NUMERICAL SIMULATION OF VORTEX-INDUCED VIBRATION OF FREE SPANNING UNDERWATER PIPELINE CLOSE TO THE SEABED

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ABSTRACT

Subsea pipelines and cables are important infrastructure in the offshore environment, particularly for conveying hydrocarbon and electricity. Failure of such lines can lead to significant environmental and economic impact, and potentially loss of life. Prediction and mitigation of the risk of failure is therefore extremely important. One of the major threats affecting the structural integrity of subsea pipelines and cables is vortex-induced vibration (VIV) which causes repeated occurrence of high values of stress which, when accumulated, can lead to fatigue failure. Existing design practice applies relatively high safety factors and is limited in application, particularly with regard to the influence of bed proximity on dynamic response and hence fatigue design.

This study aims to reduce conservatism in design practice by the development of a computationally efficient numerical method to analyse the effect of wall gap ratio (e/D) on dynamic response of free spanning pipeline. A reduced order method (ROM) has been developed for this purpose in which pipeline response is simulated using an FEA model with sectional forcing obtained from multiple CFD simulations of a 2D freely supported cylinder near a wall for a range of mass-damping ratios. This approach is validated relative to experimental measurement of response of a free span in unconstrained flow with the magnitude of peak deformation to within 5% and fatigue damage over a range of reduced velocity in the range 5-12%. This is shown to be more accurate than a widely used wake oscillator which is also computationally less costly than full two-way Fluid-Structure Interaction simulation.

The validated ROM, when applied to a cylinder pinned at both ends, it predicts a lower fatigue load, by a factor of two to three than the widely used DNVGL-RP-F105 in the industry, thus offering scope for significantly reduced design conservatism. Applied to a flexible cylinder close to a wall (e/D < 2) the method leads to fatigue load predictions that are between 12.5% to 25% of those obtained by the standard design method.
DECLARATION

No portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.
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DEDICATION

This work is dedicated to the memory of my late father
ACKNOWLEDGEMENTS

First, I thank Almighty Allah for everything I am and for what I will be. I thank HIM for the gift of life. Without HIM, this work would not be possible.

I thank my esteemed supervisor – Prof. Tim Stallard for his invaluable supervision, moral support and guidance throughout the journey of my PhD. I cannot overemphasise the level of support Prof. Tim Stallard had given me and still giving me. Words aren’t enough to express my gratitude. The extra-ordinary support you gave me and still giving me gives true meaning to my life. It was indeed a great honour to have you as my supervisor. I also thank my co-supervisor – Dr. Imran Afgan for his direction, advice and support.

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My gratitude goes to my mother for her constant prayer and support from inception of my study till this moment. I also acknowledge my late father, who was always encouraging and supporting me till his final moment.

Lastly, I thank my wife for her absolute moral support and for believing in me during the challenging period of my study, and for accommodating and patiently dealing with my frustration. You are indeed a jewel.
NOMENCLATURE

GENERAL SYMBOLS

\(\bar{p}_i, p_i\)  Mean, fluctuating pressure components in Cartesian tensors (N/m\(^2\))

\(\bar{u}_i, u_i\)  Mean, fluctuating velocity components in Cartesian tensors (m/s)

\(A_{ILj}\)  Streawise / In-line unit diameter amplitude stress for the j-th mode (N/m\(^2\))

\(C_{D_{\text{mean}}}, \bar{C}_D\)  Mean drag coefficient

\(C_{L_{\text{mean}}}, \bar{C}_L\)  Mean lift coefficient

\(f_{nL}\)  Local natural frequency of beam segment (Hz)

\(K_s\)  Stability parameter

\(S_{IL}^{\text{max}}\)  Response stress range associated with the dominant in-line mode (N/m\(^2\))

\(S_{ILj}^{p}\)  Preliminary stress range for j-th inline / streamwise mode (N/m\(^2\))

\(U_m\)  Amplitude of velocity of oscillating flow (m/s)

\(v_{rL}\)  Local reduced velocity

\(v_r\)  Reduced velocity

\(y_{\text{rms}}\)  Root mean square of displacement in the transverse direction (m)

\(z_{\text{rms}}\)  Root mean square of displacement in the stream-wise direction (m)

\(D\)  Diameter of cylinder / pipe (m)

\(E\)  Modulus of Elasticity (N/m\(^2\))

\(e/D\)  Gap ratio

\(f\)  Response frequency of vibration of beam / pipeline (Hz)
**Nomenclature**

- $f_n$ : Natural frequency of beam / pipelines (Hz)
- $I$ : Area moment of inertia ($m^4$)
- $k$ : Turbulent kinetic energy ($m^2/s^2$)
- $KC$ : Keulegan Carpenter number
- $m_a$ : Fluid added mass (kg)
- $m_s$ : Cylinder mass (kg)
- $Re$ : Reynolds number
- $St$ : Strouhal number
- $S_{ult}$ : Ultimate Tensile Strength of a material ($N/m^2$)
- $t$ : Thickness of the cylinder wall (m)
- $Y$ : Displacement amplitude in the transverse directions (m)
- $y^+$ : Dimensionless distance from the wall, $yU_r/\nu$
- $Z$ : Displacement amplitude in stream-wise direction (m)

**Greek Symbols**

- $\sigma_{eff}$ : Effective / corrected stress amplitude for fatigue on a non-sinusoidal stress cycle ($N/m^2$)
- $\sigma_m$ : Mean stress ($N/m^2$)
- $\sigma_y$ : Yield strength of material ($N/m^2$)
- $\nu$ : Kinematic viscosity, $\mu/\rho$ ($m^2/s$)
- $\rho$ : Density ($kg/m^3$)
**Nomenclature**

\[ \mu \] Dynamic viscosity (Ns/m\(^2\))

\[ \gamma \] Stall parameter

\[ \nu_t \] Turbulent kinematic viscosity (m\(^2\)/s)

\[ \mu_t \] Turbulent viscosity (Ns/m\(^2\))

\[ \Phi \] Phase difference between force and displacement (rad)

\[ \varepsilon \] Turbulent dissipation rate (m\(^2\)/s\(^3\))

\[ \omega \] Angular velocity (rad/s)

**ACRONYMS**

2-D Two Dimension

3-D Three Dimension

CFD Computational Fluid Dynamics

DNS Direct Numerical Simulation

DNV Det Norske Veritas

DOF Degree-of-freedom

EVM Eddy-Viscosity Model

FEM Finite Element Method

FSI Fluid-Structure Interaction

FVM Finite Volume Method

LES Large Eddy Simulation
Nomenclature

RANS  Reynolds Average Navier-Stokes
ROM  Reduced Order Method
RSM  Reynolds Stress Model
VIV  Vortex-Induced Vibration

GUIDELINES AND STANDARDS

DNV-RP-F105  DNV Recommended Practice for Free spanning pipelines (edition, 2006)
DNVGL-RP-F105  DNV Recommended Practice Free spanning pipelines (Revised edition, 2017)
DNVGL-ST-F101  DNV Recommended Practice for Submarine Pipeline System
CHAPTER 1

INTRODUCTION

1.1 BACKGROUND STUDY AND MOTIVATION

Subsea pipelines are important infrastructures in the offshore environment for conveying hydrocarbon. Failure of subsea pipelines can lead to environmental pollution, economic loss, operation down time and loss of lives. Therefore, prediction, elimination, and mitigation of the risk of failure of these very important infrastructures are keys to their design and operation throughout their life span. Similar challenges are posed in the design of subsea cables which are critical for telecommunications globally and for power export from offshore power plants. One of the major threats affecting the structural integrity of subsea pipelines and cables is vortex-induced vibration (VIV) which can cause high peak stresses and lead to fatigue failure (Violette et al, 2007; Rezazadeh et al., 2010). Figure 1.1(a) shows a taxonomy of piping fatigue (Keprate and Ratnayake, 2016). The highlighted regions are the path of fatigue failure due to vortex-induced vibration.

Pipeline locations that are particularly susceptible to VIV are sections that are free spanning, occurring due to local routing over trenches or between ridges during installation, or due to changes in the seabed bathymetry over time. Figure 1.2 shows a typical free spanning pipeline. The methods of installation, route and position of pipelines in the offshore environment thus play a major role in the stability of pipelines. Pipelines can be installed trenched or buried in the seabed to avoid interference with third party equipment such as trawl gear. Pipelines installed through this method have little exposure to hydrodynamic loads but are difficult to maintain or inspect for damage. Furthermore, the cost of trenching or burying pipeline in the seabed can be huge. Pipelines are also be laid directly on the seabed. This method provides an easy access for maintenance and inspection. However, in most cases, it is generally difficult to achieve perfectly laid pipelines on seabed without existence of gaps, especially as exploration is further moving into challenging environments such as deeper water locations or higher speed flows. At sites with higher average speeds particular challenges arise from the combination of
scoured beds in which trenching is impractical and higher cable forcing. Across most sites, a common occurrence is that pipelines span (unsupported sections) seabed features either due to uneven topography of the seabed, and/or due to local scouring – a phenomenon where seabed underneath pipeline is eroded leaving behind a gap (Liang et al, 2005; Yang et al, 2018; Zang et al, 2021).

![Figure 1.1: Taxonomy of piping fatigue (Keprate and Ratnayake; 2016, pp. 74)](image)

![Figure 1.2: Free spanning horizontal pipeline](image)

(a) Horizontal pipeline on rough seabed (Holden et al, 2006)

(b) A typical pipeline span showing the span length and gap

Figure 1.2: Free spanning horizontal pipeline
Crossflow of current past such an unsupported section (span) results in vortex shedding downstream of the pipeline. Depending on the length and gap ratio of the span, the free spanning section of the pipeline may behave like a flexible beam and undergo vibration induced by vortex shedding. Repeated occurrences of multiple time-varying stresses on the pipeline due to VIV over a long period of time can lead to fatigue failure. A sudden/premature catastrophic or destructive vibration can occur particularly when the shedding frequency approaches the natural frequency of the pipeline since this causes high amplitude response and peak stresses. Such synchronization can occur over a range of Reynolds number (or current velocity) known as a lock-in region. The gap length and the aspect ratio of the span are critical in determining the structural integrity of free spanning underwater pipelines.

Industrial regulations guiding the operation of free spanning underwater pipeline have undergone several reviews over the last century to address new challenges in offshore environment and conform to new research and numerical tools for analysis. The technical standards by DNV are used for 65% of world pipeline design and installation (Nair; 2016) and have undergone review from pure empirical models to semi-empirical models. These reviews range from DNV guideline of 1992 to the current DNVGL-RP-F105 formulated in 2017. DNV standard determines when a free spanning pipeline is prone to vortex-induced fatigue failure and recommend fatigue analysis check based on its aspect ratio as shown in Table 1.1. Free spans with aspect ratio greater than 30 are screened for fatigue analysis with the use of frequency criteria which are discussed in detail in further chapter. Spans whose natural frequencies in flow exceed the set frequencies require fatigue analysis while the ones whose screening fall below the frequency criteria are considered to have no threat of fatigue failure.
In the early 20th century, it was a common practice to carry out seabed intervention once a span had been identified to exceed the minimum criteria set. This was practical for operation of free spanning pipeline in shallow water and less challenging environment. However, financial implication of seabed intervention in operating free spanning pipeline in deep water environment is one of the key factors that necessitated a review of allowable span length and guidelines to reduce over-conservativeness in design. For example, Ormen Lange gas field offshore Norway triggered the review of DNV-RP-F105 of 2002 (Fyrileiv et al., 2005) because of its water depth and highly challenging environment with multiple spans.

Current DNV industrial guideline DNVGL-RP-F105 still recommend use of semi-empirical models as a means of accessing the hydrodynamic forcing loads in fatigue analysis of free spanning pipeline. This is because full analysis remains computationally intractable. For example, full 3D Computational Fluid Dynamics (CFD) for a long free span with large aspect ratio demands huge computational resources. Consideration of the potential cost of over conservativeness in the form of seabed intervention and under prediction in the form of premature fatigue failure calls for an approach that has the advantage of balancing between accuracy and computational viability in prediction.

<table>
<thead>
<tr>
<th>Aspect Ratio</th>
<th>Response Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>L/D &lt; 30</td>
<td>Very little dynamic amplification</td>
</tr>
<tr>
<td></td>
<td>- Insignificant dynamic response from environmental loads expected</td>
</tr>
<tr>
<td></td>
<td>- Required no or insignificant fatigue check</td>
</tr>
<tr>
<td>30 &lt; L/D &lt; 100</td>
<td>Response dominated by beam behavior</td>
</tr>
<tr>
<td></td>
<td>- Natural frequency sensitive to boundary conditions and effective axial force</td>
</tr>
<tr>
<td></td>
<td>- Fatigue check is required</td>
</tr>
<tr>
<td>100 &lt; L/D &lt; 200</td>
<td>Response dominated by combined beam and cable behavior</td>
</tr>
<tr>
<td></td>
<td>- Required fatigue check</td>
</tr>
<tr>
<td>L/D &gt; 200</td>
<td>Response dominated by cable behavior</td>
</tr>
<tr>
<td></td>
<td>- Natural frequency governed by deflected shape and effective axial force</td>
</tr>
<tr>
<td></td>
<td>- Very prone to fatigue failure</td>
</tr>
</tbody>
</table>
Hence, this research is focused on using a cost-effective numerical method to address the effect of gap ratio on dynamic response of free spanning pipeline and analysis of vortex induced vibration of underwater water pipeline with large aspect ratio. Important key questions to address are: to what extent does gap ratio affect the dynamic response of free spanning underwater pipeline and the stress distribution on the pipeline, and the effect of large aspect ratio on the dynamics and fatigue of unsupported marine pipelines. These questions are important to improve on design practice of free spanning pipeline.

1.2 Study Objectives

The aim of this work is to use a cost-effective numerical approach to study the dynamic response of VIV of large aspect ratio underwater pipeline close to the seabed to improve on design practice. To achieve this aim, the following are the objectives:

- To study and review the effect of wall proximity on hydrodynamic forces on rigid cylinder
- To study and review the variation of plane wall proximity on cylinder dynamics in flow
- To analyse empirical models currently used in industry to address VIV of free spanning pipeline and identify their limitations
- To develop a cost-effective predictive model that couple a computational fluid dynamic (CFD) to finite element method (FEM) in a modified one-way coupling to assess the response of flexible pipe with large aspect ratio. The CFD for characterizing the hydrodynamic forcing loads acting on long flexible cylinder and the FEM for response of beam.
- To use the predictive model to establish the effect of gap ratio on dynamic response, stress distributions along the longitudinal axis and fatigue damage accumulation on a free spanning pipeline close to the seabed near plane wall
- To make comparison between predictive model response and an established industrial guideline for improving the analysis of free spanning pipeline
Chapter 1. Introduction

1.3 SYNOPSIS OF THESIS

Chapter one introduces the background study of the problem of vortex-induced vibration of cylindrical structure in proximity to a wall and its occurrence in offshore free spanning pipeline. Key challenges of vortex-induced vibration with respect to large aspect ratio and gap ratio are highlighted. The aim and objectives of the research are summarized in this chapter and organisation of the thesis into sections and chapters are provided.

The second chapter of this thesis reviews the concept of vortex shedding in flow over blurred body, and vortex-induced vibration in details. Review of experimental and numerical work on flow around bounded and unbounded rigid cylinder in uniform flow are presented. The review is extended to flow-induced vibration of elastically supported free rigid cylinder and flexible cylinder with large aspect ratio for bounded and unbounded cases. This chapter closes with reviews of various industrial empirical and semi-empirical models for assessing vortex induced vibration of free spanning pipeline against fatigue failure, their pros and cons, and identification of knowledge gap.

In chapter three, Numerical frameworks for simulating vortex-induced vibration (VIV) are presented. Detailed explanation of the framework, divided into fluid domain and structural dynamics are provided. Description and justification for the type of Reynolds Average Navier-Stokes (RANS) turbulent modelling suitable for simulating free spanning pipeline close to the seabed are given. The chapter also presents fundamental of Finite Element Method (FEM). The coupling of Finite Volume Method (FVM) and Finite Element Method (FEM) with appropriate dynamic mesh are explained. It also includes the extensive explanation of the type of dynamic meshes suitable for wall VIV of a rigid body in proximity to a plane wall. The chapter closes with the presentation and description of the commercial codes of interest (which are ABAQUS and ANSYS Fluent) to solve the problem of fluid-structure interaction.

In Chapter four, uniform flows over stationary and elastically supported rigid cylinder in the sub-critical flow regime are studied. The uniform flow includes unconstraint and flow constraint by a plane wall. The effect of wall spacing for the constraint flow in the vicinity of plane wall are observed and validated for stationary and free cylinder with established
experimental results. Quantity prediction accuracy and key findings are highlighted to give an insight into flow over long flexible pipe, constraint and unconstraint by plane wall.

Chapter five presents a predictive model referred to as Reduced Order Method as a computational cost effective for assessing vortex-induced vibration of long flexible beam with large aspect ratio. In this chapter, the model is assessed and compared with established experimental results to ascertain its robustness and level of accuracy. Three further experimental results are used to assess and validate the applicability of the predictive model.

Chapter 6 opens with introduction and features of free spanning pipeline in deep water environment. The chapter further presents theoretical background to fatigue damage and the effect of mean stresses on damage accumulation. The validated predictive model presented in chapter 5 is used to establish the effect of gap ratio on the response of free spanning pipeline, stress distribution and fatigue damage accumulation. The results are compared with established experimental results. DNV Industrial guidelines for assessing free spanning pipeline are also compared with the predictive model of Reduced Order Methodology to ascertain the level of over-conservativeness while using an established experimental result as benchmark.

Chapter 7 presents the conclusion, summary of validations and key findings that address the objective of the research. Contribution to knowledge from the methodology used and key findings are highlighted. This chapter closes with recommendations for further works by identifying limitations and assumptions used in the present.
CHAPTER 2

LITERATURE REVIEW

2.1 PRELIMINARY REMARKS

This chapter reviews theory of vortex shedding across a cylindrical body and concept of vortex-induced vibration. Typical occurrence of vortex-induced vibration of offshore structures with focus on free spanning pipelines are identified. Experimental and numerical studies on elastically supported rigid cylinder and flexible cylinder far and near plane wall are reviewed to identify the challenges with pipeline with large aspect ratio close to the seabed. The current state-of-the-art methodologies or models for analysing vortex-induced vibration are intensively discussed to show the pros and cons of their usage in the industry. The chapter closes with the review of the different industrial method of analysing vortex-induced vibration of free spanning pipeline close to the seabed, their challenges and identification of ways of solving the challenges.

2.2 VORTEX INDUCED VIBRATION IN OFFSHORE ENVIRONMENT

Slender structures in the offshore environment are constantly under the influence of hydrodynamic loading. One of the major causes of fatigue failure of slender structures such as steel catenary riser, pipeline spanning the seabed, marine cables, tension leg platforms is vortex-induced vibration (VIV), see e.g. Hirdaris et al, 2014. As such VIV is critically considered in the design of offshore structures to avoid fatigue failure. Apart from consideration during the design phase, VIV is keenly monitored throughout the lifespan of offshore structures due to dynamic nature of the environment. For example, the length and gap of a span in an unsupported section of laid pipeline on the seabed can be altered during the operating life due to scouring caused by strong current flow. Analysis of VIV on structure is in particular very important due
to fatigue failure occurring below the yield strength of a structure undergoing VIV (Wang and Song, 2019).

Flow of current and/or waves across offshore structures causes vortex shedding downstream. Offshore structures that are flexible structure due to high aspect ratio respond to the dynamic of the fluid and vibrates as the velocity of flow changes as shown in Figure 2.1. Thus, the problem of VIV of flexible offshore structure is a Fluid-Structure Interaction problem (Lee et al, 2017). These environmental forces impose cyclic stresses on the pipeline. Even if the structures are not under the threat of sudden fatigue failure due to synchronization between the frequency of the structure and the vortex shedding frequency, long term fatigue damage which depend on the number of stress cycles and the mean stress are accumulated and can lead to unexpected failure in the future. Despite efforts to predict such phenomena typically high safety factors are used due to the uncertainties in the prediction tools and the high consequences of failure before the design life of the structure.

Quite a number of offshore structural fatigue failures due to VIV have been recorded. One-fifth of hydrocarbon release to the sea are due to vibration induced fatigue according to United Kingdom Health and Safety Executive as shown in Figure 2.1(a). Safety4sea (2019) gives a combination of high tensile overload and VIV as the root cause of mooring line failure of LNG vessel in the North Sea in 2017. Figure 2.1 (b) shows the failure of production platform in Gulf of Mexico in 2012. According to report documented by DNV (2012), excessive VIV triggered by hurricane led to the fatigue failure of the platform.
For marine pipeline spanning the seabed as shown in Figure 2.3, the gap between the seabed and the pipeline is a major factor influencing the onset and severity of vortex-induced vibration. Unlike risers and other offshore structures, the time varying stresses imposed on such pipelines are cyclic around a non-zero mean due to streamwise drag and upward lift (resulting in both streamwise and vertical bending stress) on the pipeline (Pour et al, 2021). Therefore, the study of VIV in free spanning marine pipelines is a particularly onerous design challenge due to the combination of mean and cyclic loads in both axes contributing to peak stresses in each flow regime, and hence to accumulated fatigue damage. The degree of fatigue depends on the length of the span and size of the gap. For a long gap spanned by a slender pipeline, the pipe can deform into the span under self-weight as shown in Figure 2.3 (b) and overstress the pipeline before the onset of vortex-induced vibration. The superposition of non-zero mean cyclic...
stresses due to VIV and static stresses due to dead weight of the pipeline further complicate the structural integrity of the pipe.

While early industrial regulation for the analysis of free spanning pipeline focused on the span length, the gap between the seabed and the pipe also plays major effect on the integrity of the pipeline. This is because the boundary layers developed at the seabed, result in differing velocity profiles for differing gap ratios and hence the gap ratio strongly determines the strength of the vortex shedding across the pipeline (Yang et al, 2006; Van den Abeele and Vande Voorde, 2011).

A common practise is to assess the free span for fatigue damage and determine the maximum allowable span length required against destructive vibration based on the structural properties of the pipeline (such as stiffness, aspect ratio, mass-damping ratio, natural frequency) and the flow properties (such as Reynolds number). Span lengths that exceed the maximum allowable span length are modified by seabed interventions, such as by rock dumping into the spanned gap, to reduce the span length, or eliminate the span entirely (Bai and Bai, 2014) or by application to the exterior of the pipeline of add-on vortex suppressors (Blevin, 1990; Choi et al, 2008; Rashidi et al, 2016; Skaugset, 2003). Vortex suppressors around the span section of a pipeline aim to disrupt build-up of coherent vortex shedding across the pipeline to reduce or eliminate the frequency range over which destructive vibrations due to large displacement amplitude occurs. Figure 2.4 displays several different vortex-induced suppressor.
The determination of acceptable maximum allowable span length and thus, the decision to carry out seabed intervention or application of vortex suppression depends on the critical analysis of the free span. This is extremely important because of the financial implication of overly conservative analysis. While under-prediction of VIV for a given span length can lead to premature fatigue failure, an overly conservative analysis leads to unnecessary cost. For example, Asset Integrity Engineering (2020) reassessed a 12” infield pipeline with a total length of 18.7km, and a 12” export pipeline with a length of 20 km for free spans using a combination of in-house software and DNV-RP-F105. The analysis subjected the free spans that exceeded the screening length to detailed fatigue assessment under the hydrodynamic loadings. Contrary to the initial judgement that 59 spans each of the pipelines were critical and required intervention, the detailed analysis resulted to 29 critical spans for infield pipeline and 33 for export pipeline. Studying the intrinsic properties of flow and the structural dynamics of free spans is crucial in optimizing cost and ensuring the structural integrity of offshore pipelines.

Figure 2.4: Add-on vortex-induced vibration suppressors (Blevin, 2001, pp. 78)
spanning pipeline is highly essential to making a sound judgement on mitigation of fatigue failure in free spanning pipeline.

Many studies have been carried out to determine the maximum allowable span length against potential fatigue failure. These include studies such as Bearman and Zdravkovich (1978), Lin et al (2009), Fu et al (2018) that have investigated the characteristics of vortex shedding for stationary and free cylinder in proximity to a plane wall to gain insight about the response behaviour of free spanning pipeline.

\section*{2.3 Dimensionless Quantities in VIV of Free Span}

The study of VIV of free spanning pipeline combines the interaction of fluid dynamics with structural dynamics. The following non-dimensional parameters are required for the analysis of VIV of free spanning pipeline (Summer and Fredsoe, 2002).

\textbf{Aspect ratio and the gap ratio}

The aspect ratio of a free span is the ratio of diameter to length of the span while the gap ratio is the ratio of the gap between the cylinder and plane wall/seabed to the diameter of the pipe.

\begin{equation}
\text{Aspect ratio} = \frac{L}{D} \tag{2.1}
\end{equation}

\begin{equation}
\text{Gap ratio} = \frac{e}{D} \tag{2.2}
\end{equation}

\textbf{Reynolds number (Re)}

The Reynolds number determines whether the flow is laminar or turbulent. It is the ratio of inertia force to viscous force. For a cylindrical structure, it is defined as:

\begin{equation}
\text{Re} = \frac{\rho U_{\infty} D}{\mu} \tag{2.3}
\end{equation}
Reduced velocity

The reduced velocity is a non-dimensional velocity of flow defined as the ratio of freestream velocity of flow to the product of the natural frequency of the cylinder and its diameter.

\[ v_r = \frac{U_\infty}{f_c D} \]  \hspace{1cm} (2.4)

Mass ratio

Mass ratio is the ratio of the mass of cylinder to the mass of fluid displaced by the cylinder.

\[ m^* = \frac{m}{\rho D^2} \]  \hspace{1cm} (2.5)

An important quantity in the study of VIV is obtained by combining the mass ratio with the structural damping to form a new dimensionless quantity known as mass-damping ratio \( (m^*\xi) \)

Stability Parameter

Stability parameters is the measure of structural stability of flexible pipe undergoing VIV in flow. It is defined as:

\[ K_s = \frac{4\pi m \xi}{\rho D^2} \]  \hspace{1cm} (2.6)

Strouhal Number

Strouhal number is the normalised shedding frequency around a cylinder defined as:

\[ S_t = \frac{fD}{U_\infty} \]  \hspace{1cm} (2.7)
Keulegan-Carpenter’s Number

Keulegan-Carpenter’s number is the ratio of drag force to inertia force acting on a structure in oscillating flow. It is defined as:

\[
K_C = \frac{U_m T}{D}
\]  

(2.8)

2.4 THEORETICAL BACKGROUND ON VORTEX SHEDDING AND VORTEX-INDUCED VIBRATION MECHANISM

Vortex shedding has its origin from flow around bluff bodies. The most common bluff bodies with wide application in many industries is a cylindrical body. Such bodies are found in offshore environment in the form of marine cable, mooring line, steel catenary risers, subsea pipeline. The characteristics of flow around cylindrical structures are strongly affected by the flow Reynolds number and proximity to other surfaces.

