



## Combining passivity and classical frequency-domain methods: An insight into decentralised control

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### ABSTRACT

A novel approach to tackle passivity-related issues in the frequency domain for linear multiple-input multiple-output (MIMO) cross-coupled systems is given. The aim is to design passivity-based stabilising diagonal controllers within the framework of Individual Channel Analysis and Design (ICAD). Two main results are presented. First, the ICAD is reinterpreted in terms of the passivity-related properties of either the channels or the closed-loop system. The notion of *practical passivity* is introduced. Second, for linear MIMO systems, a novel frequency-domain passification procedure is proposed. This procedure is used in the design process of the diagonal controllers. Furthermore, an indicator of how far the system is from being passive is defined. This indicator is stated in terms of gain and phase margins, with the consequent statement of robustness. Such a passivity indicator has not been established so far, and for practical applications can be more useful than setting the passivity of the system. Classical frequency-domain control techniques based on Bode and Nyquist plots are used. The results are applied to a 2-input-2-output system modelling an induction motor.

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### 1. Introduction

This paper merges the Individual Channel Analysis and Design (ICAD) framework and the dissipativity-based analysis in the frequency-domain setting for linear multiple-input multiple-output (MIMO) systems. Several benefits are obtained from this combination, mainly, robust stability, definition of coupling interaction in a natural way, adequate performance and robustness indices associated with the control design, and easy-to-implement controllers which are well-suited for engineering applications.

On the one hand, the ICAD is an analytic framework, rather than a method, for investigating and understanding the potential and limitations of the feedback design of multivariable linear-time invariant control systems [1–6]. One of the main consequences of the ICAD is that by analysing the multivariable structure function (MSF), fixed minimum-phase stable controllers which stabilise the system exist for each set of input–output channel configuration. The key issue of this result is that, as it is recently established in [7], it is not necessary to stabilise the diagonal elements of a multivariable system in order to achieve closed-loop stability.

In addition, in the ICAD, classical frequency-domain control techniques based on Bode and Nyquist plots are used. These techniques are widely used for single-input single-output (SISO) systems, however, they have not been extensively exploited for MIMO systems analysis and design. These frequency-domain methods are in general unsuitable for MIMO systems. The ICAD-based control design overcomes this drawback by means of the MSF.

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Another crucial aspect of the ICAD is that robustness for multivariable systems is expressed in terms of gain and phase margins, which are well-proven robustness measurements in engineering practice. Thus, the lack of robustness pointed out in [8,9], associated with modern techniques, is avoided.

On the other hand, it is well-known that dissipativity and its particular case of passivity give a rather intuitive interpretation of systems behaviour. In order to describe dissipative systems, two main representations are used, the input–output description of the system [10–13], or the state-space dynamical representation, based on the use of energy functions, mainly, the storage function, the supply rate and the dissipation rate function [14]. For linear systems, the equivalence between the state-space passivity properties and the frequency-domain characteristics is given by means of the Kalman–Yakubovich–Popov (KYP) Lemma or Positive Real Lemma ([15] and references therein). A complete recent review of dissipativity and passivity-related results, either in the continuous-time or in the discrete-time setting, is given in [16].

There is motivation to transform a system which is not passive into a passive one, since passive systems exhibit stability properties. Most of the previous results are focused on rendering a system passive by means of a static state feedback, which is referred to as *feedback passivity* or *passification*. In these results, the state-space system representation is used [17–23].

Most of the feedback passivity methods so far reported can be applied to systems which are minimum-phase and with relative degree less or equal to one. This is valid either in the discrete-time setting or in the continuous-time setting [18,16,24]. This restriction is overcome for linear SISO systems in [25,20] where the frequency-domain robust passification for non-minimum-phase, unstable systems or systems with relative degree greater than one is presented. These methods are extended for the MIMO case in the linear-matrix-inequalities (LMI) setting in [26,27]. As it is pointed out in [27], the problem of robust passification of linear proper MIMO systems has not drawn much attention yet. The aim of this paper is twofold. On the one hand, tackling the robust passification problem under the ICAD framework. On the other hand, in the line of Kelkar and Joshi [25,20], broadening the class of passifiable MIMO systems, by means of new feedback passivity structures.

