade is given by

$$dM_i = dM_\alpha + dM_m$$ (5.64)

where the subscripts $\alpha$ and $m$ denote the aerodynamic and inertial contributions. These contributions are given by

$$dM_\alpha = -nx dF_P,$$ (5.65a)

$$dM_m = nxma_P,$$ (5.65b)

where $x$ is the radial ordinate along the span, $dF_P$ is the out-of-plane aerodynamic force, $m$ is the mass per unit span and $a_P$ is the inertial acceleration of the element along the perpendicular direction (along $\hat{k}_{BL}$). For a point fixed in a rotating set of axes, the absolute acceleration may be calculated from the kinematic equation

$$\ddot{a} = \ddot{a}_0 + \dot{\omega} \times \ddot{p} + \ddot{\omega} \times (\dot{\omega} \times \ddot{p})$$ (5.66)

where $\ddot{a}_0$ is the linear acceleration of the origin of the rotating axes, $\dot{\omega}$ is the angular velocity of the rotating axes and $\ddot{p}$ is the position of a point in the rotating axes. In steady flight, the linear acceleration of the rotor is zero and, therefore, the term $\ddot{a}_0$ can be omitted. For the case of an articulated rotor with flapping hinges offset from the shaft axis, the linear acceleration of the flap hinge would not be zero, however, since the gimbal tilts about the shaft axis, the flap hinge is effectively located on the shaft. The out-of-plane component in Equation 5.66 is required and is expanded out by denoting the angular velocity and position vectors as

$$\ddot{\omega} = \omega_x \hat{i}_{BL} + \omega_y \hat{j}_{BL} + \omega_z \hat{k}_{BL}$$ (5.67a)

$$\ddot{p} = nx \hat{i}_{BL}$$ (5.67b)

to give the absolute out-of-plane acceleration of a blade element as

$$a_P = nx \left( \omega_x \omega_z - \Omega \omega'_y \right)$$ (5.68)

where $\Omega \omega'_y$ is time derivative of $\omega_y$. The angular velocity components are found by resolving
the angular velocity vector in Equation 5.19 into the blade-fixed axes:

\[ \omega_x = \Omega (\beta_p + \beta'_s \cos \psi - \beta'_c \sin \psi) \]  \hspace{1cm} (5.69a)
\[ \omega_y = -n\Omega (\beta'_s \sin \psi + \beta'_c \cos \psi) \]  \hspace{1cm} (5.69b)
\[ \omega_z \approx n\Omega \]  \hspace{1cm} (5.69c)

where it has been assumed that the shaft speed \( \Omega \) is an order of magnitude larger than other terms arising in \( \omega_z \). Differentiating \( \omega_y \) with respect to the azimuth angles gives the angular acceleration

\[ \omega'_y = -n\Omega [(\beta''_s + \beta'_c) \cos \psi + (\beta''_c - \beta'_s) \sin \psi] \]  \hspace{1cm} (5.70)

and the product of the angular velocities \( \omega_x, \omega_z \) is given by

\[ \omega_x \omega_z = n\Omega^2 \beta_p + n\Omega^2 (\beta'_s \cos \psi - \beta'_c \sin \psi) \]  \hspace{1cm} (5.71)

The absolute acceleration of the blade element in Equation 5.68 is then written as

\[ a_x = x\Omega^2 \beta_p + x\Omega^2 A \cos \psi + x\Omega^2 B \sin \psi \]  \hspace{1cm} (5.72)

where \( A = \beta''_c + 2\beta'_s \) and \( B = \beta''_s - 2\beta'_c \). The blade section moment about the gimbal joint is then

\[ dM_i = -nx dF_p + nmx^2 \Omega^2 \beta_p + nmx^2 \Omega^2 A \cos \psi + nmx^2 \Omega^2 B \sin \psi \]  \hspace{1cm} (5.73)

and the net out-of-plane moment is found by integration along the span axis:

\[ M_i = n \int_{nR_0}^{nR} \frac{dM_i}{dx} \, dx \]  \hspace{1cm} (5.74a)
\[ = -nM_\beta + nI\Omega^2 \beta_p + nI\Omega^2 A \cos \psi + nI\Omega^2 B \sin \psi \]  \hspace{1cm} (5.74b)

where \( I \) is the out-of-plane moment of inertia of the blade and \( M_\beta \) is the integrated flap moment:

\[ I = n \int_{nR_0}^{nR} mx^2 \, dx \]  \hspace{1cm} (5.75a)
\[ M_\beta = n \int_{nR_0}^{nR} x \frac{dF_p}{dx} \, dx \]  \hspace{1cm} (5.75b)

The tilting of the gimbal is derived with respect to the hub plane and is dependent on the out-of-plane moments from all \( b \) blades. The out-of-plane moment is resolved into the disc.
axes through Equation 5.4 to give the roll and pitch contributions:

\[ \sum_{i=1}^{b} (M_i \sin \psi_i) = 0 \]  
\[ n \sum_{i=1}^{b} (M_i \cos \psi_i) = 0 \]  

A hub spring of stiffness \( K \) maybe also be included in the rotor design and its load contribution can be added in the disc axes. Expanding out the flap moment in the disc axes and including the hub spring term gives

\[ \sum_{i=1}^{b} \left( -nM_{\beta} \sin \psi_i + nI\Omega^2 \beta_p \sin \psi_i + nI\Omega^2 A \cos \psi_i \sin \psi + nI\Omega^2 B \sin^2 \psi_i \right) + nK \beta_s = 0 \]  
\[ n \sum_{i=1}^{b} \left( -nM_{\beta} \cos \psi_i + nI\Omega^2 \beta_p \cos \psi_i + nI\Omega^2 A \cos^2 \psi_i + nI\Omega^2 B \sin \psi \cos \psi_i \right) + K \beta_c = 0 \]  

For a rotor with 3-or-more blades, the trigonometric sums in Equations 5.77a and 5.77b are simplified using the following trigonometric identities:

\[ \sum_{i=1}^{b} \sin \psi_i = \sum_{i=1}^{b} \cos \psi_i = \sum_{i=1}^{b} \sin \psi_i \cos \psi_i = 0 \]  
\[ \sum_{i=1}^{b} \sin^2 \psi_i = \sum_{i=1}^{b} \cos^2 \psi_i = \frac{b}{2} \]  

The terms \( nI\Omega^2 \beta_p, nI\Omega^2 A \) and \( nI\Omega^2 B \) are independent of the azimuth angle and, therefore, the simplified equations of motion are written as

\[ -n \sum_{i=1}^{b} (M_{\beta} \sin \psi_i)_i + nI\Omega^2 B^2 \frac{b}{2} + nK \beta_s = 0 \]  
\[ - \sum_{i=1}^{b} (M_{\beta} \cos \psi_i)_i + nI\Omega^2 A^2 \frac{b}{2} + K \beta_c = 0 \]  

Introducing the terms \( A = \beta''_c + 2\beta'_c \) and \( B = \beta''_s - 2\beta'_s \) back into Equations 5.79a and 5.79b and rearranging leads to the final equations of motion describing the the tilting of the gimbal:

\[ \beta''_s - 2\beta'_s + \frac{2K}{bI\Omega^2} \beta_c = \frac{M_s}{I\Omega^2} \]  
\[ \beta''_c + 2\beta'_c + \frac{2K}{bI\Omega^2} \beta_s = \frac{M_c}{I\Omega^2} \]
where $M_s$ and $M_c$ are the multi-blade coordinate definitions of the aerodynamic moments:

$$M_s = 2 \frac{b}{b} \sum_{i=1}^{b} (M_\beta \sin \psi)_i$$

(5.81a)

$$M_c = 2 \frac{b}{b} \sum_{i=1}^{b} (M_\beta \cos \psi)_i$$

(5.81b)

The pitch bearings of gimbal rotors are attached to both the swashplate and gimbal [77]. As the gimbal tilts an effective cyclic pitch change is applied such that the pitch angle of a blade section is given by:

$$\theta = \theta_w + \theta_0 + (\theta_s + \beta_c) \sin \psi + (\theta_c - \beta_s) \cos \psi$$

(5.82)

where $\theta_w$ is the in-built blade twist, $\theta_0$ is the collective pitch, $\theta_s$ is the longitudinal cyclic pitch and $\theta_c$ is the lateral cyclic pitch. This concludes the derivation of the equations of motion for the gimbal dynamics that are valid throughout transitioning flight.

## 5.6 Semi-Empirical Correction Factors

This section describes two correction factors that are used to improve the performance predictions of reduced-order models against experimental data. These correction factors are the tip loss factor and induced power correction, described next.

### 5.6.1 Tip Loss Factor

The loading of the rotor is concentrated towards the tip due to the increased dynamic pressure. Towards the tip, the bound circulation drops rapidly and, consequently, a strong vortex is trailed from the tip of the blade. As the bound circulation falls to zero at the tip, so does the lift force according to the Kutta-Joukowski theorem. Momentum-based blade element theory does not predict this loss of lift towards the tip and, therefore, the blade thrust is over-predicted. Vortex-based models are able to predict the bound circulation distribution along the span but are not implemented in this work due to a lack of empirical wake data and the computational cost of a free wake analysis. In this work, the loss of lift was accounted for
through the implementation of a tip loss factor. This method imposes an effective lifting span of the blade $BR$, where $B$ is the effective lifting fraction. Outboard of the effective span, $r > B$, the lift and induced drag coefficients are set to zero but the blade still produces profile drag.

Prandtl considered a rotor in axial flight with a two dimensional model of the rotor wake and derived the following tip loss factor for a low inflow rotor:

$$B = 1 - \frac{\sqrt{2C_T}}{b}$$

(5.83)

where $C_T$ is the thrust coefficient and $b$ is the blade count. Values calculated from Equation 5.83 typically give the tip loss factor between 0.95 and 0.98. This expression is not valid for forward flight when the vortex wake convects with the local convection velocity and is skewed relative to the disc. Furthermore, the thrust requirements change significantly in the transition between helicopter and aeroplane mode and the tip loss factor will not be constant throughout this transition. For simplicity in this work, a constant tip loss factor of $B = 0.97$ was implemented throughout all operating conditions which generally gives good agreement against experimental data [61].

### 5.6.2 Induced Power Factor

When using reduced-order models, computational savings are gained by using simplified induced velocity or aerodynamic models. The induced velocity distribution over the disc was expressed as a linear, first-harmonic Fourier series:

$$\nu(r, \psi) = \Omega R [\lambda_0 + r\lambda_s \sin n\psi + r\lambda_c \cos \psi]$$

(5.84)

When using simplified or prescribed induced velocity models, the induced power is typically empirically corrected to account for the nonuniform losses not captured from simplified theory. The induced power required to generate the thrust is given by

$$P_i = \sum_{i=1}^{b} \left( n \int_{nR_0}^{nR} \nu \frac{dT}{dx} \, dx \right)$$

(5.85)

where $\nu$ is the induced velocity and $dT/dx$ is the section thrust. The calculated power from the shaft torque $P = \Omega Q$ is corrected by adding a fractional increment of the induced power.
The corrected power is then given by

$$P_c = P + P_i (\kappa - 1) \quad (5.86)$$

where $\kappa$ is the induced power correction factor. The term $\kappa - 1$ is present since the induced power correction is typically presented relative to the calculated induced power, i.e $\kappa = 1.10$ represents a 10% increase in induced power. The induced power fraction is left unspecified for now and its value is determined in the validation effort described in Section 5.8.

## 5.7 Solution Methodology

The rotor loads are described by a state-space system consisting of the gimbal equations of motion and the dynamic inflow model that govern the spatial and temporal load distributions along the blades as well as the net rotor loads. In steady axial flight with no cyclic input, there is no 1/rev variation and the loads are constant with respect to the azimuth angle. However, when in non-axial flight or with cyclic control inputted, a cyclic forcing is introduced that excites the rotor into azimuthally dependent motion. The time dependent motion of the gimbal tilt and induced inflow states was integrated as a system of first-order ordinary differential equations after recasting the gimbal equations in Equations 5.81 as a system of four coupled equations. The gimbal state quantities are denoted by $\vec{x}_\beta = \{\beta_s, \beta_s', \beta_c, \beta_c'\}$ and the induced inflow state quantities are denoted by $\vec{x}_\lambda = \{\lambda_0, \lambda_s, \lambda_c\}$. The rotor state vector is then $\vec{x} = (\vec{x}_\beta^T, \vec{x}_\lambda^T)^T$. The system of state-space equations is nonlinear and expressed as

$$\frac{d\vec{x}}{d\psi} = \vec{f}(\psi, \vec{x}) \quad (5.87)$$

Several numerical integration techniques exist to integrate systems of equations, including the Euler method, predictor-corrector methods, Runge-Kutta methods amongst more. Several methods were tested but it was found the third-order Runge-Kutta scheme provided the required balance between the integration accuracy, azimuthal discretisation and computation cost (number of function evaluations per time step). The third-order Runge-Kutta method used
to integrate Equation 5.87 is given by:

\[ \vec{x}_{n+1} = \vec{x}_n + h \left( \vec{k}_1 + 4\vec{k}_2 + \vec{k}_3 \right) / 6, \]
\[ \vec{k}_1 = \vec{f} \left( \psi_n, \vec{x}_n \right), \]
\[ \vec{k}_2 = \vec{f} \left( \psi_n + h/2, \vec{x}_n + h\vec{k}_1 / 2 \right), \]
\[ \vec{k}_3 = \vec{f} \left( \psi_n + h, \vec{x}_n - h\vec{k}_1 + 2h\vec{k}_2 \right), \]

where \( h \) is the time step. The azimuth domain was discretised into \( N_\psi = 24 \) equally spaced points to give sufficient resolution of the \( b \)-per-rev loading in the nonrotating frame; the time step was therefore \( h = 2\pi/N_\psi \). In steady flight, the rotor states are periodic around the azimuth and represented mathematically as

\[ \vec{x}_{\psi=2\pi} - \vec{x}_{\psi=0} = 0. \]

The periodic solution was determined by solving Equation 5.89 using the damped Newton-Raphson scheme described in Chapter 3. A convergence criterion of \( \| \vec{x}_{\psi=2\pi} - \vec{x}_{\psi=0} \| \leq 10^{-6} \) was found to give sufficient accuracy of the induced inflow and gimbal motion to give convergence of the trim quantities in the rotor and flight mechanics module. This trim convergence criterion is comparable to that suggested in [65].

### 5.8 Performance Validation

As the rotor operates through helicopter, conversion and aeroplane mode it experiences a diverse aerodynamic environment and thrust requirements. In hover and vertical climb, the flow is axially through the disc. As the rotor enters forward flight, an in-plane velocity component is introduced that creates a one-per-rev variation of tangential velocity. When the rotor operates in propeller mode the flow is again axial but at a much larger inflow velocity. Given this large operating domain, it was necessary to validate the theoretical performance of the rotor model against experimental data through the operating domain. This section presents the validation effort of the rotor performance model through a range of operating conditions. A sensitivity study is also performed that increases the lift coefficient \( C_L \), profile drag coefficient \( C_{d_0} \) and induced drag coefficient \( C_{d_i} \) by 10% from the original data. The performance simulation is then rerun and the change in power relative to the original aerodynamic data is calculated. The sensitivity study is used to quantify how the uncertainties in the aerodynamic data affect
the predicted performance through different operating points. Validation of the predicted trim controls against experimental data is not presented due to a lack of available data in the literature and this remains as future work for the project.

5.8.1 XV-15 Blade Model

Experimental data for numerous tiltrotor-type systems are limited in the public domain largely for proprietary reasons. However, due to the research nature of the XV-15 rotor, experimental data exists for this rotor through hover, helicopter, conversion and propeller mode. The blades of the XV-15 rotor are highly-twisted with 45° of nonlinear washout twist from root to tip [57]. The original metal blade implemented five NACA 64-series aerofoils along the span; the root aerofoils were in the region of 30% thickness-to-chord ratio which tapered to an 8% thickness-to-chord ratio at the tip. These five aerofoil sections are detailed in Table 5.1. The compressible aerodynamic polars of these aerofoil sections are presented in [57], however, no reference angles of attack are detailed (the polars are of the form $C_d$ vs $C_l$ without any negative $C_l$ data) making them unsuitable for the rotor performance model developed here. Consequently, a model of the XV-15 blade was created using similar NACA 64-series aerofoil sections. This aerofoil family are designated according to NACA 64-xy, where $x$ is the design lift coefficient in tenths and $y$ is the thickness-to-chord ratio [78]. The model aerofoil sections were selected to best-match these quantities from the original blade and are detailed in Table 5.1. The table shows the outboard sections from $0.51R$ were similar to those used in the original design. However, from $0.09R$ to $0.51R$ the model aerofoils are significantly thinner and have a lower design lift coefficient. Some discrepancy between the experimental and model data is therefore likely due to the approximated aerofoil data although the contribution of the inboard loading is smaller than the outboard loading due to the proportionality to $r^2$.

Table 5.1: Blade model of the XV-15 rotor blade sections compared to the original sections from [79]. All sections are NACA 64-XXX series.

<table>
<thead>
<tr>
<th>Span Fraction</th>
<th>Original Aerofoil</th>
<th>Model Aerofoil</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>935</td>
<td>618</td>
</tr>
<tr>
<td>0.17</td>
<td>528</td>
<td>421</td>
</tr>
<tr>
<td>0.51</td>
<td>118</td>
<td>218</td>
</tr>
<tr>
<td>0.80</td>
<td>(1.5)12</td>
<td>212</td>
</tr>
<tr>
<td>1.00</td>
<td>208</td>
<td>208</td>
</tr>
</tbody>
</table>
The aerodynamic data for the selected aerofoils was taken from the incompressible experimental data presented in [80] at a Reynolds number of $6 \times 10^6$. This Reynolds number was chosen as it approximately matches the operating tip Mach number for the XV-15 rotor but is, therefore, not characteristic towards the root. This is a current limitation of the available aerodynamic data but more representative data can be easily facilitated with a new dimension in a lookup table. The experimental data for each aerofoil was presented in the low angle of attack region approximately between the positive and negative stall angles of attack. In forward flight, the freestream velocity in the plane of the disc is superimposed on the shaft speed contribution and this creates a region of reverse flow on the inboard retreating side, where the flow is from trailing edge to leading edge. This creates large inflow angles and, subsequently, large angles of attack. The high twist of the blade also contributes to local stall at some operating conditions [11]. For these reasons, the experimental data was extended into angle of attack domain $[-180^\circ, 180^\circ]$ using semi-empirical trigonometric functions [60]:

\[ C_l = A \sin 2\alpha \]  \hspace{1cm} (5.90a)
\[ C_d = B + C \cos 2\alpha \]  \hspace{1cm} (5.90b)

where $A$, $B$ and $C$ are coefficients. These coefficients were approximated from expressions in [81] for the maximum lift and drag coefficients in the post-stall region as a function of the thickness-to-chord ratio $t/c$ for a blade of infinite aspect ratio. These expressions were derived empirically from experimental data for wind turbine aerofoils and, therefore, serve only as a first-order approximation. The maximum post-stall lift and drag coefficients, $C_{l_{\text{stall}}}$ and $C_{d_{\text{stall}}}$, from [81] are given by:

\[ C_{l_{\text{stall}}} = 1.19 \left(1.10 - (t/c)^2\right) \]  \hspace{1cm} (5.91a)
\[ C_{d_{\text{stall}}} = 2.30 \exp\left(- (0.65 t/c)^{0.90}\right) \]  \hspace{1cm} (5.91b)

Assuming the maximum lift and drag coefficients in the post-stall region occur at $\alpha = 45^\circ$ and $\alpha = 90^\circ$, and the drag at $\alpha = \pm180^\circ$ is only profile drag, then the coefficients $A$, $B$ and $C$ are given by:

\[ A = C_{l_{\text{stall}}} \]  \hspace{1cm} (5.92a)
\[ B = \left(C_{d_0} + C_{d_{\text{stall}}}\right)/2, \]  \hspace{1cm} (5.92b)
\[ C = \left(C_{d_0} - C_{d_{\text{stall}}}\right)/2, \]  \hspace{1cm} (5.92c)

where $C_{d_0}$ is the profile drag coefficient determined at zero lift from the experimental data in the
low angle of attack domain. Discontinuities in the aerodynamic polars can cause convergence issues due to steep gradients. To overcome this, the low angle of attack and high angle of attack data was transitioned over a region of 10° angle of attack using a smooth spline. The lift and drag polars of the blade model are shown in Figures 5.7a and 5.7b. The aerodynamic data currently implemented was derived at a single Reynolds number for incompressible flow. It is recognised that improved performance predictions require a more comprehensive description of the aerodynamic data, ideally through 360° angle of attack, compressible Mach numbers and several representative Reynolds number in order to capture the effects of these important aerodynamic quantities.