Flow around cylindrical bodies detaches from the surface, causing free shear layers that roll up into the downstream to form vortices as the Reynolds number of flow increases. For an unbounded stationary cylinder, the characteristics of vortices produces a repeating and similar vortex shedding pattern in the wake region with a constant frequency (von Karman, 1912). This pattern, shown in Figure 2.5 (a) is known as von Karman vortex street. The shedding of alternating vortices plays a very important role on the pressure distribution around the cylinder. The differential pressure around the cylinder causes fluctuating lift and drag forces on the cylinder. While the lift on a stationary cylinder has the same frequency as the vortex shedding, the drag has twice the frequency of the vortex shedding (Achenbach, 1968; Drescher, 1956). The Strouhal number is relatively constant in the sub-critical flow regime (300 < Re < 3x10^5) with a value of 0.2 as shown in Figure 2.5.
A key interest in the nature and characteristics of vortex shedding around stationary cylinder is when the cylinder is close to a plane wall. In addition to Re influencing the vortex shedding and flow characteristics in the wake, the gap between the cylinder and the plane wall strongly affect flow dynamics in the wake and the nature of vortex shedding. This configuration in the subcritical flow regime is particularly important for understanding the dynamics of free spanning underwater pipeline close to the seabed (Salehi et al, 2018). Also, Ong et al (2010) estimated that Reynolds number of flows around subsea pipeline in deep water near the seabed ranges from $10^4$ to $10^7$ covering subcritical and transcritical flow regimes with majority falling in the subcritical. The interference (or interaction) between the plane wall boundary layer and the vortex shed at the bottom of the cylinder induces a secondary vortex which interact with the vortex shed at the top of the cylinder (Li et al, 2016) as shown in Figure 2.6.
Chapter 2. Literature Review

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The strength of the upward lift forces, dynamics of wake and pressure distribution with respect to gap ratio on stationary cylinder in flow have been investigated extensively by many researchers. One of the effects of the boundary layers interaction due to wall proximity is the formation of asymmetrical periodic pressure at the top and bottom of the cylinder leading to a mean lift that can cause a mean upward drift of flexible cylinders (Hsieh et al, 2016; Dipankar and Sengupta, 2005). To investigate the influence of wall proximity on the lift and drag force on a cylinder, Lei et al (1999) carried out experiment on flow over cylinder at $Re = 1.31 \times 10^4$ and $1.31 \times 10^4$ over a range of $e \leq 2D$. Results showed that, while the drag increased as the gap ratios increased, the lift decreased, and at $e \geq 1.5D$, the forces are constant with no influence of the plane wall. In the subcritical flow regime, many studies such as Grass et al (1984), Buresti and Lanciotti (1992), Nishino et al (2007), Taniguchi and Miyakoshi (1990) have all shown through experiment that vortex shedding for flow over stationary cylinder is suppressed for $e < 0.3D$. Sarka and Sarka (2010) presented a comprehensive CFD study, using LES, of the influence of wall proximity on the dynamics of the wake at $Re = 1440$ for $e = 0.25D, 0.5D$ and $D$. For a cylinder embedded in the plane wall boundary layer (i.e. $e \leq \delta$), the interaction of the vortices generated on the plane wall and on the cylinder wall resulted in high turbulent kinetic energy in the wake region close to the cylinder which decays in the downstream direction. The turbulent kinetic energy is closer to the cylinder in the wake for a lower value of $e/D$.

For a case where the cylinder is free to move, the frequency of vortex shedding around the cylinder may approach the natural frequency of the cylinder as the Reynold number increases and set it into vibration (Bourguet et al, 2011). In this situation, the Strouhal number is no longer constant in the sub-critical flow regime due to dynamic changes of the vortices in the wake. As

Figure 2.6: Interaction of vortices shed from the near wall cylinder with the wall boundary layer [Le et al; 2016, pp. 510]
noted by Atsavapranee et al (1998), the vortex-induced oscillation increases vortex shedding strength in the wake. A special case in the study of VIV which is the basis for this research is when the free/flexible cylinder is vibrating close to a plane wall. In addition to the flow parameters and mass-damping ratio of the cylinder affecting the dynamic response of the cylinder, the proximity between the cylinder and the plane wall is a strong determinant of the response of the cylinder. Studies such as Yang et al (2009) have shown that, unlike the corresponding stationary cylinder near wall where vortex shedding is suppressed as the gap ratio decreases, vortex shedding remains active even for zero gap ratio in free cylinder in flow. Review of experimental studies of vortex-induced vibration of cylinder near and far from wall are presented in Section 2.5

2.5 REVIEW OF EXPERIMENTAL STUDIES OF VIV

Many experimental studies have been carried out to study the effect of VIV on the dynamic response of pipeline. Majority of past and current laboratory work on VIV involves studying the interaction between elastically supported rigid cylinder and its associated surrounding fluid (Wu et al, 2012). In this case, the normal six-degree-of-freedom of flexible cylinders are reduced to either one-degree-of-freedom (typically transverse to the flow only) or two-degree-of-freedom (transverse and streamwise). The use of elastically supported rigid cylinder assumption is applicable for a moderate or fairly flexible cylindrical structure where the whole body is expected to respond as one entity (Elbanhawy, 2011).

For a highly flexible cylinder associated with high aspect ratio, the response of the cylinder varies along the longitudinal direction of the structure due to span-wise variation of vortex shedding (Hiramoto and Higuchi, 2003). Thus, a full-scale experimental procedure is ideal to assess and determine its response. However, cost of experimental study of VIV may still warrant the use of rigid body assumption even for highly flexible cylindrical structure. In such case, section of the span for a given L/D is considered. Few cases of full-scale or large-scale experimental procedures with aspect ratio exceeding 800 exist in the literature.

Generally, experimental study of VIV of flexible pipe are studied through elastically mounted rigid cylinder free to respond to VIV as a representative of flexible pipe or full-scale VIV of the flexible pipe depending on the degree of flexibility and cost.
2.5.1 Elastically mounted rigid cylinder in flow

A classical investigation of VIV of elastically rigid cylinder in uniform flow by Feng (1968) explained the intrinsic response of a free cylinder over a range of reduced velocities. The experiment with a large mass ratio, $m^*$, of 248 revealed two branches named as initial and lower branches. However further studies show that with a much lower mass-damping ratio, a upper branch / lock-in region /synchronisation region where the cylinder attains maximum displacement is revealed and lies between the initial and lower branches. The $v_r$ range over which this synchronisation, referred to as lock-in, occurs and the maximum amplitude that occurs within this range depends on the mass-damping ratio of the cylinder. Blevins and Coughran (2009) measured the lock-in to exist in the range $5.5 < v_r < 8$ for $m^*\xi$, of 0.128.

The structural behaviour and flow dynamics in the lock-in range has attracted a lot of attention because of its important to design and life cycle of a structure. Khalaak and Williamson (1997) studied VIV response in transverse direction only with focus in the lock-in range using a low $m^*\xi$ of 0.0108 to investigate the possible maximum displacement. A maximum displacement of 1D was recorded. In addition to the displacement, maximum drag and the r.m.s lift on the cylinder were 5 and 6 times those on a corresponding stationary cylinder. In a case where the cylinder is allowed to oscillate in both transverse and streamwise direction, the trajectory of the cylinder traces a figure ‘8’ in the lock-in region (Moe et al, 1994; Sarpkaya, 1995).

To further investigate the characteristics of flow in the three branches, Govardhan and Williamson (2000) used particle imaging velocimetry (PIV) for flow visualisation. The study revealed 2S mode (i.e.two single vortices per cycle) in the initial branch, and 2P mode (i.e. two pairs of vortices per cycle) in the upper and lower branches.

When the free cylinder is oscillating near a plane wall, the flow characteristics and response are altered by the boundary layer interference between the cylinder wall and the plane wall. Barbosa et al (2017) investigated the effect of plane wall proximity on the response amplitude of an elastically supported rigid cylinder with $m^*\xi$ of 0.11 in a flume over a range of flow regime $6.5\times10^3 < Re < 2\times10^4$.The gap ratios considered in the experiment were categorised into large ($e = 2D, 3D$ and $5D$), medium ($e = 0.75D, D$ and $1.5D$) and small ($e = 0, 0.25D$ and $0.5D$) with the cylinder positioned at $75D$ from the leading edge. The responses
for the large range of gap ratios showed no significant changes signifying diminished effect of plane wall gap ratios greater than 2D. For the medium and small ranges of gap ratio, less than 2D, the amplitude of response was found to decrease with decreasing gap ratios. The variation in gap ratio also has significant effect on the force on the cylinder. The mean drag values increased as the gap ratio increased while the rms value of the mean drag decreased. For the lift coefficient, decreasing gap ratio led to increase in mean lift. Contrary to a stationary cylinder where vortex shedding is suppressed for \( e \leq 0.3D \), cylinder vibration was found to occur in that range. However, for \( e \leq 0.5D \), motion of the cylinder resulted in impact with the bottom of the flume. Similar experiment by Wang et al (2013) for \( m^*\xi \) of 0.0173 in the range \( 3 \times 10^3 \leq Re \leq 1.3 \times 10^4 \) also showed cylinder vibration for \( e = 0.05D \).

For design purpose, the limiting gap ratio for touch down is highly important for free spanning pipeline because of the potential crack that can result from it which would accelerate fatigue damage.

### 2.5.2 Full scale measurements

Studies such as Price et al (2002), on flow visualisation on a stationary pipe with large aspect ratio show variation of vortex shedding along the longitudinal axis of the pipe (spanwise). Consequently, a flexible pipe with large aspect ratio undergoing VIV has response that varies along the span of the pipe. Also, a flexible pipe with constraint at both ends has multiple eigen frequencies and thus, several different modal responses can occur. Experimental investigation of spanwise variation of response of long flexible pipe requires a full-scale procedure to better understand the behaviour of the flexible pipe at different modes.

To investigate fatigue damage of marine risers due to VIV, Trim et al (2005) experimented with uniform and sheared flow over a long horizontal flexible pipe with \( m^*\xi \) of 0.0048 and aspect ratio of 1407 in the range \( 12000 \leq Re \leq 69000 \). Eight equidistant points along the spanwise direction were marked for observation. Measurements showed that the critical response lied towards the constrained end of the pipe at about 0.9 of the length of the span. The spanwise displacements and fatigue damage in the transverse direction in the range \( 18200 \leq Re \leq 60600 \) were approximately three times larger than those in the stream-wise direction. Furthermore, the dominant modes increased with increasing Reynolds number. However, the
transverse and streamwise fatigue damage were almost equal in the range $Re < 18200$. Contrary to usual practices that base design on the predicted fatigue damage in the transverse direction only, the experiment showed a significantly higher fatigue damage for streamwise direction at the highest Reynolds number studied, $Re = 66700$. Further investigation by Baarholm et al (2006) showed that streamwise response is highly significant to fatigue damage at a lower dominant mode but response at higher modes is less significant in comparison with contribution of transverse response.

Similar to elastically supported rigid cylinder near wall, wall proximity affects the spanwise response and fatigue damage accumulation on flexible pipe with large aspect ratio close to a plane wall. Tsahali (1984) investigated the effect of gap ratio at the mid-section of a flexible pipe with an aspect ratio of 112.3 over $0 \leq v_r \leq 11.6$. The study recorded a reduction in amplitude of vibration when compared the amplitude of response of $e = 1D$ with response without plane wall effect. Li et al (2011) investigated the effect of gap ratio on the significant mode of vibration for a pipeline with mass ratio of 4.30 and aspect ratio of 162.5 over a range of gap ratios of 4, 6 and 8 in the range $0 < Re \leq 9.6x10^3$. Their measurements showed that, at high reduced velocity, the dominant mode shifts from first mode to second mode as the gap ratio decreases. Also, for the same mode, the frequency of response increases linearly with reduced velocities.

2.6 State-of-the-Art Analysis and Modelling of VIV of Free Spanning Pipeline

The analysis of free spanning pipeline against fatigue damage has evolved over time due to the rapid development of computational resources, enabling state-of-the-art techniques and research that give rise to detailed information that have informed understanding of VIV. Historically, analyses were based on using empirical or simplified models in conjunction with many factors of safety due to lack of intrinsic information about VIV (Jo et al, 2015). Such methods tended to overly increase the cost of design. With the use of approaches that involve detailed analysis of the flow dynamics and the structural analysis of the pipe and their interaction, vortex induced vibration of free spanning pipeline can be efficiently modelled. These techniques allow seabed intervention to be more optimised and can thus reduce overall project costs (Knoll and Herbich, 1980; Reid et al, 2000). However, the more the level of
detailed analysis carried out on VIV of free spans, the more the computational and time resources required.

Generally, the analysis of vortex-induced vibration of free spanning underwater pipelines ranges from simplified or empirical models to semi-empirical models and to numerical simulation. Even though simple models tend to be overly conservative, they are still relatively applicable in the industry due to its fast approach and less resources involved, and most especially when the risk of fatigue failure is considered minimal. Figure 2.7 shows a decision tree by Kaneko et al (2014) that informs the use of evaluation type of analysis of VIV.

Figure 2.7: Evaluation decision of VIV based on importance [Kaneko et al, 2014, pp. 45]

2.6.1 Empirical and Simplified Models

In the past, analysis of VIV of free spanning pipeline were based on pure empirical model in conjunction with many high factors of safety. This was partly due to lack of adequate information about many uncertainties of the VIV phenomena, and partly due to exploration being limited to shallow water with less challenges associated with mitigating the onset of VIV.

Based on the observation that the detached vortex wake is cyclic downstream of the cylinder after flow separation, Thompson (1988) postulated a simple harmonic model for the wake
disturbance, and thus proposed a standard linear harmonic model for the motion of an elastically rigid cylinder to quickly obtain prediction of the cylinder response. Although Thompson’s postulated model is easy to use, it lacks in accuracy in many areas especially in the lock-in range characteristic of VIV. Crucially such linear harmonic response models fail to determine the lock-in range (synchronisation) which occurs over a range of reduced velocities.

Sumer and Fredsoe (1999) presented a simple model for the maximum displacement of one-degree-of-freedom of elastically supported rigid cylinder in uniform flow as shown by equation (2.9). The equation is based on modelling the hydrodynamic forcing load on the cylinder as sinusoidal force.

\[
\frac{y_{\text{max}}}{D} = \sqrt{2} \left( C_{l}^2 \right) \frac{v^2 r}{K_s} \tag{2.9}
\]

Comparison of the model with experimental results of King (1974) shows that despite being a very simple model, the predicted maximum displacement agrees with experimental results for stability parameter, \( K_s \geq 2 \). However, for \( K_s < 2 \), large disparity occurs.

Generally, the problem of VIV is not a small perturbation that can be defined with simple empirical model. It is an inherently nonlinear multi-degree-of-freedom phenomenon (Sarpkaya, 2004). Therefore, the major setback with simple model is that it cannot be applied alone for design purposes without the incorporation of factors of safety. Typically, large values of safety factors are required and due to the application of safety factors, the likelihood of over prediction is high and thus, lead to overly conservative model.

### 2.6.2 Semi-Empirical Models

Semi-empirical model has been developed and used for the study of vortex-induced vibration of short or long pipe in flow with different configuration. It involves applying the knowledge of established experimental results of flow over stationary cylinder to formulate a workable equation for a free cylinder in flow to predict VIV response of the cylinder. The simplicity of a semi-empirical model allows the effect of the fluid on structure motion to be modelled without consideration of the detailed characteristics of the flow around the structure (Kurushina and Pavlovskia, 2017). The most widely used among the categories of semi-empirical models is
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The wake oscillator where the wake downstream of the flow around free cylinder or elastically supported rigid cylinder is modelled using nonlinear oscillation (Xu et al, 2015; Gabbai and Benaroya, 2005).

(a) Classical Wake Oscillator

The wake oscillator solves the problem of VIV of an elastically supported rigid cylinder in flow by coupling the wake equation with the structural equation of the cylinder. Bishop and Hassan (1964) first proposed the concept of wake oscillator with the use of Van Der Pol non-linear oscillator to model the wake as shown in equation (2.9).

\[ \ddot{q} + \varepsilon \omega_s (q^2 - 1)\dot{q} + \omega_s^2 = F \]  

Where \( q \) is the wake variable defined as \( q = \frac{2C_L(\phi)}{C_{L0}} \); \( C_{L0} \) is the corresponding lift coefficient of the stationary cylinder in uniform flow from established experimental results, \( \omega_s \) is the vortex shedding frequency defined as \( \omega_s = \frac{2\pi U_{infty}}{D} \), \( F \) is the forcing term of the oscillator which can take form of displacement (\( \dot{A}y \)), velocity (\( \dot{A}y \)) or acceleration (\( \ddot{A}y \)) coupling, \( \varepsilon \) and \( A \) are tuning parameters that are user defined to fit closely to experimental results.

The careful formulation of the wake oscillator in the form of Van der Pol non-linear oscillation enables the wake equation to switch response of the free or flexible pipe from initial branch to the synchronisation (lock-in) region and then to the lower branch as the value of \( q \) changes. This self-limiting behaviour of the oscillator is achieved by the damping terms \( \{ \varepsilon \omega_s (q^2 - 1)\dot{q} \} \) in the wake equation.

The classical wake oscillator has been applied successfully in its original form to unbounded single-degree-of-freedom system with high mass-damping ratio constraint in the transverse direction. Coupling of the wake equation with the structure is achieved through displacement, velocity or acceleration coupling. Krenk and Neilson (1999) applied displacement to couple wake oscillator with the structure while Mureithi et al (2000) and Plaschko (2000) used velocity coupling. Through series of test and analysis, many studies such as Blevin (1990) and Facchinetti et al (2004) have applied acceleration coupling to achieve a more closeness to
experimental studies and considered it the most appropriate method of coupling wake oscillator with structure.

(b) Modified Wake Oscillator

Application of wake oscillator model in its original form does not cover a wide range of flow regime, low mass-damping ratios and other configuration of VIV of cylindrical structure such as two-degree-of-freedom and proximity to plane wall. Therefore, many studies have applied modifications to the classical wake oscillator to suit the Reynolds number of flow and configuration of interests. For example, Farshidianfar and Zanganeh (2010) models both elastically rigid cylinders and the wake in the form of Van der pol non-linear oscillation of equation (2.2) and coupled them with the response velocity to achieve an improved results for low mass-damping ratios in the sub-critical flow regime. The frequency response of the rigid cylinder with $m^*\xi$ of 0.0002075 over a range of reduced velocity produced a much-improved prediction than the classical wake oscillator when compared with experimental results of Khalak and Williamson (1999). It was found to be 24% more accurate than the original form.

For a two-degree-of-freedom system, the effective response of an elastically mounted rigid cylinder is strongly affected by the coupling between the streamwise and transverse response. Experimental result of Moe and Wu (1990) showed that an elastically supported rigid cylinder in uniform flow has a wider lock-in region when the cylinder is allowed to move both in transverse and streamwise direction than transverse direction only. Furthermore, Jauvtis and Williamson (2004) observed that the response of a cylinder with $m^* > 6$ in 2DOF has no significant change from the corresponding 1DOF. However, for $m^* < 6$, a secondary lower branch appears. Observation also shows that for 2-DOF, when the transverse and streamwise exhibit the same response, the cylinder trajectory traces a path forming figure ‘8’ (Moe and Wu, 1990; Moe et al, 1994; Sarpkaya, 1995) as shown in Figure 2.8.
In such cases, classical wake oscillator in its original form fails to capture this interaction especially the response in the lock-in region. To apply the wake oscillator to such system with $m^*\xi$ of 0.0002075, Kurushina and Pavlovskaia (2017) tested different combination of non-linear oscillations ranging from pure Van der pol, Rayleigh oscillations and their modified versions to model the wake in a coupled transverse/streamwise response. A combination of modified Van der pol for streamwise and modified Rayleigh for transverse produced the best fit from among the possible combinations. However, in comparison with experimental results of Stappenbelt and Lijial (2008), the modified wake oscillator over-predicted the inline response at the lock-in region to 21% maximum error and underpredicted the response at the lower region to a maximum error of 37%. For the transverse direction, over prediction of response in the lock-in region occurred within 10% error and lower region response were overpredicted to a maximum error of 35%.

For an elastically supported rigid cylinder near wall which is a cross-sectional representation of free spanning cylinder, and where boundary layer develops, the classical wake oscillator’s predicting accuracy of the response of the cylinder reduces even for a single degree of freedom because of gap ratio effect on the response. To show the influence of gap on the response of a free cylinder, Barbosa et al (2017) formulated a modification to the classical wake oscillator. It has previously been observed by experiment that close to a plane wall, the amplitude of response decreases. To capture this effect of proximity, an additional gap-dependent forcing term that has the capacity to switch to upwards and downwards direction when the cylinder responds in downward and upward motion respectively was introduced to the forcing term of
the classical wake oscillator. Overall, the gap-dependent additional forcing term acts like a damper on the response of the cylinder to lower its amplitude as the gap ratio decreases. The modified wake oscillator when applied to \( m^* \xi \) of 0.11 over a range of reduced velocity 2 – 12 for gap ratios of 1.5, 1, 0.75, 0.5 and 0.25 reflects the effect of gap ratios on the cylinder response and also captures the transition from initial to lock-in, and from lock-in to the lower region. However, the modified wake oscillator fails in the prediction of the magnitude of the amplitude response in the lower regions. Furthermore, each gap ratio requires different empirically determined tuning factors to achieve close prediction to the experimental results.

Wake oscillators have also been applied to long flexible pipelines to model the displacement of pipe undergoing VIV. However, responses are generally underpredicted, or to low accuracy, in the spanwise direction. Xu et al (2008) applied a wake oscillator to model the displacement of flexible cylinder with \( m^* \xi \) of 0.0048 at \( Re = 12,000 \) and produced symmetrical response about the centre of the span in both transverse and streamwise direction. Violette et al (2007) also obtained similar predictions and observed symmetry about the centre of flexible cylinder for pipelines with aspect ratio ranging from 100 – 2028. These results suggest the inability to model the spanwise variation of vortex shedding effect on the cylinder response with wake oscillators.

2.6.3 Numerical Simulation

Numerical simulation allows for models that capture the intrinsic properties of the structure and the fluid flow to account for the effect of VIV on free spanning pipeline. Unlike simple and semi-empirical models where non-linear oscillations are prescribed for the wake, Navier-Stokes equations describing the fluid flow over the cylinder are discretised and simulated in conjunction with equation of rigid body or finite element analysis of the cylinder using a beam model.

CFD models with Reynolds Averaged Navier Stokes (RANS) turbulence closure models, in particular the SST \( k - \omega \) model, have been widely used in modelling VIV of spring-supported rigid cylinder in 1DOF and 2DOF motion and for both unbounded and near-wall flows. Izhari et al (2017) applied SST \( k - \omega \) RANS to model the response of spring-supported rigid cylinder with \( m^* \xi \) of 0.04 in the range \( 2 \leq v_r \leq 11 \). The flow visualisation of vortex shedding showed 2S mode at \( v_r = 4 \), 2P at \( v_r = 7.2 \) and no mode at \( v_r = 10.2 \). Zhao et al (2012) also studied
the response of elastically supported rigid cylinder using SST $k-\omega$ RANS model in the oscillatory flow with $10 \leq K_c \leq 20$ and $1 \leq v_r \leq 36$. Close to a plane wall, Chen et al (2019) applied immersed boundary method developed by Peskin (1972) to couple fluid and rigid cylinder conservation equations together for a $m^*$ of 2 in the range $2 \leq v_r \leq 16$ for $0.6D \leq e \leq 6D$ to reduce computational cost. All these studies were found to relatively under-predict response in the lock-in range.

Application of Large Eddy Simulation, LES, for flow modelling has been undertaken with the aim of improving prediction of cylinder response especially in the lock-in region. Pastrana et al (2018) coupled LES turbulent modelling with structural equation of rigid cylinder with mass ratio of 26 at $Re = 3900$, 5300 and 10000 to achieve improved response prediction of VIV of cylinder in transverse and streamwise direction. The results captured the initial, upper and lower branches of VIV and compared well with experimental results of Jauvtis and Williamson (2004) with no more than 4% error. In total, the simulation used $9.3 \times 10^6$ elements in the flow domain for all Reynolds number values.

For a long flexible pipe, the computational cost is a big factor in application of numerical simulation to model VIV response. For a large aspect ratio in excess of 1000, even a more conservative RANS model is computationally impractical. For this purpose, strip-based approximations have often been applied as an economical numerical simulation method to predict VIV response of flexible structures with large aspect ratio (Huang et al, 2011; Sun et al, 2012; Yamamoto et al, 2004; Meneghini et al, 2004). In this method, 3-D flow domain is divided into several strips of 2-D flow domains and analysis at equidistant locations along the longitudinal axis of the cylinder (see Figure 2.9 (a)) employed as input to a coupled structural model. The strips are typically coupled with the flexible cylinder at the region of contact while non-contact regions are interpolated.

Figure 2.9: Strip-based method for VIV of long flexible pipe

(a) 2-D strip of fluid domain on cylinder longitudinal axis (Fu et al, 2018, pp. 176)  
(b) 3-D strip of fluid domain on cylinder longitudinal axis (Bao et al, 2016, pp. 1081)
Fu et al (2018) applied a strip-based method to model an axially tensioned cylindrical pipe with mass-damping ratio of 0.0459 and aspect ratio of 167 in oscillatory flow with $KC = 84$ and $Re = 10^5$ to study the VIV response of the pipe. For the aspect ratio considered, twenty 2D fluid domain strips were constructed on the beam equivalent to 8.35D apart. The cylinder was modelled as Euler-Bernoulli beam and discretized into 80 beam elements. Response of the cylinder at the mid-section was recorded and compared with the experimental result of Wang et al (2014). The simulation agreed in pattern and shape of the build-up, lock-in and lower region of response with the experimental result but different in amplitude magnitude to the tune of 36% underprediction error.

Although strip-based methods can be computationally efficient, the loss of 3-D vortex structures limits its accuracy (Samuel et al, 2006). To minimise the loss of spanwise variation of vortex shielding and subsequently improve prediction, Bao et al (2016) considered the use of thick strips along the longitudinal direction of pipeline of $L/D = 32\pi$ and $Re = 3900$ as shown in Figure 2.10 (b). Thick strips of fluid domain were populated on the pipeline modelled as Euler-Bernoulli beam. The thick strip tested were $L_x/D = \pi/16, \pi/8, \pi/4, \pi/2, \pi/2$. Results showed that the higher the thickness of the flow domain strip, the higher the accuracy of prediction.

### 2.7 Industrial Guidelines for VIV of Free Spanning Pipeline

Majority of industrial guidelines for the assessment of free spanning pipeline are based on a combination of empirical and numerical models. Empirical models are applied for the analysis of hydrodynamic forcing load while the structure is numerically simulated with the use of Finite Element Analysis. This method allows the analysis of free spanning pipeline to be computationally viable and less time consuming since the fluid forces are empirically determined and not simulated directly through CFD. However, since the fluid forcing loads have some uncertainties in accuracy, application of safety factor multipliers is often used to compensate for these uncertainties. This may however lead to overconservativeness in design. The most widely used industrial codes for VIV analysis of free spanning pipeline are briefly summarised below.
2.7.1 VIVANA

VIVANA is a commercial tool developed by Marintek for calculating vortex induced vibrations (VIV) of slender marine structures such as risers and free span pipelines under the influence of ocean current. Fluid forcing coefficients such as the added mass, lift coefficient, damping are evaluated through empirical method using non-dimensional frequencies as a controlling parameter (Larsen et al, 2001). For the structure, finite element analysis of 3-D beam is applied to determine the response and stress distributions.

The computational time and cost of FEA is cheaper than CFD, thus, VIVANA offers a fast method of calculating response amplitude, drag amplification and fatigue damage of a riser or free spanning pipeline. However, due to the fluid forces being assessed through empirical approach, the characteristic of flow around the cylindrical structure cannot be determined. Furthermore, Gao et al (2018) identified that one of the cons of VIVANA is the decoupling of the streamwise and the transverse response. In a typical VIV response of a cylinder as noted in the experiment of Jauvitis and Williamson (2004), overall response of a 2-degree-of-freedom has a higher amplitude than when the cylinder is constraint in one direction only. This shows the transverse and streamwise response are coupled.