This paper does not modify the ICAD. What is proposed in this paper is a new approach to deal with passivity-related properties for linear MIMO systems in the frequency-domain setting, and the use of these properties in order to design diagonal controllers within the ICAD framework. One of the main conclusions is that the passivity of the channels leads to the *practical passivity* of the closed-loop system. *Practical passivity* is used in the sense that the channels are robustly passified if they remain passive despite the channels cross-coupling. The coupling term is interpreted as a disturbance and is attenuated by maintaining the sensitivity functions low, as well as passified. This proposal will be restricted to a subclass of linear minimum-phase MIMO cross-coupled systems, whose MSF is stable. Although it is not treated in this paper, the passification procedure has potential to be applied to non-minimum-phase systems, by changing the controller and MSF structure, which is completely feasible due to the ICAD characteristics.

The passification methodology is based on Bode and Nyquist plots shaping. The goal is to obtain passive transfer functions with the phase within  $\pm 90^\circ$  for all frequencies, and Nyquist plots lying in the closed right-half complex plane. This can be carried out thanks to the fact that under the ICAD framework, the multivariable control design process is reduced to the design of a SISO controller for each channel [1,5]. The controlled system will be inherently robust due to the robustness properties of the ICAD framework.

To sum up, in this paper, three main features are introduced in the diagonal controller design process under the ICAD, which are new in comparison to ICAD-based controller designs so far proposed [1–5,28,29,49,7]:

- (1) The passivity of the MIMO closed-loop system is achieved by passifying the SISO channels through diagonal controllers.
- (2) The closed-loop system is practical or approximately passive. Closed-loop *practical passivity* is assured by attenuating the channels cross-coupling. This is achieved by maintaining the channels sensitivity functions low.
- (3) It is shown that how far the system is from being passive can be measured through the MSF. Such novel a measurement is done by means of the *dynamical structure gain* and *phase margins*.

To illustrate the controller design methodology devised, the control of an induction motor, modelled by means of a 2-input-2-output ( $2 \times 2$ ) system, is considered. The ICAD framework has been successfully used in order to control different engineering systems [28–33,49,7,50]. Although a  $2 \times 2$  system is considered in this paper, the analysis and control procedures under the ICAD framework are not restricted to  $2 \times 2$  systems. Its application to  $m \times m$  systems reduces to a nesting design procedure. The general case is reported in [5].

## 2. Preliminaries and basic definitions

Consider a  $2 \times 2$  system  $\mathbf{Y}(s) = \mathbf{G}(s)\mathbf{U}(s)$ , which can be rewritten as,

$$\begin{pmatrix} y_1(s) \\ y_2(s) \end{pmatrix} = \mathbf{G}(s) \begin{pmatrix} u_1(s) \\ u_2(s) \end{pmatrix} = \begin{pmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{pmatrix} \begin{pmatrix} u_1(s) \\ u_2(s) \end{pmatrix}, \quad (1)$$

where  $g_{ij}(s)$  are scalar proper transfer functions, and  $u_i(s)$ ,  $y_i(s)$  are the inputs and outputs of the system, respectively, with  $i, j = 1, 2$ . The relationship between representation (1) and the system state-space representation is:

$$\mathbf{G}(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}, \quad (2)$$

with  $(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$  representing a linear-time invariant (LTI) minimal realization of the system with the form,

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \quad (3)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t), \quad (4)$$

such that,  $(\mathbf{A}, \mathbf{B})$  is controllable and  $(\mathbf{A}, \mathbf{C})$  is observable, with  $\mathbf{x} \in \mathbb{R}^n$  the state vector,  $\mathbf{u} \in \mathbb{R}^2$  the input vector,  $\mathbf{y} \in \mathbb{R}^2$  the output vector, and  $\mathbf{A} \in \mathbb{R}^{n \times n}$ ,  $\mathbf{B} \in \mathbb{R}^{n \times 2}$ ,  $\mathbf{C} \in \mathbb{R}^{2 \times n}$ ,  $\mathbf{D} \in \mathbb{R}^{2 \times 2}$  constant matrices.