![Lift and Drag Polars](image)

**Figure 5.7:** Blade model lift and drag coefficients through 360° angle of attack.

In numerical analysis, the blade span is discretised into a finite number of points and the aerodynamic forces and moments calculated at each point are numerically integrated to give the net loading on the blade. It is, therefore, necessary to perform a discretisation study in order to determine the number of points at which the integrated loading becomes independent of the discretisation. In rotary-wing analysis, the blade loads are concentrated towards the tip due to the proportionality of the dynamic pressure with $r^2$. To assess the required spanwise discretisation, a typical loading distribution is integrated with a different number of radial points and spanwise spacing methods. The typical loading function is given by

$$f(r) = r^2 \sqrt{1 - r^2}$$  \hspace{1cm} (5.93)

where $r$ is the dimensionless radial ordinate and is one of the solutions of the pressure shapes describing the loading on an actuator disc [13]. This function, shown in Figure 5.8a, was integrated for several types of spanwise spacing with differing numbers of spanwise points.
The spanwise spacing models included linear spacing, cosine-spacing and sine-spacing. The cosine spacing is commonly used in fixed-wing vortex analyses to cluster the number of the points towards the wingtips where the spanwise lift gradient is greatest \cite{67, 82}. Sine-spacing clusters the spanwise points only towards the tip where the loading is greatest. The results of the discretisation analysis are shown in Figure 5.8b and were used to conclude a sine-spacing with 30 spanwise points was a sufficient discretisation of the blade.

(a) Typical loading distribution along a rotor blade.  (b) Discretisation analysis showing the value of the loading integral as a function of the number of spanwise points and spanwise spacing.

**Figure 5.8:** Typical loading distribution of a rotor blade (a) and the spanwise integration results of the discretisation study (b).

### 5.8.2 Hover

Felker, Betzina, and Signor \cite{79} presented experimental performance data for a full-scale XV-15 rotor in hover that was operated at several tip Mach numbers. The thrust and torque were reported to be accurate to within \( \pm 50\text{N} \) and \( \pm 70\text{Nm} \), corresponding to accuracies within 0.1\% and 0.3\%, respectively. Only experimental data for a single tip Mach number is presented here since its influence was observed to be small during the test. In hover, the efficiency of the rotor is measured through the figure of merit: the ratio of ideal power to actual power. The figure of merit is given by

\[
FM = \frac{1}{\sqrt{2}} C_{T}^{3/2} \frac{C_{P}}{C_{P}}
\]  

(5.94)

where \( C_{T} \) and \( C_{P} \) are the thrust and power coefficients. The rotor power is given by the torque-predicted power plus the additional induced power correction:

\[
C_{P} = C_{Q} + (\kappa - 1)C_{P_{i}}
\]  

(5.95)
Figure 5.9a shows the theoretical figure of merit as a function of the blade loading coefficient against the experimental data. Several induced power factors are also presented to illustrate their effect on the predicted figure of merit. The correlation of the predicted figure of merit behaviour with blade loading was well captured up to $C_T/\sigma = 0.14$, however, the figure of merit tends to be over-predicted compared to the experimental data and is indicative of the power being under-predicted for a given thrust setting. The difference between the datasets is not attributable to experimental error given the high accuracy of the balance system. The difference between the predicted and experimental power was found to be largest at low blade loadings coefficient and showed an under-prediction of 50% at the lowest thrust setting. At these low thrusts, this would generally suggest that the profile power was being under-predicted. However, the necessary twist of tiltrotor blades is significantly different from helicopter blades with more in-built twist required for propeller mode operation. Therefore, low blade loadings do not necessarily imply that a small thrust is created along the blade and instead is a balance of positive and negative thrust forces being generated [29]. Figures 5.9b and 5.9c show the spanwise thrust and power distributions at several blade loadings. In the figure, the sharp change in the loading towards the tip is due to the imposed tip loss factor. It can be seen that at lower blade loadings, the loading on the outboard sections was negative and these sections contributed an accelerating torque (negative power consumption). This loading distribution was due to the negative angles of attack towards the outboard region, as shown in Figure 5.9d, that generated negative lift forces. As the blade loading increased, the negative tip loading reduced and past $C_T/\sigma = 0.10$, the resultant section force contributed fully towards the thrust and deceleration of the blade. Figure 5.9d shows that towards the blade root, high angles of attack in the region of $15^\circ$ - $20^\circ$ were found which is consistent with [52] and is indicative of a region in/near stall.
Theoretical figure of merit as a function of the blade loading coefficient against the experimental data presented in [79]. Several induced power factors are also presented.

Spanwise thrust distribution at several blade loadings.

Spanwise power distribution at several blade loadings.

Spanwise angle of attack distribution at several blade loadings.

Figure 5.9: Performance validation for the rotor operating in hover.

The under-predicted rotor power at a given thrust was most likely attributable to the induced velocity model. In hover and axial flight cases, the three-state inflow model of Peters and HaQuang [44] reduces to uniform inflow since no roll and pitch moments are generated. The deficiency in the uniform induced inflow model is that it does not capture the induced velocity at a local level and instead, only gives the global performance from an actuator disc perspective. This lack of local flowfield information is also the reason for the drop in the experimental figure of merit past $C_T/\sigma = 0.14$ was not predicted due to stalled blade sections. Therefore, an improved wake model that allows for the spanwise variation of the induced velocity is required for hover predictions. This could have been incorporated using blade element momentum theory applied to an annularly discretised disc. However, this model is not well-suited for...
forward flight and relies on the azimuthal averaging of the section loads or applying the blade element momentum theory at each time step. The latter can become computationally inhibitive for large domain investigations. Furthermore, due to the time dependency of the rotor loads and the gimbal motion in forward flight, the three-state dynamic inflow model was retained.

When using a uniform induced velocity model, the induced power is usually corrected for nonuniform induced inflow losses by multiplying the induced power by an empirical correction factor. For low twist helicopter rotors, the value of the correction factor is typically 1.10 to 1.15, however, for highly twisted tiltrotor blades this is likely to be higher with Johnson [11] suggesting 1.15 - 1.17 to correlate with empirical data. The effect of the induced power factor on the predicted figure of merit is shown in Figure 5.9a. It should be noted that using a single correction factor is not representative of all operating points and this is evidence by the small effect of the induced power correction factor at low blade loadings. In the validation of the NASA Design and Analysis of RotorCraft (NDARC) code the induced power correction varies nonlinearly with operating mode, blade loading and airspeed [83]. The correction factor is a simple means to match the theoretical power to the experimental power when using reduced-order models of the rotor. The induced power factor in Figure 5.9a is shown to improve the prediction of the peak figure of merit that occurs around $C_T / \sigma = 0.10$, however, at low blade loadings makes little difference to the predicted figure of merit due to the small induced power contribution at this conditions.

The aerodynamic data for the blade was derived for similar aerofoils and, therefore, the discrepancy between the theoretical and measured rotor power could to some extent be attributed to this. The sensitivity of the predicted power to the section lift, profile drag and induced drag coefficients was found by perturbing each of the coefficients and recalculating the rotor power. The sensitivity of the predicted power to the section coefficients is shown in Figure 5.10. The figure shows a 10% increase in the section profile drag resulted in approximately a 5.5% increase in the rotor power at the lowest blade loading and decreased rapidly as the blade loading increased. This rapid decrease was due to the dominance of the induced power contribution with increasing blade loading. A similar change in power was observed with the induced drag coefficient. The perturbation of the lift coefficient at low blade loadings was found to marginally increase the predicted power. Since the lift was increased, it would have been expected that the required power for a given thrust decreased. However, since the outboard portion of the blade was negatively loaded, a larger portion of negative thrust was produced and, therefore, only a small change in the predicted power was found. Past a blade loading of $C_T / \sigma = 0.04$, the change in power showed a less than 1% reduction in the predicted
power. Due to the relatively small change in the predicted power, it can be concluded that the deficiencies in the theoretical model were largely due to the wake model.

![Figure 5.10: Sensitivity of the theoretical hover power to a 10% perturbation in the section lift coefficient, profile drag coefficient and induced drag coefficient.](image)

5.8.3 Propeller Mode

Experimental data for the rotor operating in propeller mode at several airspeed ratios were presented in [57]. The error in the balance system was reported to be within ±3%. In propeller mode, the efficiency of the rotor to produce useful thrust is the propulsive efficiency: the ratio of useful power to actual power. The propulsive efficiency is defined as

\[
\eta = \mu_{\infty} \frac{C_T}{C_P}
\]

where \( \mu_{\infty} = \frac{V_{\infty}}{\Omega R} \) is the airspeed ratio and \( C_T \) and \( C_P \) are again the thrust and power coefficients. Figure 5.11a shows the predicted propulsive efficiency as a function of the blade loading and airspeed against experimental data. The predicted efficiency was found to be largely insensitive to airspeed. The correlation between the two data sets agrees well overall, however, the propulsive efficiency was generally over-predicted through the blade loading range considered. Some discrepancy could be attributed to the uncertainty with the experimental data. The over-predicted propulsive efficiency implied that the theoretical power was under-predicted for a given thrust. The spanwise thrust, power and angle of attack
distributions for several blade loadings are shown in Figures 5.11b to 5.11d. Due to the similar efficiencies with airspeed ratio, only the loading and angle of attack distributions for $\mu_\infty = 0.341$ is shown. These figures show that approximately 50% of the span operated in an angle of attack regime that resulted in negative loading and contributed an accelerating torque. The predicted thrust and torque distributions agreed with those presented in [53]. The negative loading on the inboard sections was caused by the large inflow angles towards the blade root that arose from a combination of high forward speed and low rotational speed. At higher airspeeds, the induced velocity is small compared to the freestream $\lambda_i \ll \mu_\infty$ and the induced velocity can be well-approximated as uniform. This implies the local flow angles are reasonably well approximated and, therefore, differences in the aerodynamic data are attributed to the discrepancies between the theoretical and experimental power.

(a) Theoretical and experimental efficiency as a function of blade loading and airspeed.  
(b) Spanwise thrust distribution at several blade loadings for $\mu_\infty = 0.341$.  
(c) Spanwise power distribution at several blade loadings for $\mu_\infty = 0.341$.  
(d) Spanwise angle of attack distribution at several blade loadings for $\mu_\infty = 0.341$.  

Figure 5.11: Performance validation for the rotor operating in propeller mode.
The sensitivity of the predicted power to a 10% perturbation in the sectional lift, profile drag and induced drag coefficients is shown in Figure 5.12. The sensitivity of the predicted power was most pronounced at low blade loadings and acted to increase the required power. The peak change in predicted power from the profile and induced drag coefficients was around 7.1% and 2.9% and decreased quickly with increasing blade loading. Like the sensitivity study in hover, the perturbation of the lift coefficient showed an increase in rotor power for lower blade loadings. This was, again, due to the increased negative loading that acted to increase the required power. The predicted power was found to be fairly insensitive to perturbations in the aerodynamic data. Therefore, the discrepancy between the theoretical and experimental power could be attributed to the experimental data or a combination of effects in the aerodynamic data (stall model, compressibility and/or Reynolds number).

Figure 5.12: Sensitivity of the theoretical power in propeller mode to a 10% perturbation to the section lift coefficient, profile drag coefficient and induced drag coefficient.

5.8.4 Helicopter and Early Conversion Mode

Betzina [84] presented an experimental study of a full-scale XV-15 rotor in helicopter and early conversion mode. The experimental study reported the rotor power as a function of airspeed ratio, shaft angle of attack $\alpha_s$ and blade loading. The shaft angle of attack is defined as $\alpha_s = 0^\circ$ in propeller mode forward flight and $\alpha_s = 90^\circ$ in helicopter mode forward flight. The theoretical and experimental data presented here is for two airspeed ratios, $\mu_\infty = 0.125$
and $\mu_\infty = 0.170$, covering the lower and higher airspeeds of the test. The rotor model was configured to the parameters detailed in [84]. The standard deviations of the experimental thrust, in-plane force and torque were reported to be accurate to within 111 N, 31 N and 14 Nm. Figure 5.13 shows the theoretical power as a function of blade loading and shaft angle of attack for both airspeed ratios. No correction to the induced power has been made; $\kappa = 1.00$. The figures show that the behaviour of the rotor power was well captured throughout the operating points considered, however, the magnitude of the power was consistently under-predicted. Furthermore, the divergence of the power at higher blade loadings was not well captured. The latter was likely due to the prescribed three-state induced velocity field that did not accurately predict the local flowfield. Therefore, a higher-order wake model such as the Peters-He model [85] that retains the state-space representation is required for improved predictions. The Peters-He wake model is a generalised finite-state model that includes both radial and azimuthal states [86] and, therefore, offers an improved flowfield description compared to prescribed induced inflow distribution of the Peters and HaQuang [44] model.

![Figure 5.13](image-url)

**Figure 5.13:** Theoretical and experimental power in helicopter and early conversion mode as function of blade loading, shaft angle of attack and airspeed ratio. No correction to the induced power has been made.

The predicted performance can also be empirically corrected through the induced power factor to improve the correlation with experimental data. Johnson [11] suggests setting the induced power factor to $\kappa = 2$ for helicopter forward flight, however, the airspeed and shaft angles for ‘helicopter forward flight’ were not defined. A more comprehensive overview of the induced power factors used for tiltrotor-type rotors is presented in [83] for hover, climb, edgewise flight and propeller mode as a function of blade loading. Based on the rotor and aerodynamic model presented here, an induced power factor of $\kappa = 1.30$ is suggested for
improved correlation with the experimental data at these operating conditions.

\[ \mu_\infty = 0.125 \]

\[ \mu_\infty = 0.170 \]

**Figure 5.14:** Theoretical and experimental power in helicopter and early conversion mode as function of blade loading, shaft angle of attack and airspeed ratio. An induced power factor of \( \kappa = 1.30 \) has been added.

The predicted rotor loading for all operating points was similar and Figure 5.15 illustrates the typical radial thrust, power and angle of attack distributions at different azimuth angles for a single operating point: \( \mu_\infty = 0.125 \), \( \alpha_s = 75^\circ \) and \( C_T/\sigma = 0.10 \). Figure 5.15c shows that the angle of attack over the advancing side of the disc was in the region of \( 20^\circ \) towards the blade root and was indicative of being in/near stall. Furthermore, large negative angles of attack were found on the retreating side of the disc in the reverse flow region and, therefore, deficiencies in the stall model could have contributed to the under-predicted power over the operating conditions considered. However, the dynamic pressure at these inboard regions is small and, as shown in Figure 5.15b, contributions only a small proportion to the total power. Due to the higher dynamic pressure at the outboard sections, the under-predicted power was largely attributable to incorrect loading predictions near the tip. Figure 5.15a shows that outboard of approximately \( 0.85R \) over the advancing side of the disc the blade was negatively loaded. This negatively loaded region also contributed a significant accelerating torque which may, if incorrectly predicted, have caused a significant under-prediction of the rotor power. The incorrect loading towards the blade tip may be due to the aerodynamic data or the prescribed induced inflow distribution, or both due to their implicit relationship. The negatively loaded tip agrees with that predicted in [54] in helicopter mode. Towards higher blade loadings, the larger collective pitch input reduced the negatively loaded tip region, however, a small region was still present.
The sensitivity of the theoretical power to a 10% perturbations of the lift, profile and induced drag coefficients is shown in Figure 5.16. The perturbation of the lift coefficient was found to give a net reduction of the predicted rotor power by approximately 1.0% - 2.5%. This was due to the increased lift that acted predominately in the conventional thrust direction. The perturbation in the profile drag coefficient was more pronounced at lower blade loadings due to the lower induced power. Additionally, a higher percentage change in the predicted rotor power from the profile drag perturbation was found at higher shaft angles of attack and airspeed ratios due to the freestream component in the plane of the disc being largest at these conditions. The induced drag coefficient followed a similar trend to the profile drag coefficient perturbation, however, produced a smaller increase in the predicted rotor power. The sensitivity of the predicted power is small overall and the under-prediction of the rotor power throughout the operating domain is, therefore, largely attributable to the wake model.

Figure 5.15: Spanwise performance data at several azimuth angles at \( \mu_\infty = 0.125 \), \( \alpha_s = 75^\circ \) and \( \mathcal{C}_T/\sigma = 0.10 \).
Figure 5.16: Sensitivity of the theoretical power to perturbation in the lift coefficient (solid line), profile drag coefficient (dashed line) and induced drag coefficient (dotted line).

5.8.5 Conversion Mode

Johnson [11] presented experimental data for the XV-15 rotor operating at four shaft angles of attack that constituted conversion mode: \( \alpha_s = 15^\circ, 30^\circ, 60^\circ \) and \( 75^\circ \). Unfortunately, the data was only presented for a single airspeed ratio, \( \mu_\infty = 0.32 \). With respect to the XV-15 conversion corridor, this airspeed represents around 140kn and lies towards the max-speed boundary nearer the start of the conversion and within the middle of the corridor in aeroplane mode. Figure 5.17 shows the predicted and experimental power as a function of blade loading and shaft angle of attack. Overall, the correlation of the data sets is good with the largest discrepancy at \( C_T/\sigma = 0.04 \) at \( \alpha_s = 75^\circ \). The radial thrust and power distributions at several azimuth angles are shown in Figure 5.18 for this operating point. A significant portion of the blade on the advancing side is seen to be negatively loaded and the under-predicted power is likely due to the poor power predictions in this region emanating from the wake model as in Section 5.8.4.
Towards propeller mode, the predicted power agreed well with the experimental data throughout the blade loading range for $\alpha_s = 30^\circ$. However, at a shaft angle of attack of $\alpha_s = 15^\circ$ the power was consistently over-predicted. The over-predicted power could again be attributed to deficiencies in the aerodynamic data or uncertainties in the experimental data. This is further highlighted by the over-predicted power at $15^\circ$ shaft angle of attack compared to the under-predicted power in propeller mode from Section 5.8.3 at a similar speed. A sensitivity study is
not presented for this case as the results follow very closely from those presented in helicopter and propeller modes.

5.9 Summary

This chapter has presented the aerodynamic module of a rotor component. A kinematic description of the blades has been described that includes a novel formulation of the gimbal dynamics. The aerodynamic loads were found from spanwise integration of the section loads and summed over the blade count. The three-state Peters and HaQuang was used to model the induced inflow at all operating conditions. Two semi-empirical correction factors were incorporated into the rotor model to improve the performance predictions; a tip loss factor and induced power factor.

A validation study of the XV-15 rotor model has been presented in helicopter, conversion and propeller mode. The rotor blade was modelled using experimental data for similar aerofoil sections coupled with empirical stall models. Overall, the predicted performance of the rotor compared well to experimental data but was generally found to under-predict the required power. The predicted power could be improved using either an induced power factor or improving the wake model. The latter emanates from the complex loading of the blade towards helicopter mode consisting of positive and negative stall regions and negative loading on the advancing side. The sensitivity of the predicted power to the section aerodynamic data was presented and showed the predicted power was fairly insensitive to the data overall. However, the rotor loads and induced inflow are implicitly related and, therefore, the predicted loads be more sensitive to the aerodynamic using a different wake model. No validation of the trim control angles was presented due to a lack of available and this should be undertaken in future work.
This chapter presents the aerodynamic module of the wing. The aerodynamic module is used to determine the force and moment contributions that are substituted into the equations of motion in Chapter 3. The start of this chapter details the kinematic description of the wing components and how its orientation and geometry are established. Thereafter, the implementation of the aerodynamic data is detailed to determine the forces and moments produced on the aircraft. The wing aerodynamics can also be heavily affected by the rotor wake in certain flight conditions and, therefore, the interaction model used to quantify this effect on the aeromechanic behaviour is presented.