2.7.2 Shear7

Shear7 is a semi-empirical model based on superposition of modes of excitation. It evaluates the mode susceptible to VIV excitation and estimates VIV response. In the estimation of VIV response of pipeline, it uses hydrodynamic force coefficients database which have been derived from model testing to evaluate the forcing load on the beam (Wu et al, 2020). For the evaluation of the pipeline, set of natural frequencies of the beam are evaluated and combine with the hydrodynamic forcing to predict the response and fatigue of the beam undergoing VIV.

2.7.3 DNV-GL-F105

DNV’s recommended practice for the analysis of free spanning underwater pipeline is the DNVGL-RP-F105. The guideline is also a semi-empirical model for assessing the pipeline span for susceptibility to fatigue. The recommended practice uses the aspect ratio of the span (as shown in Table 1.1 in Chapter 1) as a screening criterion to subject the free spanning pipeline
to further analysis or not. Once the aspect ratio exceeds the minimum requirement, the natural frequency of the pipeline in conjunction with factors of safety is used to further screen the free span for fatigue analysis. In the fatigue analysis, the hydrodynamic forcing which lead to stress distribution on the pipeline are not simulated but empirically determined based.

The DNVGL-RP-F105 being the most widely used guideline for more than 70% of pipeline project (Nair, 2016) is discussed in detail in further chapter and serve as the bases for comparing with the proposed method.

### 2.8 Knowledge Gap

Offshore pipeline projects account for a large percentage of exploration in the offshore environment. Failure of marine pipeline can be detrimental to total project cost and can cause significant risks to human safety and the environment. For a pipeline spanning the seabed, the aspect ratio and the gap ratio are factors that contribute to fatigue damage caused by VIV. Closeness to the seabed causes the flexible pipe with high aspect ratio to cycle about a non-zero mean stress, thus adding to the effect of fatigue damage.

A sound design methodology or analysis must satisfy the condition of applying the minimum cost and time to achieve a high level of prediction of response and VIV of free spanning pipeline close to the seabed.

While a strongly coupled 2-way FSI simulation of VIV of free spanning pipeline to ascertain the effect of gap ratio on fatigue damage offer a reliable predictive, such analysis is however computationally impractical especially for large aspect ratio pipelines in excess of 1000. In contrast, available semi-empirical models in the literature such as wake oscillators are computationally viable, but they are limited to reducing multi-mode and six-degree-of-freedom long flexible pipe into a single mode and one or two-degree of freedom. Further application to long flexible pipe with large aspect ratio shows symmetric prediction of response about the centreline of the span. Thus, they lack accuracy in predicting the effect of VIV on the spanwise response for flexible pipe both in unbounded flows and near to a wall.

It is therefore imperative to not only investigate the effect of gap ratios on VIV response and fatigue damage on pipeline with large aspect ratio in proximity to the seabed, but also to develop a cost-effective numerical modelling that would be applicable in the industry for analysis. This
research explores the development and application of a cost-effective numerical simulation technique to investigate the effect of gap ratio on the vortex-induced vibration of long free spanning marine pipeline close to the seabed.

2.9 SUMMARY AND CONCLUSION

In this chapter, previous works on the effect of gap ratio on stationary and free cylinder in flow were reviewed. These were done as such configurations have application in free spanning marine pipeline. Change in vortex shedding as the proximity between a stationary cylinder and plane wall and the limiting gap ratio of 0.3D for which vortex shedding is suppressed were identified from previous studies.

In extension to the above, empirical model, semi-empirical model and numerical simulation for the analysis of VIV response were reviewed. While empirical models have the advantage of simplicity in their usage, they can lead to over-conservative model and increase cost of design due to the use of multiple factors of safety in their application. Also, semi-empirical models involving wake oscillators have limitation for their use for large aspect ratios. Present industrial application such as VIVANA, DNV-GL that applies other semi-empirical models do so without modelling the flow domain and thus limit their accuracy. In these guidelines, the cylinder’s structural response is numerically simulated with FEA while the gap-dependent forcing loads on the cylinder in flow are either empirically determined and/or clue taken from the established experimental studies of the corresponding stationary cylinder. As for full scale 2-way FSI numerical simulation of VIV, it is computationally non-viable due to large aspect ratio of free flexible pipeline. Strip-based method developed to reduce computational cost is lacking in predicting the change in response along the longitudinal direction of the pipe due to spanwise variation of vortex shedding.

In conclusion, a knowledge gap to improve on industrial design and analysis of VIV of free spanning pipeline has been identified as exploring the effect of proximity on the VIV of free spanning pipeline with the development of a cost-effective numerical method.
CHAPTER 3

NUMERICAL FRAMEWORK

3.1 PRELIMINARY REMARKS

The interaction between flexible marine pipeline and its surrounding fluid flow is a multi-physics problem that involves strong coupling between the governing laws of fluid dynamics and structural mechanics laws. In this chapter, the methodological strategy for modelling the VIV of pipeline with high aspect ratio in uniform flow is described. The choice of turbulent model and beam model are critically identified for the size of geometry, flow characteristics and the layout of interest. CFD and FEA numerical scheme setting and strategy for meshing the structure-flow domain which involves proximity between two walls and their interaction of boundary layers are highlighted. This chapter closes with description of commercial software - ANSYS fluent and ABAQUS adopted for solving the Multiphysics problem of vortex-induced vibration of the structure in consideration.

3.2 DESCRIPTION OF GEOMETRY SIZE AND FLOW CHARACTERISTICS OF INTEREST

The geometry layout and flow regime of interest in this research are long flexible marine pipeline vibrating close to the seabed in the subcritical flow regime. In this layout scenario, the boundary layers interaction between the seabed modelled as plane wall and the cylinder are fundamental to the dynamic response of the cylinder. Specifically, the gap ratio of interest ranges from 0.2 to 2 for a mass-damping ratio of 0.0048 with a large aspect ratio in excess of 1000 over a range of Reynolds number of 10000 to 12500. Based on previous similar experimental and numerical simulation results in the range of mass-damping ratio and flow regime such as Song et al (2011), Fu et al (2018), Xu et al (2008), the amplitude of vibration is expected in the range $\frac{Z}{D}, \frac{Y}{D} \leq 1$. 

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While a strong coupling simulation is robust for the type of FSI of the geometry and fluid whose features are highlighted above, the large aspect ratio of the cylinder leads to a very large computational flow domain and thus large computational resources. To overcome the computational resources challenge, a Reduced Order Method (ROM) is proposed that sectionalise long flexible beam into piecewise independent standalone segments.

### 3.3 Proposed Numerical Method

This proposed numerical method is based on conceptualising a long flexible cylinder as a system of infinitely connected 2-D rigid circular segments. This method uses this conceptualization to reduce approximately a fully submerged long flexible cylinder constraint at both ends in a uniform flow into a system of connected elastically supported rigid cylinder segments in uniform flow as shown in Figure 3.1. Evaluation of 2-way FSI iterative CFD simulation of each standalone rigid cylinders offers the input hydrodynamics loading for FEA analysis of the long flexible beam for response and stress distribution. The iterative process to correcting the forcing load on each segment is accessed from the constraint of adjacent segments.

![Diagram](image)

(a) Long pipe in uniform cross flow

(b) Reduction of long pipe to system of rigid cylinders

Figure 3.1: Sectionalisation of beam in uniform cross flow

The main purpose of this procedure involves reproducing the resultant forcing load exerted on the structure and its change influenced by the structural motion due to VIV. This is achieved
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by solving a two-way FSI of segments of the cylinder in 2-D domain and coupling the obtained iterative forcing load on a 3-D FEA of the flexible cylinder in one-way FSI. Figure 3.2 shows the flow chart for the proposed ROM.

![Flowchart for proposed Reduced Order Method (ROM)](image)

Figure 3.2: Flowchart for proposed Reduced Order Method (ROM)

Application of the proposed ROM to a representative mass-damping ratio and flow characteristics is evaluated and validated in Chapter 5.

3.4 **Fluid Dynamics – Choice of Turbulence Model**

The dynamics of fluid flow across a flexible pipe close to the seabed mostly falls within the subcritical flow regime. Modelling of such flow around the flexible pipe with large aspect ratio requires a balanced consideration to the computational resources and the level of accuracy of
the type of turbulence modelling. While LES has a high level of accuracy for modelling turbulence flow especially boundary layers conditions, it is computationally non-viable for flexible pipe with large aspect (Gao et al, 2017; Som et al, 2012).

From among the classes of RANS model, the choice of model requires to simulate flow around flexible cylindrical structure depends on the flow characteristics, the layout and size of the geometry. For example, a cylindrical structure close to a plane wall requires a RANS model that has the capacity to predict the interaction between the boundary layers developed on the plane wall and the cylinder wall. While \( k - \varepsilon \) produces a good predictability for internal flow as demonstrated by Elcner et al (2017) for a constricted tube, it has low predictability power for external flow around a bluff body where flow separation and adverse differential pressure are pronounced (Farhadi et al, 2018; Hossain et al, 2017) which are characteristics of flow of interest. Although Reynold Stress Model (RSM) is a high-fidelity RANS model, flow separation and complex boundary layers interaction make convergence very difficult to achieve (Vlahostergios, 2018) due to the momentum equation and turbulent stress in flow being highly coupled. For the geometry layout, size and flow of interest presented in Section 3.2, SST \( k - \omega \) RANS model is an appropriate model as it offers the capability to switch from standard \( k - \omega \) near the wall to \( k - \varepsilon \) far away from the wall. This capability enables the modelling of various flow characteristics including flow separation and boundary layers interaction which are peculiar to the simulation of interest in this research. Previous works such as Izhar et al (2017), Zhao and Cheng, (2011), Zhao et al (2012) applied SST \( k - \omega \) to study the VIV of rigid cylinder in uniform flow in the subcritical regime.

Although SST \( k - \omega \) RANS model has been identified as the appropriate model for the simulation of interest, its requirement of fine mesh resolution in the inner field is prone to negative cell volume error for a moving boundary. This challenge can be addressed by adopting a chimera mesh strategy for the near wall meshes. This strategy is extensively treated in Section 3.6.3.

### 3.4.1 Mathematical Modelling of SST \( k - \omega \) Turbulence Modelling

The incompressible flow over a cylindrical structure close to a plane wall is governed by the Navier-Stokes equations given by equation 3.1 and 3.2.
\[ \frac{\partial u_i}{\partial t} = 0 \]  \hspace{1cm} (3.1)

\[ \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + v \frac{\partial^2 u_i}{\partial x_j \partial x_j} \]  \hspace{1cm} (3.2)

Application of RANS model to equations (3.1) and (3.2) involves ensemble averaging of the instantaneous pressure and velocity components of the flow into the mean and fluctuating components given as \( u = \bar{u} + u' \) and \( p = \bar{p} + p' \). Averaging the decomposed components makes the fluctuating terms to vanish while another term called Reynold stress tensor is created as shown in equation 3.4

\[ \frac{\partial \bar{u}_i}{\partial x_i} = 0 \]  \hspace{1cm} (3.3)

\[ \frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_i}\left(\bar{u}_j u_j\right) = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + v \frac{\partial^2 \bar{u}_i}{\partial x_i \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} \]  \hspace{1cm} (3.4)

By Boussineq’s hypothesis, the Linear Eddy Viscosity model (EVM) of the Reynolds Stress Tensor gives

\[ \tau_{ij} = \frac{2}{3} k \delta_{ij} - \nu_T \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \]  \hspace{1cm} (3.5)

The SST \( k - \omega \) RANS model solves transport equation for the turbulent kinetic energy and a robust transport equation for the rate of dissipation per unit turbulence kinetic energy \( \omega = \frac{\varepsilon}{\beta k} \) that allows blending standard \( k - \omega \) in the near wall region with \( k - \varepsilon \) in the outer part of the boundary layers (Alfonso, 2009).

The turbulent viscosity in this model is defined as:

\[ \nu_T = \alpha^* \frac{k}{\omega} \]  \hspace{1cm} (3.6)

The expression for the transport equations for the \( k \) and \( \omega \) are shown in equation (3.6) and (3.7) respectively.

\[ \frac{\partial k}{\partial t} + \frac{\partial (k \bar{u}_i)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \nu + \nu_T \frac{\partial k}{\partial x_j} \right) + p_k + \varepsilon \]  \hspace{1cm} (3.7)
\[
\frac{\partial \omega}{\partial t} + \frac{\partial (\omega \vec{u}_i)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{\nu_T}{\sigma_k} \right) \frac{\partial \omega}{\partial x_j} \right] + \frac{\alpha}{\nu_T} P_k - \beta \omega^2 + S_\omega
\] (3.8)

Where the turbulent kinetic energy production, \( P_k = 2 \nu_T S_{ij} S_{ij} \) (3.9)

The strain rate tensor, \( S_{ij} \) is defined as:

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\] (3.10)

\( S_\omega \) is a term called cross-diffusion defined as:

\[
S_\omega = \frac{1.71}{\omega} (1 - F_1) \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}
\] (3.11)

Since the SST \( k - \omega \) RANS model require resolution close to the wall, the coefficients are dependent on the flow gradient and near wall distance. Therefore, the coefficient \( F_1 \) which is blending function that switches between \( k \)-\( w \) near the wall and \( k \)-\( e \) away from the wall is given as:

\[
F_1 = \tanh \left\{ \min \left[ \max \left( \frac{\sqrt{k}}{\beta^* \omega d_n}, \frac{500\mu}{\rho \omega d_n^2} \right), \frac{2k \omega}{d_n^2 \max \left( \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}, 0 \right)} \right] \right\}^4
\] (3.12)

Where \( d_n \) is the inner wall distance

Additional terms such as Kato-Lauder modification is often require with SST \( k - \omega \) RANS model to make it a robust modelling tools for certain flow characteristics such as in the case of shock, large differential pressure, and dynamic mesh motion in fluid-structure interaction. This term incorporates vorticity terms into the turbulent kinetic energy transport equation. Kato-Launder modification term is defined as:

\[
P = \mu_t S_{ij} \Omega
\] (3.13)

Where the vorticity term is defined as:

\[
\Omega = \sqrt{\frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)^2}
\] (3.14)
3.4.2 Computational Fluid Dynamics and Numerical Settings

CFD is the process of mathematically modelling the physical behaviour of fluid flow and solving them numerically. Equation (3.15) shows the generic equation of fluid flow for an arbitrary quantity $\phi$.

$$\frac{\partial}{\partial t} (\rho \phi) + \frac{\partial}{\partial x_j} (\rho u_j \phi) - \frac{\partial}{\partial x_i} (\Gamma_\phi \frac{\partial \phi}{\partial x_i}) = S_\phi$$  

(3.15)

The first term is the unsteady term, the second is the convection, third is the diffusion and the fourth term is the source term.

In order to discretise the partial differential equation of the incompressible flow of interest given by equation (3.1) and (3.2) into set of algebraic equations, Finite Volume Method (FVM) is applied to divide the flow domain into several control volumes as shown by Equation (3.16)

$$\int_{CV} \left( \int_{t}^{t+\Delta t} \frac{\partial}{\partial t} (\rho \phi) \right) dV + \int_{t}^{t+\Delta t} \left( \int_{A_f} \rho n. u_f \phi dA_f \right) dt - \int_{t}^{t+\Delta t} \left( \int_{A_f} n. \left( \Gamma_\phi \frac{\partial \phi}{\partial x_i} \right) dA_f \right) dt$$

$$= \int_{t}^{t+\Delta t} \left( \int_{CV} S_\phi dV \right) dt$$  

(3.16)

The accuracy and computational time for the solution of the discretised equations depends on the method of spatial discretisation used and how the pressure in the momentum equation is defined to satisfy the continuity equation.

(a) Spatial Discretisation

The spatial approximation at the control face centre in this work applies the second order upwind scheme that uses Taylor series expansion of the cell-centred solution at the cell centroid. With this scheme, the solution is second order accurate (Ferziger and Peric, 2002).

(b) Pressure-Velocity Coupling

The convection term in the momentum equation is non-linear, and the pressure and velocity are coupled together. The pressure, with no independent partial differential equation must also satisfy the continuity equation. To resolve the coupled pressure and velocity, segregated and pressure-based coupled algorithms are utilized. Semi-Implicit Method for Pressure Linked
Equations (SIMPLE) segregated algorithm is utilized for all cases with fixed cylinder in this research work, while pressure-based couple algorithm is used for free moving cylinder where meshes are non-stationary. The later solves the momentum equation and pressure correction equations together and it’s robust for compressible flow and cases with overset dynamic mesh (Denner, 2018; Sherma et al, 2021) which is the form of dynamic mesh applied later in this work.

3.5 SOLID MECHANICS – CHOICE OF BEAM MODEL

Simulation of a pipeline for dynamic response requires selecting appropriate beam model formulation for its governing equation. For a long pipeline with large aspect ratio, the aspect ratio is fundamental in the choice of beam model. The larger the aspect ratio, the more the degree of flexibility of the pipe and the larger the magnitude of its eigen frequencies. While Timoshenko beam theory is a shear deformable model and widely used for beam with low aspect ratio, Euler-Bernoulli beam theory is a shear non-deformable beam theory with application ideal for beam with large aspect ratio (Zhang et al, 2020). Furthermore, Euler-Bernoulli beam assumption requires the plane Section to remain unchanged and beam deformation angle (slope) to be small (Wang and Qin, 2020) such that $\sin \theta \approx \theta$ as shown in Figure 3.3.

![Figure 3.3: Plane and deformation slope of Euler-Bernoulli beam](Erochko, 2020)

The pipeline under consideration in this research whose descriptions are provided in Section 3.2 is a long aspect ratio pipe and whose expected deformation is no more than 1D to keep the deformation angle small. Thus, the choice of beam considered to model the pipeline is Euler-Bernoulli beam theory.
The governing equations for an axially tensioned Euler-Bernoulli beam as a model for pipeline with initial pre-tension in transverse and streamwise directions are given by equation (3.17) and (3.18) respectively.

\[
EI \frac{\partial^4 y(x, t)}{\partial x^4} + m \frac{\partial^2 y(x, t)}{\partial t^2} - T \frac{\partial^2 y(x, t)}{\partial x^2} + c \frac{\partial y(x, t)}{\partial t} = F_y \tag{3.17}
\]

\[
EI \frac{\partial^4 z(x, t)}{\partial x^4} + m \frac{\partial^2 z(x, t)}{\partial t^2} - T \frac{\partial^2 z(x, t)}{\partial x^2} + c \frac{\partial z(x, t)}{\partial t} = F_z \tag{3.18}
\]

### 3.5.1 Finite Element Analysis of Axially Tensioned Euler-Bernoulli Beam

The finite element analysis of Euler-Bernoulli beam involves discretising the beam into many elements connected at the nodes. As shown in Figure 3.4, the global coordinates of the beam are \(X, Y\) and \(Z\) while the local coordinate of the elements are given as \(x, y\) and \(z\). In order to determine the response of the beam under a dynamic loading, the local stiffness, mass and damping matrices of the elements are determined and assembled to form the corresponding global matrices. Application of the global matrices into the discretised equation (3.19) gives the response of the beam.

\[
[M] \ddot{U} + [C] \dot{U} + [K] U = F \tag{3.19}
\]
Equation 3.20 shows the local mass matrix of the sixth element of the beam under pre-tensioned axial loading for a constraint displacement and rotation about the axial direction.

\[
[M] = \frac{m_{67}}{420} \begin{pmatrix}
156 & 0 & 0 & 22L_{67} & 54 & 0 & 0 & 3L_{67} \\
0 & 36 & -3L_{67} & 0 & 0 & -36 & -3L_{67} & 0 \\
0 & -3L_{67} & 4L_{67}^2 & 0 & 0 & 3L_{67} & -L_{67}^2 & 0 \\
3L_{67} & 0 & 0 & 4L_{67}^2 & -3L_{67} & 0 & 0 & -L_{67}^2 \\
-36 & 0 & 0 & -3L_{67} & 36 & 0 & 0 & -3L_{67} \\
0 & -36 & 3L_{67} & 0 & 0 & 36 & 3L_{67} & 0 \\
0 & -3L_{67} & -L_{67}^2 & 0 & 0 & 3L_{67} & 4L_{67}^2 & 0 \\
3L_{67} & 0 & 0 & -L_{67}^2 & -3L_{67} & 0 & 0 & 4L_{67}^2
\end{pmatrix}
\] (3.20)

As a result of the axial loading, the overall local stiffness matrix on the element is the sum of the stiffness matrix due to the flexural rigidity of the element, and the axial tension on the element as shown by equation (3.21)

\[
K_{67} = K_{67, in} + K_{67, ax}
\] (3.21)

Where \( K_{67, in} \) and \( K_{67, ax} \) are the stiffness matrices due to flexural rigidity and axial tension respectively defined by equation (3.22) and (3.23)

\[
K_{67, in} = EI \begin{pmatrix}
\frac{12}{L_{67}^3} & 0 & 0 & \frac{6}{L_{67}^2} & -\frac{12}{L_{67}^3} & 0 & 0 & \frac{6}{L_{67}^2} \\
0 & \frac{12}{L_{67}^3} & -\frac{6}{L_{67}^2} & 0 & 0 & -\frac{12}{L_{67}^3} & -\frac{6}{L_{67}^2} & 0 \\
0 & \frac{6}{L_{67}^2} & \frac{4}{L_{67}} & 0 & 0 & \frac{6}{L_{67}^2} & 12 & 0 \\
\frac{6}{L_{67}^2} & 0 & 0 & \frac{4}{L_{67}} & \frac{6}{L_{67}^2} & 0 & 0 & \frac{2}{L_{67}} \\
\frac{12}{L_{67}} & 0 & 0 & -\frac{6}{L_{67}^2} & \frac{12}{L_{67}^3} & 0 & 0 & -\frac{6}{L_{67}^2} \\
0 & \frac{12}{L_{67}^3} & 0 & 0 & \frac{6}{L_{67}^2} & \frac{12}{L_{67}^3} & 0 & 0 \\
0 & \frac{6}{L_{67}^2} & \frac{2}{L_{67}} & 0 & 0 & \frac{6}{L_{67}^2} & \frac{4}{L_{67}} & 0 \\
\frac{6}{L_{67}^2} & 0 & 0 & \frac{2}{L_{67}} & \frac{6}{L_{67}^2} & 0 & 0 & \frac{4}{L_{67}}
\end{pmatrix}
\] (3.22)
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3.5.2 Constitutive Stress and Strain

The components of the strain at every node are related to the displacement at that node. The strain-displacement for all nodes with zero shear strain is given by Equation (3.24). Furthermore, the local stresses at the nodal points on the beam are equally shown by equation (3.25). In this case, the shear stresses are negligible.

\[
\begin{bmatrix}
\varepsilon_{y_i} \\
\varepsilon_{z_i}
\end{bmatrix} = 
\begin{pmatrix}
\frac{\partial}{\partial y} & 0 \\
0 & \frac{\partial}{\partial z}
\end{pmatrix}
\begin{bmatrix}
v_i \\
w_i
\end{bmatrix}
\]  
\tag{3.24}

\[
\begin{bmatrix}
\sigma_{y_i} \\
\sigma_{z_i}
\end{bmatrix} = \frac{E}{1 - \nu^2} \begin{pmatrix} 1 & \nu \\ \nu & 1 \end{pmatrix} \begin{bmatrix}
\varepsilon_{y_i} \\
\varepsilon_{z_i}
\end{bmatrix}
\]  
\tag{3.25}

Where \( i = 1, 2, 3, \ldots \) (Nodal points on the discretized beam)

3.5.3 Numerical Setting and Stability Criteria

A pipeline spanning the beam is modelled as a beam constraint at both ends. The use of appropriate end conditions at the point of contact on the seabed is vital to predicting the response of the pipeline under dynamic loading. Fyrileiv and Mork (2002) examined that the appropriate constraint at the touch down of a pipeline on the seabed is pin-pin end condition shown by equation (3.26) and (3.27).

\[
U_y(0, t) = U_y(l, t) = U_Z(0, t) = U_Z(l, t) = 0 
\]  
\tag{3.26}

\[
\frac{\partial^2 U_y}{\partial X^2}(0, t) = \frac{\partial^2 U_y}{\partial X^2}(l, t) = \frac{\partial^2 U_Z}{\partial X^2}(0, t) = \frac{\partial^2 U_Z}{\partial X^2}(l, t) = 0 
\]  
\tag{3.27}
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The time integration adopted in this explicit dynamic simulation of Euler-Bernoulli beam is the Newmark-beta method given by equation (3.28) and (3.29)

\[
\ddot{U}_{n+1} = \dot{U}_n + \Delta t \left[(1 - \gamma)\ddot{U}_n + \gamma \dot{U}_{n+1}\right]
\] (3.28)

\[
U_{n+1} = U_n + \Delta t \ddot{U}_n + \Delta t^2 \left[\left(1 - \frac{2\beta}{2}\right)\ddot{U}_n + \beta \dddot{U}_{n+1}\right]
\] (3.29)

Where \(\gamma\) and \(\beta\) are stability-controlling parameters.

To further ensure the stability of the explicit dynamic simulation of Euler-Bernoulli beam, Courant-Friedrichs-Levy criterion given by equation (3.30) to determine the minimum time-step is considered.

\[
\Delta t \leq \frac{h_{\text{min}}}{c}
\] (3.30)

Where \(\Delta t\) is the required time step for stable numerical solution, \(h_{\text{min}}\) is the minimum element size, and \(c\) is the speed of sound in the material.

### 3.6 Coupled Fluid and Beam

Coupled fluid and beam resulting to vortex-induced vibration is a two-way FSI involving transfer of pressure force to deform/displace the beam, and transfer of strain from the deformed beam to alter the state of the fluid (Benra et al, 2011; Ezkurra et al, 2018). The equation of the coupled configuration in transverse and streamwise direction are given by equation (3.31) and (3.32) respectively. The mass of the beam in the inertia term of the equation has additional added mass of displaced fluid.

\[
EI \frac{\partial^4 y(x, t)}{\partial x^4} + (m + m_a) \frac{\partial^2 y(x, t)}{\partial t^2} - T \frac{\partial^2 y(x, t)}{\partial x^2} + \left(c_s + c_{f,y}\right) \frac{\partial y(x, t)}{\partial t} = F_L(x, y, t)
\] (3.31)

\[
EI \frac{\partial^4 z(x, t)}{\partial x^4} + (m + m_a) \frac{\partial^2 z(x, t)}{\partial t^2} - T \frac{\partial^2 z(x, t)}{\partial x^2} + \left(c_s + c_{f,z}\right) \frac{\partial z(x, t)}{\partial t} = F_D(x, z, t)
\] (3.32)

At the interface between fluid and structure, the kinematic equilibrium must be satisfied to ensure a no slip at the interface. Also the traction between the fluid and structure must be equal and opposite (Hewitt, 2019). These conditions are expressed by equation (3.33) and (3.34)
\[ \mathbf{u}^f = \frac{\partial \mathbf{U}^s}{\partial t} \]  
\[ \sigma^f \cdot \mathbf{n} = \sigma^s \cdot \mathbf{n} \]

Where \( \mathbf{u}^f \) is the fluid velocity at the interface, \( \mathbf{n} \) is unit normal on the interface, \( \sigma \) is the stress at the interface with the superscript \( s \) and \( f \) denoting fluid and structure respectively.