A diagonal controller  $\mathbf{K}(s)$  is considered, such that  $\mathbf{U}(s) = \mathbf{K}(s)\mathbf{E}(s)$  or,

$$\begin{pmatrix} u_1(s) \\ u_2(s) \end{pmatrix} = \begin{pmatrix} k_{11}(s) & 0 \\ 0 & k_{22}(s) \end{pmatrix} \begin{pmatrix} e_1(s) \\ e_2(s) \end{pmatrix},$$

with  $e_i(s) = r_i(s) - y_i(s)$  and  $r_i(s)$  are the plant references, for  $i = 1, 2$ . The open-loop input–output relationships, referred to as channels, are:

$$C_i(s) = k_{ii}(s)g_{ii}(s)[1 - \gamma(s)h_i(s)], \quad (5)$$

where

$$\gamma(s) = \frac{g_{12}(s)g_{21}(s)}{g_{11}(s)g_{22}(s)}, \quad h_i(s) = \frac{k_{ii}(s)g_{ii}(s)}{1 + k_{ii}(s)g_{ii}(s)}, \quad (6)$$

for  $i, j = 1, 2$  and  $i \neq j$ . The transfer function  $C_i(s)$  denotes the open-loop channel  $i$ , that is, the pairing of reference  $r_i(s)$  with output  $y_i(s)$ . The function  $\gamma(s)$ , which is referred to as the MSF, can be interpreted as a measurement of the system internal coupling.  $h_i(s)$  defines the impact of controller  $k_{ii}(s)$  in the  $j$ th control loop (with  $i \neq j$ ). The internal interaction between the channels is given by  $\gamma(s)h_i(s)$ .

The closed-loop system is [29,50],

$$y_i(s) = T_i(s)r_i(s) + S_i(s)Q_i(s)r_j(s), \quad (7)$$

with,

$$T_i(s) = \frac{C_i(s)}{1 + C_i(s)}, \quad S_i(s) = \frac{1}{1 + C_i(s)}, \quad Q_i(s) = \frac{g_{ij}(s)}{g_{jj}(s)}h_j(s), \quad (8)$$

for  $i, j = 1, 2$  and  $i \neq j$ . The term  $S_i(s)Q_i(s)r_j(s)$  in (7) can be considered as the cross-coupling term between channel  $i$  and channel  $j$  (with  $i \neq j$ ). Functions  $S_i(s)$  and  $T_i(s)$  play the role of the  $i$ th-channel sensitivity function and complementary sensitivity function, respectively. The influence of the reference  $r_j(s)$  on the output  $y_i(s)$  can be considered as a disturbance and is given by,

$$d_i(s) = \frac{g_{ij}(s)}{g_{jj}(s)}h_j(s)r_j(s), \quad (9)$$

for  $i, j = 1, 2$  and  $i \neq j$ . In order to reject the external disturbances and the coupling between the channels,  $S_i(s)$  must be small in the range of operating frequencies, specially, in those at which the disturbances are dominant [1,2].

Recently [7,50], it has been shown that in order to stabilise the closed-loop system, it is only necessary to stabilise the channels  $C_i(s)$ . Consequently, the  $2 \times 2$  multivariable design can be decomposed into the two equivalent SISO control designs depicted in Fig. 1.