6.1 Kinematics

The aerodynamic forces and moments produced by the fixed-wing components are derived from strip theory, analogous to blade element theory for rotary-wing analysis. The aerodynamic forces and moments are modelled as point loads acting at the aerodynamic centre of the section. The orientation of the lifting-line describing the fixed-wing is arbitrary and allows for both in-plane and out-of-plane displacements arising from sweep and dihedral angles. The lifting-line of the wing is defined in a Cartesian coordinate system, termed the wing frame and denoted by $O_{Wxyz}$, whose axes are initially parallel to the body axes. It is assumed the wing is symmetric about the wing $O_{Wxz}$ in order to simplify the kinematics. The origin of the wing frame is located at the wing root aerodynamic centre (quarter-chord) and its position is supplied as input data in the wing configuration file. The location used in this work for the GTRS model of
the XV-15 is detailed in Table 3.2. The unit vectors of the frame are denoted by \( \hat{i}_W \), \( \hat{j}_W \) and \( \hat{k}_W \). In this frame, the wing spans parallel to \( \hat{j}_W \) and the aerofoil sections lie in the plane \( O^Wxz \).

Now consider the effect of wing dihedral. In this work, the dihedral angle is treated as a rotation about the wing \( x \)-axis. The dihedral effect rotates the orientation of the aerofoil plane relative to the \( O^Wxz \)-plane. The dihedral angle \( \Gamma \) is measured from the wing frame \( y \)-axis and is taken positive for the starboard wing upwards, as shown in Figure 6.1. In this work, the dihedral geometry is simplified by assuming a constant value along the span. More complex geometries such as box and gull wings cannot, therefore, be modelled but current tiltrotor designs employ simpler straight wings. Treating the dihedral as a rotation introduces some complexity into the kinematics due to the opposing rotation directions on the starboard and port side. However, since the wing has been assumed to be symmetric the dihedral angle rotation for either side of the wing can be generalised as

\[
\gamma = \text{sgn}(\bar{y})\Gamma
\]

where \( \text{sgn} \) is the sign operator, \( \bar{y} \) is the spanwise location along \( \hat{j}_W \) and \( \Gamma \) is the dihedral angle of the starboard wing.

![Figure 6.1: Dihedral geometry of the wing showing the positive definition of the dihedral angle.](image)

Now define a Cartesian coordinate system that is fixed to the aerodynamic centre of a wing section. This frame is denoted \( O^Cxyz \) and termed the chord axes. The unit vectors of the coordinate system are denoted by \( \hat{i}_C \), \( \hat{j}_C \) and \( \hat{k}_C \) and are initially parallel to the wing axes. The
The position vector of a point on the wing, accounting for both the dihedral rotation and sweep angle shear, is given by

\[ \vec{p} = -|\gamma| \Lambda \hat{i}_W + \gamma \hat{j}_W - |\gamma| \Gamma \hat{k}_W \]  

(6.4)

where the absolute value of the spanwise location is used due to the symmetry of the wing.
The calculate of the aerodynamic moments about the centre of gravity requires the position of the aerofoil section relative to the body-fixed origin. The axes of the wing and body frames are parallel and, therefore, only the offset between the axes needs to be considered. The position of the centre of gravity and wing origin are supplied in the operating point file and the configuration file of the wing component. Both positions are given in the aircraft axes and are transformed to the give the relative position vector in the body-fixed axes:

\[
\vec{p} = \begin{cases} 
-(l_W - l_{CG}) - |y| \Lambda \\
|y| \\
-(h_W - h_{CG}) - |y| \Gamma 
\end{cases} 
\]

(6.5)

where \(l\) and \(h\) are the station and waterline coordinates of the subscripted quantities.

The aerodynamic forces and moments produced by a wing section are proportional to its dynamic pressure and, therefore, its velocity. The velocity kinematics are now described. In the general case of aircraft motion, the aircraft is free to translate and rotate such that the absolute velocity of the wing strip is given by the kinematic relationship:

\[
\vec{u} = \vec{u}_0 + \vec{\omega} \times \vec{p} 
\]

(6.6)

where \(\vec{u}_0\) is the translational velocity vector, \(\vec{\omega}\) the rotational velocity vector and \(\vec{p}\) is the position vector relative to the centre of gravity. For the steady trim problem considered here, the rotational velocity is \(\vec{\omega} = \vec{0}\) and, therefore, the absolute velocity of the wing strip is simply the translational velocity vector \(\vec{u}_0\). To account for aerodynamic interaction between various components, the aerodynamic velocity vector, \(\vec{u}_\alpha\), is defined as the sum of the absolute velocity

\[
\vec{u}_\alpha = \vec{u}_0 + \vec{\omega} \times \vec{p} 
\]
vector and the interaction velocity vector $\vec{u}_i$:

$$\vec{u}_\alpha = \vec{u} + \vec{u}_i$$  \hspace{1cm} (6.7)

This method assumes the interactional velocities can be superimposed onto the freestream which is a significant simplification of the real flowfield. In reality, the rotor wake is extremely complex with an unsteady and vortical nature that is related to the loading along the blades and the wake itself. However, to make the interaction problems tractable and applicable to an aeromechanics analysis, the complex interactions are simplified using reduced-order theory and/or empirical models [13, 14], as done so here. The interactional velocity vector is expressed in the body-fixed axes as

$$\vec{u}_i = u_i \hat{i}_B + w_i \hat{k}_B$$  \hspace{1cm} (6.8)

where the lateral interaction velocity has been omitted due to the steady-level flight conditions considered in this work. In general, the interaction velocities can be a function of the spanwise wing position. The quantification of the interaction vector is detailed in Section 6.3. The velocity of a wing section in the local chord axes is found by rotating the body-fixed aerodynamic velocity vector through the dihedral angle $\gamma$. The aerodynamic velocity vector expressed in the local chord axes is denoted by

$$\vec{u}_\alpha = V_x \hat{i}_C + V_y \hat{j}_C + V_z \hat{k}_C$$  \hspace{1cm} (6.9)

Transforming the body-fixed aerodynamic velocity vector into the chord axes gives the directional components:

$$V_x = u + u_i$$  \hspace{1cm} (6.10a)

$$V_y = -\gamma [w + w_i]$$  \hspace{1cm} (6.10b)

$$V_z = w + w_i$$  \hspace{1cm} (6.10c)

These velocity components can then be used to determine the aerodynamic forces and moments produced by each aerofoil section.
6.2 Aerodynamic Forces and Moments

The aerodynamic forces and moments on the aircraft are now calculated. The section lift, drag and pitching moment produced by the aerofoil are given by:

\[
dL = \frac{1}{2} C_l \rho V^2 c \, dy
\]

\[
dD = \frac{1}{2} C_d \rho V^2 c \, dy
\]

\[
dM = \frac{1}{2} C_m \rho V^2 c^2 \, dy
\]

where \( C_l \) is the lift coefficient, \( C_d \) is the drag coefficient, \( C_m \) is the moment coefficient, \( \rho \) is the air density, \( V \) is the resultant velocity, \( c \) is the chord length. The coefficients \( C_l, C_d \) and \( C_m \) are tabulated functions of angle of attack and Mach number. The angle of attack of the section is defined from the velocity components in the plane \( O^Cxz \), as shown in Figure 6.3, and is given by

\[
\alpha = i_0 - i_w + \tan^{-1} \left( \frac{V_z}{V_x} \right)
\]

where \( i_0 \) is the setting angle at the wing root (\( \gamma = 0 \)) and \( i_w \) is the washout angle with respect to the span. The angle of attack is wrapped to the domain \([-180^\circ, 180^\circ]\) to comply with the lookup table. It is conventional for the wing twist to washout towards the tip to ensure the root sections stall first and roll control is still attainable. In terms of commonly-adopted fixed-wing nomenclature, the angle of attack in Equation 6.12 would be considered the effective angle of attack. The geometric angle of attack would be obtained by omitting the interaction component in \( \tan^{-1}(V_z/V_x) \). The induced angle of attack can be considered as the interaction component in \( \tan^{-1}(V_z/V_x) \), however, is not strictly the induced angle of attack used in fixed-wing nomenclature since a model of the wake is not included here (e.g. a horseshoe vortex model).
In strip theory, the spanwise velocity component $V_y$ is generally neglected as the adjacent wing sections are assumed to operate independently to each other. The effect of wing sweep is to produce a spanwise flow component that does not accelerate over the chord. The resultant velocity of the section is then defined normal to the leading edge. Resolving the velocity components $V_x$ and $V_y$ normal to the leading edge gives $V_x \cos \Lambda - V_y \sin \Lambda$, which, for small $\Lambda$ is approximately $V_x - V_y \Lambda$. The resultant velocity of the section in a plane normal to the leading edge is then

$$V = \sqrt{(V_x - V_y \Lambda)^2 + V_z^2}.$$  

(6.13)

The section Mach number is then found from

$$M = \frac{V}{c_0}$$  

(6.14)

where $c_0$ is the speed of sound determined from the altitude defined at the operating point using an ISA model. In this formulation of simple sweep theory, no corrections are made to the section angle of attack, chord length or resultant drag force direction.

The angle of attack and Mach number are used to linearly interpolate a supplied lookup table of compressible aerodynamic data. A lookup table is supplied for each predefined flap/flaperon setting for the aircraft model. The flap/flaperon setting at the given operating point is detailed in the operating point file that is read by the flight mechanics module and aerodynamics module of the wing to interpolate the correct lookup table. The lift and drag forces of the section are now rotated into the body-fixed axes for substitution in the equations of motion. The section forces are defined parallel and perpendicular to $\sqrt{V_x^2 + V_z^2}$ as shown in
Figure 6.3. The spanwise force component is zero due to the assumed two-dimensionality of the strip theory. The section forces are rotated into the body-fixed axes through the local wind angle

$$\phi = \tan^{-1} \frac{V_z}{V_x}$$  \hspace{1cm} (6.15)

and dihedral angle $\gamma$ to give the contributions to the body forces as:

$$dX = dL \sin \phi - dD \cos \phi$$  \hspace{1cm} (6.16a)

$$dY = -\gamma(dL \cos \phi + dD \sin \phi)$$  \hspace{1cm} (6.16b)

$$dZ = - (dL \cos \phi + dD \sin \phi)$$  \hspace{1cm} (6.16c)

Due to the symmetry of the problem considered here, the lateral force on the starboard and port wings will cancel and, therefore, the $Y$ force is dropped herein. The net forces produced by the wing are then found by integrating along the span axis ($\hat{j}_W$) from tip-to-tip:

$$X = \int_{-b}^{b} \frac{dX}{dy} dy$$  \hspace{1cm} (6.17a)

$$Z = \int_{-b}^{b} \frac{dZ}{dy} dy$$  \hspace{1cm} (6.17b)

where $b$ is the semi-span of the wing.

The aerodynamic moments about the centre of gravity are found in a similar manner. The total moment produced by a wing section about the centre of gravity contain the contributions from both the lift and drag forces, and the section pitching moment. The moment produced by a wing section about the centre of gravity is found from

$$d\vec{M} = \vec{p} \times d\vec{F} + d\vec{M}_s$$  \hspace{1cm} (6.18)

where $\vec{p}$ is the position of the wing section relative to the centre of gravity, $d\vec{F} = \{dX, 0, dZ\}^T$ are the section forces and $d\vec{M}_s = \{0, dM, 0\}^T$ is the section pitching moment. Expanding Equations 6.18 about the pitch axis of the body frame ($\hat{j}_B$) gives the section pitching moment about the centre of gravity as

$$dM = zdX - xdZ + dM_s$$  \hspace{1cm} (6.19)

where $x$ and $z$ are the longitudinal coordinates of the section from Equation 6.5:

$$x = -(h_W - h_{CG}) - |\gamma|\Lambda$$  \hspace{1cm} (6.20a)

$$z = -(h_W - h_{CG}) - |\gamma|\Gamma$$  \hspace{1cm} (6.20b)
Roll and yaw moments about the centre of gravity are also produced by the section, however, due to the symmetry of the problem about the plane $O^Bxz$, are equal and opposite on the starboard and port sides. The net pitching moment from the wing about the centre of gravity is found by again integrating along the span axis:

$$M = \int_{-b}^{b} \frac{dM}{dy} dy$$

(6.21)

In the normal operation of tiltrotor flight, the wing sections will generally experience angles of attack in the domain $[-90^\circ, 25^\circ]$. However, angles of attack outside this range may still be encountered, for example, in rearward or vertical flight. The extrema in this range are due to the rotor wake downwash over the wing in hover and the large pitch angles required in low-speed forward flight. Therefore, the aerodynamic data for the wing sections of generic tiltrotor aircraft should include a full aerodynamic description through $360^\circ$ angle of attack. Furthermore, the trailing-edge of the wing is flapped to provide precise aerodynamic control and reduce the projected wing area under the rotor wake in hover. The aerodynamic data for the wing should, therefore, also facilitate the influence of any trailing-edge, and/or leading-edge geometries.

Aerodynamic data for flapped aerofoils through large, stalled angles of attack does not generally exist in literature and the gathering of such data was beyond the scope of this thesis.

In this work, the reference aircraft was the GTRS model of the XV-15 and the incompressible aerodynamic data for $C_l$, $C_d$ and $C_m$ was taken directly from [45]. The data is given for each flap/flaperon setting (4 in total) with the data for each flap/flaperon setting being a function of the fuselage angle of attack and rotor tilt angle. Compressibility effects were accounted for through the Prandtl-Glauert correction up to stall conditions. The validation of the XV-15 GTRS model presented in [29] implemented two flap/flaperon settings through transitioning flight: a flap/flaperon deflection of $\xi_f/\xi = 40^\circ/25^\circ$ in helicopter and early conversion mode; a flap/flaperon deflection of $\xi_f/\xi = 25^\circ/12.5^\circ$ later in conversion mode and aeroplane mode. The aerodynamic lift and drag polars as functions of angle of attack and rotor tilt are shown in Figure 6.4. The increased lift-curve slope in Figures 6.4a and 6.4b is due to the increased end-plating effect of the nacelles as the rotors are tilted towards aeroplane mode [31]. The data is presented for the entire wing and, therefore, is not strictly the two-dimensional data used in strip theory. Nonetheless, the data is implemented into the tiltrotor model but remains a current caveat in the absence of other data sources.
Figure 6.4: Lift and drag polars for different flap/flaperon settings as function of angle of attack and rotor tilt angle.

6.3 Rotors-on-Wing Interaction

The immersion of the wing in the downwash of the rotors adversely affects the lifting capability of the rotors, particularly in hover, by creating a download on the airframe. The required thrust and power of the rotors then increase to overcome the download produced by the wakes of the rotors. The interaction of the rotor wakes with the wing creates a complex flowfield, as illustrated in Figure 6.5. At the wing root, the flow is largely in the spanwise-inboard direction partially due to the swirl of the rotor wake. When this flow meets the opposing flow at the fuselage centreline from the opposite rotor, a vertical jet is formed, referred to as fountain flow. This can result in flow being recirculated through the rotor disc which has been estimated to contribute 15%-20% of the total download [12]. The large negative angles of
attack of the immersed wing sections, around \( \alpha = -90^\circ \), create a bluff body stall with a large region of separation below the wing [22]. For tiltrotor configurations, the rotor radius is largely that of the wing semi-span to minimise the disc loading and induced power. This represents a significant area of the wing projected under the rotor disc, which for the XV-15 rotor, amounts to approximately 78%. The induced velocity at the rotor disc is proportional to the square-root of the disc loading and, therefore, due to the smaller disc area of tiltrotor aircraft has a larger dynamic pressure in the wake. The download force on the wing can be as large as 10%-15% of the rotor thrust in hover [12], corresponding to a significant increase in the required hover power.

![Flowfield of the rotor wake interaction with the wing showing the download flow, formation of the fountain flow, recirculation of flow through the rotor and the spanwise flow component along the wing. Image from [12].](image)

**Figure 6.5:** Flowfield of the rotor wake interaction with the wing showing the download flow, formation of the fountain flow, recirculation of flow through the rotor and the spanwise flow component along the wing. Image from [12].

In the fixed-wing analysis implemented here, a two-dimensional representation of the flowfield is used over each aerofoil section. The three-dimensional presence of the wing, coupled with the swirl of the rotor wake, creates a spanwise flow component that is not considered in a two-dimensional model. Modelling the highly three-dimensional and unsteady rotor wake over the wing using two-dimensional strip theory is a significant simplification of the real flowfield. To capture the real flowfield and mutual interference between the rotors and wing, higher-order models such as panel, vortex or CFD methods are required for good analytical predictions of the wing download. However, these methods still present a difficult numerical problem due to the large unsteady and separated flowfield. The computational cost
associated with these physics-richer models can inhibit their use in large domain investigations, such as the prediction of the conversion corridor. In such large-scale investigations, reduced-order models that capture the gross effects of the interaction through empirical or simplified theoretical approaches are more applicable.

As the aircraft gains forward speed the rotor wakes are swept downstream and may still interact with the wing due to the wake vorticity. In low-speed flight up to approximately 30 kn, experiments showed the outboard wing sections experienced a download from the rotor washes but changed to an upload on the inboard wing sections [32]. The typical download/upload distribution created is highlighted in Figure 6.6. In reduced-order aeromechanics models, it is necessary to use less computationally expensive methods, usually correlating the mean induced velocity at the rotor disc with a representative drag coefficient to determine the interactional loads. This method has been used in the real-time tiltrotor simulation models of [28, 30, 45].

![Figure 6.6: Effect of the rotor wake on the typical download/upload pattern along the wing span. The simplified theoretical approximation used to model download on the immersion wing sections is also shown. Image from [28].](image)

To approximate the effects of the rotor wakes on the wing, the problem is collapsed into two-dimensions by considering the effects of the rotor wakes on each aerofoil section. The vorticity in the rotor wakes is not modelled directly and, therefore, the locally induced velocities inside and outside the rotor wakes are not captured in the actuator disc model implemented here. To this extent, the force and moment distributions along the wing cannot be accurately determined and simplified distributions, such as the rectangular distribution seen in Figure 6.6, are adopted that replicate the net forces and moments. In this work, the simplified distribution model is implemented since the rotor wakes are modelled as cylindrical streamtubes. This implementation is valid for generic tiltrotor aircraft, however, requires calibrating the model parameters (detailed subsequently) to match the loads produced by the simplified and real flowfield.
The rotor wake is modelled as a cylindrical streamtube that is skewed relative to the shaft axis as shown in Figure 6.7. The skew angle of the rotor wake was calculated from the mean induced inflow component $\lambda_0$ from the Peters and HaQuang [44] dynamic inflow model superimposed with the freestream velocity. From idealised momentum theory, the induced velocity in the wake approaches twice the velocity at the rotor disc far downstream. To conserve mass flow in the streamtube, the wake radius must contract and this reduces the projected wing area immersed in the rotor wake. McCormick [89] estimated the ratio of the downstream induced velocity to that at a propeller disc using a prescribed helicoidal wake geometry. The prescribed wake pattern did not include the contraction of the wake or any self-induced distortion but gives a simple expression to approximate the downstream wake velocity from the induced velocity at the disc:

$$\frac{v(\zeta)}{v(\zeta = 0)} = 1 + \frac{\zeta}{\sqrt{1 + \zeta^2}},$$

(6.22)

where $\zeta$ is the downstream distance from the rotor disc normalised by the rotor radius $R$ and $v$ is the dimensional mean induced velocity. Equation 6.22 is plotted in Figure 6.8 and shows how the induced velocity tends towards twice the induced velocity at the disc as $\zeta \to \infty$. The rotor wake is seen to quickly contract with the induced velocity approaching almost twice that at the rotor disc by one diameter downstream. The expression derived by McCormick was for a propeller in steady axial flight with a wake pattern analogous to a rotor in hover or steady vertical climb. Equation 6.22 can, therefore, be used to approximate the influence of the rotor wake velocity on the wing in axial flight conditions.

![Skewed cylindrical model of the rotor wakes.](image)
In forward flight, the rotor wake is no longer axisymmetric about the shaft axis and the blades experience cyclic loading from the one-per-rev variation of the in-plane freestream velocity. A cyclic variation of vorticity is, therefore, shed into the wake. The rotor wake is also free to distort under the influences of its own shed and trailed vorticity, a so-called free wake geometry. In the momentum approximation of the induced velocity, the wake is modelled as a rigid cylinder that is skewed by some angle relative to a reference axis. In this work, the tip-path-plane axis is used as the reference axis and the wake skew angle $\chi$ is given by

$$\chi = \tan^{-1} \left( \frac{\mu}{\lambda} \right)$$  

(6.23)

where $\mu$ and $\lambda$ are the advance ratio and inflow ratio, both referenced to the disc axes. The influence of the rotors-on-wing interaction in forward flight is generalised by assuming that Equation 6.22 is still approximately valid as the wake is convected downstream. The interaction components in the body axes are then given by the corrected induced velocity at the rotor disc resolved through the rotor tilt angle:

$$u_i = (\nu_{RoW} \nu) \sin \tau$$  

(6.24a)

$$w_i = - (\nu_{RoW} \nu) \sin \tau$$  

(6.24b)

where $\nu_{RoW}$ is the correction factor applied to the calculated induced velocity. Note in

Figure 6.8: Induced velocity ratio as function of the downstream distance normalised by the rotor radius.
Equation 6.24 the induced velocity has been assumed to be parallel to the rotor shaft and neglects the small components due to the gimbal tilt angles.