The damping term in a beam vibrating in fluid flow is a combination of structural and fluid damping. The damping matrix in a discretized beam being a combined effect of more than one damping component, and mode dependent will not be accurate with the use of linear model but Rayleigh damping model (Song and Su, 2017). Hence, Rayleigh damping model presented in equation (3.35) is adopted in this simulation to provide the equivalent combined effect of the structural damping and viscous damping on the response of the beam.

\[ [\mathcal{C}] = \alpha [M] + \varrho [K] \]  

Where \( \alpha \) and \( \beta \) are the mass and stiffness proportional damping coefficients defined as:

\[ \begin{bmatrix} \alpha_i \\ \varrho \end{bmatrix} = \frac{2 \omega_i \omega_j}{\omega_i^2 - \omega_j^2} \begin{bmatrix} \omega_j & -\omega_i \\ -\frac{1}{\omega_j} & \frac{1}{\omega_i} \end{bmatrix} \begin{bmatrix} \xi_i \\ \xi_j \end{bmatrix} \]  

Where \( \omega_i \) and \( \omega_j \) are the eigen frequencies of the beam at the lower and upper participating modes. Similarly, \( \xi_i \) and \( \xi_j \) are the damping ratio at the modes in consideration.

### 3.7 Mesh Strategy

The geometry of interest and its layout require adoption of a strategic method of meshing both the geometry and the flow domain due to the large aspect ratio and the interaction of boundary layers between the plane wall and the cylinder wall.

Due to the large aspect ratio of the geometry, beam element is adopted in the discretisation of the beam. For the flow domain, overset mesh is used for the dynamics mesh to allow displacement of the beam in the flow. The justifications for these types of meshes and elements for the geometry of interest are provided in section 3.7.1, 3.7.2 and 3.7.3.
3.7.1 Meshing of Euler-Bernoulli Beam for long flexible pipeline

The effective use of mesh/elements in the discretisation of Euler-Bernoulli beam model for a structure depends on the aspect ratio of the structure and the significant stress distribution in the directional axis on the beam. For a structure that has one of its dimensions far larger than the other two axes, application of beam elements simplifies the discretization and provides no significant change in stress distribution from solid elements (Alisson, 2020; Sun, 2021). The use of beam elements for such geometry leads to computational time savings (Lopez, 2013). Application of beam elements for discretising beam whose one axis is far greater than the other axes are utilised in Mironova et al (2017), Nirgude and Venugopalan (2015) and Lee (2010).

The geometry of interest in this research is a long pipe with large significant aspect ratio. Therefore, beam elements are used in the discretisation of the pipe to offer simplicity and less computational resources in the FEA analysis of the pipeline.

3.7.2 Mesh Type and Near-wall Treatment in Flow Domain

The sizes and types of grids in a fluid domain especially the ones near wall are essential to the quality of result of CFD simulation. Structured grids come in the form of rectangular element in 2D-domain and hexahedral in 3-D domain, while unstructured appear as tri-element in 2D and either tetrahedral or polyhedral in 3-D domain. In this research, structured quadrilateral grids are applied in the fluid domain due to its advantage of higher degree of control to capture the flow physics at the critical region. Also, in structured grids, each element has equal number of neighbours, and thus do not require interpolation for explicit cell connection (Tunstall, 2016)

![Figure 3.5: Wall velocity boundary layer](Engineering & design, 2021, pp. 1)
Due to no-slip effect of the fluid elements directly in contact to the wall, a turbulent boundary layer has a thin layer of viscous sublayer as shown in Figure 3.5. Capturing of the physics in this thin region with RANS model depends on whether the model is high or low RANS turbulence model. As stated in Section 3.4, the choice of turbulence model adopted for the layout of geometry of interest is SST $k - \omega$ RANS model which is a low Reynolds modelling. This requires resolution in the inner domain near wall to capture the flow physics in the viscous sublayer region without the use of wall function. The requirement for grid spacing at the inner region adjacent to the wall for this low Reynold modelling is that $y^+$ defined by equation (3.37) must be equal or less than a unity (Pope, 2000). All near wall $y^+$ value in this work conform to this requirement.

\[ y^+ = \frac{\delta}{v} \sqrt{\frac{\tau_w}{\rho}} \quad (3.37) \]

Where $\delta$ is the distance from the wall, $\tau_w$ is the wall shear stress, $v$ is the kinematic viscosity.

### 3.7.3 Dynamic Mesh Implementation

Simulation of VIV of a flexible structure requires non-stationary meshes in the fluid domain to respond to the deformation or displacement of the structure. The use of Arbitrary Lagrangian-Eulerian (ALE) formulation where meshes deform and reform to match the displacement of the structure is applicable to solve VIV problem. This application of ALE formulation to deform meshes for FSI simulation requires a substantial volume of the cells at the region of moving boundary for remeshing and smoothing to accommodate structural movement (Le Tallec and Mouro, 2001; Tezduyar et al, 2008). However, the layout of structure of interest involves varying the distance between the plane wall and the cylinder. In this case, with the implementation of SST $k - \omega$, fine meshes are required between the cylinder wall and the plane wall. These meshes are prone to negative volume of cell error for ALE formulation for dynamic mesh especially for large deformation or displacement of the cylinder as shown in Figure 3.6. Therefore, ALE formulation for dynamic meshes are not suitable for the layout of interest.
A chimera/overset mesh is therefore employed as an ideal mesh strategy for the case of interest. Chimera mesh approach allows fine meshes to be resolved between the two walls while preventing the formation of negative cells error. It involves creating a separate body fitted mesh around the structure (known as overset mesh) and overlaying it onto a background mesh as depicted in Figure 3.7. The quality and accuracy of results with chimera mesh is dependent on achieving a smooth transition from the background mesh to the overset mesh. A good practice is to use very fine elements close to the cylinder and grows gradually outwards such that, the smallest elements around the cylinder wall is smaller than the elements in the background mesh, and the larger elements far from the cylinder in the overset mesh is approximately equal or not more than 1.2 times the elements in the background mesh. By this approach, most of the cells are solving and the number of donor and receptor cells at the interface (where the solution is interpolated from one mesh to the other) are minimized.

Figure 3.6: Fluid domain mesh deformation due to structural displacement (Elabbasi, 2013)

Figure 3.7: Structured overset mesh on background mesh


**3.8 NUMERICAL SOLVER DESCRIPTION**

In this study of vortex-induced vibration of flexible pipeline with large aspect ratio in uniform flow, commercial solvers ANSYS Fluent and ABAQUS are used to solve the FSI problem. While the ANSYS Fluent is the solver for the CFD of spring-supported rigid cylindrical segments in uniform flow, ABAQUS is used for the FEA of the Euler-Bernoulli beam. The external dynamic forcing loads on the beam are mapped from iterative CFD of cylindrical segments of the beam.

ANSYS Fluent is adopted for the CFD part because of its ability to solve the forcing loads and response of spring-supported rigid body in flow in a monolithic approach (single matrix) and ease of implementation of overset dynamic mesh.

For the structural analysis, ABAQUS would be used for the static, modal and dynamic response of the Euler-Bernoulli beam.

**3.9 CONCLUSION**

The description of the proposed methodology for simulating vortex-induced vibration of free spanning underwater pipeline close to the seabed with large aspect ratio to reduce computational cost and time has been stated. This method, called Reduced Order Method (ROM) is proposed to apply the use of theory of beam bending to sectionalise the pipe modelled as Euler-Bernoulli beam into a system of connected elastically supported 2-D rigid cylinders in uniform flow. With this approach, individual elastically rigid cylinder in flow is iteratively simulated to characterise forcing load on the beam. Subsequently, the response and stress distribution on the beam with informed loading from CFD is simulated though FEA.

For the layout of geometry of interest in this research, which involves proximity to plane wall, chimera mesh is chosen as the mesh type to model the FSI of elastically rigid cylinder. Also, due to flow separation and boundary layer effect, SST $k-\omega$ RANS model is adopted as the turbulent model in the flow domain.

For the flow domain simulation, ANSYS fluent is used while ABAQUS is utilised for the Finite Element Analysis simulation of the pipeline.
CHAPTER 4

FLOW OVER STATIONARY AND FREE CYLINDER BOUNDED AND UNBOUNDED BY PLANE WALL

4.1 PRELIMINARY REMARKS

This chapter focuses on investigating and validating the effect of cylinder proximity to a plane wall on both the hydrodynamic forcing on a stationary cylinder and the response of elastically supported cylinder, each in uniform flow. The intent is to obtain force coefficients for use in the reduced order model of the response of a flexible pipe that is introduced in Section 3.4. This requires sectionalising the pipe into system of connected elastically supported 2-D cylinders in flow. It is therefore necessary to investigate and validate flow over elastically rigid cylinder in flow over the range of Reynolds number of interests for later studies. The flow regime considered in this numerical simulation is sub-critical Reynold number range as this is typical of the Reynolds number of flow over a pipeline close to the seabed. Established experimental studies of the loading and response of circular cylinders are used to validate the CFD models over the Reynolds number of interests.

4.2 OVERVIEW OF NUMERICAL SIMULATION

In order to understand the complex behaviour associated with flexible pipe with large aspect ratio in flow and develop a numerical method for the response of the pipe, insight is taken from studying the dynamic response of a simple spring supported rigid cylinder in flow and its corresponding stationary case. This is because the fundamental driver of VIV of flexible pipe is the mechanism of vortex shedding from the cylinder. The flow of work in this chapter shown in Figure 4.1 involves setting up domain and meshes for stationary and free cylinder near and
far from wall to validate effect of gap ratio on cylinder dynamic in flow. The CFD model developed is validated by comparing with experimental results. The meshes and domain in the validated model are used in later chapter for a much higher level of VIV study involving response and fatigue of flexible pipe with high aspect ratio in uniform flow without further mesh independent study. This is due to this present and later simulation occurring within the same Reynolds number which ranges from $1.0 \times 10^4$ to $3.0 \times 10^4$.

Figure 4.1: Flowchart of numerical simulation procedure for flow over stationary and free cylinder

Flow over stationary cylinder (near and far from wall)  
Flow over free cylinder (near and far from wall)  
Domain and mesh construction  
Numerical simulation and validation of results  
Application to assessment of long flexible pipe in flow (far and near wall)
4.3 **Flow past Stationary Cylinder near Wall**

In this numerical study, the effects of gap ratios on the hydrodynamic forcing on cylinder in uniform flow near wall and flow dynamics especially in the wake are observed. The experimental set up for model validation is briefly described in Section 4.3.1

### 4.3.1 Description of Experimental Benchmark for Model Validation

The numerical simulation for the study of effect of gap ratio on uniform flow over stationary cylinder follows the experimental procedure of Lei et al (1999) whose geometry is shown in Figure 4.2. In the experiment, uniform flow was passed across a cylinder with diameter of 50mm and aspect ratio of 26 in a wind tunnel. The gap between the cylinder and the bottom plate were varied, as well as the position of the cylinder from the leading edge to produce gap ratio and boundary layer thickness. A total of six boundary layers were generated in the experiment. In this numerical study, two boundary layers with 3D and 11D as leading-edge distance from the cylinder corresponding to $\delta = 0.25D$ and $0.48D$ respectively are considered.

![Figure 4.2: Layout of model set-up in the wind tunnel (Lei et al; 1999; pp 266)](image)
The gap between the flat plate and the bottom of the cylinder and the range of Reynolds numbers in the experiment are presented in Table 4.1.

In addition to the shear flow performed in the experiment, shear free is also studied in this simulation to investigate the effect of variation of boundary layers on the hydrodynamic forcing and pressure distribution on the cylinder.

Table 4.1: Experimental test case data

<table>
<thead>
<tr>
<th>Experimental</th>
<th>Gap ratio (e/D)</th>
<th>Boundary layer thickness (δ)</th>
<th>Distance from leading edge</th>
<th>Re (x 10⁶)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear flow near wall</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lei et al (1999)</td>
<td>0.2, 0.5, 0.75, 1, 2</td>
<td>0.25D</td>
<td>3D</td>
<td>1.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.48D</td>
<td>11D</td>
<td>1.31</td>
</tr>
</tbody>
</table>

### 4.3.2 Numerical Simulation Framework for Stationary Cylinder

Following the experiment described above, the geometric configuration for the simulation is provided in Figure 4.3.

Ensembled Incompressible RANS turbulent models presented in equations (3.3) and (3.4) in Chapter 3 are applied as the model of unsteady flow over the cylinder near wall shown in Figure 4.3
A computational domain of size $25D + x$ by $15.5 + e_1$ with boundary conditions is constructed for the simulation of flow over the 2D cylinder near wall as shown in Figure 4.4.

The outlet plane from the mid-section of the cylinder is set to 25D, long enough to allow fully developed vortex shedding in the wake to avoid back flow of pressure. The values of $x$ and $e_1$ correspond to cylinder distances from the leading edge, and gap between the plane wall and cylinder respectively. No slip boundary condition is imposed on the fixed cylinder wall and the lower bottom of the domain to model velocity boundary layer from the surface of the plane wall upwards as flow moves along the surface resulting to shear flow. The distance of the cylinder from the leading edge is matched with the experimental setup by setting $x$ to 3D and 11D for boundary layer thicknesses 0.25D and 0.48D respectively. For the inlet plane, a constant freestream velocity as dictated by the Reynolds number of flow is specified while the outlet has zero pressure and zero normal velocity gradients. Two Reynolds numbers with value $Re = 1.31 \times 10^4$ and $1.38 \times 10^4$ are applied for $\delta=0.25D$ and $0.48D$ respectively to conform with the experimental conditions.

For the case of shear-free flow, the bottom plane is changed to slip boundary condition for $\delta = 0$ over the gap ratios under considerations.
SST $k - \omega$ defined by equation (3.7) – (3.14) in Chapter 3 is applied as closure for the Reynolds Stress Tensor of the incompressible RANS equations.

(a) Mesh generation and resolution

Structured meshes are used throughout the computational flow domain. To resolve the near wall viscous sub-layer and its effect on forcing on cylinder, fine meshes are biased towards the cylinder wall and the plane wall (lower bottom of the domain) as shown in Figure 4.5. Successive cell sizes from the first layer on the wall are no more than 1.2 times the next layer to allow smooth transition and avoid error due to irregular meshes. A constant time step of 0.0005s is used throughout the simulation. In conjunction with the time-step, the distance of cell first layer from the cylinder and plane wall (lower bottom of domain) are specified such that the maximum $y^+$ value and courant number in the domain are kept below 1.

Mesh independence study is carried out by increasing the number of cells by 15%. Accuracy is assessed on the basis of the mean drag and the rms lift coefficients at highest Reynolds number of flow. This is because the higher Reynolds number require finer mesh resolution than lower Reynolds number for the same geometry and numerical domain (Xu et al, 2009). Therefore, the mesh selected for $Re = 1 \times 10^5$ is utilized for $Re < 1 \times 10^5$. The results of mesh independence study are presented in Figure 4.6.

The optimised meshes for shear flow cases are utilised for the shear-free cases of the same gap ratios. This is because no boundary effect exists for shear-free cases, and thus require lesser meshes.
From the result of mesh independent studies, the optimized meshes for the gap ratios under consideration and their corresponding maximum $y+$ value and courant numbers are provided in Table 4.2. The simulation is run until at least 8 cycles of steady state are achieved.

Figure 4.6: Mesh independence study for stationary cylinder near wall in uniform flow
(Mean drag − , Root mean square of lift − − − )
4.3.3 Result and Validation of Numerical simulation

In this section, the effect of gap ratio on the hydrodynamic forcing on stationary cylinder, vortex shedding frequency and pressure distribution are observed and validated.

(a) Effect of gap ratio on Drag and Lift coefficient

The time histories of drag and lift coefficients for $e = 0.2D$, $0.5D$, $0.75D$ and $D$ at $Re = 1.31 \times 10^4$ are presented in Figure 4.7. The remaining force-time histories for all gap ratios, boundary layer thickness and Reynolds number of interests are available in Appendix A.

Table 4.2: Cell optimization for flow over cylinder near wall

<table>
<thead>
<tr>
<th>$e/D$</th>
<th>$\delta = 0.25D$</th>
<th>$\delta = 0.48D$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cylinder wall $y^+$</td>
<td>Plane wall $y^+$</td>
</tr>
<tr>
<td>0.2</td>
<td>0.242</td>
<td>0.243</td>
</tr>
<tr>
<td>0.5</td>
<td>0.387</td>
<td>0.217</td>
</tr>
<tr>
<td>0.75</td>
<td>0.396</td>
<td>0.310</td>
</tr>
<tr>
<td>1</td>
<td>0.370</td>
<td>0.207</td>
</tr>
<tr>
<td>2</td>
<td>0.512</td>
<td>0.398</td>
</tr>
</tbody>
</table>
Figure 4.8 shows the variation of mean drag coefficients with gap ratios at Re = \(1.38 \times 10^4\) and \(1.31 \times 10^4\) for \(\delta = 0.25D\) and \(\delta = 0.48D\) respectively. The mean values are obtained by averaging the complete cycles of the steady portion of the time histories while the transient parts are neglected. For Re = \(1.31 \times 10^4\), prediction of the mean drag agrees with experimental result of Lei et al (1999) with a maximum error of 6.5% for \(e = 0.5D\) (shown in Figure 4.8b) while other gap ratios fall within 5% error. The predictions of mean drag for Re = \(1.38 \times 10^4\) for all gap ratios also fall within 5% error except for \(e = 2D\) with percentage error of 5.5%. This variation is consistent in pattern with the effect of boundary layer thickness presented by other studies such as Zdravkovich (1985) and Prsic et al (2016) but different in magnitude due to different Reynolds numbers. Close to the plane wall, the cylinder is enclosed within the boundary layer thickness of the plane wall and causes a decrease in drag coefficient. As the gap ratio increases, the effect of the boundary layer on the cylinder diminishes and the drag force increases. At a very large gap ratio above 1, the cylinder falls in the potential flow, outside the boundary layer thickness of the plane wall. Hence, the plane wall has no effect on the cylinder and the drag coefficient becomes constant in this region.
Chapter 4. Flow over Stationary and Free Cylinder Bounded and Unbounded by Plane Wall

The influence of gap ratio on the lift coefficient is shown in Figure 4.9. For $Re = 1.38 \times 10^4$, prediction accuracy of the mean lift for $e = 0.5D$ has 6.1% error, while for other gap ratios, prediction falls under 5% error. The corresponding fluctuating component of the lift is predicted to within 5% error except for $e = 0.2$ where prediction is 6.6%. In Figure 4.9 (b), the predictions accuracy of the mean lift for $Re = 1.31 \times 10^4$ are within 5% except for $e = 0.5D$ and $1.0D$ that have under-prediction of 6.4% and over prediction 6.6% for $e = 0.5D$ and $D$ respectively.

(a) Lift coefficient with $e/D$ at $Re = 1.38 \times 10^4$ and $\delta = 0.25D$

(i) Coefficient of mean lift against gap ratio

(ii) Coefficient of rms lift against gap ratio
At a very small gap ratio as seen in 4.9 (ai) and (bi), a positive mean lift occurs, and reduces as the gap ratio increases, approaching zero at e/D = 2. This is due to uneven pressure distribution at the top and bottom of the cylinder due to proximity to the plane wall. A narrower gap leads to a greater pressure at the bottom of the cylinder compared to the top, and hence positive mean upwards force. However, vortex shedding a suppressed towards the plane wall, leading to reduced $C_{\text{L}_{\text{rms}}}$ as gap ratio reduces. A value of $C_{\text{L}_{\text{rms}}} = 0.03$ and 0.05 are recorded for Re = $1.38 \times 10^4$ (Figure 4.7aii) and Re = $1.31 \times 10^4$ (Figure 4.7bii) respectively for the lowest gap ratio (e = 0.2D). For a larger gap ratio, the plane wall boundary layer effect diminishes, and the pressures becoming evenly distributed at the top and bottom of the cylinder, hence, the mean lift coefficient approaches zero, and the $C_{\text{L}_{\text{rms}}}$ increases. These trends of lift coefficient agree with many studies, some of which are Price et al (2002) and Ong et al (2010).

**b)** Variation of shear stress and stagnation points on cylinder with gap ratios

At the stagnation point, the resultant force is normal to the tangential direction on the cylinder, hence zero shear force exists at the point (Sin and Tong, 2009). Furthermore, at the separation point, the fluid detaches from the cylinder and no force acts on the cylinder (Summer and Fredsoe, 1999). Therefore, the shear forces at the stagnation and separation points are zero. Figure 4.10 presents the effect of gap ratio on shear stress and pressure distribution around the cylinder.
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In Figure 4.9(a), the stagnation points are $344^0$, $354^0$ and $356^0$ for gap 0.2D, 0.75D and D for a boundary layer thickness 0.25D. The separation points for the three gap ratios all lie at $92^0$ and $265^0$ points on the cylinder. This shows that, for a given boundary layer thickness, as the gap ratio is reduced, the stagnation point shifts to the lower region of the cylinder but has no significant effect on the separation points. This was also observed in the experimental result of Lei et al (1999). For easy comparison, points around the cylinder are calibrated from $0^0$ to $180^0$ for the upper half of the cylinder and $0^0$ to $-180^0$ for the lower half. This means the stagnation points $344^0$, $354^0$ and $356^0$ correspond to $-16^0$, $-6^0$ and $-4^0$ respectively. The prediction for the gap ratios under consideration (i.e., $e = 0.2D$, 0.75D and D) are accurate to within 5%. The shift in stagnation points due to cylinder proximity to a plane wall can further be identified as a factor responsible for producing mean lift on the cylinder. The stagnation point at the lower region of the cylinder produces unbalanced pressure around the cylinder leading to positive lift force. The smaller the gap ratio, the lower the stagnation points below the lower half of the cylinder. This explains why smaller gap ratio produces a large mean lift on a stationary cylinder in uniform flow.

To isolate and observe the specific effect of boundary layer thickness on the cylinder, a corresponding result from uniform onset flow is presented in comparison to the results obtained for a cylinder in shear flows with boundary layer thickness, $\delta = 0.25D$ and 0.48D for $e = 0.75D$, see Figure 4.11.
The Figure shows an unsymmetric pressure distribution around the cylinder with the lower half of the cylinder ($180^\circ - 360^\circ$) having a higher coefficient of pressure ($C_p$). A higher stagnation pressure is obtained for uniform shear free flow compared to its corresponding shear flow. This means the mean lift in shear free flow near wall is higher than shear flow for the same gap ratio. The variation in the cylinder pressure distribution with wall boundary layer thickness agrees with Nishino (2007) and Bimbato et al (2011). This is as a result of freestream velocity for shear free flow remaining unaltered on the cylinder at the stagnation point.

(c) **Effect of gap ratio on the flow dynamics in the wake**

Proximity of the plane wall has significant on the dynamics of flow in the cylinder wake. As the proximity between the cylinder and the plane wall is reduced, vortex shedding is suppressed as shown in Figure 4.12 for $Re = 1.31 \times 10^4$ and $\delta = 0.48D$. No vortex shedding occurs for $e = 0.2D$ and instead a recirculation region occurs. For $e = 0.5D$ and $0.75D$, vortices are shed but form an inclined vortex pattern downstream, with vortex pairs moving away from the wall with increasing downstream distance. The vortex shedding is fully developed at $e = 2D$. At this gap ratio, the vortices shed are harmonically symmetrical in the wake.
In addition to plane wall affecting the formation and suppression of vortex shedding, the turbulence kinetic energy of the flow in the wake are strongly influenced by this proximity. Figure 4.13 shows the visualisation of the evolution of turbulent kinetic energy in the wake of the cylinder at Re = 1.31x10^4 for small and large gap ratios. For e = 0.2D, production of maximum mean turbulent kinetic energy occurs immediately after flow separation which extends to about 2D into the wake as shown in Figure 4.13 (a). The maximum mean turbulent kinetic energy for e = 2D case is lower and only extend to about 0.5D. Variation in the turbulent kinetic energy is due to the formation of secondary vortex shedding on the plane wall. For the case of e = 0.2D, the secondary vortex on the plane rolls up and interact with the vortex shed at the top and bottom of the cylinder and causes lead to high level of vortical fluctuation. Hence, a higher turbulent kinetic energy. This is not the case for e = 2D where the secondary vortex formed on the plane is isolated from interacting with the primary vortices shed on the cylinder. The maximum turbulent kinetic energy in this case which is lower to the corresponding e =
0.2D is limited to the interaction between the vortex shed at the top and bottom of the cylinder only. Far away into the wake, the turbulent kinetic energy diminishes for both cases.

(a) Mean turbulent kinetic energy for e = 0.2D  
(b) Mean turbulent kinetic energy for e = 2D

Figure 4.13: Flow visualization of average turbulent kinetic energy in the wake at Re = 1.31x10^4

(d) Effect of gap ratio on vortex shedding frequency

The simulation predicts the effect of proximity between the cylinder and plane wall on the Strouhal number to within 5% maximum error for all gap ratios at Reynolds number of 1.38x10^4 as shown in Figure 4.14. The Fast Fourier Transform (FFT) for e = 0.2D and e = 0.5D are shown in Figure 4.10 (a) and (b) respectively. FFT for other gap ratios and flow Regime are presented in Appendix A
A distinct variation occurs in the frequency of vortex shedding as the gap ratio changes. This variation can be categorized into three. The first category is the shedding frequency at $e = 0.2D$. The Strouhal number for this case is $1.921$.

The second category is Strouhal number in the range $0.2D < e \leq 1$. The Strouhal number increases with a maximum at $e = 0.5D$ and decreases slightly at $e = D$. Within this range, the vortex shedding at the top and bottom are not symmetrical due to the plane wall as shown in Figure 4.12 for $e = 0.5D$ and $e = 0.75D$. The vortex shedding is characterised by multiple frequencies and thus lead to a slight high value of Strouhal number, and in this case $St = 0.23$ for $0.5D$. 

Figure 4.14: Frequency of vortex shedding and at $Re = 1.38 \times 10^4, \delta = 0.25D$
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For $D < e \leq 2D$, the vortex shedding profile behave like that of cylinder in infinite flow with a regular frequency. The Strouhal number is this category is approximately 0.2.

4.4 Uniform Flow past an Elastically Supported Rigid Cylinder near and Far from Wall

The effect of wall proximity plays a very important role on the forces and dynamic of fluid flow past a cylinder. Section 4.3 had been presented to show the relationship between near wall proximity on flow over stationary cylinder in uniform flow. For a case of free cylinder (i.e. where the cylinder is supported elastically) where VIV is set in, the dynamic response of the cylinder is also dependent on the closeness of the cylinder to a plane wall. Observation of this effect and its validation are presented in this section.

4.4.1 Numerical Simulation Framework Free Cylinder

Two experimental test cases are used for the validation of these simulations – Khalak and Williamson (1997) and Barbosa et al (2017). While the former is an experiment on free cylinder in an infinite uniform flow (far from wall), the latter is the case with near wall.

In Khalak and Williamson’s, an elastically mounted cylinder was restricted to vibrate in transverse direction only in uniform flow. The unidirectional movement was made possible by the air bearing configuration shown in Figure 4.15. The combined test cylinder and the spring system in the uniform flow produced a mass-damping ratio of 0.0108. Other relevant parameters for the experiment are shown in Table 4.3

A test cylinder of diameter 0.04m was mounted on four elastic springs connected in parallel and the whole set up immersed in a uniform flow in a tank in Barbosal et al’s study. Softer and stiffer set of springs were used to mount the cylinder. Only the softer springs with spring constant value of 50N/m each are considered in this present simulation. The position of the cylinder was varied from the bottom of the tank to produce gap ratios. The natural frequency of the elastically (soft springs) mounted cylinder in fluid was measured to be 1.3Hz. In the set-up, the cylinder was placed at 3m (75 x diameter of the cylinder) upstream of the test section throughout the experiment as shown in Figure 4.8. The velocity profile at this point without the
cylinder in place at Re = 12134.83 was measured, and the profile is given in Figure 1(c). Damping source in the system with an estimate of 2% was due to friction in the bearings. The gap ratios of interest in this study, other mechanical properties of the elastically mounted cylinder and fluid properties are displayed in Table 4.3.