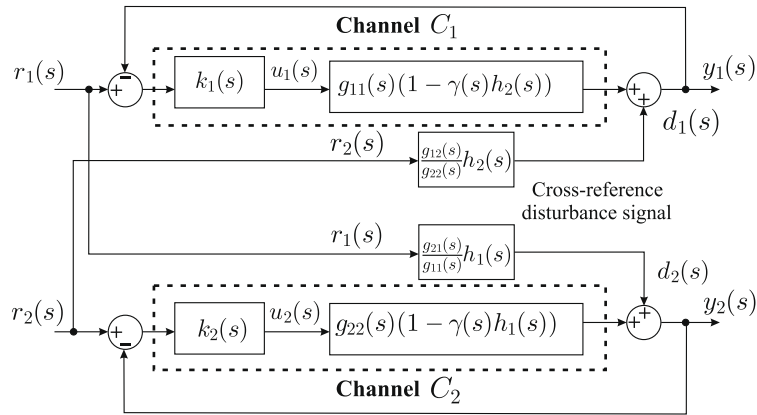
If  $m \times m$  systems, with  $m > 2$ , were considered, the analysis would reduce to the analysis of  $m$  SISO systems, that is the reason why the framework is referred to as Individual Channel Analysis and Design. Individual refers to individual SISO channels. For  $m \times m$  systems, instead of having two functions  $C_i(s)$  as defined in (5) and (6), there would be  $m$  functions  $C_i(s)$ . Thus, the ICAD leads to modular analysis and design.

The number of right-hand side poles (RHP's) and right-hand side zeros (RHZ's) is usually referred to as the *dynamical structure* and is a key issue in the system design [51,1,52]. A *structural stable* system is defined as a system whose dynamical structure is preserved under structured and unstructured uncertainties.

Provided no pole–zero cancellation occurs within the channels, the open-loop channel poles and zeros are given in Table 1 [1,29]. From Table 1, the poles of channel  $i$  are defined by individual transfer functions, while its zeros are defined by the zeros of

$$1 - \gamma(s)h_j(s),$$

with  $i \neq j$ . The number of RHZ's of  $\mathbf{G}(s)$  can be determined by applying the Nyquist criterion to  $\gamma(s)$ . Moreover, it is possible to measure how far the system is from becoming non-minimum-phase and assess if it is possible to maintain the dynamical structure. Such an assessment is done by introducing gain and phase margins referred to as *dynamical structure margins* (DYNSMAR).  $DYNSMAR_g$  and  $DYNSMAR_p$  denote the gain DYNSMAR and the phase DYNSMAR, respectively. From (5) and (6), it is obtained that the expression  $\gamma(s) = 1$  is equivalent to  $\det(\mathbf{G}(s)) = g_{11}(s)g_{22}(s) - g_{12}(s)g_{21}(s) = 0$ . Consequently, if  $\gamma(s) = 1$ , the matrix  $\mathbf{G}(s)$  is singular, which implies that the system is non-minimum-phase.



**Fig. 1.** Equivalent representation of a 2-input-2-output system with a diagonal controller. Individual channels  $C_1(s)$  and  $C_2(s)$  with cross-reference disturbance signals ( $d_1(s)$  and  $d_2(s)$ ).

**Table 1**

Zero-pole structure of the open-loop channels.

Transmittance	Zeros	Poles
Channel 1	Zeros of $1 - \gamma(s)h_2(s)$	Poles of $g_{11}(s), g_{12}(s), g_{21}(s), h_2(s)$
Channel 2	Zeros of $1 - \gamma(s)h_1(s)$	Poles of $g_{22}(s), g_{12}(s), g_{21}(s), h_1(s)$

Since the study is focused on LTI systems, passivity can be studied by means of its equivalent concept of positive realness of a transfer function, and the following lemma is used.

**Lemma 1** [15]. *The LTI minimal realization (3) and (4) with  $G(s) = C(sI - A)^{-1}B + D$  is passive if  $G(s)$  is positive real.*

Furthermore, due to the fact that the design of the multivariable controller  $K(s)$  reduces to the design of SISO controllers for the channels, the following lemma can be used.

**Lemma 2** [15]. *A proper rational transfer function  $g(s)$  is positive real if the following conditions are met:*

- (i)  $g(s)$  has no poles in the open right-half complex plane.
- (ii)  $\text{Re}[g(j\omega)] \geq 0, \forall \omega \in \mathbb{R}$ , with  $j\omega$  not a pole of  $g(s)$ .
- (iii) Every pure imaginary pole  $j\omega$  of  $g(s)$ , if any, is simple and the associated residue has positive real part.