The contraction of the streamtube reduces the projected area of the wing immersed in the rotor wake. The expression derived by McCormick does not account for the wake contraction, however, combining Equation 6.22 with the continuity equation approximates the contraction ratio as [90]:

\[
\frac{R(\zeta)}{R(\zeta = 0)} = \sqrt{\frac{\sqrt{1 + \zeta^2}}{\zeta + \sqrt{1 + \zeta^2}}}
\] (6.25)

where $\zeta$ is again the normalised downstream distance. The contracted rotor disc is now projected downstream along the wake centreline $\chi$ and any wing sections that are bounded within the contracted wake radius accrue the wake velocity superimposed on the freestream velocity.

To implement Equation 6.22, the downstream position of the wing sections relative to the hub are required. This introduces some complexity since the downstream distance is relative to any point on the disc. To circumvent this ambiguity, the downstream distance was taken to be the perpendicular distance from the conversion axis to the rotor hub; the shaft length $\ell$. Given the already simplified model of the flowfield, this approximation is sufficient. In the GTRS model of the XV-15, the distance from the conversion axis to the rotor hub is $\ell = 1.42$ m and gives an induced velocity ratio of 1.35. In the GTRS model, the induced velocity multiplier and wing aerodynamics are calibrated to give the expected download. The induced velocity multiplier in the GTRS model is 1.60 which is significantly larger than that predicted by Equation 6.22. Some discrepancy between these values may be attributed to the neglect of the wake contraction in McCormick’s model and the fact the GTRS parameters are calibrated for the expected download. Since the GTRS XV-15 is used as the reference aircraft here, the induced velocity multiplier was made consistent with this model.

To calculate the wing sections immersed in the rotor wake, the sections bounded within the projected streamtube must be determined. The streamtube is defined two-dimensionally in the disc-wind axes (a resultant in-plane velocity and normal velocity) and, therefore, the position of the wing sections relative to this frame is required. The position vector of a wing section is calculated from

\[
\vec{p} = \vec{W} - \vec{H}
\] (6.26)

where $\vec{W}$ and $\vec{H}$ are the position vectors of the wing section and rotor hub, respectively. The positions of a wing section and rotor hub are readily expressed in the aircraft axes. The position
of a wing section in the aircraft axes is

\[ \mathbf{\hat{W}} = (l_W + \mid y \mid \Lambda) \mathbf{\hat{i}}_A + y \mathbf{\hat{j}}_A + (h_W + \mid y \mid \Gamma) \mathbf{\hat{k}}_A. \quad (6.27) \]

The position of either rotor hub in the aircraft axes is

\[ \mathbf{\hat{H}} = (l_P - \ell \sin \tau) \mathbf{\hat{i}}_A \pm d_P \mathbf{\hat{j}}_A + (h_P + \ell \cos \tau) \mathbf{\hat{k}}_A \quad (6.28) \]

where the subscript \( P \) denotes the pivot, \( \ell \) is the shaft length from the conversion axis to the rotor hub and \( \tau \) is the rotor tilt angle. The directional components of the position vector in the aircraft axes then are given by:

\[ x^A = (l_W + \mid y \mid \Lambda) - (l_P - \ell \sin \tau) \quad (6.29a) \]
\[ y^A = y \mp d_P \quad (6.29b) \]
\[ z^A = (h_W + \mid y \mid \Gamma) - (h_P + \ell \cos \tau) \quad (6.29c) \]

where the superscript denotes the coordinate system. The skew angle of the rotor wake is expressed with respect to the disc-wind axes and, therefore, the position of the wing sections are required in these axes. The position vectors are rotated into the disc axes through the rotor tilt angle and gimbal longitudinal and lateral tilt angles described in Equations 5.1 and 5.4. In the disc axes, the wing sections relative to the rotor hub are denoted by \( \mathbf{\hat{p}} = \{x^D, y^D, z^D\}^T \).

The final transformation is through the disc-wind angle \( \Delta \) as shown in Figure 6.9 to give the position components \( \{x^{DW}, y^{DW}, z^{DW}\}^T \).

\[ \textbf{Figure 6.9: Disc-wind axes transformation through the disc-wind angle } \Delta. \]

On the contracted rotor disc, the condition for the wing section to be bounded by the rotor
streamtube and accrue the additional velocity from the rotor wake is

\[
(x^{DW})^2 + (y^{DW})^2 \leq R^2
\]  \hspace{1cm} (6.30)

where \( R \) is the contracted wake radius at the downstream location \( \zeta = \ell/R \). This expression can be generalised to the case where the wake cylinder is skewed downstream of the disc along the resultant in-plane velocity direction. The effect of the skewed cylinder is to displace the centreline downstream by \(-z^{DW} \tan \chi\) relative to the disc plane. The wake skew angle does not consider the change in skew angle caused by the increasing wake velocity downstream. The generalised condition for a wing section to be immersed in the rotor wake is then

\[
(x^{DW} + z^{DW} \tan \chi)^2 + (y^{DW})^2 \leq R^2 \hspace{1cm} \chi \neq 90^\circ.
\]  \hspace{1cm} (6.31)

This completes the interaction model for the rotors-on-wing.

### 6.4 Summary

This chapter has presented the aerodynamic module for the wing. The kinematic description of the wing geometry was detailed that facilitates dihedral, sweep and variable pitch along the span. The calculation of the aerodynamic forces and moments produced by the wing were calculated by integrating the section loads along the span. This chapter has contributed a generic model to account for the rotors-on-wing interaction based on a skewed cylindrical geometry of the rotor wake. The model is valid for both axial and nonaxial flight conditions but does require calibration of the wake velocity and wing section aerodynamics to match experimental data. The interaction model can be used to give a first-order approximation of the rotors-on-wing interaction but is not suitable for higher-order analysis due to the simplification of the rotor wake geometry.
Chapter 7

Aerodynamic Module: Empennage

This chapter details the aerodynamic module used to calculate the aerodynamic forces and moments of the empennage. The empennage consists of several smaller fixed-wings that are used as control surfaces to provide control moments throughout the operational domain. This chapter first describes the aerodynamic interaction associated with the empennage due to the rotors and wing. This work only considers longitudinal behaviour and, therefore, attention is focussed on the longitudinal effects of the interactions and not the lateral or directional effects. Following a description of the interaction models, a kinematic description of an arbitrary empennage surface is presented, including the interactions models. The final section in this chapter details the calculation of the aerodynamic forces and moments.

7.1 Rotors-on-Empennage Interaction

In forward flight, the convection of the rotor wakes downstream can cause aerodynamic interaction with the empennage surfaces. For conventional helicopters in forward flight, the tailplane centreline is usually parallel with the main rotor and the tailplane operates in the rotor downwash. For tiltrotor aircraft, however, the interaction of the rotor wakes with the empennage is different due to the lateral-tandem configuration. The interaction of the rotor wakes is a complex mechanism and depends on several parameters such as airspeed, rotor incidence, blade loading and empennage position relative to the rotors. Experimental tests have shown this interaction was most pronounced for the XV-15 aircraft in low-speed forward flight up to approximately 40 kn [15].
Early experimental studies of the interaction of the rotor wakes with the empennage in [15] showed the tip vortices quickly rolled-up downstream. At these conditions, the wake was found to be characteristic of a low aspect ratio wing [32]. Figure 7.1 shows the flowfield at the empennage for airspeeds ranging from 16 kn - 40 kn. The increasing forward speed and small fuselage angle of attack show the concentrated rolled-up tip vortex directly above the tailplane at 40 kn. The interaction of the rotor vortices with the tailplane was to create a net upwash. Additionally, the dynamic pressure at the empennage was found to be approximately double that of the freestream [20]. The upwash was most pronounced for tilt angles of the rotors around helicopter mode. The interaction of the rotor wakes at the empennage resulted in undesirable aircraft handling qualities, namely a stick reversal in this low-speed operating region.
Figure 7.1: Flowfield at the H-tail empennage in low-speed flight showing the roll-up of the rotor vortices above the tailplane. In descending order: 1) $V_{\infty} = 16\, \text{kn}$, $\alpha = 0^\circ$; 2) $V_{\infty} = 20\, \text{kn}$, $\alpha = 0^\circ$; 3) $V_{\infty} = 30\, \text{kn}$, $\alpha = 0^\circ$; 4) $V_{\infty} = 40\, \text{kn}$, $\alpha = -2^\circ$. The empennage is viewed from tail to nose. Image from [15].

Predicting the aerodynamic interaction at the tailplane is difficult due to the complex rotor wake geometry. Higher-order methods such as free wake vortex models, VPM or CFD are best suited to predict such flows and the empennage forces and moments. Unfortunately, the computational cost required by these higher-order methods does not make them suitable for analysing a large number of operating points. Reduced-order models are, therefore, more suitable but require either empirical data or flowfield approximations from higher-order methods.
methods to accurately predict the empennage loads. Unfortunately, the higher-order models in literature have not been used to develop reduced-order models and early empirical data remains the best source of data for rotors-on-empennage interaction. The effect of the rotor wakes at the empennage was determined experimentally in [15, 20, 31] by analysing pitching moment data at tail-on and tail-off configurations. This data was used to formulate an empirical model that could be used in mathematical models to simulate the effect of the rotors-on-empennage interaction.

In the early real-time aeromechanics model of the Bell 301 tiltrotor aircraft in [28], a curve fit was used to calculate the ratio of the rotor-induced velocity at the empennage to the mean induced velocity at the disc. The polynomial fit was derived trimmed operating points as a function of rotor tilt angle and forward speed. It was, therefore, only an approximation to the influence of the rotor wakes on the empennage throughout the operating domain. In the improved GTRS model of the XV-15, the interaction of the rotor wakes with the empennage was based on the experimental data in [15, 31]. The data was used to formulate a more comprehensive model of the interaction as a function of airspeed, angle of attack and rotor tilt angle. The interaction is facilitated in two steps:

1. Calculate an upwash velocity at the empennage by multiplying the mean induced velocity at the rotor disc by an empirical factor \( \upsilon_{RoE} \). This is used to calculate the angle of attack of the tailplane from the corrected velocity components. Figure 7.2a shows \( \upsilon_{RoE} \) as a function of angle of attack and airspeed in helicopter mode. Similar curves exist for each rotor tilt angle.

2. Calculate the dynamic pressure at the empennage by multiplying the freestream dynamic pressure by an empirical factor, \( q_{RoE} \), to give the ‘correct’ lift and pitching moment. Figure 7.2b shows \( q_{RoE} \) as a function of angle of attack and airspeed. Similar curves exist for each rotor tilt angle.
These empirical models are used only to give the expected lift and pitch moments. This is a convenient way to incorporate the wake interaction but is a significant simplification of the complex interactions. Additionally, the data is empirically based on the XV-15 H-tail configuration. It is likely the interaction models are not representative of different empennage configurations and positions. However, configuration dependent data can be facilitated in a lookup table if experimental, empirical or theoretical data is available.

### 7.2 Wing-on-Empennage Interaction

The velocity induced by the bound and trailed vortices produced by a lifting wing creates a downwash at the empennage. This downwash can have a significant effect on the aerodynamic environment at the empennage and, therefore, the aerodynamic forces and moments produced. The influence of the vortex wake is to reduce the geometric angle of attack of the tailplane by the downwash angle. Consequently, the effect of the wing wake is to reduce the lift of the tailplane. A first-order analysis of a horseshoe vortex system attached to the main wing gives the downwash angle at the centre of the tailplane as [17]

\[
\epsilon = \frac{8}{\pi^3 A R} C_L \left(1 + \sec \beta \right),
\]

where \( C_L \) is the three-dimensional wing lift coefficient, \( A R \) is the aspect ratio of the wing and \( \beta \) is the angle between the trailing leg of the horseshoe vortex and tailplane centreline.
From this expression, the downwash is found to be proportional to the wing lift and inversely proportional to the aspect ratio. The lift coefficient in compressible flow prior to the onset of the critical Mach number can be approximated using the Prandtl-Glauert correction factor. In compressible flow, the downwash is then

\[ \varepsilon = \frac{8}{\pi^3 A R} \frac{C_L}{\sqrt{1 - M^2_\infty}} (1 + \sec \beta) , \]  

(7.2)

where \( M_\infty \) is the freestream Mach number.

For typical tiltrotor aircraft, the installation of the wingtip mounted rotors and the tilting actuation system restricts the wing to a relatively low aspect ratio for structural and whirlflutter considerations [91]. Furthermore, wing flaps are used to alleviate the rotor download on the airframe and improve the lifting capability of the wing at low forward speeds. This has the effect of helping to widen the conversion corridor. The effect of the flaps is to increase the wing lift coefficient at a given angle of attack, which, coupled with the low aspect ratio of the wing tends to increase the downwash angle from Equation 7.2. On the other hand, the tip-mounted nacelles have an end-plating effect on the wing aerodynamics and reduce the nonuniformity of the induced downwash at the wing. This was shown experimentally in [31] by an increase in the wing lift gradient with respect to the angle of attack as the rotors were tilted towards aeroplane mode increasing the end-plating effect. From these considerations, it is evident that the tailplane aerodynamics may change significantly and affect the predicted conversion corridor boundaries.

A second effect of the main wing on the empennage is a change in its dynamic pressure from the freestream value. This change in dynamic pressure is usually a reduction relative to the freestream and, therefore, reduced empennage effectiveness. An increase in dynamic pressure can be found if, for example, the empennage is in the slipstream of a propeller. In the GTRS model of the XV-15, a constant dynamic pressure loss of 0.80 \( q_\infty \) is used. However, this is not specifically referenced to the wing-on-empennage interaction. Since the change in tailplane dynamic pressure from the rotors-on-empennage was accounted for separately in the aeromechanics model, this constant dynamic pressure loss was assumed to be from the wing-on-empennage interaction.

The wing downwash data used in the GTRS model is a function of the flap/flaperon setting, rotor tilt angle and angle of attack. The data is derived from wind-tunnel experiments up to wing stall and approximated thereafter. The downwash data for flap/flaperon settings used in the validation study of the model in [29] are presented in Figure 7.3. The figures show the
significant effect of the flap settings on the downwash angle with maximum values in the region of 9° to 11°.

![Figure 7.3: Wing downwash angle at the tailplane as a function of angle of attack and rotor tilt for two flap/flaperon settings. Discrete data points from [45].](image)

7.3 Kinematics

The aerodynamic forces and moments produced by an empennage surface are modelled from strip theory where the aerodynamic loads are applied at the aerodynamic centre of each empennage section. The empennage consists of a series of smaller control surfaces that are used to provide control moments predominantly in pitch and yaw; roll control is obtained through differential aileron deflections on the main wing or rotor control inputs. In order to provide this pitch and yaw control, a collection of surfaces are implemented with their respective span axis typically along the required control axis. Hybrid control methods also exist whereby the pitch and yaw controls are implemented by a single pair of control surfaces orientated between the required control axes, e.g. a V-tail configuration. In all cases, the orientation of the empennage surface relative to the body can be arbitrary and the orientation of the respective span axis is, therefore, a user-defined input for the empennage component. The geometry of an empennage surface is simplified compared to the main wing and consists of only sweep and twist; the dihedral angle relative to the span axis is assumed to be zero. This assumption is justified since the dihedral of the main wing is implemented to provide roll stability which is not the primary function of the empennage.

The lifting line of the empennage surface is defined in a Cartesian coordinate system
denoted $O^E_{xyz}$ and termed the empennage frame. The unit vectors of the empennage axes are denoted by $\hat{i}_E$, $\hat{j}_E$ and $\hat{k}_E$ and are initially parallel to the body axes. The location of $O^E$ for the tailplane and vertical fins used in the GTRS model of the XV-15 are detailed in Table 3.2. The span axis of the empennage surface is $\hat{j}_E$ and is orientated relative to the longitudinal body-axis through the span rotation angle $\Psi$ as shown in Figure 7.4. The aerofoil sections then lie parallel to the plane $O^E_{xz}$. The span rotation angle is defined positive for the empennage span axis upwards about $\hat{i}_B$. The orientation of the empennage axes relative to the body axes is described by the rotation matrix

$$
\begin{pmatrix}
\hat{i}_E \\
\hat{j}_E \\
\hat{k}_E
\end{pmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos\Psi & -\sin\Psi \\
0 & \sin\Psi & \cos\Psi
\end{bmatrix}
\begin{pmatrix}
\hat{i}_B \\
\hat{j}_B \\
\hat{k}_B
\end{pmatrix}
$$

(7.3)

The geometry of the empennage may also incorporate sweep. The sweep angle $\Lambda$ is again taken positive in the aft direction and is treated as a shear transformation rather than a rotation. Therefore, the plane of the aerofoil section remains parallel to the plane $O^E_{xz}$ after the transformation. The sweep angles of empennage surfaces are often of sufficient magnitude that the small-angle approximation is not valid and, therefore, is not implemented here. The position vector of an aerofoil section in the empennage axes is then given by

$$\vec{p} = -|y| \tan\Lambda \hat{i}_E + y \hat{j}_E$$

(7.4)
where $\gamma$ is the spanwise ordinate and $|\gamma|$ accounts for aft sweep on either side of the span axis.

The calculation of the aerodynamic moments about the centre of gravity requires the position of the aerofoil section relative to the body-fixed origin. The location and orientation of the empennage axes are, in general, both offset and angled relative to the body axes. The position offset is determined from the predefined position of the centre of gravity and empennage origin in the aircraft axes. The orientation of the empennage axis is determined from the span rotation angle. The position vector of a point on the empennage surface relative to the centre of gravity, expressed in the body axes, is:

$$\vec{p} = \begin{cases} 
-l_E - |\gamma| \tan \Lambda \\
\frac{d_E + |\gamma| \cos \Psi}{\cos \Psi} \\
-h_E - |\gamma| \sin \Psi
\end{cases}$$

(7.5)

where $l$ and $h$ are the station and waterline coordinates in the aircraft axes of the subscripted quantities.