The two experiments are limited to transverse direction only. For further study, this simulation also considers two-degree-of-freedom for one of the gap ratios.

Figure 4.15: Schematic of experimental set up (Khalaak and Williamson; 1999; pp. 826)

Figure 4.16: Schematic of free vibration set up near wall (Barbosa et al, 2017, pp. 384)
4.4.2 Configuration and Governing Equation

The configurations for two-degree of freedom of elastically mounted cylinder in uniform flow near wall is shown in Figure 4.17.

![Figure 4.17: Configuration of rigid cylinder supported by spring in uniform flow near wall](image)

The elastically supported cylinder fully submerged in uniform fluid flow is under the influence of time varying forces in stream-wise and transverse direction due to oscillating pressure field developed by vortex shedding in the wake. In addition to the forces, the cylinder experiences additional inertial mass of the fluid and viscous damping. Therefore, the governing equations
Chapter 4. Flow over Stationary and Free Cylinder Bounded and Unbounded by Plane Wall

of the cylinder in the set up shown in Figure (4.17) in stream-wise and transverse directions are given by equations (4.1) and (4.2) respectively.

\[(m + m_a) \frac{\partial^2 z}{\partial t^2} + (c + c_f) \frac{\partial z}{\partial t} + k_z = F_D(t) \quad (4.1)\]

\[(m + m_a) \frac{\partial^2 y}{\partial t^2} + (c + c_f) \frac{\partial y}{\partial t} + k_y = F_L(t) \quad (4.2)\]

Where \(m_a\) is the fluid-added mass per unit length defined by equation (4.3), \(c_f\) is the fluid-damping coefficient, \(F_D\) and \(F_L\) are time varying drag and lift forces on the cylinder.

\[m_a = \frac{\pi}{4} C_a \rho D^2 \quad (4.3)\]

For a fully submerged cylinder in fluid, \(C_a\) has a value of unity (Blevins, 1990).

The time varying drag and lift forces are resolved by solving Navier-Stokes equation using RANS model presented in Equation 3.5 in Chapter 3.

4.4.3 Computational Domain and Boundary Conditions

The computational domains for elastically mounted cylinder in infinite flow and shear flow are given in Figure 4.18. Two domains are constructed – one for unbounded cylinder and another for cylinder bounded by plane wall. The sizes of the domains are 30D by 35D and 15.5D+e by 35D respectively with e being the gap between the cylinder and the plane wall.

![Figure 4.18: Numerical domain for flow over elastically mounted](image)
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The top and outlet of both domains have symmetry and zero pressure boundary conditions respectively. While the bottom of the unbounded cylinder has symmetry boundary condition (Figure 4.18 (a)), no slip condition is assigned to the bottom of the domain and the cylinder for near wall case for velocity gradient development (Figure 4.18 (b)). A log velocity profile presented in Prsic (2011) and shown by equation (4.4) is adopted and specified as the inlet in Figure 4.15 (b) at section Y1-Y1. This allows the computational domain to begin at 65D from the leading edge as shown in Figure 4.19. For the unbounded cylinder in flow (Figure 4.18 (a)), the inlet is the free stream velocity.

The reactional forces of the spring-damper system to the flow over the cylinder is user defined. Appendix B1 presents the user defined function (UDF) for these forces.

\[ u(y) = U^* = \min \left\{ \frac{u^*}{k} \ln \left( \frac{y}{x_w} \right), U_\infty \right\} \quad (4.4) \]

Where \( u^* \) is defined as:

\[ u^* = \frac{k U_\infty}{\ln \left( \frac{y}{x_w} \right)} \quad (4.5) \]
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$U_\infty$ is the free stream velocity, $k$ is the von karman constant with a value of 0.41, $\delta$ is the boundary layer thickness at section Y1-Y1, $y$ is the vertical direction, $z_w$ is the absolute roughness of the cylinder.

The relative roughness ($z_w/D$) of a smooth circular cylinder from the moody chat is $1 \times 10^{-6}$ (Moody, 1944).

(a) Inet velocity profile test for shear flow

Simulation of flow on a flat plate is carried out first to determine the robustness of the velocity profile given by equation (4.4) and matched with the given experimental velocity profile. A rectangular fluid domain of size 35D by 30D shown in Figure 4.16 (b) with structured meshes is used. The velocity profile is set at section Y1-Y1, 65D from the leading edge in Figure 4.19 (a), and corresponding to 10D from the mid-section of the cylinder. A no-slip boundary condition is imposed on the surface of the lower bottom of the domain. Fine and dense meshes are created towards the lower bottom of the domain to model the viscous sub-layer and capture the velocity gradient. A total number of 802385, 914550, and 970452 cells are tested in the domain until flow is fully developed and no changes in velocity profile is achieved with the optimized cell being 914550. The velocity profile at section Z1-Z1 compares well with the given experimental velocity profile as shown in Figure 4.20. With a good comparison achieved, the velocity profiles for the number of reduced velocities of interest ($v_r = 3, 4, 5, \ldots, 11$) are applied at section Y1–Y1.

![Graph](image)

Figure 4.20: Velocity profile at section Z1-Z1 from the leading edge
4.4.4 Mesh Generation and Computational Procedure

The grid with the cylinder in place uses the structured chimera mesh approach. In this meshing approach, a circular domain of size 2.5D is constructed harbouring the overset mesh around the cylinder and overlays on a structured rectangular background mesh. A cut-out section of the overset and background meshes is presented in Figure 4.21 (a). In the overset mesh, fine and dense meshes are produced for velocity gradient development and viscous sub-layer detection. Similarly, for the cases with cylinder near wall, meshes are also biased towards the plane wall (bottom of the rectangular domain). The size of the first layer close to the walls are chosen such that the maximum \( y^+ \) value is less than one. The interface of the meshes between the overset and the background are refined such that the outermost layer of the cells in the overset meshes are not more than 1.2 times the background cells they intercept. This enables smooth transition between the overset cells and the background cells at the interface. Figure 4.21 (b) shows the intercept between the background and the overset mesh.

(a) A cut-out section of overset mesh overlayed on background

(b) Mesh for flow over elastically supported unbounded cylinder in uniform flow

(c) Mesh for flow over elastically supported cylinder near wall in uniform flow

Figure 4.21: Mesh generation for flow over elastically mounted cylinder
In order to resolve the Reynold Stress Tensor of the RANS model, SST $k-\omega$ turbulence model is used for closure in this simulation of the flow. The ratio of Reynolds number to reduced velocity ratio ($\text{Re}/v_r$) of interest for unbounded and bounded cases are 1064.83 and 30337.08 respectively.

Sensitivity studies are done by increasing number of successive meshes by 15%. Four time-steps which are 0.001s, 0.0005s, 0.0002s and 0.0001s; and the highest Reynolds number of flow are used in the study to observe the transverse response of the cylinder as shown in Table 4.4. Mesh ID’s are assigned to each of the gap ratio to show the number of cells for sensitivity study. The optimized number of cells from the sensitivity study with their measure of accuracy for which further mesh refinement has no changes on the transverse displacement is presented in Table 4.5.

In addition to single degree of freedom of shear flow over the elastically supported cylinder, two degree of freedom is also considered with the same optimized cells and time-step for $a = 1D$.

Table 4.4: Mesh and time-step independent studies

<table>
<thead>
<tr>
<th>Gap ratio ($e/D$)</th>
<th>Mesh ID</th>
<th>Number of cells</th>
<th>Time steps</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Overset mesh</td>
<td>Background mesh</td>
<td>$1\times10^{-3}$</td>
</tr>
<tr>
<td>0.75</td>
<td>MSH01</td>
<td>21667</td>
<td>492413</td>
</tr>
<tr>
<td></td>
<td>MSH02</td>
<td>24917</td>
<td>566275</td>
</tr>
<tr>
<td></td>
<td>MSH03</td>
<td>28654</td>
<td>651216</td>
</tr>
<tr>
<td></td>
<td>MSH04</td>
<td>32964</td>
<td>748897</td>
</tr>
<tr>
<td></td>
<td>MSH11</td>
<td>34681</td>
<td>695195</td>
</tr>
<tr>
<td>1</td>
<td>MSH12</td>
<td>30157</td>
<td>799476</td>
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<tr>
<td></td>
<td>MSH13</td>
<td>33310</td>
<td>919397</td>
</tr>
<tr>
<td></td>
<td>MSH21</td>
<td>29113</td>
<td>857927</td>
</tr>
<tr>
<td>2</td>
<td>MSH22</td>
<td>33480</td>
<td>986617</td>
</tr>
<tr>
<td></td>
<td>MSH23</td>
<td>38502</td>
<td>1134609</td>
</tr>
<tr>
<td>$\infty$</td>
<td>MSH31</td>
<td>23374</td>
<td>355283</td>
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<tr>
<td></td>
<td>MSH32</td>
<td>26880</td>
<td>408576</td>
</tr>
<tr>
<td></td>
<td>MSH33</td>
<td>30912</td>
<td>469862</td>
</tr>
</tbody>
</table>
4.4.5 Results and Discussion

One of the challenges of predicting VIV of elstically supported rigid cylinder with RANS turbulent model is the relatively low measure of prediction accuracy in the lock-in region. Under-prediction in the lock-in region are recorded in many studies as such Izhar et al (2017), Lucor et al (2005) and Zhao et al (2014). The poor prediction can be attributed to the sudden jump (or shock) in displacement due to synchronisation between the shedding frequency and the natural frequency of the free spring supported cylinder. This sudden jump causes large distortion or disturbance of fluid over the range of velocity ratio in the lock-in region.

With the use of Kato-Launder modification term to the SST k-\omega turbulence modelling, prediction of maximum displacement in the lock-range improves by approximately 50% as shown in Figure 4.22. Therefore, all other simulations for the gap and mass-damping ratios of interest apply SST k-\omega RANS model incorporating Kato-Launder modification term.
Figure 4.22: Effect of Kato-Lauder vorticity and production limiter in the lock-in region

The time histories of displacements and forces on elastically supported rigid cylinder for gap ratio \( \frac{e}{D} = \infty, 0.75 \) and 1 in uniform flow for a sample number of reduced velocity \( (v_r) \) are shown in Figure 4.23 and 4.24 respectively. Other time histories are presented in Appendix B2.
(a) Time history of displacement for $e = 0.75D$

(b) Time history of displacement for $e = D$

Figure 4.23: Time history of displacement and force for $\frac{e}{D} = \infty, m^*\xi = 0.0048$

Figure 4.24: Variation of displacement of cylinder with time for $e \leq 2D$
(a) Displacement and forces

The amplitude of displacement of the cylinder response for the gap ratio of interest are presented. The displacement results are divided into two categories – one with the cylinder in infinity flow that involves no wall proximity, and the other bounded by plane wall with influence of gap ratios.

The amplitudes of response from the steady state displacement time histories are determined by the product of the rms response and root of 2 (i.e. $Y = y_{rms} \sqrt{2}$)

(i) Unbounded cylinder ($\frac{L}{D} = \infty$).

As shown in Figure 4.25, the predicted displacement of the cylinder captures the three branches of VIV of rigid cylinder – the two lower regions (the first where vorticies are beginning to gain momentum, and the other lower region where vortex shedding frequency unlock from the cylinder) and the lock-in region where the frequency of vortex shedding synchronizes with the natural frequency of the spring supported rigid cylinder. The lock-in region prediction occurred in the range $4 < v_r < 8$ with a maximum percentage error of 9.8%. Outside the lock-in range, prediction of the transverse displacement falls within 5%. In comparison with other studies such as Lin and Wang (2019) based on discrete vortex method and wake oscillator, the simulation performs better.

Figure 4.25: VIV response of unbounded rigid cylinder in uniform flow
(ii) Bounded cylinder near wall \((e \leq 2D)\)

For all the gap ratios, underprediction occurs at the lock-in range. These underpredictions recorded are however an improvement from when the standard SST \(k - \omega\) without Kato-Launder terms produces. At the initial and lower branches of the displacement response and the mean drag, and also the frequency response, prediction accuracies are within 5% error as shown in Figure 4.26, 4.27 and 4.28. In the lock-in range, the maximum underprediction accuracy error of 11% occurs for mean drag for \(e = 0.75D\).

As shown in Figure 4.26 to 4.28, the amplitude of response decreases as the gap ratio increases. This is further explain in Section 4.4.5 (b) by combining the response on a single graph.

Implementation for a gap ratio of 0.5D is also considered but result not presented. This is due to abrupt error in simulation which can be attributed to contact between the cylinder and the plane wall.

![Graphs showing VIV response of bounded rigid cylinder in uniform flow for \(e = 0.75D\)]

Figure 4.26: VIV response of bounded rigid cylinder in uniform flow for \(e = 0.75D\)
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Figure 4.27: VIV response of bounded rigid cylinder in uniform flow for $e = D$

(a) Amplitude ratio against reduced $V_r$

(b) Mean drag against reduced velocity

(c) Frequency ratio vs $V_r$
Figure 4.28: VIV response of bounded rigid cylinder in uniform flow for $e = 2D$

(b) Effect of gap ratio on synchronization band width and mean lift

The gap ratio has a significant effect on the mean lift of the cylinder. Figure 4.29 (b) shows the effect of gap ratio on the mean lift on the free cylinder. Similar to the effect on stationary cylinder, positive mean lift also occur on free cylinder as the gap ratio decreases. The narrow path between the cylinder and the fluid coupled with strong interaction between the velocity gradients on the cylinder wall and the plane wall generates unsymmetrical pressure at the top
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and bottom of the cylinder. The lower the gap ratio, the stronger the differential pressure, and the more the cylinder drifts away from the plane wall.

![Graph showing effect of wall proximity on VIV displacement and mean lift](image1)

**Figure 4.29:** Effect of wall proximity on the VIV displacement and mean lift on free cylinder

A very important and useful effect of the gap ratio on the free cylinder is on the lock-in region. Figure 4.29 (a) shows that the synchronization occurs in the range $5 < v_r < 9.5$ for gap $e = 0.75D$, $5 < v_r < 8.3$ for $e = D$, and $5 < v_r < 7.6$ for $e = 2D$. Therefore, this means that, as the gap ratio decreases, the lock-in region shrinks. The narrow effect on the lock-in region however affect only the upper band while the lower band remain unchanged.

**c) Effect of gap ratio on vorticity**

The flow visualisation in the wake at the lock-in region and lower region for $e = \infty$ as shown in Figure 4.30 (a) and (b) for different flow time shows ‘2P’ mode of vortex shedding i.e. two alternate vortices are shed per cycle.
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(b) Vorticity contour at $V_r = 8$ (lower branch)

(i) $BN_H = 280$

(ii) $BN_H = 301.6$

(iii) $BN_H = 322.4$

(a) Vorticity contour at $V_r = 5$ (lock-in branch)

Figure 4.30: Vorticity contour of flow over free cylinder for $e = \infty$

However, as the gap ratio is reduced for $e \leq 2D$, the mode switches to ‘2T’ (i.e. three vortices are shed at the top and bottom) as shown by Figure 4.31(a) for $e = 0.75D$ in the lock-in region. This is due to the shedding occurring on the plane which rolls up and interact with that of the cylinder thereby producing three vortices per cycle. In the lower region as shown in Figure 4.31 (b), the strength of the shedding on the plane reduces and form a combination of 2P+ 2T (i.e. two vortices at the top and three vortices at the bottom per cycle).
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In this chapter, the effects of gap ratio and velocity gradient on hydrodynamic forces and displacement of cylinder have been studied and validated with established experimental results. The study begins with stationary cylinder and end with elastically supported rigid cylinder in flow. The results obtained are consistent with other previous studies and key information from the study are highlighted below.

4.5 CONCLUSION

The change in mode of vortex shed as the gap ratio reduces is responsible for the variation in the width of lock-in region identified in Section 4.4.5 (b).
4.5.1 Stationary Cylinder

i. The study validates that a lower the gap between the cylinder and the plane wall causes a higher pressure at the bottom of the cylinder, and thus, drift the cylinder in upwards direction resulting in a positive mean lift coefficient. However, while lower gap ratio increases the mean lift, the vortex shedding downstream of the cylinder is suppressed leading to a reduced r.m.s value of the lift.

ii. The opposite is the case for mean drag coefficient. The lower the gap ratio, the smaller the drag force acting on the stationary cylinder.

iii. The upper limit for the influence of the gap ratio have been identified as 2D. Above this value, the cylinder is assumed to be in the potential flow outside the boundary influence and behave like a cylinder in infinite flow. Thus, zero mean lift occurs and the mean drag is only affected by Reynolds number only.

iv. The lower limit is 0.2D and below this value, vortex shedding is completely suppressed for all flow regime.

v. Decreasing gap ratio pushes the stagnation point to the lower half of the cylinder but has no significant effect on the separation of the fluid around the cylinder.

4.5.2 Free Cylinder

i. Application of the Kato-Laaunder and production limiter correction to the SST $k - \omega$ RANS formulation improves the prediction of VIV at the lock-in region. It means these combination of correction factors are suitable for large displacement and sudden distortion of fluid

ii. While decrease in gap ratio leads to decrease in cylinder amplitude, it however increases the mean displacement and transverse force on the cylinder.

iii. Closeness to a plane narrows the lock-in bandwidth by reducing the upper band of the lock-in. The lower band remain unchanged.

iv. For the mass-damping ratio ($m^*\xi = 0.11$) under consideration, simulation for case with $e/D \leq 0.5$ results to collision of the cylinder with the plane wall.

The study and validation of stationary and elastically rigid cylinder in close proximity to a plane wall in uniform flow would help and be utilised in the study of flexible beam far and in close proximity to a plane wall.
CHAPTER 5
MODEL DEVELOPMENT FOR VORTEX-INDUCED VIBRATION OF LONG FLEXIBLE CYLINDER IN UNIFORM FLOW

5.1 PRELIMINARY REMARKS

The variation of response of a flexible cylinder with large aspect ratio along the longitudinal direction to a flow is strongly affected by the spanwise variation of vortex shedding.

In this chapter, the Reduced Order Method (ROM) briefly described in Section 3.3 is detailed and the predictive accuracy of the model is assessed for the analysis of VIV of flexible pipe with large aspect ratio for a representative mass-damping ratio in the subcritical flow regime. Three established experimental results are used as benchmarks for this assessment and comparison is also drawn with other numerical methods.

5.2 OVERVIEW OF ASSESSMENT OF ROM

The governing equations of the fluid-structure interaction of an axially tensioned flexible pipe and the fluid flow depicted by Figure 5.1 in transverse and streamwise direction have been provided in Chapter 3 as Equations 3.31 and 3.32 respectively.

Figure 5.1: Schematic diagram of a horizontal subsea pipeline
Application of ROM to Figure 5.1 follows three stages which are: beam sectionalisation, forcing load characterisation and FEA as shown by the flow chart in Figure 3.2 in Chapter 3.

To apply beam sectionalisation, it is assumed that flow across a long flexible cylinder is gradual. Therefore, before the onset of vortex shedding, the beam is in dynamic equilibrium with the fluid in the streamwise direction and static equilibrium in the transverse direction. Under these conditions, the beam is undergoing pure bending only in transverse and streamwise directions. Equations (5.1) and (5.2) display the bending equations in transverse and streamwise directions respectively.

\[
EI \frac{\partial^4 y(x)}{\partial x^4} - T \frac{\partial^2 y(x)}{\partial x^2} = F_y(x) \tag{5.1}
\]

\[
EI \frac{\partial^4 z(x)}{\partial x^4} - \ell \frac{\partial^2 z(x)}{\partial x^2} = F_z(x) \tag{5.2}
\]

Choi (2001) provides the solutions for Equations (5.1) and (5.2) for different end conditions. For pinned-pinned end condition, the equivalent spring stiffness along the spanwise direction of the beam is given by Equation (5.3).

\[
k(x) = l \left[ \frac{1 - \cosh \beta l}{\beta^2 \ell \sinh \beta l} \sinh \beta x + \frac{1}{\beta^2 \ell} \cosh \beta x + \frac{1}{2 \ell} x - \frac{1}{2 \ell \beta^2} \right]^{-1} \tag{5.3}
\]

Where \( \beta = \sqrt{\frac{\ell}{EI}} \tag{5.4} \)

The local spring stiffness is symmetric about the mid-section of the beam and becomes large at the end points – points of no displacement.

With this approach of cylinder segments, new dimensionless quantities which are localised reduced velocity and frequency with respect to the localised equivalent spring stiffness and position of cylinder segment are defined as:

\[
f_L(x) = \frac{1}{2\pi} \sqrt{\frac{k(x)}{m_i}} \tag{5.5}
\]

\[
v_{rL}(x) = \frac{U_\infty}{f_L(x) D} \tag{5.6}
\]
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Figure 5.2: Reduction of long flexible cylinder in uniform flow into elastically supported rigid cylinder segments

This assumption enables the long flexible cylinder to be sectionalised into a finite number of connected cylinder segments as shown in Figure 5.2 with spring stiffness at location of cylinder segment obtained from Equation (5.3). The equations of motion of VIV of flexible cylinder presented in Chapter 3 as Equations (3.31) and (3.32) are therefore reduced to Equations (5.7).

\[
\begin{align*}
\frac{m_i}{m_{si}} + \frac{m_9}{L_o} \frac{\partial^2 y_i}{\partial x^2} + \left( c_i + c_{f_i} \right) \frac{\partial y_i}{\partial x} + k_i y_i &= F_{L_i} \\
\frac{m_i}{m_{si}} + \frac{m_9}{L_o} \frac{\partial^2 z_i}{\partial x^2} + \left( c_i + c_{f_i} \right) \frac{\partial z_i}{\partial x} + k_i z_i &= F_{D_i} \\
\end{align*}
\]

(5.7)

Where \( n \) is the number of segments in the flexible beam, and \( k \) is the stiffness at location \( i \), given as a function of the position on the beam, \( x_i \), by Equation (5.3).

The forcing loads on each segment of the flexible cylinder are characterised by CFD simulation of rigid cylinder segments supported by equivalent local spring stiffness. This analysis is undertaken in standalone CFD analysis rather than a coupled FEA-CFD analysis. The accuracy of the forcing loads must account for the mechanical constraint of the adjacent segments which provide additional damping to the motion of the cylinder segment under consideration. Therefore, in addition to the structural damping of the segment, and the viscous damping of the fluid, coulomb damping due to constraints from adjacent segments are iteratively added to improve the prediction of the hydrodynamic forcing loads on the beam.
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The mechanical constraint-induced damping is determined by the relative displacements of the adjacent segments and is given by Equation (5.8)

$$\xi_c = \frac{1}{\pi} \left\{ \frac{Y_{i+1} - Y_i}{Y_i - Y_{i-1}} - 1 \right\}$$  \hspace{1cm} (5.8)

With forcing loads corrected through iterative FSI simulation of elastically supported cylinder segments, the response of the beam is determined by FEA analysis of the long flexible pipe modelled as Euler-Bernoulli with time-varying forcing of each segment informed from CFD as shown in Figure 5.4 and expressed by Equations (5.9) and (5.10).

$$F_{y,i} = F_{z,i} = \frac{1}{2} \rho u^2 \left[ C_{D_{mean,i}} + C_{D_{amp,i}} \sin(\omega_i t + \phi_{y,i}) \right]$$  \hspace{1cm} (5.9)

$$F_{z_i} = F_{D_i} = \frac{1}{2} \rho u^2 \left[ C_{D_{mean,i}} + C_{D_{amp,i}} \sin(\omega_i t + \phi_{y,i}) \right]$$  \hspace{1cm} (5.10)
5.3 Numerical Simulation Procedure

The experimental study used as benchmark for the proposed reduced order method is performed by Trim et al (2005) in a tow tank. The study observed the dynamic response of a horizontal riser with an aspect ratio of 1407 undergoing vortex induced vibration in a uniform and linearly sheared velocity flow profile. The configurations of risers used in this study were bare and straked-covered risers. The set-up involved pre-tensioning the riser and placing it fully submerged in horizontal orientation parallel to the bed of a towing tank. The riser was towed with a constant velocity in the tank to achieve a uniform velocity flow profile while the linearly sheared velocity profile is achieved by fixing the riser at one end and towing at the other end in a circular path. The use of U-joint attached to a moving crane allowed the towing of the riser in the tank. The complete set up subjected the riser to respond in two degree of freedom – transverse and streamwise direction while the torsional motion constrained. Eight strain gauges and accelerometers were attached equidistant to the riser along its longitudinal axis to measure response. The geometric set up of the experiment and principal parameters are shown in Figure 5.5 and Table 5.1 respectively. For the purpose of this present simulation and validation, the experimental data for a bare pipe in uniform flow is considered.
5.3.1 Beam Sectionalisation

By the assumption that flow around the riser is in dynamics equilibrium with the riser before the onset of vortex shedding, the riser modeled as Euler-Bernoulli beam is reduced (divided) to a number of elastically mounted rigid cylinders in conformity with explanation offered in Section 5.2 and presented in Figure 5.6 for each segment.

5.3.2 Forcing Loads Characterisation

To characterise the positional forcing load on the beam, a rectangular domain of size 30D by 35D shown in Figure 5.7 is constructed to analyse the hydrodynamic forcing elastically...
Chapter 5. Model Development for Vortex-Induced Vibration of Constraint Long Horizontal Pipeline

supported rigid cylinders over a representative range of mass-damping and stiffness ratios. This domain has identical dimensions and boundary conditions to the numerical simulations presented and validated against experimental data of Khalak and Williamson (1999) in Section 4.4.3 in Chapter 4.

Simulations are conducted for cylinders constrained to both one-degree of freedom and two-degrees of freedom. The one-degree case allows the cylinder to move in transverse direction (y-axis in CFD and y- in FEA) while the stream-wise direction (z-axis in CFD and z- in FEA) is fixed. All the parameters are kept constant except the spring constant and the coefficient of damping which are specified for each simulation within the range calculated for the set of rigid cylinder segments. RANS turbulent model presented in Equation (3.3) and (3.4) in Chapter 3 are applied to resolve the local forcing loads on each segment of the beam. SST $k - \omega$ RANS turbulent model is utilized for transport closure scheme for the Reynolds stress tensor of the RANS model.

![Figure 5.7: Computational flow domain and grid](image)

(a) Flow domain showing Boundary conditions  (b) Chimera mesh generation

The mesh strategy applied for the grid is an overset mesh (Chimera method) with the resolution and time-step as used for simulating Khalak and Williamson (1999) in Section 4.4.4 in Chapter 4 adopted without further mesh independent study. This is because, the former was considered for a large Reynold number and high mass-damping ratio and so the parameters used for those numerical solutions are sufficient for the lower Reynolds number and mass-damping ratios for
the cylinder segments considered in this analysis. Thus, the cell counts utilised are 26880 and 408576 for the overset and background meshes respectively, time step ratio of 0.0002 and the first layer cell to cylinder diameter ratio \( (\Delta x/D) \) being 0.000662. This corresponds to a maximum courant number of 0.604 in the flow domain.

The CFD of elastically rigid cylinder segments predicts the forcing load on the cylinder segments, displacement of the cylinder segments and the phase difference between the force and displacement. The accuracy of the force components due to spanwise variation of vortex shedding which is absent in this 2-D simulation are achieved by iterating the forcing load on the cylinder using the damping effect of adjacent segments constraints presented in Equation (5.8). The procedure involves initially simulating flow over each elastically supported segment with no effect of adjacent constraint, and then correcting the forcing load obtained by applying the effect of adjacent constraints.