Condition (ii) in Lemma 2 is held when the Nyquist plot of  $g(s)$  lies in the closed right-half complex plane. Let  $r_d(\cdot)$  denote the relative degree of a transfer function. Condition (ii) can be satisfied only if the relative degree of  $g(s)$  is zero or one, that is, when  $r_d(g(s)) \leq 1$  [15].

For control purposes, it is not necessary to require passivity for all the frequency range. A more practical concept of passivity is passivity within a finite frequency interval [53,25]. These systems are usually referred to as band-limited positive real or approximately positive real. In this paper, the term practical passive or practical positive real is preferred in order to highlight the convenience of this concept in practice.

**Definition 3.** A transfer function  $g(s)$  is said to be practical positive real or practical passive if it satisfies conditions (i)–(iii) of Lemma 2 for a set of frequencies  $\Omega := \{\omega \in \mathbb{R} : \omega_{\min} \leq \omega \leq \omega_{\max}\}$ .

$\Omega$  can be identified from the Bode plot, and it is the bandwidth for which the phase of the system is within  $\pm 90^\circ$ .

### 3. Closed-loop system passification and stabilization

In this section, a diagonal controller design within the ICAD framework is carried out by considering the passivity-related properties of the channels. The basic idea of the method is that the stability of the closed-loop system is straightforwardly achieved by means of the passification of each open-loop channel.

The MSF-based design procedure proposed in this paper is only valid for systems with  $\gamma(s)$  with no RHP's, and with the Nyquist plot of  $\gamma(s)$  to the left of the critical point (1,0). The fact that the Nyquist plot of  $\gamma(s)$  is to the left of (1,0) means that the system  $G(s)$  is minimum-phase according to the Nyquist criterion.

The design methodology proposed for the stabilization and the passification of the system shown in Fig. 1 consists of three steps:

- Step 1** To study the channels dynamical structure for  $k_{ii}(s) = 1$ ,  $i = 1, 2$ , through the MSF in order to ensure the basic conditions for passification.
- Step 2** To stabilise and passify each open-loop channel  $i$  by designing  $k_{ii}(s)$ . The controller  $k_{ii}(s)$  consists of two controllers in cascade, one for the channel stabilization, another for the channel passification.
- Step 3** To ensure the practical passivity of the closed-loop system through the analysis of the sensitivity and complementary sensitivity functions of each channel.

Each of the following subsections are devoted to each of these steps.

Finally, according to the properties of the ICAD, the achieved passivity of each open-loop channel ensures the stability of the closed-loop system. This will be explained in Section 3.4.

### 3.1. Passification conditions through the channels dynamical structure

In order to analyse and determine the existence of passifying controllers, it can be assumed as a starting point that  $k_{ii}(s) = 1$  with  $i = 1, 2$ . Thus, according to (5) and (6), the following transfer functions are defined:

$$h'_i(s) = \frac{g_{ii}(s)}{1 + g_{ii}(s)}, \quad C'_i(s) = g_{ii}(s) [1 - \gamma(s)h'_j(s)],$$

with  $i, j = 1, 2$ ,  $i \neq j$ .  $C'_i(s)$  denotes the open-loop channel  $i$ .

Well-known conditions for the passification of continuous-time systems [18,25,20,54,16,21,23] will be rewritten under the ICAD framework. The analysis of conditions for the channels to be passifiable is based on investigating the dynamical structure of the open-loop channels. Table 1 is used for this purpose.

Channel  $C'_i(s)$  is considered to be passifiable under the ICAD framework by means of controller  $k_{ii}(s)$  if the following conditions are satisfied:

- (i)  $\gamma(s)$  and  $h'_j(s)$  have no RHP's;
- (ii)  $1 - \gamma(s)h'_j(s)$  has no RHZ's (the channel is minimum-phase);
- (iii)  $r_d(C'_i(s)) \leq 1$  (the channel relative degree is no greater than one),

for  $i, j = 1, 2$  and  $i \neq j$ .