The aerodynamic forces and moments produced by an empennage section are a function of the section angle of attack, dynamic pressure and control input. For the interaction models described in [45], the angle of attack of an empennage section is determined from the aerodynamic velocity at the tailplane. On the other hand, the dynamic pressure of an empennage section is determined from the freestream dynamic pressure that is corrected for the effect of the wing and rotors. In steady-level flight, the aerodynamic velocity of an empennage section is written in the body axes as

$$\vec{u}_\alpha = (u + u_i) \hat{1}_B + (w + w_i) \hat{k}_B$$

(7.6)

where $\{u, 0, w\}^T$ are the freestream velocity components and $\{u_i, 0, w_i\}^T$ are the interaction velocity components. No lateral interaction component is included due to the symmetry of the problem in steady-level flight. The interaction velocities are based on an empirical correction to the mean induced velocity calculated at the rotor disc:

$$u_i = \nu_{RoE} \nu \sin \tau$$

(7.7a)

$$w_i = -\nu_{RoE} \nu \cos \tau$$

(7.7b)

where $\nu_{RoE}$ is the rotors-on-empennage correction factor, $\nu$ is the mean induced velocity and $\tau$ is the rotor tilt angle. The correction factor is determined from a user-supplied lookup table as
a function of airspeed, fuselage angle of attack and rotor tilt angle; \( \psi_{RoE} = \psi_{RoE}(V_\infty, \alpha_F, \tau) \). The table is linearly interpolated at the required arguments. The section aerodynamics are calculated in the local empennage axes. The aerodynamic velocity in these local axes is denoted by

\[
\overline{u}_\alpha = V_x \hat{i}_E + V_y \hat{j}_E + V_z \hat{k}_E
\]  
(7.8)

where the components \( \{V_x, V_y, V_z\}^T \) are found through Equation 7.3:

\[
\begin{align*}
V_x &= u + u_i \quad (7.9a) \\
V_y &= -(w + w_i) \sin \Psi \quad (7.9b) \\
V_z &= (w + w_i) \cos \Psi \quad (7.9c)
\end{align*}
\]

The local angle of attack \( \alpha \) is defined between the section chordline and the resultant velocity in the plane \( O^E_{xz} \) as shown in Figure 7.5. The orientation of the resultant velocity is corrected for the downwash angle and is calculated from

\[
\alpha = i_0 - i_w + \tan \left( \frac{V_z}{V_x} \right) - \varepsilon
\]  
(7.10)

where \( i_0 \) is the setting angle at the root (\( \gamma = 0 \)), \( i_w \) is the washout angle with respect to the span and \( \tan(V_z/V_x) - \varepsilon \) is the local wind angle. The first term of the local wind angle is the freestream contribution and the second term, the downwash angle, is the compressibility-corrected contribution from the wing wake. The incompressible downwash angle \( \varepsilon_0 \) is determined from user-supplied lookup tables as a function of airspeed, fuselage angle of attack and rotor tilt angle; \( \varepsilon_0 = \varepsilon_0(V_\infty, \alpha_F, \tau) \). One table for each flap/flaperon setting is supplied and is linearly interpolated at the required arguments. From Equation 7.2, the downwash angle from the main wing is dependent on its lift coefficient and, therefore, the Mach number. This compressibility effect was accounted for using the Prandtl-Glauert correction factor:

\[
\varepsilon = \frac{\varepsilon_0}{\sqrt{1 - M_\infty^2}}
\]  
(7.11)

where \( M_\infty \) is the freestream Mach number of the wing. The angle of attack is wrapped to the domain \([-180^\circ, 180^\circ]\) to comply with the limits of the lookup table. For the vertical fins, the angle of attack is effectively the sideslip angle due to the rotation of the span axis. To properly account for the downwash on the tailplane alone, the vertical fins in this work are defined as noninteracting components and, therefore, the downwash angle is set to \( \varepsilon = 0 \).
7.4 Aerodynamic Forces and Moments

The aerodynamic forces and moments on the aircraft are now calculated. The local lift, drag and pitching moment produced by an aerofoil section are given by:

\[ dL = Cl \left( q_\infty q_{RoE} q_{WoE} \right) c dy \]  
\[ dD = Cd \left( q_\infty q_{RoE} q_{WoE} \right) c dy \]  
\[ dM = Cm \left( q_\infty q_{RoE} q_{WoE} \right) c^2 dy \]

where \( Cl, Cd \) and \( Cm \) are the sectional lift, drag and pitching moment coefficients, \( q_\infty \) is the freestream dynamic pressure, \( q_{RoE} \) is the dynamic pressure correction due to the rotor wake, \( q_{WoE} \) is the dynamic pressure correction due to the wing wake and \( c \) is the local chord.

The dynamic pressure correction due to the rotors is determined from user-supplied lookup tables as a function of the airspeed, fuselage angle of attack and rotor tilt angle: \( q_{RoW} = q_{RoW} (V_\infty, \alpha_F, \tau) \). The table is then linearly interpolated at the required arguments. The dynamic pressure correction due to the wing is a constant quantity defined in the configuration file of the empennage component. The lift, drag and pitching moment coefficients are user-supplied lookup tables that are functions of the sectional angle of attack, Mach number and control.
input:

\[ C_l = C_l(\alpha, M, \Delta) \]  \hspace{1cm} (7.13a)
\[ C_d = C_d(\alpha, M, \Delta) \]  \hspace{1cm} (7.13b)
\[ C_m = C_m(\alpha, M, \Delta) \]  \hspace{1cm} (7.13c)

where \( M \) is the Mach number normal to the leading edge and \( \Delta \) is the control input. The tables are again linearly interpolated at the required arguments where the control input is determined from the pilot stick position as described in Chapter 3. The Mach normal normal to the leading edge is found by resolving the section velocity components through the sweep angle:

\[ M = \frac{1}{c_0} \sqrt{(V_x \cos \Lambda - V_y \sin \Lambda)^2 + V_z^2}. \]  \hspace{1cm} (7.14)

The contributions of the section lift and drag forces to the body-fixed forces are now calculated. The lift and drag forces are defined perpendicular and parallel to the local wind vector along \( \tan(V_z/V_x) - \epsilon \) as shown in Figure 7.5. The contributions of the section lift and drag forces to the body-fixed \( X \) and \( Z \) forces are then found from successive rotations through the local wind angle, denoted by \( \phi \), and span rotation angle \( \Psi \):

\[ dX = dL \sin \phi - dD \cos \phi \]  \hspace{1cm} (7.15a)
\[ dZ = -(dL \cos \phi + dD \sin \phi) \cos \Psi \]  \hspace{1cm} (7.15b)

The net forces produced by the empennage surface are found by integrating the section forces along the span between the appropriate integration limits. The contribution of the section loads to the pitching moment about the centre of gravity is found to be:

\[ dM = z dx - xdz + dM \cos \Psi \]  \hspace{1cm} (7.16)

where the position coordinates of the aerofoil section in the body-fixed axes are:

\[ x = -(l_E - l_{CG}) - |y| \tan \Lambda \]  \hspace{1cm} (7.17a)
\[ z = -(h_E - h_{CG}) - y \sin \Psi \]  \hspace{1cm} (7.17b)

The total pitching moment produced by the empennage surface is then found by the same integration along the span. This completes the calculation of the aerodynamic forces and moments of an empennage surface.
7.5 Summary

This chapter has presented the aerodynamic module of the empennage. A kinematic description of a generic empennage surface has been given that facilitates arbitrary orientation relative to the body as well as twist and sweep geometries. The interaction models for the rotors-on-empennage and wing-on-empennage interactions have been presented based on the GTRS model of the XV-15 aircraft. The rotors-on-empennage interaction implements two correction factors, the first to the induced velocity of the rotor and the second to the freestream dynamic pressure. The wing-on-empennage interaction implements a downwash angle at the tailplane and changes the freestream dynamic pressure. The calculation of the empennage forces and moments are found from spanwise integration of the section forces and resolved into the body-fixed axes for substitution into the equations of motion.
Chapter 8

Aerodynamic Modules: Spinner and Wing-Nacelle

This chapter describes the aerodynamic modules of the spinners and wing-nacelle interfaces. These additional aerodynamic modules were not accounted for directly in the discretisation of the major aerodynamic components, however, are present in the GTRS model of the XV-15 to improve the performance prediction [29]. Since the GTRS model is a validated aeromechanics model, it was decided to include these additional aerodynamic modules in order to better predict the aircraft trim behaviour and performance throughout the conversion corridor. This chapter details the aerodynamic models of the rotor spinners and wing-nacelle interfaces used in the GTRS model [45]. Some simplifications are made here since only steady-level flight was considered.

8.1 Spinner Drag

The spinner is an aerodynamically efficient shape on the front of a propeller to streamline the oncoming flow through the propeller disc. In helicopter design, a similar fairing exists on the rotorhead, sometimes called a hub cap or beanie, and can be used to deflect the rotor downwash to reduce interference and vibration at the tail [13]. The hub cap can also be used to reduce the dynamic pressure deficit in the rotor wake and improve the tail effectiveness [61]. Good high-speed performance for tiltrotor aircraft requires a fairing on the rotorhead similar to a propeller spinner. The drag force of both rotor spinners is modelled collectively from the
expression:

\[ D = 2q \left( k_1 + k_2 \sin^3 \alpha_{SP} \right) \]  

(8.1)

where \( q \) is the dynamic pressure at the spinner, \( k_1 \) and \( k_2 \) are empirical coefficients and \( \alpha_{SP} \) is the angle of attack of the shaft. The value of the coefficients used in the GTRS model of the XV-15 are \( k_1 = 0.09 \text{ m}^2 \) and \( k_2 = 0.51 \text{ m}^2 \). The dynamic pressure at the spinners is given by

\[ q = \frac{1}{2} \rho V^2 \]  

(8.2)

where \( \rho \) is the air density and \( V \) is the resultant velocity at the spinner from the freestream and rotor induced velocity:

\[ V = \sqrt{u_H^2 + (v - w_H)^2} \]  

(8.3)

where \( \{u_H, 0, w_H\}^T \) are the components of the freestream velocity in the hub axes and \( v \) is the uniform induced velocity component of a rotor. The rotor induced velocity model used in this work was the three-state dynamic inflow model of Peters and HaQuang [44] and is formulated in disc axes. When this uniform induced velocity component is rotated into the hub axes it has components along the \( x \)- and \( y \)-axes due to the steady gimbal tilt angles \( \beta_c \) and \( \beta_s \). These components are small compared to the uniform component and, therefore, neglected. Furthermore, the spinner drag in Equation 8.1 is proportional to the resultant dynamic pressure which is substantially larger in high-speed flight when the induced velocity is much smaller than the freestream velocity [13]. The angle of attack of the spinner is defined relative to the shaft axis and is given by

\[ \alpha_{SP} = \arctan \left( \frac{u_H}{v - w_H} \right) \]  

(8.4)

The expression in Equation 8.4 neglects the velocity of the spinners due to the airframe rotation rates and rotor tilting but is applicable to the steady flight conditions in this work.

The drag force of the spinners is expressed in the hub axes by rotating the resulting drag force through the spinner angle of attack:

\[ -D \sin \alpha_{SP} \hat{i}_H + D \cos \alpha_{SP} \hat{k}_H \]  

(8.5)

The equations of motion are derived in a body-fixed coordinate system and, therefore, the drag force of the spinners expressed in the body axes are found by rotating the components in Equation 8.5 through the rotor tilt angle. The contribution of the spinner drag to the body
forces is then found to be:

\[
X = -D \sin(\alpha_{SP} + \tau) \quad (8.6a)
\]
\[
Z = D \cos(\alpha_{SP} + \tau) \quad (8.6b)
\]

The drag forces of the spinners is applied at the hub and, therefore, exert a pitching moment about the centre of gravity. This pitching moment in the body axes is given by

\[
M = zX - xZ \quad (8.7)
\]

where \(x\) and \(z\) are the longitudinal and vertical position components of the rotor hub relative to the centre of gravity:

\[
x = -(l_P - \ell \sin \tau - l_{CG}) \quad (8.8a)
\]
\[
z = -(h_P + \ell \cos \tau - h_{CG}) \quad (8.8b)
\]

where \(\{l_P, 0, h_P\}^T\) and \(\{l_{CG}, 0, h_{CG}\}^T\) are the pivot and centre of gravity coordinates in the aircraft axes and \(\ell\) is the shaft length from the conversion axis to the rotor hub.

### 8.2 Wing-Nacelle Interference Drag

During the reconversion from aeroplane mode towards helicopter mode, flight test behaviour of the XV-15 exhibited a pronounced loss of airspeed [29]. In aeroplane mode, the nacelles act similarly to endplates at the wing-tips that help reduce the strength of the tip vortex. As the nacelles are reconverted, a region of the wing tip becomes exposed to the flow and forms a strong vortex at the nacelle-wing interface. The highly turbulent flow at the nacelle-wing interface was found to substantially increase the drag and was the cause of the loss of airspeed. This behaviour was not originally demonstrated by the GTRS model but was later accounted for by introducing the empirical relation [29]

\[
D = \frac{1}{2} \rho V^2 f(\tau) \quad (8.9)
\]

where \(D\) is the drag force, \(\rho V^2 / 2\) is the dynamic pressure at the nacelle-wing interface and \(f(\tau)\) is a user-supplied lookup table interpolated at the required rotor tilt angle. The dynamic pressure at the nacelle-wing interface accounts the increased induced velocity due
to the contraction of the rotor wake downstream from the disc. The resultant velocity at the
nacelle-wing interface is calculated as:

\[ V = \sqrt{u_H^2 + (v_{RoW} v - w_H)^2} \] (8.10)

where \( v_{RoW} \) is the induced velocity multiplier due to the wake contraction and was derived in
Chapter 6. In the GTRS model, the drag force at the nacelle-wing interface is resolved into the
body-fixed axes through the same angle as the spinner drag forces:

\[ X = -D \sin(\alpha_{SP} + \tau) \] (8.11a)

\[ Z = D \cos(\alpha_{SP} + \tau) \] (8.11b)

The nacelle-wing drag is applied at the rotor pivot location and, therefore, exerts a pitching
moment about the centre of gravity given by

\[ M = zX - xZ \] (8.12)

where the position components are

\[ x = -(l_P - l_{CG}) \] (8.13a)

\[ z = -(h_P - h_{CG}) \] (8.13b)

8.3 Summary

This chapter has presented the aerodynamic modules of the rotor spinners and wing-nacelle
interface. These modules were included in the GTRS model of the XV-15 to improve the
performance predictions and match flight behaviour. The spinner drag is included in the
rotor aerodynamics module and the wing-nacelle drag in the wing module. The inclusions
of these models in this work were derived from their proportionality to the dynamic pressure
and, therefore, may significantly affect the power requirements at high speed.
Chapter 9

Conversion Corridor Methodology

This chapter presents the methodology used to determine the conversion corridor. Three methods are rationalised that could be used to determine the min-speed and max-speed boundaries and, also, the trim behaviour through transition. The benefits of each approach are detailed and critically examined with the most viable approach selected. This chapter also presents the methodology and calculation of the initial guesses for the unknown trim quantities in the flight mechanics module.

9.1 Modelling Approach

The conversion corridor is a unique regime of flight belonging to convertible aircraft transitioning between rotor-borne and wing-borne flight. The corridor bounds the aircraft flight envelope and presented as a function of rotor tilt against airspeed. The corridor is usually constructed for a fixed configuration, i.e. weight and flap/flaperon setting [4, 6]. The corridor boundaries are established by identifying operating points at which trimmed flight is not possible or, alternatively, when the trim solution breaks any of the imposed constraints on the aircraft. Factors that affect the viability of a trim solution include the stall of the wing, control input limitations, dynamic constraints (flapping or gimbal angle limits) and the installed power limit. When determining the conversion corridor, any additional power or control margin required for manoeuvring or climb is neglected.

In literature, no discussion exists regarding the different approaches used to determine
the conversion corridor or its associated trim behaviour and, therefore, this thesis contains a contribution to this topic. In this work, three approaches were rationalised to predict the conversion corridor that are illustrated in Figure 9.1:

1. A constrained optimisation problem to seek the boundaries.
2. A simple trim sweep over the required domain.
3. A systematic trim sweep over the required domain using the history of the trim solution as it becomes available.

The first method constructs the problem of finding the conversion boundaries as a constrained optimization problem to search for the min-speed and max-speed boundaries at each discrete rotor tilt angle. This method does not require the discretisation of both the airspeed and rotor tilt domains and instead, just the rotor tilt domain needs to be discretised. The predicted min-speed and max-speed boundaries at each rotor tilt can then be pieced together to form the conversion corridor. This approach is simple to implement, however, a disadvantage to this approach is that trim information may not be readily available over a wide range of airspeeds. Therefore, this method may potentially offer little insight into the aircraft behaviour through the conversion corridor. Furthermore, the constraints for the optimisation problem must be prescribed \textit{a priori} and this does not offer the flexibility to change the constraints without rerunning the simulation of the corridor.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure9_1.png}
\caption{Methods to determine the conversion corridor of tiltrotor aircraft. Trim points are represented by the circles corresponding to viable (green) and not viable (red) operating points. The corridor boundary is indicated by the dashed line.}
\end{figure}
The trim sweep methods require the discretisation of the airspeed and rotor tilt angle domains into a finite number of points. The simple trim sweep attempts to find a trim solution at every point in the conversion corridor domain and can be readily parallelised for large domain investigations and/or optimisation routines. On the other hand, the systematic trim sweep works progressively through the conversion corridor domain and uses the available trim history to improve the initial guesses at subsequent trim points. This method is best implemented by sweeping the airspeed domain at a selected rotor tilt angle; once the min-speed and max-speed boundaries for the selected rotor tilt angle have been found, the airspeed sweep can be terminated and thus save computational cost by not evaluating unnecessary trim points. The rotor tilt angle can then moved to the next discrete value. If the conversion corridor is attempted from helicopter to aeroplane mode then the initial airspeed at the new rotor tilt angle can be set to the min-speed boundary at the previous rotor tilt angle since the min-speed boundary increases towards aeroplane mode.

The major drawback to the systematic trim sweep is that it relies on the history of the trim solution to improve the initial guess at the current trim point. The chance of convergence at the current trim point is then theoretically improved provided the operating points are similar. However, this method does rely on the trim behaviour being smooth with no discontinuities; both criteria are applicable to trim curves. Furthermore, the discretisation of the airspeed and rotor tilt angle domains should provide sufficient spacing that neighbouring trim points have similar trim states. From experience, an airspeed spacing and rotor tilt angle spacing of $\Delta V_\infty = 15\text{kn}$ and $\Delta \tau = 15^\circ$ were sufficient. Since the operational domain for tiltrotor aircraft is large, it is preferable to parallelise the trim sweep over several processors. Both the simple and systematic trim sweep methods were implemented and predicted identical conversion corridors and trim behaviour. The evaluation of a single trim point was not expensive¹ and, therefore, both trim sweeps ran quickly with the parallelised trim sweep around 50% faster than the systematic trim sweep. Another advantage of the simple trim sweep method is that the performance constraints can be applied during post-processing, whereas to take full advantage of the systematic trim sweep the constraints should be prescribed a priori. Given the limitations of the constrained optimisation and systematic trim sweeps, the method chosen here was the simple trim sweep. The downside to this trim sweep method is that the airspeed boundaries are not computed directly, instead the boundary is identified by two discrete points either side of it. The exact boundary could be bisected if the trim point spacing is large, else, taken at the

¹Computer architecture used for simulations: Intel(R) Core(TM) i5-7200U CPU @2.50GHz; 8GB RAM; 2 cores.
appropriate side. The latter was used in this work.

### 9.2 Initial Guess of the Unknown Trim Quantities

The solution of the trim problem involves solving a system of nonlinear equations. Due to the complexity of the rotorcraft problem, a numerical method is required for the solution of both the rotor loads and the aircraft trim problem. From Chapter 3, the unknown trim quantities are the aircraft pitch angle $\theta$, collective pitch input $\theta_0$ and pilot stick position $\delta$. The Newton-Raphson scheme used to find the trim solution requires an initial guess and the convergence of the iteration routine can be largely dependent on this initial guess.

The inflow through the rotor disc largely dictates the required twist of the blades. The transition from low-speed to high-speed flight greatly increases the inflow through the rotor due to its change of incidence through the conversion corridor. As a result, a significant change in the collective pitch is required and is as large as 30° for the XV-15 between helicopter and aeroplane mode. The nonlinearity in the aerodynamic tables of the blade sections can compound the divergence of the trim iterations if the initial guess is far from the trimmed value. Due to this fact, a method to determine a sufficiently good approximation to the collective pitch was devised. To calculate a good initial guess, an initial blade loading target was set. The blade loading target is set to a nonzero quantity and, therefore, necessitates a positive rotor thrust at all operating points. In general, this value can be a function of airspeed and rotor tilt angle, however, the blade loading target used in this work was set simply to $(C_T/\sigma)^* = 0.02$. It is worth noting that if the target blade loading is set to a constant value, this value should be chosen to be representative of the blade loading in aeroplane mode when the blade loading is significantly smaller than in helicopter mode. Setting the target blade loading too high can cause convergence problems in aeroplane mode where the target blade loading may be unattainable.