### 5.3.3 Finite Element Analysis

The three-dimensional axially tensioned long flexible riser (structure) is modelled as Euler-Bernoulli beam subjected to instantaneous flow-induced forces. In a one-way FSI, the forcing transverse and stream-wise loads informed from the simulation of the segments of the cylinders (from Section 5.5.3) are applied at their respective local position along the longitudinal axis of the beam as shown in Figure 5.4.

The U-joint connection at the end point of the riser in the experiment is modelled as pinned-pinned end conditions given by Equations (3.26) and (3.27) in Chapter 3.

The beam is meshed using beam element in the discretised Equation (3.19) as described in Section 3.7.1 in Chapter 3.

The simulation of the response of the beam under the forcing follows three sequential procedure which are static, modal and steady state analysis.

The static analysis is used to account for the initial stiffening or pretensioning on the beam. This is done by imposing axial tension at the end of the beam. The stiffening of the beam causes an increase of the natural frequencies of the beam and also prevents large deformation of the
beam under bending loads. The static analysis reduces the discretized equation (3.19) in Chapter 3 to equation (5.11).

\[
[K][u] = \{F\}
\]  \hspace{1cm} (5.11)

The set of eigen frequencies of the fully submerged stiffened beam is determined by solving the discretised Equation (3.19) in Chapter 3 for zero force vector and damping matrix. The eigen value problem requires \( U = \omega^2 U \). Therefore, the eigen values and frequencies of the beam are obtained by reducing the generalised discretised equation of the beam to equation (5.12).

\[
U(\omega^2[M] + [K]I) = 0
\]  \hspace{1cm} (5.12)

The response of the beam to the effect of hydrodynamic loading on it is both time-varying and steady. The longtime effect of the time-verying loads result to steady response which is the response for the long time fatigue damage of the beam depending on the number of stress cycles. Therefore, the steady state response of the beam is simulated and accounted for in the dynamic response of the beam. The damping matrix in the descritised equation is modelled using Rayeigh damping whose model has been presented in equation (3.19) in Chapter 3 in accordance to Song and Su (2017) to account for the combined effect of structural and fluid damping on the response of the beam.

Nine equidistant points (a spacing of \( L/9 \)) designated as points 1 – 9 along the longitudinal axis of the beam are selected as reference points for the observation of the response of the beam.

5.3.4 Sensitivity studies

Use of a smaller aspect ratio for each of the beam segments is expected to lead to more accurate prediction of beam response but requires a higher number of beam segments along the span. Furthermore, there is a limit to the beam division for which the predictions will remain unchanged for further division. The meshes for the optimized segments uses a beam element and are increased by 20% in succession. The numbers of beam elements used are 160, 192, 230 and 276. The sensitivity results are shown in Figure 5.8 with the optimized mesh count being 192 beam elements. The sensivity results are based on contraining the beam to excite in transverse direction only with loading informed from CFD simulation of rigid cylinder
segments at the first iteration. The sensitivity study for the number of beam segments shows that the optimized aspect ratio of the beam segment is $L = 155D$, equivalent to nine segments along the span. Further reduction of the beam into smaller segments above nine have no more than 3% difference in root mean square of displacement from successive divisions.

5.3.5 Iteration of Forcing Loads

To account for the effect of the spanwise variation of vortical structure on the forcing loads on the beam, iteration of the forcing load is carried out between the CFD simulation of rigid segments and the FEA of the beam as shown in the cut section of the ROM flowchart in Figure 5.7. The mechanical constraint of the adjacent segments presented in Equation (5.8) to achieve this iteration.

![Figure 5.8: Optimization of mesh and beam segments study](image)

![Figure 5.9: Iteration for forcing loads correction](image)
The iteration with the mechanical constraints in the form of coulomb damping gradually correct the forcing load until the upper limit of $|\xi_c| \leq 1$ is attained in equation (5.8). A value above the upper limit put excessive damping into the system and so is not valid.

5.4 MODEL VALIDATION RELATIVE TO BENCHMARK EXPERIMENT

The numerical simulation is validated against experimental result of Trim et al (2005) whose description is provided in Section 5.3. The presentation of the results follow simulation and testing at various stages ranging from 1DOF to 2DOF with and without the effect of constraint from adjacent segments. These are categorised into cases.

5.4.1 Response of Beam in Transverse Direction Only

The response of beam in the transverse direction only with and without mechanical constraint of the adjacent cylinder segments are presented. The determination of the effect of mechanical constraint is obtained from the relative displacement of the connected segments.

(a) Forcing and response of beam in transverse direction without mechanical constraint effect of adjacent segments

The local forcing loads and displacement with respect to local reduced velocity obtained from CFD simulation of free elastically rigid cylinder allowed to move in the transverse direction (y-axis) only are displayed in Figure 5.10. Without the effect of constraint from the adjacent cylinder, the forcing load is symmetric about the center of the span with $v_{rL} = 2.2$ with only half of the result shown. The local lift and drag are maximum at $v_{rL} = 1.01$ but the corresponding local displacement increases towards $v_{rL} = 2.2$. This is due to the large local stiffness towards the edge of the flexible beam i.e segment in the region of the edge has a larger stiffness and produces higher resistance to motion.
The steady state response of the beam with the loading and phase difference from Figure 5.10 applied on the beam is presented in frequency domain in Figure 5.11.
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Figure 5.11: Frequency response at 9 equidistant points on the beam without effect of mechanical constraint

By Parseval’s theorem shown by Equation 5.14, the rms transverse displacement are determined from the frequency domains. Figure 5.12 compares the rms value of the transverse displacement with Trim et al. (2005).

\[ y_{rms} = \sqrt{\frac{SR/2}{2} \sum_{f=0}^{SR/2} |Y(f)|^2} \]  \hspace{1cm} (5.14)

Where \(|Y(f)|\) is the modulus of the complex components of the Fast Fourier Transform at each spectral line and SR is the sample rate.

Figure 5.12: Transverse displacement of beam with loading informed from free cylinder segment without adjacent constraint effect

The result of Figure 5.12 shows symmetry about the the centre of the beam when the phase differences in Equation (5.9) and shown in Figure 5.9 (c) are applied in the loading. With the
inclusion of phase differences, response is non-symmetry about the centre of the span. The large dispersity in the prediction with experimental result can be attributed to non-introduction of the mechanical constraint of the adjacent beam and restricting the beam to transverse direction only.

(b) Forcing and response of beam in transverse only with mechanical constraint of adjacent cylinder segments

The mechanical constraint on each cylinder segment with respect to adjacent segments in the form of coulumb damping is shown in Figure 5.13. These are obtained from equation (5.8) with the use of the relative displacement of the standalone segments shown in Figure 5.10 (a)

![Figure 5.13: Coulomb damping on cylinder segment (0 ≤ \( \frac{x}{L} \) ≤ \( \frac{1}{2} \), \( \frac{1}{2} \) ≤ \( \frac{x}{L} \) ≤ 1)](image)

Figure 5.13: Coulomb damping on cylinder segment (0 ≤ \( \frac{x}{L} \) ≤ \( \frac{1}{2} \), \( \frac{1}{2} \) ≤ \( \frac{x}{L} \) ≤ 1)

Figure 5.14 (a) and (b) show respectively the local displacement and hydrodynamic forcing on cylinder segments with the effect of adjacent beam segments’ constraints included in the model in the form of coulumb damping of Figure 5.13 (via Equation 5.8). Unlike Section 5.4.1 (a), the response of, and hence local forcing on each segment is non-symmetric about the span with \( v_{rL} = 2.2 \). The phase difference between the local forcing and the displacement, and the frequency of response of each segment obtained from CFD for the distribution of forcing loads for FEA are equally presented in Figure 5.14 (c).
Chapter 5. Model Development for Vortex-Induced Vibration of Constraint Long Horizontal Pipeline

Figure 5.14: input parameters from CDF simulation of rigid segments
The steady state displacement response in frequency domain at 9 equidistant points along the longitudinal axis of the beam under the application of the forcing in Figure 5.14 is provided in Figure 5.15.

By the application of Parseval’s theorem of equation (5.14), the root mean square of the displacement in the transverse direction from the frequency domain of response is shown in Figure 5.16 along the longitudinal axis of the beam.
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The maximum rms displacement of the beam occurs at 0.87L of the beam similar to the result of the experiment. The relative improvement in the prediction of the rms displacement of the beam with respect to Figure 5.12 is due to the forcing load applied on the beam which has taken into consideration the mechanical constraint of adjacent segments. However, response is still over-predicted due to constraint of beam to transverse direction only.

5.4.2 Response of Beam in Transverse and Streamwise Direction

The response of beam with motion allowed in transverse and streamwise directions with and without mechanical constraint of the adjacent cylinder segments are presented in this section and compare with experimental results in succession.

(a) Forcing and response of beam in transverse and streamwise direction without mechanical constraint of effect of adjacent segments

The forcing coefficients and phase difference obtained from CFD simulation of free segments of beam (rigid cylinder) with no mechanical constraint are shown in Figure 5.17 for transverse and streamwise directions. Without mechanical constraint from adjacent segments of the beam, the local response and forcing on cylinder segments are symmetric about the centre of the beam with \( v_{rL} = 2.2 \). Therefore only half the result is provided.
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Figure 5.17: Local displacement of beam segments against local reduced velocity

(a) Local displacement of beam segments against local reduced velocity

(i) Streamwise displacement vs $v_{rL}$

(ii) Transverse displacement vs $v_{rL}$

(b) Local forcing load on beam segments against local reduced velocity

(i) Streamwise force coefficient vs $v_{rL}$

(ii) Transverse force coefficient vs $v_{rL}$

(c) Frequency and phase difference on beam segments against local reduced velocity

(i) Local frequency ratio vs $v_{rL}$

(ii) Phase difference vs $v_{rL}$

Figure 5.17: Local displacement of beam segments against local reduced velocity
Figure 5.18 shows the response of the beam in frequency domain at 9 equidistant points apart under the forcing load and phase difference of Figure 5.17 via Equations (5.9) and (5.10).

The corresponding root mean square of the response from frequency domain is presented in Figure 5.19 through the application of Parseval’s theorem on the frequency responses.
Due to symmetric loading about the centre of the beam, the response for zero phase difference is also symmetry about the centre of the beam with \( v_{rl} = 2.2 \). However, a slight difference occurs when the phase difference of the forcing load is accounted for. In comparison with experimental benchmark, the response is poorly predicted.

Similar to Figure 5.12, the lack of implementation of coulumb damping due to the constraint from adjacent segments in the evaluation of the forcing lead to over-prediction of the response of the beam.

(b) Forcing and response of beam in transverse and streamwise direction with mechanical constraint of effect of adjacent segments

The mechanical constraint in the form of coulumb damping on each cylinder segment due to relative displacement of adjacent segments (shown by Figure 5.17 (a)) using equation (5.9) is presented in Figure 5.20 for both transverse and streamwise direction.
The corrected streamwise loads, transverse loads and phase difference between the local forcing and displacement from CFD simulation of each cylinder segments incorporating the additional damping of Figure 5.20 are provided in figure 5.21, 5.22 and 5.23. Further iteration exceed the limit $|\xi_c| \leq 1$. Therefore no further iteration is required.
Chapter 5. Model Development for Vortex-Induced Vibration of Constraint Long Horizontal Pipeline

Figure 5.21: streamwise displacement and local forcing on beam segments

(a) Local transverse displacement vs $v_{rl}$ for range $0 \leq \frac{x}{L} \leq \frac{1}{2}$
(b) Local transverse displacement vs $v_{rl}$ for range $\frac{1}{2} \leq \frac{x}{L} \leq 1$

(c) Local streamwise force vs $v_{rl}$ for range $0 \leq \frac{x}{L} \leq \frac{1}{2}$
(d) Local streamwise force vs $v_{rl}$ for range $\frac{1}{2} \leq \frac{x}{L} \leq 1$

Figure 5.22: transverse displacement and local force on beam segments

(a) Local transverse force vs $v_{rl}$ for range $0 \leq \frac{x}{L} \leq \frac{1}{2}$
(b) Local transverse force vs $v_{rl}$ for range $\frac{1}{2} \leq \frac{x}{L} \leq 1$
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The steady state response of the beam with loading from the streamwise force coefficient in Figure 5.21, transverse force coefficients in Figure 5.22 and phase and frequency response in Figure 5.23 are presented in Figure 5.24 in frequency. Frequency responses at 9 points from 11 equidistant points along the beam are shown.
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Figure 5.24: Frequency response at along the longitudinal axis of the beam (––– Y/D, ——— Z/D)

The corresponding root mean square value of displacement are shown on Figure 5.25. the rms transverse displacement compares with less than 8% maximum error in the tranverse direction. However, in the streamwise direction, at about 0.86L along the longitudinal axis of the beam, a maximum error of 13% occurs.
5.5 Comparison of ROM to Alternative Predictive Models

The predicted rms displacement along span of the beam is compared with two published models: a wake oscillator model and CFD model as shown in Figure 5.25.

In the wake oscillator, Xu et al (2008) applied the same experimental data of Trim et al (2005) using the application of Van der Pol’s non-linear equation to model the wake and coupled with beam equation of the flexible cylinder. For the 2-way FSI simulation of Holmes et al (2006), highly coarse moving meshes were used in the 3-D flow domain and coupled with beam equation. The flow was solved with $k - \varepsilon$ RANS turbulent model with wall function applied for the velocity gradient at the wall of the flexible cylinder.

The result of Holmes et al (2006) deviates so much from the established experimental result at $x = 0.36L$ and $0.635L$. This is due to low quality of mesh usage especially at the wall of the beam. The wake oscillator and coarsed mesh-based CFD show symmetry about the mid section of the beam. This deviates from the effect of spanwise variation of vortices along the longitudinal axis of a flexible beam. By the influence of spanwise variation of vortex shedding, displacement of a flexible pipe varies along the spanwise direction. This present ROM uses the mechanical constraint of the each segments to determine the spanwise variation of response of the flexible beam.
In terms of accuracy, the present ROM is about 18.6% more accurate than the wake oscillator of Xu et al (2008) at the point of maximum error i.e. at \( x = 0.2L \) and \( 0.5L \).

5.6 **EVALUATION FOR PIPELINES OF DIFFERING ASPECT RATIO**

To further validate and ascertain the applicability of the developed ROM, the methodology is applied to experimental work of Song J (2011) and Lehn (2003) whose data are presented in Table 5.2. Each experiment is for a different aspect ratio to the pipeline with \( L = 1407 \) D considered in the previous section. For the experiment of Lehn (2003), comparison is also drawn to numerical models developed by Huang et al. (2011) and Lin & Wang (2019).
Table 5.2: Benchmark experimental data

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<tr>
<td>Outer diameter (m)</td>
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<td>0.02</td>
</tr>
<tr>
<td>Inner diameter (m)</td>
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<td>0.0191</td>
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<tr>
<td>Aspect ratio</td>
<td>L/D</td>
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<td>482</td>
</tr>
<tr>
<td>Axial Pre-tension (N)</td>
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<td>817</td>
</tr>
<tr>
<td>Structural damping (%)</td>
<td>ξ</td>
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</tr>
<tr>
<td>Young modulus (N/m²)</td>
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<td>1.025x10¹¹</td>
</tr>
<tr>
<td>Bending stiffness (Nm²)</td>
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<td></td>
</tr>
<tr>
<td>Mass ratio</td>
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<td>2.23</td>
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<tr>
<td>Material density (kg/m³)</td>
<td>ρ</td>
<td>7930</td>
<td></td>
</tr>
</tbody>
</table>

In these test cases, the lengths of the beams are reduced to eleven and three elastically supported rigid cylinder for Song et al (2011) and Lehn (2003) respectively. This is based on optimized division of L = 155D equal division along the longitudinal axis of the beam presented in section 5.3.4. The equivalent local stiffnesses according to Equation 5.3 at the centroid of each rigid cylinder segment for the two experimental cases are presented in Appendix C.

The transverse and streamwise force coefficients and phase differences between force and response are determined by CFD simulation of flow over each elastically-supported rigid cylinder along the span. The computational domain and cell counts shown in Figure 5.7 are utilized without any further sensitivity studies. This is because the domain and Reynolds number are large enough to accommodate the Reynolds numbers of the present cases.
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To ascertain the transverse and span-wise response of the riser under the influence of the uniform flow, the model of Section 5.4.2 is used. The force coefficients, frequency and phase difference are determined in the same manner.

The present ROM predicts the room mean square of transverse displacement of the experimental result of Song et al to an accuracy of 5.8% error at x = 0.2L while other position along the beam are within 5% error as shown in Figure 5.27.

Comparing to the experimental result of Lehn et al., the present ROM over predicts the rms displacement in the transverse direction by 6.4 % at the midpoint of the beam as shown in Figure 5.28 (a). Predictions at other position along the beam are within 5% along the span. In comparison with other methods, ROM is 18.3% and 26.2% more accurate in the prediction of rms displacement in the transverse and streamwise direction respectively than Strip Discrete Vortex Method coupled with FEM (SDVM-FEM).

Figure 5.27: Response of beam at Re = 11429

Figure 5.28: Root mean square of displacement along the longitudinal direction of a beam at Re = 10,000
5.7 SUMMARY AND CONCLUSION

A numerical technique has been developed to solve vortex induced vibration of flexible pipe with high aspect ratio in uniform flow. The major feature of this technique is reducing a multi-mode flexible pipe in uniform flow into a number of connected single-mode elastically supported 2-Dimensional rigid cylinders in uniform flow using theory of beam bending. The optimization study of the number of division shows that, for aspect ratio in excess of 500, the local aspect ratio of the piecewhile rigid cylinder required is \( L < 155D \). The technique applies the forcing loads informed from CFD simulation of the elastically mounted rigid cylinders in FEA simulation of the flexible pipe modelled as Euler-Bernouli beam. This method offers the advantage of computational viability which is a paramount challenge in 2-way FSI method of addressing VIV of flexible pipe with aspect ratio in excess of 1000.

Several steps have been taken to develop a final acceptable technique, with the prediction accuracy of each case evaluated first against the along-span variation of response obtained from the experimental study of Trim et al. (2005). It is shown that accurate prediction of response along the entire span can be obtained with a reduced order model in which: the mechanical constraint applied to each segment of the pipeline by adjacent rigid cylinders is modelled as a Coulomb damping, hydrodynamic force coefficients, and the phase between excitation force and cylinder motion, are obtained from 2D CFD simulations of flow over each elastically supported cylinder segment. The advantage this method offers is the low computational cost which is a big issue with full-scale 2-way FSI of the beam and flow in 3-D.

To establish the performance and predictive accuracy of this method, three experimental test cases with aspect ratio of 1407 (trim et al.; 2005), 1750 (Song et al 2011) and 482 (Lehn et al, 2003) have been used as benchmarks. The results show that the present ROM match with the established experimental results with prediction of rms displacement along the span in the transverse direction for \( L/D = 1407 \) to an accuracy of 8% maximum error at \( x = 0.88L \). Other positions on the span falls within 5%. In the streamwise direction, the rms displacement is predicted to within 5 % along the span except at \( x = 0.9L \) where prediction has 13% error.

The accuracy of the ROM is considerably higher than previous methods evaluated against the same experimental data such as wake oscillators, coupled strip-FEM method and CFD (based on coarse meshes). The wake oscillator model fails to predict the along-span variation of form,
this is captured to an extent by the strip-based FEM and CFD methods but neither provide accurate prediction of the magnitude. The performance of the current ROM can be attributed to the inclusion of Coulomb damping to model the constraint between adjacent segments along the span, and the phase-difference between forcing and response; accuracy is reduced, and predictions limited to symmetric along the span, if these terms are excluded from the model. Although CFD simulations are required, these are not coupled with the reduced order model in 2-way FSI and so the computational cost of the response predictions remains low. This method is thus considered to produce good results that balance between accuracy and computational viability in simulating VIV of flexible pipe with large aspect ratio in a uniform flow.
CHAPTER 6
APPLICATION OF REDUCED ORDER METHOD (ROM) TO FREE SPANNING SUBSEA PIPELINE

6.1 PRELIMINARY REMARKS

In this chapter, the established and validated Reduced Order Model (ROM) presented in chapter 5 is applied to study the dynamic response and fatigue of free spanning pipeline in uniform flow. Analysis is presented for a cylinder in unconstrained flow, as in the previous chapter, and for a cylinder close to the seabed. The effect of seabed proximity on the dynamic response, stress distributions and fatigue damage are assessed in the subcritical flow regime. To ascertain the value of the Reduced Order Model for practical design, and also establish the level of conservativeness in the widely used industrial semi-empirical models, the results are compared with DNV-GL-F105 guidelines that are widely used in industry for analysing the response and fatigue of free spanning subsea pipelines. An established experimental result of Trim et al (2005) is used as a benchmark for comparison.

6.2 GOVERNING EQUATIONS OF VIV OF FREE SPANNING PIPELINE

A free spanning pipeline has a gap between the seabed and the pipeline and is subjected to dynamic loadings in the form of VIV due to flow across the gap. Unlike riser, the velocity of flow around free spanning pipeline in a deep-water environment is dominated by current as wave head diminishes with respect to water depth.
Chapter 6. Application of ROM to free spanning subsea pipeline

The equations of motion of a free spanning pipeline due to external flow are modelled as Euler-Bernoulli and have been presented in equation (3.31) and (3.32) in Chapter 3 for motion in two-dimension; in the streamwise and transverse directions. The only change to the non-linear equations are the dependency of the forcing load on an additional variable, which is the gap between the pipeline and the seabed. The force coefficients are thus characterised by the non-dimensional parameters as shown in equation (6.1)

\[ C_D, C_L = \phi \left( \frac{e}{D}, \frac{z}{D}, \frac{\gamma}{D}, \text{Re}, v/\gamma \right) \] (6.1)

### 6.3 Fatigue Damage of Free Spanning Pipeline

VIV of free spanning sections of a pipeline can lead to sudden fatigue failure in the lock-in region or a long-time fatigue damage accumulation outside the lock-in range. The fatigue damage accumulation of VIV of free spanning pipeline can be assessed by Palmgren-Miner’s summation rule shown by equation (6.2)

\[ D = \sum_{i=1}^{N} \frac{n_i}{N_f} = \frac{1}{a} \sum_{i=1}^{N} n_i (\Delta \sigma_i)^m \] (6.2)

Where a and m are constants obtainable on the S-N curve of the pipeline material, \( n \) is the number of external stress cycle, and \( N_f \) is the number of stress cycle to failure, and \( D \) is the accumulated fatigue damage.

In Section 4 of Chapter 4, it has been established that a free cylinder close to a plane wall in flow has a non-zero mean lift. Therefore, the transverse stress distribution along the longitudinal axis of a free spanning pipeline cycles around a non-zero mean value. The effect of the non-zero mean stress on the stress amplitude correction in equation (6.2) through different models is shown in Figure 6.1 and mathematically provided by Equations (6.3) to (6.6).

![Figure 6.1: Mean stress effect correction on fatigue prediction of metal (Zhu et al, 2017)](image-url)
Goodman model: \[ \sigma_{ef} = \frac{\sigma_a}{1 - \frac{\sigma_m}{S_{ut}}} \] (6.3)

Gerber model: \[ \sigma_{ef} = \frac{\sigma_a}{1 - \frac{\sigma_m}{S_{ut}}}^2 \] (6.4)

Morrow model: \[ \sigma_{ef} = \frac{\sigma_a}{1 - \frac{\sigma_m}{S_f}} \] (6.5)

Solderberg model: \[ \sigma_{ef} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_y}} \] (6.6)

The ultimate strength of steel is related to its yield strength by equation (6.7) (Clifton Steel; 2021)

\[ S_{ut} = \sigma_y \left[ 1 + 2 \left( \frac{150}{\sigma_y} \right)^{2.5} \right] \] (6.7)

### 6.4 DNV-GL-F105 ASSESSMENT OF FREE SPANNING PIPELINE FOR DYNAMIC RESPONSE AND FATIGUE DAMAGE

DNV assesses the characteristic of free span and fatigue damage using RP-F105. Free span outside the acceptable range undergoes seabed intervention to reduce or eliminate the gap between the pipeline and the seabed.

In the analysis of free span under DNV regulation, the first task is to identify the classification of the response of the free span based on the aspect ratio as presented in Table 1.1 in Chapter 1. The flow chart for the analysis of VIV of free spanning pipeline using DNV-GL-F105 is shown in Figure 6.2
6.4.1 Screening Criterion

The screening checks use natural frequency in streamwise (or inline) and transverse (cross flow) as the bases for acceptance or rejection of free span. The check must satisfy the condition of both inline and cross flow criteria for acceptability.

The inline and cross flow span screening criteria are given by equation (6.8) and (6.9) respectively.

\[
\frac{f_{n,IL}}{\gamma_{IL}} > \frac{U_{c,100-year}}{V_{R,\text{onset}}^{IL}} \left( 1 - \frac{L}{D} \right) \frac{1}{\alpha}
\]

\[
\frac{f_{n,CF}}{\gamma_{CF}} > \frac{U_{c,100-year} + U_{w,1-year}}{V_{R,\text{onset}}^{CF}} \cdot D
\]

Where \(\gamma_{IL}\) and \(\gamma_{CF}\) are the screening factor for inline and cross flow respectively whose values are 1.4 each, \(L\) is the span length, \(V_{R,\text{onset}}^{CF}\) is the cross flow onset reduced velocity, \(V_{R,\text{onset}}^{IL}\) is
the inline flow onset reduced velocity, $U_{c,100\text{-year}}$ is the 100 year return period current flow at pipe level, $U_{w,1\text{-year}}$ is the one year return wave, and $\alpha$ is the current flow ratio defined as:

$$\alpha = \frac{U_c}{U_c + U_w} \quad (6.10)$$

In a deep-water environment where flow is dominated by current, $\alpha = 1$.

The onset values of the reduced velocities in the inline/streamwise and crossflow/transverse direction are related to the gap ratio of the span, and are defined as:

$$V_{CF,k\_onset} = \frac{3 \varphi_{proxi\_onset} \varphi_{trench\_onset}}{\gamma_{CF}} \quad (6.11)$$

$$V_{IL,k\_onset} = \begin{cases} 
1 & for \ K_s < 0.4 \\
\frac{0.6 + K_s}{\gamma_{IL}} & for \ 0.4 \leq K_s \leq 1.6 \\
\frac{2.2}{\gamma_{IL}} & for \ K_s \geq 1.6 
\end{cases} \quad (6.12)$$

Where $\varphi_{proxi\_onset}$ is the correction factor due to seabed proximity, $\varphi_{trench\_onset}$ correcton factor due to trench depth, $K_{sd}$ is the stability parameter with safety factor defined as $K_{sd} = K_s/\gamma_{CF}$.

Equation 6.13 and 6.14 presents the expression for the correction factors.

$$\varphi_{proxi\_onset} = \begin{cases} 
\frac{1}{5} \left(1 + 1.25 \frac{e}{D}\right) & for \ \frac{e}{D} < 0.8 \\
1 & else 
\end{cases} \quad (6.13)$$

$$\varphi_{trench\_onset} = 1 + 0.5 \frac{\Delta}{D} \quad (6.14)$$

Where $\Delta$ is the trench depth

### 6.4.2 Fatigue Criterion

Free span with natural frequency outside the acceptable range of equation (6.8) and (6.9) undergoes fatigue screening to determine the amount of fatigue damage that would be
accumulated on the pipeline and the time it takes to fracture. The fatigue check uses Palmgren-Miner’s rule of equation 6.2.