Condition (ii) can be evaluated according to the *DYNSMAR* concept. That is, it is possible to define how far  $C'_i(s)$  is from being non-minimum-phase (and consequently, from being passive) by applying the Nyquist stability criterion to  $\gamma(s)h'_j(s)$ , and thus, obtaining the number of RHZ's ( $Z$ ) of  $1 - \gamma(s)h'_j(s)$ . From condition (i), which establishes that  $h'_j(s)$  has no RHP's, and from the fact that  $\gamma(s)$  has no RHP's,  $\gamma(s)h'_j(s)$  is concluded to have no RHP's ( $P = 0$ ) then the number of RHZ's of  $1 - \gamma(s)h'_j(s)$  is  $Z = P + N$ , with  $N$  being the number of clockwise encirclements to the point  $(1, 0)$  of  $\gamma(s)h'_j(s)$ . If the Nyquist plot of  $\gamma(s)h'_j(s)$  is to the left of  $(1, 0)$  then  $N = 0$  and  $Z = 0$ . Therefore, condition (ii) is accomplished. The further the Nyquist plot is to the left of  $(1, 0)$ , the further away the channel is to become non-minimum-phase, and therefore the closer is to be passifiable.

This can be quantified by means of the Bode plot of  $\gamma(s)h'_j(s)$ , as in the usual definition of stability margins, which are jointly referred to as *DYNSMAR* in this paper.

**Remark 4.** The compensation configuration obtained through  $k_{ii}(s)$  in the system shown in Fig. 1 has a hybrid structure (following the nomenclature used in [25]). From the form of  $C'_i(s)$ ,  $k_{ii}(s)$  is included in  $h_j(s)$  through a feedback loop, in addition to the series pre-compensation. Due to this fact, the control configuration proposed can allow the passification of a wider family of channels. The passification method has potential to be applied to non-minimum-phase open-loop channels, by changing the controller and MSF structure, which is completely feasible due to the ICAD characteristics. Nevertheless, minimum-phase open-loop channels are passified for the sake of simplicity in the presentation of the main ideas of the passivity-based design methodology proposed.

### 3.2. Passification of the open-loop channels

The stabilising-passifying controller  $k_{ii}(s)$  is proposed to have the following form for  $i = 1, 2$ :

$$k_{ii}(s) = k_{ii}^t(s)k_{ii}^p(s), \quad (10)$$

where  $k_{ii}^t(s)$  is designed in order to stabilise and meet the design specifications in transient and steady regimes, and  $k_{ii}^p(s)$  is the passifying controller. Both  $k_{ii}^t(s)$  and  $k_{ii}^p(s)$  can be designed by means of classical techniques of Bode–Nyquist plots shaping.

The main steps in order to design  $k_{ii}(s)$  are the following ones:

- Step 1** Design of  $k_{ii}^t(s)$  by Bode-plot shaping of the channels in order to meet the design specifications (e.g. bandwidth, zero steady-state error, transient response, and so on) for both channels. This step can be divided into the following substeps:

- (1) Design of  $k_{11}^t(s)$  in order to meet the design specifications for  $C_1(s)$ , considering  $k_{22}(s) = 1$  and  $h_2'(s) = 1$ . The Bode plot of  $C_1'(s)$  is shaped. The analysis of the MSF is of key importance in this step.
- (2) Design of  $k_{22}^t(s)$  in order to meet the design specifications for  $C_2(s)$ , including in  $h_1(s)$  the controller  $k_{11}^t(s)$  obtained from Step 1.1. That is, the Bode plot of  $C_2'(s) = g_{22}(s)[1 - \gamma(s)h_1(s)]$  is shaped, with  $h_1(s) = \frac{k_{11}^t(s)g_{11}(s)}{1+k_{11}^t(s)g_{11}(s)}$ .
- (3) Include controller  $k_{22}^t(s)$  obtained from Step 1.2 in  $h_2(s)$  of  $C_1(s)$ .
- (4) Repeat Steps 1.2 and 1.3 until the design specifications are satisfied.