The calculation of the collective pitch from the specified blade loading is now described. Firstly, the rotor was treated as a propeller and, therefore, the gimbal degree of freedom was locked-out. This simplification removed the necessity to integrate the gimbal dynamics through time helping to reduce the computation cost of the rotor model at the initial guess stage. The induced velocity was then modelled as constant over the disc based on the steady rotor thrust. Given the target blade loading $(C_T/\sigma)^*$, the induced inflow was calculated from momentum.
theory:

\[ \lambda_i = \frac{1}{2\sqrt{\mu^2 + \lambda^2}} \left( \frac{C_T}{\sigma} \right)^* \sigma \]  

(9.1)

where \( \mu \) and \( \lambda \) are the advance and inflow ratios in the hub axes and \( \sigma \) is the solidity ratio. The velocity components of the blade from Section 5.2 can then be determined by omitting the gimbal terms. The section lift and drag coefficients are modelled from the simplified expressions:

\[ C_l = \frac{a}{\sqrt{1 - M^2}} (\theta_{tw} + \theta_0 - \phi) \]  

(9.2a)

\[ C_d = 0.01 \]  

(9.2b)

where \( a = 5.7 \text{rad}^{-1} \) is the incompressible lift curve slope commonly used in rotary-wing literature [60, 61] and is compressibility corrected using the Prandtl-Glauert factor; the term \( \theta_{tw} + \theta_0 - \phi \) represents the angle of attack as a function of the collective pitch. The steady thrust is found by averaging the rotor thrust over a revolution:

\[ \frac{\bar{C}_T}{\sigma} = \frac{1}{\pi \rho \Omega^2 R^4 \sigma} \frac{1}{2\pi} \int_0^{2\pi} T(\psi) \, d\psi \]  

(9.3)

where the azimuth-dependent thrust is found by spanwise integration and summation over the blade count:

\[ T(\psi) = n \sum_{i=1}^{b} \left( \int_{nR_0}^{R} \frac{dT}{dx} \, dx \right)_i \]  

(9.4)

A form of the Newton-Raphson scheme described in Section 3.4 was implemented until the collective pitch gives the blade loading target:

\[ \frac{\bar{C}_T}{\sigma} (\theta_0) - \frac{C_T^*}{\sigma} = 0 \]  

(9.5)

The linearity in the blade section aerodynamics improves the convergence of the Newton-Raphson scheme to a suitable collective pitch by removing the nonlinear stall behaviour. A small drag coefficient was implemented to represent a reduction in the thrust due to the profile drag, however, its contribution was small and could have been omitted with very little effect on the predicted collective pitch. The unknown pitch attitude and pilot stick position were set to neutral quantities for the initial guess \( \theta = \delta = 0 \) and showed no issues with convergence.
9.3 Summary

This chapter has included a contribution to the modelling approach of the conversion corridor of tiltrotor aircraft. Three approaches were rationalised to determine both the conversion corridor and the associated trim behaviour. These approaches consisted of a constrained optimisation routine, simple trim sweep and a systematic trim sweep. The simple trim sweep was adopted since it allowed the trim points to be evaluated using parallel computing and also allowed the constraint parameters to be implemented during post-processing. Additionally, a contribution to how the initial guess of the unknown trim quantities can be determined through the conversion corridor was also been presented. These unknown trim quantities are used at the start of the Newton-Raphson scheme in the flight mechanics module. A nonzero blade loading target was specified that was representative of aeroplane mode (this work used \((C_T/\sigma)^* = 0.02\)) and was used to compute an approximation of the required collective pitch. In doing so, the rotor was modelled as a propeller by omitting the gimbal dynamics and implementing a linear representation of the blade section aerodynamic data.
Chapter 10

Results and Discussion

This chapter presents the simulated conversion corridors and their associated trim behaviour. The results are presented for several cases that consider different aerodynamic interactions between the components: the case of no interaction (a baseline case); rotors-on-wing case; rotors-on-empennage case; wing-on-empennage case; all interactions case. The trim behaviour of the interaction cases are also compared to the baseline case. A discussion of the predicted conversion corridors, trim behaviour and aircraft performance is provided that gives insight into the aeromechanics behaviour of tiltrotor aircraft.

The configuration parameters of the aeromechanics model developed in this work are largely derived from the GTRS model of the XV-15 in [45]. The GTRS model was designed to run in real-time and implemented experimental and flight test data to improve the fidelity and trim behaviour exhibited. A validation study of the GTRS model for the XV-15 was given in [29] and showed good correlation to the flight test data presented. Trim results were also presented for the aircraft states, controls and rotor performance through the corridor. These results were, however, limited to flight at sea-level altitude with no climbing. Within the literature, a lack of validation data for tiltrotor aircraft exists in the public domain and this is largely for proprietary reasons. Therefore, comparisons are made between the trim results exhibited by the aeromechanics model develop in this thesis to the trim results from the GTRS model of the XV-15. These comparisons are used to verify the predictions of the current aeromechanics model and assess the impact of the interactional aerodynamics on the predicted conversion corridor and trim behaviour. It is realised, however, that the GTRS model is only a simulation model and further validation efforts are required against experimental and flight
data when publicly available.

The conversion corridor domain was discretised into airspeed intervals of $\Delta V_\infty = 5\text{ kn}$ and rotor tilt intervals of $\Delta \tau = 15^\circ$. The minimum and maximum simulation airspeeds as a function of the rotor tilt angle are summarised in Table 10.1. The trim results of the XV-15 GTRS model in [29] were presented at design gross weight with an aft centre of gravity. Furthermore, the flap/flaperon scheduling was dependent on the rotor tilt angle and is summarised in Table 10.2. All these parameters are defined in the operating point file.

**Table 10.1:** Discrete rotor tilt angles and airspeeds used to compute the conversion corridor.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>160</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>180</td>
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<tr>
<td>30</td>
<td>0</td>
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<tr>
<td>45</td>
<td>50</td>
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<td>60</td>
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<td>240</td>
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<tr>
<td>75</td>
<td>70</td>
<td>260</td>
</tr>
<tr>
<td>90</td>
<td>80</td>
<td>280</td>
</tr>
</tbody>
</table>

**Table 10.2:** Flap/flaperon schedule during the conversion corridor.

<table>
<thead>
<tr>
<th>Rotor Tilt, $\tau$ [deg]</th>
<th>Flap/Flaperon [deg/deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq \tau \leq 15$</td>
<td>40/25</td>
</tr>
<tr>
<td>$\tau &gt; 15$</td>
<td>20/12.5</td>
</tr>
</tbody>
</table>

The predicted conversion corridors are compared to the published corridor presented in [4] shown in Figure 10.1. The configuration parameters used to define the published corridor are not presented and, therefore, the boundaries are used only as a reference to check the predicted boundaries are realistic. The trim solution of the XV-15 is also constrained by two design factors, namely, the magnitude of the gimbal tilt angle and the installed power. Therefore, the validity of the predicted trim solution cannot break these imposed design parameters. The maximum gimbal tilt amplitude and installed power are reported to be $\beta_{\text{max}} = 12^\circ$ and $P_{\text{max}} = 930\text{ kW}$ [6, 28]. No attitude constraint was imposed due to a lack of available data, however, based on the predicted trim results an approximate attitude constraint is derived later.
in the chapter. An induced power factor of \( \kappa = 1.15 \) is used throughout the conversion corridor. This factor was derived in Chapter 5 to give good correlation of the predicted peak figure of merit. It was shown that a larger induced power factor was required in helicopter mode forward flight to correlate with experimental data. However, a constant value was used here for simplicity to avoid calibrating the induced power factor at each operating point and, therefore, the total power in helicopter mode is likely to be under-predicted.

Figure 10.1: XV-15 conversion corridor. Image from [4].

10.1 Case: No Interactions

The predicted conversion corridor accounting for no aerodynamic interaction between the components is presented in Figure 10.2. The published conversion corridor presented in [4] is illustrated by red lines. Hovering flight was found viable at rotor tilts from helicopter mode to 30° which over-estimated hover viability compared to the published corridor. As the rotors were tilted forward past 30°, the min-speed boundary was consistently over-predicted by around 10 kn. On the other hand, the max-speed boundary was largely under-predicted from helicopter mode to 75° rotor tilt. This suggested the predicted power was substantially higher than in the published corridor since the max-speed boundary is typically power limited. The design cruise speed through conversion mode was 170 kn [6] and, therefore, the max-speed boundary was over-predicted past this speed. Beyond this, structural limits on components
become important which are not considered here. The correlation of the predicted corridor against the published corridor showed the low-speed region near helicopter mode was over-predicted. This could be from the attitude constraint that was not imposed. Additionally, the early conversion corridor predicted in [6] compared to the more modern corridor in [4] shows a reduced low-speed operating region. This could be from a flight handling perspective evaluated from flight testing and such considerations are beyond the scope of this work. The max-speed boundary was consistently under-predicted around 10 kn up to the design cruise speed of 170 kn. Compared to the GTRS model, the predicted power was conservative in this region which would suggest the boundary is similarly under-predicted by the GTRS model. The exact configuration used to determine the corridor in [4] is not given and, therefore, definitive conclusions for why the boundaries differ cannot be made.

**Figure 10.2:** Predicted conversion corridor as a function of airspeed. The red lines show the published conversion corridor for the XV-15 from [6].

The blade loading through the conversion corridor is presented in Figure 10.3. The figure shows the blade loading correlation to the GTRS data is good overall and demonstrates the required drag between the models agrees well. The blade loading in hover was under-predicted by approximately 13% which was due to the download from the rotor wakes on the wing being omitted in this no interaction case. The blade loading as the forward speed increased up to 40 kn was also under-predicted and was again due to the low-speed rotors-on-wing interaction being omitted. In helicopter mode, the blade loading towards the max-speed boundary was under-predicted but agreed well for rotor tilt angles of 15° and 30°. Figure 10.4 shows the power
required through conversion. The correlation of the data sets is again good overall with the trends matching. From hover to 40 kn, the power was under-predicted due to the omitted rotors-on-wing interaction. For rotor tilt angles of 0°, 15° and 30°, the power was under-predicted compared to the GTRS model, particularly at the minimum power airspeeds where the thrust was slightly over-predicted from Figure 10.3. This was consistent with the performance results of the rotor model from Section 5.8.4 and could be attributed to the induced power correction being too small and also deficiencies in the wake model. However, for these tilt angles the predicted airspeeds at which the power was a minimum agreed well, occurring around 60 kn. From Figure 10.4, it was found the conversion corridor was power limited at all rotor tilt angles other than helicopter mode. The predicted power at a rotor tilt angle of 60° agreed well with the GTRS model above 120 kn. This matched the predicted conversion mode performance correlations from Section 5.8.5 where the power was well predicted over the blade loading range at \( \mu_{\infty} = 0.32 \) (≈150 kn). Below 120 kn the power was under-predicted despite the blade loading being over-predicted compared to the GTRS model. In aeroplane mode, the blade loading was well predicted but the predicted power was substantially lower than the GTRS model. This under-predicted power was the cause of the max-speed boundary in aeroplane mode being near 250 kn. However, this result was consistent with the predicted propulsive efficiency from Section 5.8.3 where the power was under-predicted for the corresponding blade loading around \( C_T/\sigma = 0.01 \).

**Figure 10.3:** Blade loading through the conversion corridor for no aerodynamic interaction. The filled circles correspond to the GTRS model.
Figure 10.4: Power through the conversion corridor for no aerodynamic interaction. The filled circles correspond to the GTRS model.

The power curves from Figure 10.4 demonstrated the typical rotary-wing and fixed-wing behaviours. In hover, the induced power was largest and can be expected to be higher for tiltrotor aircraft due to the proportionality to the square-root of the disc loading. The power quickly decreased with increased airspeed due to the reduction in induced power, before rising again at higher airspeeds due to the parasitic power of the airframe. Towards aeroplane mode, the power required at low speed was mostly to overcome the lift-induced drag from the wing for the given flap/flaperon deflections. The parasitic power became dominant at high speed due to the proportionality to $V^3_\infty$. The pitch attitude through the conversion corridor is presented in Figure 10.5. The correlation between the data sets is good overall, with the same trends being predicted through transitioning flight. For rotor tilts angles of 0°, 15° and 30°, the trim curves followed the typical trends in helicopter performance: as the airspeed increased, the attitude tended more nose-down with a proportionality $\theta \propto -V^2_\infty$ to overcome the drag of the fuselage [13, 14]. On the other hand, the typical trim curve for fixed-wing aircraft is distinctly different. The angle of attack was largest at low speed to increase the wing lift coefficient and decreased proportionally with $\alpha \propto 1/V^2_\infty$ due to the increased freestream dynamic pressure. This fixed-wing behaviour was observed for rotor tilts from 45° through to aeroplane mode. At these rotor tilts, the pitch attitude did not exceed $\theta = 13^\circ$ which corresponded to the stall angle of the wing as shown in Figure 6.4. With the rotors at 30°, a steady transition from helicopter to aeroplane behaviour was observed from the inflection of the trim curve between 60 kn and
Hovering flight was found viable for rotor tilts from helicopter mode to 30°. As the rotors were tilted forwards, the pitch behaviour was to oppose the rotor tilt, i.e. forward rotor tilt was reacted by pitching nose-up. In hover and low-speed forward flight, the freestream dynamic pressure was small and the aircraft weight was overcome primarily by the lift of the rotors. As the airspeed increased, the attitude tended more nose down to provide the propulsive thrust and thus reduced the wing angle of attack and lift available. In hover and low speed forward flight, as the rotors were tilted forwards the aircraft pitched nose-up to reduce the angle between the thrust and weight vectors, \( \theta - \tau \), which was the cause of the pitch attitude at 30° rotor tilt being so large. From Figure 1.3, the estimated conversion corridor for the XV-15 was attitude-limited up to 50 kn past a rotor tilt of 15°. Since no attitude constraint was imposed here, hovering flight was found viable through to 30° tilt. No data for the XV-15 attitude limit could be found in the literature but cross-correlating the corridors would suggest an attitude limit in the region of 15°. This roughly agrees with that presented in [92] for a tiltwing conversion corridor where an attitude limit of 12° was imposed. However, realistic pitch trim attitudes were found throughout the conversion domain without the pitch attitude constraint. If the thrust is referenced to the tip-path-plane (disc axes) which is tilted through \( \bar{\beta}_c \) relative to the hub plane, then the angle between the thrust and weight vector is \( \theta - (\tau + \bar{\beta}_c) \). The gimbal degree of freedom is required to provide the pitch control authority in the absence of fixed-wing controls at low dynamic...
pressure. The pitch attitude is, therefore, dependent on the required gimbal tilt to trim.

The longitudinal stick position in trimmed flight is presented in Figure 10.6. Whilst the pitch trim showed generally good correlation to the GTRS results, the predicted stick position for the case of no interaction correlated poorly and suggested the aerodynamic interactions are important. The predicted stick position was mostly aft of the GTRS model except for helicopter mode. The increased aft stick compared to the GTRS model was found to increase towards aeroplane mode and implied a larger pitch-up moment was required from the tailplane. In helicopter mode past an airspeed of 20 kn, the stick reversal observed in the GTRS model was not exhibited. Instead, the stick position moved progressively forward to give the nose-down pitch attitude required to trim. The observed stick reversal in the GTRS model was due to the upwash at the empennage from the rotor wakes [15] and, therefore, was not captured without considering this interaction. In helicopter mode, the forward stick position was found to be the constraining parameter at the max-speed boundary. At rotor tilt angles $\tau \geq 45^\circ$, the aft stick position was found to be an imposing constraint at the min-speed boundary. The aft stick position was required to hold the nose-up pitch attitude and freestream angle of attack at low speed. The stick travel was not found to impose any constraints at the max-speed boundary for these rotor tilt angles.

![Figure 10.6: Longitudinal stick position through the conversion corridor for no aerodynamic interaction. The filled circles correspond to the GTRS model.](image)

The predicted aft stick position compared to the GTRS model past helicopter mode implied
differences in either the predicted rotor dynamics or component moments: a larger nose-down moment would require more aft stick travel and vice versa. Due to the omitted aerodynamic interactions, discrepancies between the predicted component moments compared to the GTRS model are likely. In this work, the rotor dynamics were represented using a gimballed rotorhead that allowed the plane of rotation to tilt with respect to the hub plane whereas, in the GTRS model, a first-order approximation of the flapping dynamics was used due to the requirement to run in real-time. Some discrepancy could, therefore, be caused by differences between these rotor dynamics models. Furthermore, the GTRS rotor was modelled as an actuator disc and used a uniform induced velocity model. The rotor model used in this work was based on an individual blade model with the induced velocity described by a linear, first harmonic distribution. For these reasons, the predicted rotor dynamics with respect to cyclic control inputs, airspeeds and/or rotor incidence angles may have been different. The predicted rotor dynamics and trim controls of the current rotor model were not validated against experimental data in the literature and this remains as future work of the project. However, the overall trends between the two stick curves do show good agreement except for low-speed flight in helicopter mode where the stick reversal is exhibited. In hover, a small forward stick input was required to trim the weight moment from the forward position of the rotors relative to the centre of gravity at the aft setting. As the forward speed increased, the natural tendency of the rotors was to tilt the gimbal rearwards due to the increased dynamic pressure on the advancing side and the gyroscopic response of the blades. The rearward gimbal tilt angle tended to produce a nose-up moment on the airframe and was controlled by the longitudinal cyclic pitch input. At higher airspeeds, the rise in freestream dynamic pressure shifted the control authority to the elevator.

Near the min-speed boundary towards aeroplane mode, the longitudinal stick gradient with respect to airspeed was largest and reduced steadily as the forward speed increased. This was caused by the required fuselage angle of attack being largest at low speed to increase the wing lift coefficient and help support the weight. To trim the nose-down moments on the airframe that arose from both the rotors and wing lift, the tailplane was required to produce a counter-acting moment in the nose-up sense. At positive fuselage angles of attack and with no tailplane setting angle or washout (and neglecting the wing downwash), the tailplane angle of attack was identical to the fuselage angle of attack and, therefore, the natural tendency was to produce a nose-down moment from the lift. To create the required nose-up trim moment, the elevator was deflected trailing-edge up by aft travel of the stick. As the airspeed increased, the fuselage trim angle of attack reduced due to the rising freestream dynamic pressure over the wing and the associated increase in lift. Consequently, the elevator deflection required to give the nose-up trim moment decreased due to the reduced angle of attack of the tailplane and, therefore, the
stick gradient became shallower with airspeed.

The magnitude of the gimbal tilt was a constraining parameter to the trim solution and was given by

\[ \bar{\beta} = \sqrt{\bar{\beta}_s^2 + \bar{\beta}_c^2} \]  

(10.1)

where \( \bar{\beta}_s \) and \( \bar{\beta}_c \) are the steady lateral and longitudinal tilt angles. Figure 10.7 shows the magnitude of the gimbal tilt throughout the conversion corridor and, from the figure, was not found to be a constraining parameter. The largest tilt angle was around \( \bar{\beta} = 9^\circ \) and was due to the longitudinal tilt component as shown in Figure 10.8. These large gimbal angles occurred at low speeds to tilt the rotor thrust vector against the weight vector and trim the pitching moments on the airframe. The longitudinal tilt in helicopter mode was always forwards and tended to produce a nose-down moment. The magnitude of the gimbal tilt angle in helicopter mode was small throughout the permissible airspeeds and was well-within the constraining value. At low speed, the dynamic pressure due to the freestream was small and, therefore, so was the wing lift due to the proportionality. To balance the weight at these airspeeds, the thrust of the rotors was required to be orientated against the gravity vector. As the rotors were tilted forwards, the aircraft pitched nose up to reduce the angle between the thrust and weight vectors, \( \theta - \tau \). At these fuselage attitudes, the wing and tail moments acted in the nose-down sense and, therefore, the stick was moved aft to tilt the thrust force backwards to generate the trim moment. Hence, the rearward gimbal tilt increased as the rotors were tilted forwards. The combination of the rotor tilt angle and pitch attitude resulted in a freestream velocity that was orientated largely in the plane of the rotor disc. This acted to excite the rotor to and cause the gimbal to naturally tilt rearwards. The increased aft stick travel as the rotors were tilted forwards was also exemplified by the washout of the longitudinal cyclic pitch with respect to the rotor tilt:

\[ \theta_s \propto \cos \tau \]  

(10.2)

For rotor tilt angles past 45° towards aeroplane mode, the longitudinal gimbal angle was found to steadily increase from a steady rearward tilt to a forward tilt. The change in the steady orientation of the gimbal was due to the rotor incidence that decreased the in-plane velocity component and also due to the washout of the longitudinal cyclic pitch input. The predicted longitudinal tilt angle was rearward of the GTRS model, however, was consistent with the increased rearward stick position. When the rotors operated in propeller mode, there was no cyclic pitch input due to the displacement of the pilot stick. However, the gimbal degree of freedom was not locked-out and, therefore, the gimbal dynamics were free to adopt their equilibrium orientation. In aeroplane mode, the predicted gimbal tilt was found to agree very
well with the GTRS model as shown in Figure 10.8.

Figure 10.7: Gimbal tilt angle spectrum through the conversion corridor for no aerodynamic interaction.

Figure 10.8: Longitudinal gimbal tilt angle through the conversion corridor for no aerodynamic interaction. The filled circles correspond to the GTRS model.