In the application of equation (6.2) for fatigue analyses, the streamwise and transverse stress range are determined with the use of equation (6.15) and (6.16) respectively.

$$S_{ill,j}^p(x) = 2A_{ll,j}(x)\left(\frac{A_L}{D}\right)_j \psi_{a,ill}y_s$$  \hspace{1cm} (6.15)

$$S_{ill}^{max} = \max_{1\leq j\leq n}(S_{ill,j}^p)$$  \hspace{1cm} (6.16)

In the DNV guideline, the safety factors in Table 6.1 are applied as a measure against uncertainties that could not be captured by the semi-empirical model.

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<thead>
<tr>
<th>Safety factor</th>
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</tr>
<tr>
<td>$\gamma_{on,CF}$</td>
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</tr>
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</table>

### 6.5 Assessment of Reduced Order Method (ROM) for Design

The present ROM is first applied to the same geometric and mechanical features as the pipeline considered in Trim et al (2005) presented in Section 5.1 of Chapter 5 to assess stress distributions and fatigue damage on the pipeline. The assessment is then extended for the case of the same pipeline in proximity to a flat seabed. The gap ratios (e/D) considered for this analysis are e = 0.75D, 1D, 2D and infinity. The infinity here implies that the plane wall is far
away from the cylinder such that the velocity profile developed near the wall has negligible effect on the flow around the cylinder. Although, in practice, the gap within the span varies due to irregularities of the seabed, this study assumes a constant gap ratio within the span as shown in Figure 6.3. The pipeline is considered to be smooth with no discontinuities on its surface to prevent stress concentration at a local region that would otherwise affect the fatigue damage prediction. The surface of the seabed is considered to be smooth, and thus can be modelled as a plane wall by neglecting roughness factor effect.

![Figure 6.3: Free span with uniform gap ratio and smooth seabed](image)

The Reynolds number of interest of the uniform flow around the span is in sub-critical Reynolds number spanning from 5000 to 12700.

For the purpose of validating the fatigue damage with experimental measurement, the S-N fatigue properties of the pipeline material uses the same properties as Trim et al (2005) which is the D-curve of NORSOK standard (1998) presented in Appendix G. This corresponds to $\log a = 11.687$ and $m = 3$ in equation (6.2)

### 6.5.1 Pipeline Discretisation

In conformity with the validated ROM in Section 5.4.2 of Chapter 5, the flexible cylinder (pipeline) is reduced to nine equally spaced segments with an aspect ratio of 155D each in uniform flow. The segments are supported elastically as shown in Figure 6.4 (one of the segments) by applying the principle of beam bending to determine the equivalent spring stiffness of each segment of the span as presented in Equation 5.3 of Chapter 5. The values of the local spring stiffness used herein for each segment are listed in Appendix C1.1.
6.5.2 Computational Domain and Boundary Conditions

To characterise the forcing loads on flexible cylinder, a computational domain of size 35D by 15.5D+e is created for each gap ratio and each elastically supported rigid cylinder as shown in Figure 6.5. The gap e between the cylinder and the plane wall is varied from 0.75D, D and 2D corresponding to the gap ratios of interest. The centre of the cylinder is located at 25D from the outlet to ensure the wake develops fully and to prevent back pressure flow. Symmetry boundary condition is assigned to the top horizontal length of the domain while the bottom horizontal length of the domain has no slip condition. This allows velocity gradient to develop on the bottom plane wall outwards towards the cylinder. A no slip boundary condition is specified on the lower wall of the domain and on the cylinder. The cylinder is allowed to move in two degrees of freedom (streamwise and transverse direction) while torsion is constrained. All the parameters are kept constant except the spring constant and the coefficient of damping which determine variation in local position of the rigid cylinder segments. The analysis of flow over each elastically rigid cylinder is carried out as a standalone simulation, with the initial simulation having no mechanical constraint to, and from, the adjacent segments. The characterised forcing loads are then updated and corrected successively by the introduction of mechanical constraint of adjacent segments using the coulomb damping of Equation (5.8) in Chapter 5 based on the calculated response of each segment.
Chapter 6. Application of ROM to free spanning subsea pipeline

The CFD parameters are as used in Section 5.3.2 in Chapter 5. Incompressible Navier-Stokes equations with SST \( k - \omega \) RANS turbulence closure model used is applied to resolve the uniform flow around the cylinder. This is because SST \( k - \omega \) has been widely used to give good prediction of adverse pressure gradient flows (Zhao and Cheng, 2011) which is typical of flow over cylinder near wall. The mesh size and time step for the gap ratio cases use the set of criteria in Table 4.5 of Chapter 4 which have been validated without further mesh resolution.

6.5.3 Finite Element Analysis

Commercial Finite Element Analysis software ABAQUS is used in the simulation of the pipeline subjected to instantaneous forcing load. The free spanning pipeline is modelled as Euler-Bernoulli beam with loading informed from CFD simulation of sectional segment of the beam distributed at their respective positions. Figure 5.4 in Chapter 5 depicts the distribution of the loading along the longitudinal direction of the beam. Mesh generation follows the same procedure as Section 5.3.3 in Chapter 5. However, input forcing differed in that the instantaneous loading distributed on the beam is, for the cases in this chapter, dependent on the gap between the pipeline and the seabed. 9 equidistant positions along the longitudinal direction of the beam are selected as the points of observation of beam response.
6.6 RESPONSE AND FATIGUE PREDICTIONS USING ROM

In the presentation of the results, the flow velocity is normalized using the first global natural frequency of the beam as shown by equation 6.17. The set of eigen frequencies of the beam are presented in Figure C1 in Appendix C. This corresponds to reduced velocity range $4 \leq v_r \leq 10$ for the Reynolds number of interest $(5000 \leq Re \leq 12700)$. Also, the frequency ratio is the ratio of response frequency to the first natural frequency of the beam.

$$v_r = \frac{U_\infty}{f_{n_1}D} \quad (6.17)$$

To compare the response of the free span with DNVGL-RP-F105 guideline, some variables are neglected in the DNV analysis to conform to the experimental benchmark of Trim et al (2005) whose data are present whose data are presented in Table 5.1 in Chapter 5. These include eliminating wave quantities in equations (6.9) and (6.10) to conform to deep water environment where velocity is dominated by the current $(i.e. U_w = U_{w,1-year} = 0)$. Also, in equations (6.8) and (6.9), the initial and return values of the current velocity are constant and no statistical model is required for it $(i.e. U_{c,1-year} = U_{c,100-year} = u_\infty)$. The shoulder of the seabed where the free span is resting is considered rigid and modelled as pin-pined end condition. Therefore, soil stiffness at the shoulder is neglected.

6.6.1 Displacement of Pipeline

The frequency response and root mean square of transverse and streamwise displacement for the case involving $e = \infty$ and $v_r = 10$ had been studied and used to validate the ROM in Section 5.4 of Chapter 5. For $e = 0.75$ and $2D$ at $v_r = 4$ and 10 respectively, the frequency responses of displacement at three sample points $(x = 15.2L, 19L$ and $22.8L)$ on the beam are shown in Figure 6.6. Other frequency domains of the response are presented in Appendix D.
Figure 6.7 and 6.8 to show the effect of proximity to the seabed on the mean and time-varying displacement along the span of the pipeline.

(a) Transverse displacement

In the transverse direction, with respect to the flow speed, the fluctuating component of the displacement increases as the flow speed increases (i.e. as \( v_r \) increases). Figure 6.7 (ai), (bi) and (ci) show that, for each of the gap ratios of interest, the rms of displacement increases as \( v_r \) increases. The same results occur for the mean component of the displacement except for \( e = \infty \) where the mean component of the displacement remains zero. This is due to generation of symmetric dynamic pressure distribution above and below the surface of the pipeline. Thus, the pipeline undergoes time-varying vortex induced vibration about zero mean displacement.

The maximum transverse displacement occurs towards the end of the pipeline at about \( x = 0.88L \) for all the gap ratios and for all reduced velocities. Contrary to static loading that
produces maximum deformation at the mid-section, the spanwise variation of vortex shedding coupled with the flexibility of the span due to large aspect ratio produces asymmetric displacement of the span section with the maximum tending towards one end of the pipeline. This pattern agrees with prior observations including by Duamu et al (2017), Wang and Xiao (2016) and Huang et al (2011).

The rms of the transverse displacement reduces with reduction to gap ratio. At $v_r = 4$, the peak rms value of the displacements are 0.056D, 0.065D, 0.135D and 0.18D for $e = 0.75D$, $D$, $2D$ and $\infty$ respectively. However, mean displacement along the span increases as the gap ratio decreases. For $v_r = 10$, the maximum mean displacement recorded are 0.013D, 0.011D, 0.0008D for $e = 0.75D$, $D$, $2D$ respectively. These peak values occur between $x = 0.8L$ to $0.83L$. Cases with $e = \infty$, has zero mean displacement for all reduced velocity ratios.

(a) Transverse displacement at $v_r = 4$

(b) Transverse displacement at $v_r = 8$
(b) Streamwise displacement

Similar to the transverse displacement, the rms and mean displacement in the streamwise direction increases as the reduced velocity increases for all the gap ratios as shown in Figure 6.8. Furthermore, the maximum streamwise displacement occurs towards the edge of the span. However, for the same reduced velocity, it is observed that a change in gap ratio has no significant effect on the mean displacement as shown in figure 6.8 (aii), (bii) and (cii). The peak mean displacement at \( v_r = 4, 8 \) and 10 are 0.0023D, 0.0094D and 0.013D respectively for all the gap ratios. These peak mean displacements occur between \( x = 0.8L \) and 0.83L on the span length. The rms of the displacement slightly increases as the proximity increases (Figure 6.8 (ai), (bi) and (ci)). This means that proximity of the pipeline to the seabed principally affects the fluctuating displacement of the pipeline in the streamwise direction about its constant mean position at any reduced velocity.
Chapter 6. Application of ROM to free spanning subsea pipeline

6.6.2 Strain along Pipeline Span

The concentration of stresses on the span due to mean deformation and VIV is one of the most important indicators in design analysis of the free span to ascertain their potential life span (Jhingran, 2008). The stresses along the span due to VIV are non-uniform dynamic stresses and vary along the span. These stresses are non-dimensionless to the strain using the material Poisson ratio and modulus of elasticity of the material. The strain along the beam is presented at 9 equidistant points on the beam for \( e = \infty \) at \( v_r = 10 \) are shown in Figure 6.9. By the application of Parseval’s theorem, the strain amplitude shown in Figure 6.10 are determined from the frequency domains.
Figure 6.9: Strain in frequency domain at ten equidistant points on the longitudinal axis of the beam for $v_r = 10$ and $\frac{e}{D} = \infty$ (–– Transverse, —— Streamwise)

(a) Transverse distribution of strain

For all the gap ratios, the amplitude and mean dynamic strain on the span increases as the reduced velocity increases as shown in Figure 6.10. With respect to proximity to the seabed, the amplitude of the transverse strain decreases as the gap ratio reduces. However, the mean strain increases as the proximity decreases. For all reduced velocity, the free span cycles about zero mean strain for cases with $e = \infty$ as shown in Figure 6.10 (aii), (bii) and (cii).

In Figure 6.10 (ai), (bi) and (ci), for all the gap ratios, the position of the maximum amplitude of strain lies in the range $0.09L < x < 0.18L$ along the span except for $e = 2D$ at $v_r = 10$, $e = \infty$ at $v_r = 10$, $e = 0.75D$ at $v_r = 4$ and $c = D$ at $v_r = 4$ where maximum amplitude occurs within
the range $0.8L < x < 0.9L$. This shows that the maximum amplitude of the strain occurs towards a constrained end of the span for all the gap ratios and reduced velocities. Therefore, in the event of fatigue damage occurring, it takes place towards the edge of the beam. This prediction is in agreement with observations of Huang et al (2011).

Figure 6.10: Stress distribution in the transverse direction
(b) Streamwise strain distribution

Figure 6.11 shows the strain distribution along the span in the streamwise direction. The amplitude and mean strain increase with increase in the reduced velocity just as the cases with transverse direction presented in Section 6.5.2 (a). For each reduced velocity as shown in Figure 6.11 (ai), (bi) and (ci), the maximum amplitude along the span occurs at 0.091L of the span for $v_r = 4$ and 8. However, for $v_r = 10$, it lies at 0.91L. Change in proximity of the pipeline to the seabed has very little or no significant effect on the mean streamwise strain along the span, this is shown in Figure 6.11 (aii), (bii) and (cii).
6.6.3 Variation of Peak Strain with Gap Ratios

In DNV guideline, only the maximum stress amplitude is obtained as it is one of the requirements for design. The disparity in maximum strain amplitude between ROM and DNV guidelines in log-log form in the transverse and streamwise directions are shown by Figure 6.12 (a) and 6.13 (a) respectively. The result shows that, while DNV caps the effect of proximity to 1 and considers that proximity has no effect on the response of pipeline for \( e \geq 1 \), the present ROM shows effect of proximity above \( e = 1 \).

The disparity between the design standard and the ROM are quantified in Figure 6.12 (b) and 6.13 (b). In the transverse direction, at \( v_r = 4 \), DNV predicts the maximum stress amplitude 2.68 times the values predicted by the present ROM for \( e = D \). Also, at \( v_r = 8 \), the ratios of the maximum amplitude predictions are 1.79 and 1.94 for \( e = 0.75D \) and \( D \). Furthermore, at \( v_r = 10 \), the ratios are 2.33 and 1.66 for \( e = D \) and 2D respectively. Similarly, in the streamwise direction, DNV predictions of the maximum stress amplitude are 1.21 and 1.39 times more than the present ROM for \( e = 0.75D \) and \( D \) respectively at \( v_r = 4 \). At \( v_r = 8 \), it is 1.23, 1.33, 1.27 and 1.16 times more for \( e = 0.75D \), \( D \), 2D and \( \infty \) respectively. Lastly, for \( v_r = 10 \), the predictions are 1.28, 1.8, 1.3 and 1.2 times more than present ROM predictions of the amplitude on the span.

Although for uniform flow over stationary cylinder, gap between the cylinder and a plane wall has a limit of influence at \( e = 1D \) and behave like cylinder in an unbounded flow (Lei et al, 1999),
this is not the case for a free cylinder. Barbosa et al (2017) shows influence of gap ratio on free vibrating cylinder as far as $e = 4D$. Therefore, it can be inferred that, for the gap ratio under consideration, the DNV guidelines are conservative for gap ratio $e = D$ and $2D$.

![Figure 6.12](image1)

(a) Comparison of maximum strain amplitude on the span

(b) Ratio of maximum strain amplitude on the span

Figure 6.12: Maximum strain amplitude on span in the transverse direction

![Figure 6.13](image2)

(a) Comparison of maximum strain amplitude on the span

(b) Ratio of maximum strain amplitude on the span

Figure 6.13: Maximum strain amplitude on span in the transverse direction
Chapter 6. Application of ROM to free spanning subsea pipeline

(a) Peak strain along free span

The peak stress or strain is the superposition of the stress or strain amplitude onto the mean stress or strain. The peak strains in the transverse and streamwise direction are shown in Figure 6.14. For all the gap ratios, the peak stresses increase as the flow speed increases as shown in Figure 6.14 (a).

In the transverse direction shown in Figure 6.14 (a), at \( v_r = 4 \), the maximum peak strain (i.e. 4.2x10\(^{-6}\)) occurs for \( e = 0.75D \) while the minimum occurs for \( e = \infty \). However, as \( v_r \) increases, the peak strain for \( e = \infty \) increases and has a maximum value of 8.26x10\(^{-6}\) at \( v_r = 10 \) compared to other gap ratios. This is because, \( v_r = 4 \), the peak strain for \( e = 0.75D \) is dominated by the mean strain with less contribution from the strain amplitude due to the larger amplitude response that occurs near the seabed. Whereas the peak strain for \( e = \infty \) is completely dependent on the amplitude of the pipeline response, with negligible mean strain (as shown in Figure 6.10 aii, bii and cii). Also, as \( v_r \) increases, the rms of strain increases, and subsequently the strain amplitude increases.

Similar behaviour is observed for the streamwise direction. The peak strain increases as flow speed increases for all gap ratios in the streamwise direction as shown in Figure 6.14 (b). At a given value of reduced velocity, the peak strain increases as the gap ratio increases. This variation is exclusively due to the increase in strain amplitude as gap ratio increases (shown in Figure 6.10) since change in gap ratios has negligible effect on the strain distribution on the span for any reduced velocity.

![Figure 6.14: Peak strain on along the span length](image)

(a) Peak strain in the transverse direction  
(b) Peak strain in the streamwise direction
6.6.4 Fatigue damage on the span

Fatigue damage is accumulated at all sections along the beam. However, since a structure is as strong the point of maximum stress concentration (Dowling, 2012), it is sufficient to account for the damage of the structure at the critical point which is the point of highest stress concentration. The peak stresses and subsequently the peak strain on the span have been presented in Figure 6.14. However, the gap ratio with the peak strain does not translate to highest fatigue damage accumulation. The contribution of the mean strain and rms strain within the peak value and the number of stress cycles are essential for fatigue damage. The fatigue damage of the span at the critical point (towards the edge) on the span are determined using Palmgren-Miner’s summation rule of Equation (6.2) with mean stress correction factor of Goodman’s model shown in Equation 6.3.

The fatigue damage for $\epsilon/D = \infty$ in the transverse and streamwise directions are shown in Figure 6.15 and compared with experimental result of Trim et al (2005) as a benchmark for accuracy and validation. The calculation code for the fatigue damage is presented in Appendix E. Standard DNV guidelines results for fatigue damage over the range $4 \leq \nu_r \leq 15$ are also compared. With respect to the experimental result, ROM over predicts the measured fatigue damage by 4.8%, 12.4%, 6.8% and 6.2% at $\nu_r = 8, 10, 13,$ and $15$ respectively in the transverse direction. In the streamwise direction, the ROM again over-predicts, by 9.9%, 3.9%, 8% and 12.2% error at $\nu_r = 8, 10, 13$ and $15$ respectively. In contrast, application of the DNV guidelines without the incorporation of the standard factor of safety to allow direct comparison, results in significantly higher over-prediction. Predictions are 2.0, 2.1, 2.1 and 2.2 times the experimental result in the transverse direction at $\nu_r = 8, 10, 13,$ and $15$ respectively. Similarly, in the streamwise direction, DNV guideline predictions are 2.1, 1.95, 2.4 and 2.5 times at $\nu_r = 8, 10, 13$ and $15$ respectively. With the introduction of standard fatigue factor of safety (high class) shown in Table 6.1, DNV prediction increases by a factor of 2.34 and 2.48 in the transverse and streamwise direction respectively.

This analysis shows that, in general the DNVGL-RP-F105 predictions are more conservative than the present ROM with fatigue damage predictions typically a factor of two greater than experimental data, rather than over-predicted by around 4 – 12% with the present ROM.
The cause of the disparities between DNV and present ROM predictions of fatigue damage for wall spacing can be considered in two categories based on the DNV treatment of wall-proximity either side of a threshold value of wall proximity at \( e = 1 \). These categories shown in log-log plot in Figure 6.16 are; fatigue damage for \( e/D < 1 \) and \( e/D \geq 1 \).

In the transverse direction, for \( e = 0.75D \), DNV predicts fatigue damage with a factor of 3.8, 3.6 and 3 times higher than present ROM at \( v_r = 4, 8 \) and 10 respectively. A higher prediction is also recorded in the streamwise direction with a factor of 3.6, 3.1 and 3.5 for \( v_r = 4, 8 \) and 10 respectively. These are shown in Figure 6.17 (a) with the chain continuous line signifying damage ratio in the transverse direction while the thick continuous line denotes damage ratios in the streamwise direction.
Figure 6.17: Fatigue damage ratio over a range of reduced velocity
Generally, the fatigue damage in both experimental result and the predicted result are very small. This can be attributed to the initial stiffening of the pipe through the application pre-axial tension load. This causes a high cycle fatigue of the material and thus, the stress cycles close to the endurance limit of the material (i.e. the stress below which a material has infinite number of stress cycles to failure)

### 6.7 Summary and Discussions

The standard DNV calculation is found to be applicable for wall spacing e/D > 1.0. There is good agreement with the DNV method for e/D = 2 and \( v_r \geq 8 \), for the transverse strain. However, for the streamwise strain and for all other wall spacing and reduced velocities the proposed reduced order method provides a considerably lower prediction of fatigue load than the DNV standard. For e/D = infinity, fatigue load from proposed method is approximately a factor of two less than by the DNV calculation except for \( v_r = 10 \) for which the prediction is a factor of three smaller, in the streamwise direction. For wall spacing of e/D = 1, the strain is considerably lower than DNV, by a factor between 5.5 to 7.8. The DNV standard uses a modified expression for e/D < 1, and this provides closer agreement with the proposed method for e/D = 0.75 than for e/D =1, with predictions with the proposed method a factor of 3.5 to less than the design standard. This indicates that the DNV standard is conservative for wall gap ratios e/D < 2 and there is potential to modify the approach to better capture the variation of peak strain for e/D < 2 rather than only e/D < 1 as currently used in design practice.
CHAPTER 7

CONCLUSION AND SUGGESTION FOR FURTHER WORK

7.1 CONCLUSIONS

In this thesis, the study of effect of gap ratio on free spanning underwater pipeline close to the seabed have been examined with the intent of reducing conservatism in design practice. A reduced order method (ROM) has been developed for this purpose in which pipeline response is simulated using an FEA model with sectional forcing obtained from multiple CFD simulations of a 2D freely supported cylinder located near a wall and for a representative mass-damping ratio. This approach is validated relative to experiment for unconstrained flow, and shown to predict a lower fatigue load, by a factor of two to three, relative to a widely used design standard. For a cylinder close to a wall the method leads to fatigue load predictions that are between 12.5% to 25% of those obtained by the standard design method.

For uniform flow over a stationary cylinder, RANS CFD with SST $k-\omega$ turbulence model is used to predict and validate hydrodynamic forces on the cylinder and vortex shedding frequency of flow in subcritical flow regime. For a cylinder within a boundary layer close to a plane wall over Reynolds number of $1.38 \times 10^4$ and $1.31 \times 10^4$ for boundary layer thickness of 0.25D and 0.48D respectively, drag coefficient predictions are within 5.5% error for all gap ratios ($e = 0.2D, 0.5D, 0.75D, D$ and $2D$) at Reynolds number of $1.38 \times 10^4$. At a flow regime of Reynolds number of $1.31 \times 10^4$, the mean drags are predicted within a range 4.2 – 6.5% error. The predictions of the mean lift and root mean square of the lift at Reynolds number of $1.31 \times 10^4$ and $1.38 \times 10^4$ are accurate to a range of 4.3 – 6.6% error. For the stagnation point prediction on the cylinder at Reynolds number of $1.38 \times 10^4$, it is accurate to within 5% for the three gap ratios considered ($e = 0.2D, 0.75D$ and $D$).

Vortex-induced vibration of a 2D cylinder is modelled using an overset mesh method. For a cylinder far from a wall with a mass-damping ratio ($m^*\zeta$) of 0.0108 in the transverse axis only,
displacement is predicted to within 5% over most of the response range at the lower branches (3 \(< v_r < 4 \) and \( 8 < v_r < 11 \)). However, in the lock-in range (\( 4 < v_r < 8 \)), the displacement is somewhat under-predicted with a maximum error of 19.8%. Other studies also record under prediction with large error in the lock-in range. Prediction in this range is improved to within 9.8% of experiment with inclusion of the Kato-Launnder correction to the k-w turbulent model.

The same model provides similar accuracy for a cylinder freely responding in the transverse direction and close to plane wall. A maximum under prediction error of 9.9 \% is recorded at the synchronization region for the displacement of a cylinder of mass-damping ratio of 0.11 close to a wall at \( Re/v_r = 30300 \) for \( e = 0.75D, D, \) and 2D. Outside the lock-in range, the error falls within 5%.

One of the findings from this simulation is improved prediction in the lock-in region that seems to be the challenges from other studies. Kato-Launnder inclusion improves prediction in this range and thus, useful in modelling sudden and large distortion of fluid particles. It is also found that the synchronization bandwidth narrows at the upper limit as the gap ratio decreases for the same mass-damping ratio and Reynolds numbers. The recorded bandwidths are \( 5 \leq v_r \leq 8, \ 5 \leq v_r \leq 9.4 \) and \( 5 \leq v_r \leq 9.8 \) for \( e = 0.75D, D, \) and 2D respectively. This is due to development of large differential dynamic pressure at the top and bottom of the cylinder which otherwise lead to decrease (suppression) in vortex shedding frequency which tends to unlock from the natural frequency of the cylinder at a lower reduced velocity. In addition to the narrowing of the lock-in range, a positive increasing mean lift and mean displacement occur. Therefore, the vortex-induced time varying response of the cylinder occurs with a non-zero mean lift which causes a mean displacement, altering the gap ratio from the original position. This mean lift increases as the gap ratio decreases.

A predictive model, referred to as a Reduced Order Method (ROM), which leverages on the study of simulation of VIV of 2-D free cylinder in two-degree of freedom is developed. The model, involving CFD simulation of segmented sections of beam incorporated mechanical constraint between adjacent segments and phase variation to inform loading. The coupled CFD loading with FEA steady state analysis of the beam predicts fluctuating component of displacement of the beam pinned at both ends to within 5% and 13% error in transverse and streamwise direction respectively. This prediction has been established for an experimental result with Reynolds number 1.21x10^5, mass-damping ratio (m*ζ) of 0.0048 and a pipe with aspect ratio of 1407. Further validations are established for two additional experimental results,
Chapter 7. Conclusions and Suggestions for Further Work

The root mean square (rms) component of displacement is also predicted within the range 4 - 11% error along the entire span in streamwise and transverse direction. In comparison with Wake oscillator, pure CFD with wall function, and Strip based Discrete Vortex Method coupled with Finite Element Method (SDVM-FEM), the predictive model is more accurate. It is 18.6% more accurate than Wake oscillator in comparison with experimental benchmark. With respect to SDVM-FEM, the present ROM is 18.3% and 26.2% more accurate in the transverse and streamwise directions respectively for the same beam mechanical configuration and flow properties.

The ROM is applied to study the effect of wall spacing on the response and fatigue of free spanning pipeline. The ROM is first validated for fatigue for e = infinity over a range of reduced velocities to ascertain its robustness for fatigue damage using established experimental fatigue result of Trim et al (2005) as benchmark. Fatigue damage is predicted to accuracy within the range 4.8 - 12.4%. On the other hand, DNVGL-RP-F105 without standard factor of safety over-predicted fatigue damage by a factor ranging from 1.95 – 2.5 more than experimental results. Comparison of ROM and DNV guideline shows that the two model’s predictions of fatigue damage agree for e = 2D at v_r ≥ 8 in the transverse direction. However, for all other gap ratios and reduced velocities in the transverse and streamwise directions, ROM predicts lower fatigue damage. Quantitatively, ROM prediction of fatigue damage is approximately by a factor ranging from 2 – 3 times less than DNV guideline for e = ∞. The highest disparity occurs for e/D = 1 where fatigue damage prediction ranges from 5.5 to 7.8 times less than DNV calculation. The DNV standard uses the same expression for e/D ≥ 1 and a modified expression for e/D < 1. Thus, reason for closer agreement with the proposed method for e/D = 0.75 than for e/D = 1. This indicates that the DNV standard is conservative for wall gap ratios in the range 1 < e/D < 2, and there is potential to modify the approach to better capture the wall spacing effect on response of free spanning pipeline for e/D < 2 rather than only e/D < 1 as currently used.