**Step 2** Design of  $k_{ii}^p(s)$  by Bode and Nyquist plots shaping in order to passify both  $C_1(s)$  and  $C_2(s)$  obtained from Step 1:

- (1) Design of  $k_{11}^p(s)$  by Bode and Nyquist plots shaping of  $C_1(s) = k_{11}^t(s)g_{11}(s)[1 - \gamma(s)h_2(s)]$  considering  $h_2(s) = \frac{k_{22}^t(s)g_{22}(s)}{1+k_{22}^t(s)g_{22}(s)}$ , with  $k_{11}^t(s)$  and  $k_{22}^t(s)$  obtained from Step 1. Special attention will be paid to the passification of  $g_{11}(s)$  by means of  $k_{11}^p(s)$ .
- (2) Design of  $k_{22}^p(s)$  by Bode and Nyquist plots shaping of  $C_2(s) = k_{22}^t(s)g_{22}(s)[1 - \gamma(s)h_1(s)]$  considering  $h_1(s) = \frac{k_{11}^t(s)k_{11}^p(s)g_{11}(s)}{1+k_{11}^t(s)k_{11}^p(s)g_{11}(s)}$ , with  $k_{11}^p(s)$  obtained from Step 2.1. Special attention will be paid to the passification of  $g_{22}(s)$  by means of  $k_{22}^p(s)$ .
- (3) Include  $k_{22}^p(s)$  obtained from Step 2.2 in  $h_2(s)$  of  $C_1(s)$ .
- (4) Repeat Steps 2.2 and 2.3 until both channels are passified.

**Remark 5.** For minimum-phase systems  $G(s)$  with  $\gamma(s)$  stable and such that

$$20\log_{10}(|\gamma(j\omega)|) < 0 \text{ dB}, \forall \omega$$

(i.e., the phase margin is infinite), the passivity of  $C_i(s)$  can be achieved by passifying  $g_{ii}(s)$  by means of  $k_{ii}(s)$ . That is, if  $k_{ii}(s)g_{ii}(s)$  is passive for  $i = 1, 2$ ,  $h_i(s)$  is passive, stable and minimum-phase [18,55]. Thus, if the gain of  $\gamma(s)$  is low, the term  $\gamma(s)h_i(s)$  is negligible and the passivity of  $C_i(s)$  is determined by the passivity of  $k_{ii}(s)g_{ii}(s)$ . In addition, the facts that  $\gamma(s)$  does not contain any RHP's and its Nyquist plot does not encircle (1,0) imply that functions  $h_i(s)$  do not influence the channel dynamical structure at high frequency values. That is, the number of encirclements to the point (1,0) by  $\gamma(s)h_i(s)$  and  $\gamma(s)$  coincides.

**Remark 6.** The case of having a weak cross-coupling between the channels would imply  $\gamma(s)$  to have a non-significant magnitude, and  $\gamma(s)h_i(s)$  would be close to zero. In other words, the MIMO system would be equivalent to two decoupled SISO systems, which would lead to a more simple control design.

### 3.3. Practical passivity of the closed-loop system: the role of the sensitivity functions

The practical passivity of the closed-loop system (7) is achieved by means of the passivity of  $T_i(s)$  and by shaping the sensitivity functions  $S_i(s)$ . This is summarized in the following two propositions.

**Proposition 7.** If  $C_i(s)$  is passive then the sensitivity ( $S_i(s)$ ) and the complementary sensitivity ( $T_i(s)$ ) functions are passive. Consequently,  $S_i(s)$  and  $T_i(s)$  are stable and minimum-phase. Furthermore,  $r_d(S_i(s)) = 0$  and  $r_d(T_i(s)) = r_d(C_i(s))$ .