The share of the rotor and wing lift relative to the aircraft weight through the conversion corridor is shown in Figure 10.9. In hover, with the absence of any download forces, the rotor
lift supported all the aircraft weight. As the airspeed increased in helicopter mode, the rotor lift fraction decreased due to the increased dynamic pressure over the wing. In this flight mode, the wing lift increased to a peak value of 9% at approximately 80 kn. Past this speed, the required thrust quickly increased and the fuselage attitude tended more nose-down to give the required propulsive force. The same general trend was observed for rotor tilt angles of 15° and 30° with the unloading and reloading of the rotor lift. At these rotor tilts, the peak share of the wing lift was found to be approximately 28% and 48% at 100 kn and 110 kn, respectively. This showed that at 30° rotor tilt the lift loading of the rotors and wing was approximately equal. At 45° rotor tilt, the min-speed boundary had increased to 85 kn and the wing began to generate more lift than the rotors through the entire airspeed range at around 55% - 65%. As the rotors were tilted more towards aeroplane mode, the unloading of the wing lift at higher airspeeds was not observed since the rotor thrusts were acting increasingly in the propulsive direction. In aeroplane mode, the rotor shaft axes were parallel to the body \( x \)-axis and the orientation of the thrust axis relative to the horizon was given by \( \theta - \bar{\beta}_c \). This orientation was found to give a downward component of the rotor thrusts that acted against the lift of the wing. As the airspeed increased, the pitch attitude of the fuselage and tilt of the gimbal progressed more nose-down. Therefore, the wing lift in aeroplane mode was found to increase to 125% of the aircraft weight at the highest airspeed.

**Figure 10.9:** Rotor and wing lift ratios through the conversion corridor for no aerodynamic interaction. The rotor lift fraction is given by the solid line and the wing lift fraction by the dashed line.
10.2 Case: Rotors-on-Wing Interaction

The predicted conversion corridor for the rotors-on-wing case is shown in Figure 10.10 against the no interaction case. From the figure, it is evident that the influence of the rotors-on-wing interaction on the predicted boundaries was small with a 5 kn increase of the min-speed aeroplane mode boundary and 5 kn decrease of the max-speed boundary at 60° rotor tilt. The influence of the rotor wakes on the wing varied with the operating condition and was largely dependent on the airspeed, rotor inclination and wing position relative to the rotor. The detrimental effect of the rotors-on-wing interaction was the download imposed on the airframe in hover and low-speed flight. A significant portion of the main wing was immersed in the rotor wakes at these conditions and the strong induced velocity field produced by the highly loaded rotor discs can create download forces as large as 10% - 15% of the rotor thrust [12]. Flight test data of the wing download for the XV-15 showed a download of approximately 13% of the rotor thrust [4]. The required increase in rotor thrust to overcome the download was realised as an increase in the required power and thus degraded payload capability and overall performance. As the forward speed increased, the freestream velocity quickly swept the wakes downstream and the problem of the rotor wake interaction was shifted to the empennage. For tiltrotor aircraft, the rotor tilt degree of freedom caused the rotor wakes to again immerse the wing towards aeroplane mode. However, at these operating conditions, the effect of the rotor wakes was less significant but the additional velocity component from the rotor wakes generally acted to reduce the angle of attack of the immersed wing sections.
The influence of the rotors-on-wing interaction on the trimmed pitch and longitudinal stick position are shown in Figures 10.11 and 10.12. The influence of the rotors-on-wing interaction was small with only minor differences compared to the no interaction case. In helicopter mode, the influence of the rotors-on-wing interaction was seen to affect the trim solutions from hover up to 50 kn. Above this airspeed, the rotor wakes were predicted to have swept rearwards off the wing and did not influence the wing aerodynamics. This airspeed was over-predicted compared to experimental data which showed the nonuniform induced velocity in the rotor wake and the strong upwash it created at the wing swept the rotor wakes off the wing by 20 kn [32]. This discrepancy was attributed to the simplified model of the rotor wakes that prescribes the geometry and does not allow it deform under the induction of itself or the presence of the wing. For rotor tilt angles of 15° and 30°, the rotor wakes were again found to affect the trim solutions from hover up to a forward speed of 50 kn. However, as the forward speed increased and the attitude of the rotor changed, the rotor wakes were predicted to reimmerse the wing. This is shown in Figures 10.11 and 10.12 by the small changes to the pitch and stick trims above 100 kn. Throughout the conversion corridor, the influence of the rotors-on-wing interaction on the pitch and stick trim was found to be small. The positions of the immersed wing section were slightly forwards of the centre of gravity due to the forward sweep of the wing and, therefore, the resultant download on the immersed portion of the wing created a
small nose-down moment. This moment was counter-acted with a small aft displacement of the stick position compared to the no interaction case that increased the rearward tilt angle of the gimbal. The ‘hump’ in the pitch trim curve that occurs around 40 kn at 30° rotor tilt was due to the aerodynamic data in the lookup table. At this operating point, the angle of attack from the freestream velocity and superimposed rotor wake velocity corresponded to the peak negative $C_l/C_d$ and hence, a noticeable pitch change due to the moment exerted by the download force.

Figure 10.11: Pitch attitude through the conversion corridor including the rotors-on-wing interaction against the no interaction case shown by dashed lines. The filled circles correspond to the GTRS model.
Figure 10.12: Longitudinal stick position through the conversion corridor including the rotors-on-wing interaction against the no interaction case shown by dashed lines. The filled circles correspond to the GTRS model.

The skew angle of the rotor wake centrelines from the shaft axes is shown in Figure 10.13. The skew angle was found to rapidly increase with forward speed from hover up to 60 kn and reached a maximum value in the region of 70° - 75° depending on the rotor tilt angle. Past the respective peaks and continuing towards aeroplane mode, the wake skew angle then decreased. This was caused by the rotor incidence that reduced the in-plane velocity component. This component of velocity determines the skew velocity away from the rotor shaft and reduces to zero in axial flight. The reduction of the rotor skew angle with airspeed and rotor tilt has the effect of reimmersing the outboard wing sections in the rotor wake. Figure 10.14 shows the fraction of the total wing area immersed in the rotor wake through the conversion corridor. The lack of smoothness in Figure 10.14 was due to the discretisation of the wing such that as the projected area of the wake changed with the operating point the number of immersed wing sections did not necessarily change. The discretisation of the wing was determined from the number of spanwise points to make the integration of the section loads constant over the span; 50 linearly spaced spanwise points were found to be sufficient for the fixed-wing components. The immersed wing area in the rotor wakes from Figure 10.14 showed that as the wake skew angle increased, so did the downstream displacement of the wake and consequently, the area of the wing immersed in the wake decreased. The convection of the rotor wakes rearwards past the wing beyond an airspeed of 50 kn and the partial reimmersion of the wing after 90 kn is
shown in Figure 10.14. The skew angle of the rotor wakes at which the wakes are swept past the wing was given approximately by

\[ \chi \approx \tan \frac{R}{\ell} \]  

(10.3)

where \( \ell \) is the shaft length and \( R \) is the contracted wake radius. Equation 10.3 represents the angle between the contracted wake radius at the front of the disc and the rotor pivot point. For the XV-15, the shaft length and wake radius are given by \( \ell = 1.42 \text{ m} \) and \( R = 0.79 \text{R} \), and the approximate skew angle for the wake to be swept over the wing is then \( \chi \approx 65^\circ \). This value agreed with the results in Figures 10.13 and 10.14 and showed no wing area immersed in the rotor wakes above this skew angle.

**Figure 10.13:** Skew angle of the cylindrical wake centreline relative to the shaft axis through the conversion corridor.
In literature, the download on the wing due to the rotor wakes is typically referenced to the rotor thrust [12, 19, 23]. However, this work defines the download on the aircraft as the component of the wing forces acting parallel to the gravity vector. The body axis components of the wing force, $X_W$ and $Z_W$, are resolved through the fuselage pitch angle to give the lift force $L$ acting against the gravity vector. The lift fraction of the wing is then defined by the ratio of this vertical lift component to the aircraft weight $mg$:

$$\frac{L}{mg} = \frac{X_W \sin \theta - Z_W \cos \theta}{mg}$$  \hspace{1cm} (10.4)

A negative wing lift fraction then implies a net download on the airframe due to the wing force. The wing lift fraction through the conversion corridor is presented in Figure 10.15. The download in hover was dependent on the rotor tilt since this affected the orientation of the induced velocity relative to the wing. In helicopter mode, the download was calculated at 10.8% which represented a significant increase in the thrust and power required. With the rotors tilted forwards, the download was reduced slightly to 9.2% and 7.2% at 15° and 30° rotor tilt.

**Figure 10.14:** Fraction of the total wing area immersed in the rotor wake through the conversion corridor.
The blade loading and power with the rotors-on-wing interaction is shown in Figures 10.16 and 10.17. Compared to the GTRS model, both the blade loading and power were under-predicted but did show a significant increase compared to the baseline trim case. The differences from the GTRS model were likely due to the fact the induced velocity and wake contraction were calculated differently between the two aeromechanics models. In the present model, the wake contraction was prescribed \textit{a priori} whereas in the GTRS model it was based on the total instantaneous rotor force. Additionally, the GTRS rotors-on-wing interaction model was calibrated based on the uniform induced velocity model. The calculated induced velocity was likely larger than that calculated using the Peters and HaQuang model since the download was under-predicted. This highlights the reliance of these reduced-order models on calibration to match experimental data. Figure 10.17 shows that the increased required power was easily accommodated by the available power and did not constrain the conversion corridor. The increased rotor power to overcome the download was, however, completely undesirable.
In forward flight, the download reduced due to the smaller projected area of the wing immersed in the rotor wakes from the increased skew angle. By 40 kn the net effect of the wing was a positive lift force and by 100 kn the outboard sections of the wing were calculated
to be reimmersed in the rotor wakes. This had the effect of inducing a downwash over the immersed wing sections and, therefore, reduced the section angle of attack. This effect was more pronounced towards helicopter mode when the angle between the rotor shaft axes and wing chord line was large. To compensate for the reduced lift of the outboard wing sections, the aircraft pitched up and also increased thrust (since the rotor and wing lift was approximately evenly split at 30° and 45° rotor tilt). The reimmersion of the outboard wing sections was not considered in the GTRS model past 30° rotor tilt which, from the results of this interaction case, made little difference to the trim solutions. The small increase in required power due to the rotor downwash was the cause of the 5 kn reduction of the max-speed boundary at 60° rotor tilt.

Overall, the rotors-on-wing interaction was small on the overall aircraft trim behaviour and predicted conversion corridor. The primary effect of the rotors-on-wing interaction was a rise in the required thrust to overcome the induced download and consequently a rise in the required power. For the XV-15, the required hover power was approximately 50% of the installed power and thus the download power was easily accommodated, though completely undesirable. The effect of the download power would be most pronounced on aircraft with low excess power in hover. The effect of hovering at a higher altitude would also increase the required power and thus, the download effect would be detrimental to the hover ceiling. Research into the estimation and reduction of the download phenomenon has been ongoing through experimental and theoretical studies. The computational expense of the higher-order models that improve the flow physics and load estimations makes their implementation into large analysis domains unsuitable. Empirical and reduced-order models are, therefore, required for a first-order estimation at a preliminary design and analysis stage, as used here. However, these are likely to require case-by-case calibration or empirical correlations to determine suitable values of the induced velocity multiplier, wake contraction ratio, drag coefficient and other corresponding parameters.

10.3 Case: Rotors-on-Empennage Interaction

The rotors-on-empennage interaction was characterised by empirically correcting the velocity and dynamic pressure at the tailplane. Firstly, the rotor induced velocity was corrected and superimposed on the freestream to give the tailplane angle of attack. Secondly, the freestream dynamic pressure was corrected to give the expected lift and pitching moment. The predicted conversion corridor for the rotors-on-empennage case is shown in Figure 10.18.
From the figure, the effect of the rotors-on-empennage interaction was small and showed only an increase in the max-speed boundary in helicopter mode.

![Figure 10.18: Predicted conversion corridor as a function of airspeed for the rotors-on-empennage interaction case (filled green region) against the no interaction case (dashed black lines). The red lines show the predicted limits for the XV-15 from Maisel (2000).](image)

The trimmed pitch attitude and stick position are presented in Figures 10.19 and 10.20 and show a much more pronounced effect of the rotors-on-empennage interaction. The effect of the rotors-on-empennage interaction was apparent for airspeeds from around 10 kn - 100 kn. The pitch attitude for the rotors-on-wing case showed a more nose-down tendency compared to the no interaction case. For the stick position in Figure 10.20, the rotors-on-wing interaction showed a significantly more aft stick position was required to trim. This was the cause of the increased max-speed boundary in helicopter mode since, for the no interaction case, this was control limited. A shallow stick reversal, characterised by an inflection of the stick gradient with respect to the airspeed, was found in helicopter mode and 15° rotor tilt. This was expected since the GTRS model was calibrated to display this behaviour. The effect of the rotors-on-empennage interaction was to induce an upwash at the empennage from the trailing rotor vortices due to the lateral offset of the rotors. These vortices were visualised experimentally when operating in-and-around helicopter mode and found to quickly roll-up downstream of the rotors. Depending on the airspeed and attitude, these vortices where convected downstream and located directly above the tailplane and induced a net upwash [15]. This acted to increase the
tailplane angle of attack, increasing the lift and producing a nose-down moment. The upwash effect was amplified by an increase in dynamic pressure, almost twice that of the freestream.

**Figure 10.19:** Pitch attitude through the conversion corridor including the rotors-on-empennage interaction against the no interaction case shown by dashed lines. The filled circles correspond to the GTRS model.

**Figure 10.20:** Longitudinal stick position through the conversion corridor including the rotors-on-empennage interaction against the no interaction case shown by dashed lines. The filled circles correspond to the GTRS model.
The increased nose-down moment from the upwash at the tail required a larger nose-up moment to trim. At low airspeed, the rotor provided both the propulsive thrust, lift and control moment. The control moment was obtained by cyclic inputs that tilted the rotor tip-path-plane relative to the hub plane in the desired direction. To counter the increased nose-down moment from the tailplane, the stick moved aft to increase the aft gimbal tilt as shown in Figure 10.21. From the figure, the rearward gimbal tilt compared to the no interaction case was increased by approximately 2° around 50 kn airspeed. At these airspeeds, the rotor provided the propulsive thrust, lift and control moment and the pitch attitude was coupled to the trim moment through the gimbal tilt. The angle of the thrust vectors relative to the vertical was $\theta - (\tau + \bar{\beta}c)$ and, therefore, to compensate for the increased aft gimbal tilt the aircraft pitched more nose-down. This reoriented the thrust against the weight vector to provide the required lift. Figure 10.22 presents the effect of the rotors-on-empennage interaction on the predicted blade loading. The figure shows between 60 kn and 120 kn in helicopter mode and 15° rotor tilt, the blade loading was increased slightly to provide the trim control moment. Figure 10.23 shows the wing lift fraction compared to the no interaction case. From the figure, the decreased pitch attitude due to the empennage upwash reduced the wing lift fraction. This was compensated for by an increase in rotor lift and hence, blade loading, as shown in Figure 10.22. The effect of the rotors-on-empennage interaction on the required power is shown in Figure 10.24. The increased blade loading showed only a small increase in the required power which was easily accommodated within the available power.
**Figure 10.21:** Longitudinal gimbal tilt angle through the conversion corridor including the rotors-on-empennage interaction against the no interaction case shown by dashed lines. The filled circles correspond to the GTRS model.

**Figure 10.22:** Blade loading through the conversion corridor including the rotors-on-empennage interaction against the no interaction case shown by dashed lines. The filled circles correspond to the GTRS model.
Figure 10.23: Wing lift fraction through the conversion corridor including the rotors-on-empennage interaction against the no interaction case shown by dashed lines.

Figure 10.24: Power through the conversion corridor including the rotors-on-empennage interaction against the no interaction case shown by dashed lines. The filled circles correspond to the GTRS model.

The angle of attack at the tail was determined from the superposition of the freestream and empirically corrected rotor induced velocity. The induced velocity was based on the mean
induced velocity from momentum theory; $\lambda_0 \Omega R$ from the prescribed three-state induced inflow distribution. The use of the mean induced inflow is usually necessitated by real-time and reduced-order simulations due to the inexpensive computation time of momentum theory. The mean induced inflow through the conversion corridor is shown in Figure 10.25 and followed the expected trend. The largest downwash velocity was found in hover to be $0.07 \Omega R$ and corresponded to a dimensional velocity of approximately $v = 16 \text{m/s}$ at the disc. As the forward speed increased, the induced velocity quickly decreased due to the larger inflow and was approximately halved by a forward speed of 50 kn. The induced velocity decreased with rotor tilt since the thrust required became that to overcome the airframe drag rather than provide lift. The correction factors applied to the induced velocity and freestream dynamic pressure are shown in Figures 10.26 and 10.27. The negative sign in Figure 10.26 indicated the upwash contribution from the rotor wake velocity. It should be noted that these corrections were derived semi-empirically to correlate the empennage lift against the mean induced velocity at the rotors and did not represent the actual rotor wake at the empennage [20]. Areas in Figures 10.26 and 10.27 where the correction factors were zero indicated the tailplane was not affected by the rotor wakes.

![Figure 10.25: Induced inflow ratio through the conversion corridor.](image)

<table>
<thead>
<tr>
<th>Rotor Tilt [deg]</th>
<th>0</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>75</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Induced Inflow Ratio, $\lambda_0$</td>
<td>0.08</td>
<td>0.06</td>
<td>0.04</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>KTAS</td>
<td>0</td>
<td>50</td>
<td>100</td>
<td>150</td>
<td>200</td>
<td>250</td>
<td></td>
</tr>
</tbody>
</table>
The experimental data in [15, 31] was used to empirically model the rotors-on-empennage interaction for the XV-15 with H-tail empennage configuration. The tailplane was approximately the same waterline position as the main wing. In the present aircraft model, the rotors-on-empennage interaction had only a small effect on the predicted conversion boundaries, however, did have a strong influence on the pitch and stick trim. The successor to
the XV-15, the V-22 Osprey, has a disc loading around $1.5 \times$ that of the XV-15 (100 kg m$^{-2}$ compared to 65 kg m$^{-2}$). Consequently, the impact of the rotors-on-empennage interaction is likely to be more pronounced since the rotor wake is stronger. The V-22 Osprey does use a negative tailplane setting [88] which would reduce the effect of the upwash on the tailplane. Furthermore, the interaction is largely dependent on the airspeed, rotor attitude and empennage position. The Leonardo AW 609 civil tiltrotor and Bell V-280 Valor both utilise different empennage configurations, a T-tail and V-tail, respectively. Therefore, the empirical model developed for the XV-15 is not likely to be directly transferable and would need calibrating to specific configurations.

10.4 Case: Wing-on-Empennage Interaction

The wing-on-empennage interaction was characterised by a reduction of the tailplane angle of attack due to the vortex wake of the lifting wing and reduced dynamic pressure. The predicted conversion corridor for the wing-on-empennage case is shown in Figure 10.28. The overall change of the determined boundaries was 5 kn - 10 kn. The inclusion of the wing-on-empennage interaction reduced the min-speed boundary towards that from [4] but made little difference to the correlation at the max-speed boundary.
Figure 10.28: Predicted conversion corridor as a function of airspeed for the wing-on-empennage interaction case (filled green region) against the no interaction case (dashed black lines). The red lines show the predicted limits for the XV-15 from Maisel [6].

The pitch attitude through the conversion corridor is shown in Figure 10.29 against the case for no interaction. The figure shows the pitch attitude was most affected by the wing-on-empennage interaction from hover to 100 kn forward flight. Past approximately 120 kn, the influence of the wing downwash on the pitch attitude was negligible. Throughout the corridor, the effect of the wing downwash was to increase the pitch attitude compared to the no interaction case. In helicopter mode, the increased pitch attitude reduced the correlation to the GTRS model. For 15° and 30° rotor tilt the wing-on-empennage interaction still correlated well to the GTRS model. A sharp increase in pitch was found at 45° rotor tilt at the min-speed boundary and peaked around 20°. At this attitude, the wing was stalled and the loss of wing lift was compensated by an increased pitch to align the rotor thrust towards against the weight. The increased blade loading is shown in Figure 10.30 where the loading increased to around 50% compared to the no interaction case. In helicopter mode and 15° and 30° rotor tilts, the wing-on-empennage interaction reduced the blade loading towards higher airspeeds. This was due to the increased pitch angle at trim compared to the no interaction case and hence, a larger wing angle of attack and lift coefficient. This allowed the rotor lift to be offloaded slightly as shown in Figure 10.31. Figure 10.32 shows that the reduced blade loading decreased the required power slightly, but not significantly throughout the conversion corridor.
Figure 10.29: Pitch attitude through the conversion corridor including the wing-on-empennage interaction against the no interaction case shown by dashed lines. The filled circles correspond to the GTRS model.