The reduced order model developed in this study has been shown to provide response and fatigue load prediction to reasonable accuracy for a benchmark experimental case, and to higher accuracy than typical design practice. This is achieved at low computational cost providing a useful approach for assessing the magnitude and location of peak strain due to VIV of flexible pipes with large aspect ratio. The model is informed by 2D CFD which has been used to
investigate the impact of bed proximity on pipeline fatigue design. Constant bed proximity and mass-damping has been considered but the approach could be extended to account for varying bed separation, sectional changes to mechanical properties and potentially to assess vortex induced vibration mitigation measures. The approach, and developments of this type, would further inform industrial practice to reduce conservativism in the design of such critical offshore infrastructure.

## 7.2 Recommendation for Further Works

In this present analysis, several assumptions have been made which should be addressed through further analysis. These are briefly discussed.

This analysis which involves sectionaising a 3-D beam into series of cylinder segments. Then modelling flow over the elastically supported cylinder segments iteratively in 2-D to obtain the forcing load has only been verified and validated in over a range $5000 \leq Re \leq 12700$ in the sub-critical flow regime. More analysis for a wider flow regime is further required to fully ascertain the applicability of the model over all flow regimes.

Vortex induced vibration of free spanning pipeline close to the seabed has many uncertainties which are yet to fully explored in this analysis. This thesis has only studied cases where the pipeline is closed to the seabed but not close enough to have impact on the seabed. The seabed is also modelled as a plane wall in this study. The lower the proximity of the pipeline to the seabed, the more the potential of the free span having impact on the seabed. During impact, the segment of the free span is prone to crack. Therefore S-N curve is not ideal in assessing fatigue damage under this condition but Fracture Mechanics due to discontinuity in the structure of the pipeline (crack) which accelerates fatigue damage. Fracture mechanics is applicable to analyse the crack initiation, propagation and the potential fatigue failure in such situation.

Therefore, one of the recommendations is the study of effect of seabed impact on the fatigue damage of free spanning pipeline. The seabed textures are in varieties such as clay, silt. Impact study on the VIV by varying the mechanical structure of the seabed is also a very important study in deep water environment.
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APPENDIX A

TIME HISTORY OF FLOW OVER STATIONARY CYLINDER NEAR WALL

A1 Flow Regime Re = 1.38 x 10^4 and δ = 0.25D

Figure A.1: Time history of forces on cylinder in uniform flow near wall at Re = 1.38 x 10^4, δ = 0.25D (C_D, C_L)
Appendix A. Time history of flow over stationary cylinder near wall

Figure A.2: Fast Fourier Transform (FFT) of cylinder in uniform flow near wall at $Re = 1.38 \times 10^4$, $\delta = 0.25D$

**A2  Flow Regime $Re = 1.31 \times 10^4$ and $\delta = 0.48D$**

Figure A.3: Time history of forces on cylinder in uniform flow near wall at $Re = 1.31 \times 10^4$, $\delta = 0.48D$ ($-- C_D$, $-- C_L$)
Figure A.4: Fast Fourier Transform (FFT) of cylinder in uniform flow near wall at \( \text{Re} = 1.31 \times 10^4, \delta = 0.48D \)
APPENDIX B

USER DEFINED FUNCTION AND TIME HISTORY OF FLOW OVER ELASTICALLY SUPPORTED RIGID CYLINDER

B1. USER DEFINED FUNCTION FOR FLOW OVER ELASTICALLY MOUNTED RIGID CYLINDER IN UNIFORM FLOW

B1.1 C++ code for single degree of freedom of cylinder in uniform flow

```c
#include "udf.h"

#define spring_constant 35.06764
#define original_cylinder_cg_y 0
#define zeta 0.0045
#define mass 0.371016
DEFINE_SDOF_PROPERTIES(rigid_5_udf, PROP, dt, time, dtime)
{
  real cylinder_movement_y;
  real damping_coeff;
  real net_force_y;
  real spring_force_y;
  real damping_force_y;

  PROP[SDOF_MASS] = mass;
  PROP[SDOF_IDX] =0.0;
  PROP[SDOF_IDY] = 0.0;
  PROP[SDOF_IDZ] = 0.0;
  PROP[SDOF_ZERO_TRANS_X] = TRUE;
  PROP[SDOF_ZERO_TRANS_Y] = FALSE;
  PROP[SDOF_ZERO_TRANS_Z] = TRUE;
  PROP[SDOF_ZERO_ROT_X] = TRUE;
  PROP[SDOF_ZERO_ROT_Y] = TRUE;
  PROP[SDOF_ZERO_ROT_Z] = TRUE;

  /** CHANGE IN VERTICAL POSITION OF THE CENTRE OF THE CYLINDER FROM EQUILIBRIUM POSITION
  cylinder_movement_y = (DT_CG(dt)[1] - original_cylinder_cg_y);```
Appendix B. User defined function and time history of flow over elastically supported rigid cylinder

/** CALCULATION OF DAMPING COEFFICIENT
   damping_coeff = 2*zeta*sqrt(mass*spring_constant);

/** CALCULATION OF SPRING FORCE AT THE PRESENT POSITION OF CYLINDER CENTRE
   spring_force_y = -1*spring_constant*cylinder_movement_y;

/** COMPUTING THE DAMPING FORCE AT THE PRESENT POSITION OF CYLINDER
   damping_force_y = -1*damping_coeff*DT_VEL_CG(dt)[1];

/** COMPUTING THE TOTAL FORCES COUNTERBALANCING THE LIFT FORCE DUE TO VORTEX SHEDDING
   net_force_y = spring_force_y + damping_force_y;
   PROP[SDOF_LOAD_F_Y] = net_force_y;

/** WRITING THE NEW POSITION OF CYLINDER CENTRE FOR NEXT ITERATION
   Message("\nNew Location = %g",DT_CG(dt)[1]);
   Message("\n net external force = %g", net_force_y }

B1.2 C++ code for two-degree of freedom of cylinder in uniform flow

/***************************************************************************/
/* CODE FOR HOOKING THE DAMPING AND SPRING FORCES IN FLOW OVER ELASTICALLY MOUNTED CYLINDER */
/******************************************************************************/
#include "udf.h"
#define spring_constant 62.85101
#define original_cylinder_cg_x 0
#define original_cylinder_cg_y 0
#define zeta 0.02
#define mass 0.797104
DEFINE_SDOF_PROPERTIES(rigid_udf, PROP, dt, time, dtime)
{
   real cylinder_movement_x;
   real cylinder_movement_y;
   real damping_coeff;
   real net_force_x;
   real net_force_y;
   real spring_force_x;
   real spring_force_y;
   real damping_force_x;
   real damping_force_y;
   PROP[SDOF_MASS] = mass;
   PROP[SDOF_IXX] =0.0;
   PROP[SDOF_IYY] = 0.0;
   PROP[SDOF_IZZ] = 0.0;
   PROP[SDOF_ZERO_TRANS_X] = FALSE;
   PROP[SDOF_ZERO_TRANS_Y] = FALSE;
   PROP[SDOF_ZERO_TRANS_Z] = TRUE;
   PROP[SDOF_ZERO_ROT_X] = TRUE;
   PROP[SDOF_ZEROROT_Y] = TRUE;
   PROP[SDOF_ZERO_ROT_Z] = TRUE;
Appendix B. User defined function and time history of flow over elastically supported rigid cylinder

```c
/** CHANGE IN POSITIONS OF THE CENTRE OF THE CYLINDER FROM EQUILIBRIUM POSITION */
cylinder_movement_x = (DT_CG(dt)[0] - original_cylinder_cg_x); /* Change in horizontal position of the centre of the cylinder from equilibrium position*/
cylinder_movement_y = (DT_CG(dt)[1] - original_cylinder_cg_y); /* Change in vertical position of the centre of the cylinder from equilibrium position*/

/** CALCULATION OF DAMPING COEFFICIENT */
damping_coeff = 2*zeta*sqrt(mass*spring_constant);

/** CALCULATION OF SPRING FORCE AT THE PRESENT POSITION OF CYLINDER CENTRE */
spring_force_x = -1*spring_constant*cylinder_movement_x; /* computing the formular F = -k* delta x */
spring_force_y = -1*spring_constant*cylinder_movement_y; /* computing the formular F = -k* delta y */

/** COMPUTING THE DAMPING FORCE AT THE PRESENT POSITION OF CYLINDER */
damping_force_x = -1*damping_coeff*DT_VEL_CG(dt)[0]; /* computing the formular Fd = -c*x(dot) */
damping_force_y = -1*damping_coeff*DT_VEL_CG(dt)[1]; /* computing the formular Fd = -c*y(dot) */

/** COMPUTING THE TOTAL FORCES COUNTERBALANCING THE LIFT FORCE DUE TO VORTEX SHEDDING */
net_force_x = spring_force_x + damping_force_x;
net_force_y = spring_force_y + damping_force_y;
PROP[SDOF_LOAD_F_X] = net_force_x;
PROP[SDOF_LOAD_F_Y] = net_force_y;
```

B2. TIME HISTORY AND FREQUENCY RESPONSE OF FREE CYLINDER IN UNIFORM FLOW

B2.1 Cylinder with \( \frac{e}{D} = \infty \) and \( m*\xi = 0.00048 \)
Appendix B. User defined function and time history of flow over elastically supported rigid cylinder

(c) Displacement time history at $v_r = 6$

(d) Force time history at $v_r = 6$

(e) Displacement time history at $v_r = 8$

(f) Force time history at $v_r = 8$

(g) Displacement time history at $v_r = 9$

(g) Force time history at $v_r = 9$
Appendix B. User defined function and time history of flow over elastically supported rigid cylinder

![Displacement time history at $v_0 = 10$](image)

![Displacement time history at $v_0 = 6$](image)

![Displacement time history at $v_0 = 4$](image)

![Displacement time history at $v_0 = 3$](image)

![Displacement time history at $v_0 = 2$](image)

![Displacement time history at $v_0 = 10$](image)

Figure B.1: Time history and frequency response for $\frac{e}{D} = \infty$, $m^*\xi = 0.0048$

**B2.2 Cylinder with $\frac{e}{D} \leq 2$ and $m^*\xi = 0.11$ (near wall)**

**B2.2.1 Cylinder with $e = 0.75D$ and $m^*\xi = 0.11$**

![Displacement time history at $v_0 = 6$](image)

![Displacement time history at $v_0 = 3$](image)

![Displacement time history at $v_0 = 10$](image)

Figure B.2: Time history of transverse displacement for $e = 0.75D, m^*\xi = 0.11$
Appendix B. User defined function and time history of flow over elastically supported rigid cylinder

**B2.2.2 Cylinder with e = 1D and m^*ξ = 0.11**

(b) Displacement time history at v_r = 3

(a) Displacement time history at v_r = 4

![Graphs](image)

Figure B.3: Time history of transverse displacement for e = 0.75D, m^*ξ = 0.11
APPENDIX C

EULER-BERNOULI BEAM MODEL

C1. BEAM SECTIONALISATION

The beam sectionalisation of Euler-Bernoulli into series of elastically supported rigid cylinders presented in Section 5.3 and shown in Figure D.1 has localized stiffness along the longitudinal axis of the beam as presented in equation of Chapter 5 and equation (D.1)

\[
k(x) = l \left[ \frac{1 - \cosh \beta l}{\beta^2 T \sinh \beta l} \sinh \beta x + \frac{1}{\beta^2 T} \cosh \beta x + \frac{1}{2T} x - \frac{1}{\beta^2 T} - \frac{1}{2T} x^2 \right]^{-1}
\] (C.1)

Where \( \beta = \sqrt{\frac{T}{EI}} \)  (C.2)

The equivalent local spring stiffness of each beam segment (L=155D each) using the formula in equation (C.1) for Trim et al (2005), Song et al (2011) and Lehn (2003) are shown in Table C2, C4 and C6 respectively.
### C1.1 Pipeline in Trim et al (2005) with $L = 1400D$

Table C1: Flow and structural properties of beam

<table>
<thead>
<tr>
<th>Data</th>
<th>Symbols</th>
<th>Units</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Riser’s mechanical properties</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>$L$</td>
<td>m</td>
<td>38</td>
</tr>
<tr>
<td>Outer diameter</td>
<td>$D_o$</td>
<td>m</td>
<td>0.027</td>
</tr>
<tr>
<td>Inner diameter</td>
<td>$D_i$</td>
<td>m</td>
<td>0.021</td>
</tr>
<tr>
<td>Modulus of elasticity</td>
<td>$E$</td>
<td>Nm$^2$</td>
<td>$3.62\times10^{10}$</td>
</tr>
<tr>
<td>Mass ratio</td>
<td>$m^*$</td>
<td></td>
<td>1.6</td>
</tr>
<tr>
<td>Structural damping</td>
<td>$\zeta$</td>
<td></td>
<td>0.003</td>
</tr>
<tr>
<td>Axial pre-tension</td>
<td>$T$</td>
<td>N</td>
<td>5000</td>
</tr>
<tr>
<td>End constraint</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Fluid properties</em></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flow characteristics</td>
<td></td>
<td></td>
<td>Uniform</td>
</tr>
<tr>
<td>Current velocity</td>
<td></td>
<td></td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table C2: Equivalent local stiffness and local reduced velocity of beam segment with respect to position on the flexible beam

<table>
<thead>
<tr>
<th>Segment ID</th>
<th>$x$</th>
<th>$k_L(x)$</th>
<th>$f_L(x)$</th>
<th>$v_{rl}(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segment 1</td>
<td>2.11</td>
<td>5016.65</td>
<td>14.70</td>
<td>1.01</td>
</tr>
<tr>
<td>Segment 2</td>
<td>6.33</td>
<td>1895.17</td>
<td>9.03</td>
<td>1.6</td>
</tr>
<tr>
<td>Segment 3</td>
<td>10.56</td>
<td>1312.03</td>
<td>7.52</td>
<td>1.97</td>
</tr>
<tr>
<td>Segment 4</td>
<td>14.78</td>
<td>1107.56</td>
<td>6.91</td>
<td>2.14</td>
</tr>
<tr>
<td>Segment 5</td>
<td>19.0</td>
<td>1052.86</td>
<td>6.73</td>
<td>2.20</td>
</tr>
<tr>
<td>Segment 6</td>
<td>23.22</td>
<td>1107.56</td>
<td>6.91</td>
<td>2.14</td>
</tr>
<tr>
<td>Segment 7</td>
<td>27.44</td>
<td>1312.03</td>
<td>7.52</td>
<td>1.97</td>
</tr>
<tr>
<td>Segment 8</td>
<td>31.67</td>
<td>1895.167</td>
<td>9.03</td>
<td>1.64</td>
</tr>
<tr>
<td>Segment 9</td>
<td>35.89</td>
<td>5016.65</td>
<td>14.70</td>
<td>1.01</td>
</tr>
</tbody>
</table>
Appendix C. Beam Model

C1.2 Pipeline in Song et al (2011) with $L = 1750D$

Table C3: Flow and structural properties of beam

<table>
<thead>
<tr>
<th>Properties</th>
<th>Symbols</th>
<th>Song et al (2011)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material and mechanical properties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outer diameter (m)</td>
<td>D</td>
<td>0.016</td>
</tr>
<tr>
<td>Inner diameter (m)</td>
<td>D_i</td>
<td>0.015</td>
</tr>
<tr>
<td>Aspect ratio</td>
<td>L/D</td>
<td>1750</td>
</tr>
<tr>
<td>Axial Pre-tension (N)</td>
<td>T</td>
<td>600</td>
</tr>
<tr>
<td>Structural damping (%)</td>
<td>$\xi$</td>
<td>2</td>
</tr>
<tr>
<td>Young modulus (N/m$^2$)</td>
<td>E</td>
<td>$2.10 \times 10^{11}$</td>
</tr>
<tr>
<td>Bending stiffness (Nm$^2$)</td>
<td>EI</td>
<td>153.71</td>
</tr>
<tr>
<td>Mass ratio</td>
<td>$m^*$</td>
<td>1.0</td>
</tr>
<tr>
<td>Material density (kg/m$^3$)</td>
<td>$\rho$</td>
<td>7930</td>
</tr>
<tr>
<td>Flow properties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flow type</td>
<td>Uniform</td>
<td></td>
</tr>
<tr>
<td>Flow velocity (m/s)</td>
<td>v</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Table C4: Equivalent local stiffness and local reduced velocity of beam segment with respect to position on the flexible beam

<table>
<thead>
<tr>
<th>Segment ID</th>
<th>$x$</th>
<th>$k_L(x)$</th>
<th>$f_L(x)$</th>
<th>$v_{rl}(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segment 1</td>
<td>2.15</td>
<td>10987.11</td>
<td>19.53</td>
<td>1.14</td>
</tr>
<tr>
<td>Segment 2</td>
<td>6.44</td>
<td>4047.91</td>
<td>11.86</td>
<td>1.87</td>
</tr>
<tr>
<td>Segment 3</td>
<td>10.74</td>
<td>2714.52</td>
<td>9.71</td>
<td>2.29</td>
</tr>
<tr>
<td>Segment 4</td>
<td>15.04</td>
<td>2197.51</td>
<td>8.74</td>
<td>2.54</td>
</tr>
<tr>
<td>Segment 5</td>
<td>19.33</td>
<td>1972.17</td>
<td>8.28</td>
<td>2.69</td>
</tr>
<tr>
<td>Segment 6</td>
<td>23.63</td>
<td>1907.052</td>
<td>8.14</td>
<td>2.73</td>
</tr>
<tr>
<td>Segment 7</td>
<td>27.92</td>
<td>1972.36</td>
<td>8.28</td>
<td>2.69</td>
</tr>
<tr>
<td>Segment 8</td>
<td>32.22</td>
<td>2197.98</td>
<td>8.74</td>
<td>2.54</td>
</tr>
<tr>
<td>Segment 9</td>
<td>36.52</td>
<td>2715.59</td>
<td>9.71</td>
<td>2.29</td>
</tr>
</tbody>
</table>
C1.3 Pipeline in Len et al (2003) with $L = 482D$

Table C5: Flow and structural properties of beam

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Material and mechanical properties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outer diameter (m)</td>
<td>D</td>
<td>0.02</td>
</tr>
<tr>
<td>Inner diameter (m)</td>
<td>D_i</td>
<td>0.0191</td>
</tr>
<tr>
<td>Aspect ratio</td>
<td>L/D</td>
<td>482</td>
</tr>
<tr>
<td>Axial Pre-tension (N)</td>
<td>T</td>
<td>817</td>
</tr>
<tr>
<td>Structural damping (%)</td>
<td>$\xi$</td>
<td>1.5</td>
</tr>
<tr>
<td>Young modulus (N/m²)</td>
<td>E</td>
<td>1.025x10^{11}</td>
</tr>
<tr>
<td>Mass ratio</td>
<td>m*</td>
<td>2.23</td>
</tr>
<tr>
<td>Material density (kg/m³)</td>
<td>$\rho$</td>
<td></td>
</tr>
<tr>
<td>Flow properties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flow type</td>
<td>Uniform</td>
<td></td>
</tr>
<tr>
<td>Flow velocity (m/s)</td>
<td>v</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Table C6: Flow and structural properties of beam

<table>
<thead>
<tr>
<th>Segment ID</th>
<th>x</th>
<th>$k_i(x)$</th>
<th>$f_i(x)$</th>
<th>$v_{rl}(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segment 1</td>
<td>1.61</td>
<td>720.1</td>
<td>14.29</td>
<td>1.47</td>
</tr>
<tr>
<td>Segment 2</td>
<td>4.82</td>
<td>400.12</td>
<td>10.66</td>
<td>1.97</td>
</tr>
<tr>
<td>Segment 3</td>
<td>8.04</td>
<td>720.1</td>
<td>14.29</td>
<td>1.47</td>
</tr>
</tbody>
</table>
C2. Mode Shape and Eigen Frequencies of Submerged Pipeline Modelled as Euler-Bernoulli Beam

Figure C.1: First five natural frequencies of pipeline in Trim et al (2005) shown in Table C1

\[ \begin{align*}
    f_1 &= 1.55\text{Hz} \\
    f_2 &= 3.10\text{Hz} \\
    f_3 &= 4.66\text{Hz} \\
    f_4 &= 6.22\text{Hz} \\
    f_5 &= 7.81\text{Hz}
\end{align*} \]

Figure C.2: First five natural frequencies of pipeline in Song et al (2011) shown in Table C3

\[ \begin{align*}
    f_1 &= 3.18\text{Hz} \\
    f_2 &= 6.52\text{Hz} \\
    f_3 &= 10.16\text{Hz} \\
    f_4 &= 14.24\text{Hz} \\
    f_5 &= 18.85\text{Hz}
\end{align*} \]

Figure C.3: First five natural frequencies of pipeline in Song et al (2011) shown in Table C3

\[ \begin{align*}
    f_1 &= 1.32\text{Hz} \\
    f_2 &= 2.96\text{Hz} \\
    f_3 &= 5.12\text{Hz} \\
    f_4 &= 7.94\text{Hz} \\
    f_5 &= 11.45\text{Hz}
\end{align*} \]
APPENDIX D

FREQUENCY RESPONSE OF FREE SPANNING PIPELINE (L/D = 1407)

D1. FREQUENCY RESPONSE OF DISPLACEMENT FOR $e = 0.75D$ AT $v_r = 4$

Figure D1: Frequency response of displacement along the longitudinal direction of beam for $e = 0.75D$ at $v_r = 4$ ($\cdash$ Y/D, $\dash$ Z/D)
D2. Frequency response of displacement for $e = 2D$ at $v_r = 10$

![Graphs showing frequency response of displacement for various positions](image)

Figure D2: Frequency response of displacement along the longitudinal direction of beam for $e = 2D$ at $v_r = 10$ (--- Y/D, --- Z/D)
APPENDIX E

FATIGUE ANALYSIS OF FREE SPANNING PIPELINE

E1. FATIGUE PROPERTIES OF A CORROSION-FREE STRUCTURAL STEEL IN SEAWATER

The S-N curve of a material is a plot of the stress range against the number of stress cycles to failure of the material. Norsork standard (1998) presents the fatigue criteria of a number of corrosion-free structural steel with yield stress less than 500MPa in seawater. Figure E.1 and Table E.1 show the S-N of the steels and their coefficient of fatigue properties respectively.

Figure E.1: S-N curves in seawater for corrosion-free structural steel
Table E1: Fatigue properties of structural steels with yield strength less than 500 in seawater

<table>
<thead>
<tr>
<th>S-N curve</th>
<th>log ( \bar{\sigma} )</th>
<th>Thickness exponent k</th>
</tr>
</thead>
<tbody>
<tr>
<td>For all cycles ( m = 3.0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>12.436</td>
<td>0</td>
</tr>
<tr>
<td>B2</td>
<td>12.262</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>12.115</td>
<td>0.15</td>
</tr>
<tr>
<td>C1</td>
<td>11.972</td>
<td>0.15</td>
</tr>
<tr>
<td>C2</td>
<td>11.824</td>
<td>0.15</td>
</tr>
<tr>
<td>D</td>
<td>11.687</td>
<td>0.20</td>
</tr>
<tr>
<td>E</td>
<td>11.533</td>
<td>0.20</td>
</tr>
<tr>
<td>F</td>
<td>11.378</td>
<td>0.25</td>
</tr>
<tr>
<td>F1</td>
<td>11.222</td>
<td>0.25</td>
</tr>
<tr>
<td>F3</td>
<td>11.068</td>
<td>0.25</td>
</tr>
<tr>
<td>G</td>
<td>10.921</td>
<td>0.25</td>
</tr>
<tr>
<td>W1</td>
<td>10.784</td>
<td>0.25</td>
</tr>
<tr>
<td>W2</td>
<td>10.630</td>
<td>0.25</td>
</tr>
<tr>
<td>W3</td>
<td>10.493</td>
<td>0.25</td>
</tr>
<tr>
<td>T</td>
<td>11.687</td>
<td>0.25 for SCF ≤ 10.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.30 for SCF &gt;10.0</td>
</tr>
</tbody>
</table>

E2. MATLAB CODE FOR FATIGUE DAMAGE OF A STRUCTURE

```matlab
% This code determines the fatigue damage and service life of a structure
% using rainflow counting for stress range and Miner's rule for damage
% accumulation.
% The time history of stress is stored as an excel file under the name
% Stress_history.xlsx
%
% READING STRESS TIME HISTORY FROM A FILE
[X] = xlsread('stress_history.xlsx'); % Calling the excel file of stress
- time history
  t = X(:,1); % A vector of time
  stress = X(:,2); % A vector of stresses (Pa)
  stress = stress*10^-6; % Stress (MPa)

% FATIGUE PROPERTIES OF MATERIAL
v = 0.3; % Poisson ratio
a = 10^11.687; % log a is the intercept of log N axis on S-N curve
m = 3; % Negative inverse slope of the S-N curve
yield_str = 500; % Yield strength of material (MPa)
s_ultimate = yield_str*(1+2*(150/yield_str)^2.5) % Ultimate strength of
material
true_fracture_stress = 650;

% RAINFLOW COUNTING OF STRESS RANGES
c = rainflow(stress,t);
```
stress_range = c(:,2); % a vector of stress ranges determined by rainflow
n = c(:,1); % counts of corresponding stress ranges
mean_stress = c(:,3); % corresponding mean stress
stress_amplitude = stress_range/2;

% EFFECT OF MEAN STRESS ON STRESS RANGE

%***Goodman equation
eff_stress_good=2*stress_amplitude./(1-(mean_stress/s_ultimate));
%***Gerber equation
eff_stress_gerber=2*stress_amplitude./(1-(mean_stress/s_ultimate).^2);
% Effective stress range
%***Solderberg equation
eff_stress_solder=2*stress_amplitude./(1-(mean_stress/yield_str));
%***Morrow equation
eff_stress_mor=2*stress_amplitude./(1-(mean_stress/true_fracture_stress));
%***Elliptic equation
eff_stress_ellip=2*stress_amplitude./sqrt(1-(mean_stress/s_ultimate).^2);

%FATIGUE DAMAGE CALCULATION USING MINER’S RULE
for i = 1:height(c)
% based on Goodman correction for stress cycle amplitude
N_gd(i)=10^(log10(a)-m*log10(eff_stress_gd(i))); % Allowable number of cycles (N)
Damage_gd(i) = n(i)/N_gd(i);

% based on Gerber correction for stress cycle amplitude
N_gb(i) = 10^(log10(a) - m*log10(eff_stress_gb(i)));
Damage_gb(i) = n(i)/N_gb(i);

% based on Solderberg correction for stress cycle amplitude
N_sd(i) = 10^(log10(a) - m*log10(eff_stress_sd(i)));
Damage_sd(i) = n(i)/N_sd(i);

% based on Morrow correction for stress cycle amplitude
N_mor(i) = 10^(log10(a) - m*log10(eff_stress_mor(i)));
Damage_mor(i) = n(i)/N_mor(i);

% based on Elliptic correction for stress cycle amplitude
N_ellip(i) = 10^(log10(a) - m*log10(eff_stress_ellip(i)));
Damage_ellip(i) = n(i)/N_ellip(i);
end
cummulative_damage = [sum(Damage_gd) sum(Damage_gb) sum(Damage_sd)
sum(Damage_mor) sum(Damage_ellip)]; % damage per second
cummulative_damage = cummulative_damage*60*60*24*365; % cumulative damage per year
fprintf('Total fatigue damage = %f
', cummulative_damage)

%SERVICE LIFE
service_life = 1./cummulative_damage; % Service life in years
fprintf('Service life = %f
', service_life)