Figure 10.30: Blade loading through the conversion corridor including the wing-on-empennage interaction against the no interaction case shown by dashed lines. The filled circles correspond to the GTRS model.
Figure 10.31: Wing lift fraction through the conversion corridor including the wing-on-empennage interaction against the no interaction case shown by dashed lines.

Figure 10.32: Power through the conversion corridor including the wing-on-empennage interaction against the no interaction case shown by dashed lines. The filled circles correspond to the GTRS model.

The wing-on-empennage interaction acted in the opposite sense to the rotors-on-empennage by creating a downwash rather than an upwash on the tailplane. The reduced angle
of attack at the tail decreased the lift and acted to give a larger nose-up moment compared to the no interaction case. At low speed, the elevator control authority was small and the pitch moment was trimmed from the gimbal tilt of the rotors. Figure 10.33 shows the longitudinal stick position through the conversion corridor. The stick position for the wing-on-empennage case was always forward of the no interaction case. This was caused by the larger nose-up moment at the tail compared to the no interaction case. When operating towards helicopter mode, the increased nose-up moment was countered by moving the stick forward and causing the gimbal to tilt nose-down. Figure 10.34 shows the increased forward tilt angle of the gimbal in trim compared to the no interaction case. For all cases except aeroplane mode, the forward gimbal tilt increased due to the forward stick position. No difference was found in aeroplane mode since the rotor controls are completely phased out. When the rotors provided the force and moment requirements for the aircraft, the pitch attitude was coupled to the trim moment. The angle of the thrust vector relative to the vertical was $\theta - (\tau + \bar{\beta}_c)$ and hence to compensate for the increased forward tilt of the gimbal, the aircraft pitched nose-up.

**Figure 10.33:** Longitudinal stick position through the conversion corridor including the wing-on-empennage interaction against the no interaction case shown by dashed lines. The filled circles correspond to the GTRS model.
Figure 10.34: Longitudinal gimbal tilt angle through the conversion corridor including the wing-on-empennage interaction against the no interaction case shown by dashed lines. The filled circles correspond to the GTRS model.

As the rotors are tilted more towards aeroplane mode, the change in stick position was considerable and amounted to $\approx 30\%$ of the total stick travel towards the min-speed boundary. Figure 10.35 shows the tailplane lift coefficient as a function of the angle of attack and elevator input. The figure shows that for a given elevator input, the reduced angle of attack caused by the downwash reduces the tailplane lift. For static stability, the aircraft centre of gravity was forward of the wing aerodynamic centre and required the tail to produce a nose-up moment. At lower speeds, the fuselage was pitched nose-up to attain the required angle of attack to derive the lift from the wing. At these attitudes, the tail tended to pitch the aircraft nose-down which was overcome by deflecting the elevator trailing edge-up. The downwash from the wing acted to reduce the tailplane angle of attack and, therefore, required less elevator input to provide the negative lift required. This was the cause of the forward migration of the trimmed stick position. At $45^\circ$ rotor tilt below 90 kn, the forward travel of the stick compared to the no interaction case gave enough control availability to pitch the aircraft nose-up, past the wing stall, and derive the required lift from the thrust of the rotors.
Figure 10.35: Tailplane lift coefficient as a function of the angle of attack and elevator deflection (positive trailing-edge down). Discrete data points from [45].

The wing downwash angle through the conversion corridor is shown in Figure 10.36. The sharp increase in downwash angle from hover to forward flight was due to there being no freestream velocity in hover and, therefore, the tailplane was obsolete. The figure shows the largest downwash angles occurred for 15° rotor tilt at low speed. This was caused by the large nose-up pitch attitudes required to maintain sufficient lift as the rotors were tilted forwards. The downwash was then exemplified by the large flap/flaperon deflections used to reduce the wing download and resulted in a larger downwash angle compared to a clean wing configuration (due to the higher lift coefficient of the wing). These large downwash angles are, therefore, characteristic of tiltrotor aircraft operating at low airspeeds with the rotors tilted forwards and the wing-on-empennage interaction is an important interaction at these operating points. The large fuselage pitch attitudes at rotor tilts of 30° and 45° below approximately 100 kn exceeded the wing angle of attack and, from Figure 10.36, was the cause of the reduced downwash angle in this region.
10.5 Case: All Interactions

The predicted conversion corridor for all interactions considered is shown in Figure 10.37. The conversion corridor for all interaction showed the min-speed boundary was reduced by 5 kn - 10 kn past a rotor tilt of 30°. The low-speed region in early conversion mode was still over-predicted compared to the corridor presented in [4], however, this was likely due to the absence of a pitch attitude constraint. In helicopter mode, a 5 kn increase at the max-speed boundary was predicted and, from Section 10.3, was due to the rotors-on-empennage interaction that generated an upwash at the tailplane and reduced the required forward stick position. The difference in the predicted max-speed boundary towards aeroplane mode was small and arose due to small changes in the predicted power as a result of the rotors-on-wing interaction. Since the parasitic power increased proportionally to the cube of the airspeed, small changes in the power-limited max-speed boundary were found when the airspeed was sampled at $\Delta V_{\infty}$, 5 kn intervals. Overall, the effect of the aerodynamic interactions on the predicted corridor boundaries was found to be fairly small for the current aircraft configuration and could have been well approximated without considering any interactions. However, the singular and combined interactions did have a significant impact on the predicted trim behaviour and aircraft performance as discussed next.
Figure 10.37: Predicted conversion corridor as a function of airspeed for all interactions (filled green region) against the no interaction case (dashed black lines). The red lines show the predicted limits for the XV-15 from Maisel [6].

The pitch attitude through the conversion corridor is shown in Figure 10.38. In helicopter mode, the inclusion of the wing-on-empennage downwash coupled with the rotors-on-empennage upwash resulted in a small nose-down attitude compared to the no interaction case. This implied the rotors-on-empennage interaction was more pronounced since this tended the pitch attitude nose down. The pitch attitude in helicopter mode agreed well with the GTRS model with both the rotors-on-empennage and wing-on-empennage interactions included. For 15° and 30° rotor tilt there was a small increase in pitch attitude compared to the no interaction case. Therefore, at these rotor tilts the wing-on-empennage downwash was more pronounced. This was due to the higher pitch attitude required in hover and low speed to compensate against the forward tilt of the rotors which caused the wing angle of attack, and hence downwash, to increase. The predicted pitch trim agreed well with the GTRS model throughout the airspeed range equally well with and without the aerodynamic interactions. At 45° and 60° rotor tilt, trimmed flight was found viable above the wing stall limit. This was because the share of the rotors and wing lift was fairly close, around 50% - 60%, as seen in Figure 10.39. The increased nose-up attitude and increased thrust provided the additional lift. Since the wing was stalled, there was a sharp increase in blade loading and power required to overcome the additional drag, as seen in Figures 10.40 and 10.41. Nearer aeroplane mode, the inclusion of all the interaction models showed little effect on the pitch trim.
**Figure 10.38:** Pitch attitude through the conversion corridor with all interactions against the no interaction case shown by dashed lines. The filled circles correspond to the GTRS model.

**Figure 10.39:** Wing lift fraction through the conversion corridor with all interactions against the no interaction case shown by dashed lines.
Figure 10.40: Blade loading through the conversion corridor with all interactions against the no interaction case shown by dashed lines. The filled circles correspond to the GTRS model.

Figure 10.41: Power through the conversion corridor with all interactions against the no interaction case shown by dashed lines. The filled circles correspond to the GTRS model.

The inclusion of all the aerodynamic interactions showed significant changes to the
trimmed stick position as shown in Figure 10.42. In hover, the stick position moved slightly aft to tilt the gimbal nose-up against the nose-down moment caused by the wing download. In helicopter mode, the upwash from the rotors-on-wing interaction was dominant and a shallow stick gradient was found from 20 kn and 60 kn. A stick reversal was not predicted compared to the rotors-on-empennage only case. Since the upwash velocity at the tailplane was calculated from the rotor induced velocity it is likely the different induced velocity models between the current model and GTRS model was the cause of this. Despite the absence of the stick reversal, the trend between the current model and GTRS model agreed well but was predicted more forward from 20 kn. The increased aft stick position compared to the no interaction case was the reason for the small increase of the max-speed boundary since, for helicopter mode only, the boundary was control limited. For 15° rotor tilt the stick position agreed fairly well and exhibited an almost linear increase with airspeed. Similar behaviour was shown at 30° rotor tilt where the predicted stick position correlated well with the GTRS results but was slightly aft. For rotor tilts from 45° to aeroplane mode the stick behaviour was almost identical to the wing-on-empennage case and was, therefore, dominated by the wing downwash. The increased forward stick position from the wing-on-empennage interaction at 45° and 60° rotor tilt provided enough control margin to trim at the min-speed boundary above the wing stall angle of attack.

Figure 10.42: Longitudinal stick position through the conversion corridor with all interactions against the no interaction case shown by dashed lines. The filled circles correspond to the GTRS model.
10.6 Summary

This chapter has presented the results of the simulated conversion corridors for the different aerodynamic interactions. The predicted corridor boundaries have been compared to published data and the predicted trim behaviour and aircraft performance have been compared to the GTRS validation data. It was found the conversion corridor boundaries could have been well-approximated without the considerations of the aerodynamic interactions. The wing-on-empennage interaction had the largest effect on the predicted boundaries and acted favourably to reduce the min-speed boundary and widen the corridor. The high-speed boundary was over-predicted towards aeroplane mode due to the power being under-predicted and the absence of any structural constraints. Similarly, the low-speed region of the conversion corridor was also over-predicted compared to the published data but may have been due to the absence of a pitch attitude constraint.

On the other hand, the trim behaviour and aircraft performance were significantly affected by the aerodynamic interactions. The main effect of the rotors-on-wing interaction was the significant download imposed that increased the rotor thrust and required power. Only a small effect was found on the pitch and stick trim behaviour and this was due to the small offset of the download forces from the centre of gravity. This interaction would be more pronounced when the download exerts a significant pitching moment on the airframe. The wing-on-empennage and rotors-on-empennage interactions acted against other. The upwash caused by the rotors-on-empennage interaction was found to be dominant in helicopter mode and pitched the aircraft more nose-down and introduced an undesirable stick reversal at low speed. Conversely, the increased pitch attitude during early conversion mode and the large flap settings used to alleviate the wing download both contributed to significant downwash angles at the tailplane. The wing downwash was found to be more dominant on the trim behaviour and aircraft performance at such operating conditions. Since the rotors-on-empennage and wing-on-empennage interactions oppose each other at lower speeds, both interactions must be included in an aeromechanics analysis to avoid trim predictions that are biased towards a single interaction. Furthermore, the wing-on-empennage interaction significantly reduced the aft stick input required towards aeroplane mode and, without considering such an interaction, can cause the predicted corridor to be stick-limited towards aeroplane mode.
Chapter 11

Conclusions and Future Work

This thesis has documented the development of a reduced-order tiltrotor model suitable for aeromechanics investigations. Of interest here was the conversion corridor; the flight envelope encompassing the transition from helicopter to aeroplane mode where the lift is off-loaded from the rotors to the wing (or vice-verse for reconversion). The combined capabilities of rotary-wing and fixed-wing aircraft into a single aircraft creates complex interactions between different components. The aeromechanics model was used to investigate the effects of these interactions on the predicted conversion corridor, trim behaviour and aircraft performance. The rotors-on-wing, rotors-on-empennage and wing-on-empennage interactions were investigated using reduced-order and empirical interaction models given the large operating domain considered. This chapter concludes the presented work and details suggested future work for the project.

11.1 Conclusions

This section concludes the general objectives of this thesis that were given in Chapter 1:
1. Construct a generic model for an aeromechanics analysis of tiltrotor aircraft.

A generic framework to determine the aerodynamic loads on tiltrotor aircraft has been presented by decomposing the aircraft into the main aerodynamic components: fuselage, rotors, wing and empennage. The aeromechanics analysis was programmed in MATLAB using an object-oriented approach and, therefore, easily facilitates the construction of generic tiltrotor designs. The input data is read from text files and is readily reconfigurable to geometry/design changes as well new/updated aerodynamic data. Each component type has an associated aerodynamic module that calculates the aerodynamic forces and moments on the airframe as is called by the main flight mechanics module when calculating the trim solution.

2. Implement theoretical and/or empirical interaction models into the relevant component aerodynamic models to predict the effects of the rotors-on-wing, rotors-on-empennage and wing-on-empennage interactions on both the predicted conversion corridor and its associated trim behaviour and performance.

A general model to account for the rotors-on-wing interaction was presented based on the wake geometry of a skewed cylinder. This model allows the rotors-on-wing interaction to be accounted for at arbitrary operating points through the conversion corridor domain. The interaction model does require calibration of the relevant parameters to match experimental data, however, can easily be reconfigured for generic tiltrotor aircraft. The rotors-on-empennage interaction was modelled empirically due to the complex interaction mechanisms and lack of reduced-order models in the literature. The model was based on two correction factors; one to the induced velocity of the rotors and the other to the dynamic pressure at the empennage. The wing-on-empennage interaction was modelled using a downwash angle on the tailplane angle of attack and an associated dynamic pressure loss.

3. Determine a suitable and robust numerical method to solve the aircraft equations of motions for all operating conditions.

The general trim problem of tiltrotor aircraft is under-determined; there are more unknown trim quantities than trim equations. To close the trim problem, the pitch trim controls were expressed parametrically as a function of the pilot stick position. The control scheduling was made consistent with the GTRS model with the longitudinal cyclic being washed out with rotor tilt and the elevator being constant. The unknown trim quantities were then the fuselage pitch attitude, rotor collective pitches and pilot stick position. Due to the large
variation of collective pitch throughout the conversion corridor, a robust methodology to find a ‘good’ initial guess was required. This work suggests trimming a linearised aerodynamic model of a propeller to a specified blade loading target to find the required collective pitch. The trim solution of a given operating point is then found by using a damped Newton-Raphson scheme with a variable damping factor. The variable damping factor reduces the step size when the norm of the step vector is large and accelerates the convergence towards the trim solution. Both of these strategies showed robust and favourable convergence characteristics at all operating points.

4. **Rationalise and critically examine several approaches that could be used to determine the conversion corridor and decide on the most practical method.**

Three approaches were rationalised that could be used to evaluate the conversion corridor domain and solve for the conversion boundaries: a constrained optimisation, simple trim sweep and system trim sweep. The simple trim sweep was adopted in this work as the conversion constraints could be imposed during post-processing and the conversion domain can be readily parallelised. Such advantages make the developed aeromechanics model ideally suited for large domain and optimisation investigations.

5. **Verify the predicted trim behaviour and performance of the aeromechanics model through simulated conversion corridors. In the first instance, the aerodynamic interaction is neglected to construct a baseline case.**

The developed aeromechanics model was first simulated through a conversion corridor neglecting all aerodynamic interactions. The predicted conversion corridor was compared with the published literature corridor and showed good correlations overall. The low-speed region was over-predicted but this was likely attributable to the absence of a pitch attitude constraint. The max-speed boundary towards aeroplane mode was also over-predicted due to the power being under-predicted and no consideration of structural constraints. The predicted trim behaviour and aircraft performance did, however, show good correlations to the validation data.

6. **Present an aeromechanics analysis of tiltrotor behaviour through the conversion corridor. Discuss the changes to the trim quantities, performance and conversion boundaries for each of the interaction cases and conclude the importance of each interaction to determine the conversion boundaries and trim behaviour.**
The effects of the aerodynamic interactions on the predicted conversion boundaries were fairly small overall and these boundaries could have been well approximated without interactional considerations. The wing-on-empennage interaction was found to have the largest influence on the predicted conversion boundaries by alleviating the constraining aft stick position towards aeroplane mode.

The trim behaviour and aircraft performance, however, were significantly affected by the aerodynamic interactions. The rotors-on-wing interaction created a substantial download on the aircraft resulting in a significant increase in required thrust and power. Such an interaction can be a constraining factor for aircraft with low excess power in hover when the required download is largest. This interaction also reduces the hover ceiling. The rotors-on-wing interaction was not found to significantly affect the trim behaviour due to the small pitching moment exerted by the imposed download. This interaction would be more pronounced when the download exerts a significant pitching moment on the airframe (due to, for instance, the location of the centre of gravity).

The wing-on-empennage and rotors-on-empennage interactions created a downwash and upwash at the tailplane. These interactions, therefore, acted against each other through the conversion corridor. In helicopter mode, the rotors-on-empennage interaction was found to be more dominant and resulted in a more nose-down trimmed pitch attitude. Furthermore, this interaction created an undesirable shallow stick gradient at low speed. A stick reversal was not exhibited in the current model but this was likely due to differences in the induced velocity models of the current work and the GTRS model. As the rotors were tilted forwards, the pitch trim behaviour was found to oppose this motion resulting in increased nose-up trimmed pitch angles. The large fuselage angles of attack and large flap/flaperon settings generated large downwash angles at the tailplane at low speeds. The wing downwash, therefore, had a considerable impact on both the pitch and stick trim and was found to be the dominant interaction past helicopter mode. Towards aeroplane mode, the downwash at the tailplane migrated the trimmed stick position more forwards and, compared to the no interaction case, prevented the aft stick position being a constraining parameter at the low-speed boundary. Since the rotors-on-empennage and wing-on-empennage interactions oppose each other, both interactions should be considered to accurately predict the aircraft trim behaviour. Including only one of the interactions would bias the trim behaviour towards the included interactions. The wing-on-empennage interaction had the largest effect on the predicted trim behaviour throughout the airspeeds considered and, therefore, should be
included in an aeromechanics analysis.

This thesis has also presented a new formulation to predict the dynamics of a rigid-bladed gimballed rotor. The equations of motion were derived by balancing the inertial, aerodynamic and hub spring moments on the rotorhead from the blades on the rotor. The dynamics were derived for either sense of rotor rotation owing to the counter-rotation of the lateral-tandem tiltrotor configuration. The equations of motion are valid through helicopter, conversion and aeroplane mode provided the small-angle approximation is valid.

11.2 Future Work

The current aeromechanics model has been constructed to robustly find the longitudinal trim solution of the whole aircraft. The aerodynamic forces and moments on the airframe are derived from aerodynamic modules of generic component types with interactions accounted for from predetermined interaction models. The future work of this project can be categorised into four topics:

1. **Improvements of the aerodynamic modules.**

   The calculation of the aerodynamic forces and moments on the aircraft is paramount to accurate trim and performance predictions. Therefore, validation of each of the aerodynamic modules against a larger data set is required to ensure the aerodynamic loads are consistently accurate. An improved wake model is suggested for the rotor module due to the under-predicted rotor power throughout the operating domain. The Peters-He [85] wake model was suggested as this retains the finite-state representation with reduced computational cost compared to higher-order methods such as free wake analyses. Furthermore, a validation study of the rotor trim controls is required. Based on the results of the validation effort, an improved database of induced power corrections factors could be derived that can be incorporated into reduced-order codes.

2. **Extension of the flight mechanics modules in lateral/directional flight.**

   The current aeromechanics model was derived for longitudinal flight which is typically only considered for the construction of the conversion corridor. For more general aeromechanics
considerations, the model should be extended to include a description of the lateral and directional motion. This would, however, require changes to the aerodynamic modules to account for the lateral/directional forces and moments.

3. **Incorporation of a flight dynamics module to assess stability and control methods.**

   The flight mechanics module was used to find a trim solution of the aircraft. Once this trim solution is found, the stability of the trim point and the dynamic behaviour of the aircraft can be investigated through the development of a flight dynamics module. Such a module can be used to investigate control methods to improve handling qualities and dynamic stability through the conversion domain.

4. **Control scheduling through the conversion corridor.**

   The trim problem for tiltrotor aircraft is under-determined. To close the trim problem in this work the pitch controls were parametrically related to the pilot stick position. This under-determined trim problem also extends to the case of lateral/directional flight. The reduced-order nature of the current aeromechanics model is ideally suited to investigate control strategies as a function of airspeed and rotor tilt angle. This can be undertaken through an optimisation routine with the aim to, for example, minimise the required power or pilot workload through the conversion domain.
References