**Proof.** From results given in [18,55], the negative feedback interconnection of passive systems is passive, which is the case of  $S_i(s)$  and  $T_i(s)$ . Moreover, passive systems are stable and minimum-phase. This can be obtained from conditions of Lemma 2. The following sets are defined:

$$P_S = \{\text{poles of } S_i(s)\}, \quad P_T = \{\text{poles of } T_i(s)\},$$

$$Z_S = \{\text{zeros of } S_i(s)\}, \quad Z_T = \{\text{zeros of } T_i(s)\}.$$

It is obtained that,

$$P_S = P_T = \{\text{zeros of } 1 + C_i(s)\}, \tag{11}$$

$$Z_S = \{\text{poles of } C_i(s)\}, \quad Z_T = \{\text{zeros of } C_i(s)\}. \tag{12}$$

From (11), if  $C_i(s)$  is passive (i.e., stable and minimum-phase), both  $S_i(s)$  and  $T_i(s)$  have no RHP's and are stable. From (12), if  $C_i(s)$  is passive,  $S_i(s)$  and  $T_i(s)$  have no RHZ's and are minimum-phase. From the above relationships and the fact that  $r_d(C_i(s)) \leq 1$  (from condition (ii) in Lemma 2), it is obtained that  $r_d(S_i(s)) = 0$  and  $r_d(T_i(s)) = r_d(C_i(s))$ .  $\square$

**Proposition 8.** Let  $k_{ii}(s)$  be such that passifies  $C_i(s)$  for  $i = 1, 2$ . If  $S_i(s)$  is such that,

$$20\log_{10}(|S_i(j\omega)|) < 0 \text{ dB}, \forall \omega < \omega_s; \tag{13}$$

$$20\log_{10}(|S_i(j\omega)|) = 0 \text{ dB}, \forall \omega \geq \omega_s, \tag{14}$$

for some  $\omega_s > 0$ , then the closed-loop system is practical passive, that is, passive for the set of frequencies  $\Omega = \{\omega \in \mathbb{R} : \omega \geq \omega_s\}$ .

**Proof.** Let the closed-loop output  $y_i(s)$  as given in (7) and the system configuration shown in Fig. 1. On the one hand, from Proposition 7,  $T_i(s)$  is concluded to be passive.

On the other hand, from the characteristics of the MSF-based design, the disturbance  $d_i(s) = Q_i(s)r_j(s)$  is attenuated by means of the sensitivity function  $S_i(s)$ , which is passive from Proposition 7. If the magnitude of  $S_i(s)$  is maintained low, specifically, if the gain  $20\log_{10}(|S_i(j\omega)|) < 0 \text{ dB}, \forall \omega < \omega_s$ , the term  $d_i(s)$  in  $y_i(s)$  is negligible. Moreover, if  $20\log_{10}(|S_i(j\omega)|) = 0 \text{ dB}, \forall \omega \geq \omega_s$ , the term  $d_i(s)$  is completely attenuated, and the system is passive  $\forall \omega \geq \omega_s$ . The term  $d_i(s)$  represents the cross-coupling between the channels. Consequently, from (7), if the cross-coupling term is negligible, the system is decoupled and the term  $T_i(s)r_i(s)$  is dominant. In conclusion, the closed-loop system is practical passive for the set of frequencies  $\omega \geq \omega_s$ .  $\square$

3.4. Stability of the closed-loop system: a consequence of the channels passivity

The stability of the closed-loop system can be established by achieving the passivity of both open-loop channels.

**Proposition 9.** Consider the closed-loop system (7). If  $k_{ii}(s)$  passifies  $C_i(s)$ , and the references  $r_i(s)$  and  $r_j(s)$  are bounded, then  $k_{ii}(s)$  stabilizes the channel closed-loop output  $y_i(s)$ , where  $i, j = 1, 2$  and  $i \neq j$ .

**Proof.** The proof is based on the result reported in [2] which is later proven in [7,50]. This result establishes that if  $k_{ii}(s)$  stabilizes  $T_i(s)$  then it also stabilizes the closed-loop system output.

The controller  $k_{ii}(s)$  is designed in such a way that passifies  $C_i(s)$  for  $i = 1, 2$ . From Proposition 7,  $T_i(s)$  is passive, stable and minimum-phase, then from the above-mentioned ICAD-stability result,  $k_{ii}(s)$  stabilizes (7) with  $i, j = 1, 2$  and  $i \neq j$ .  $\square$

**Remark 10.** The passification and stabilization procedure proposed under the ICAD framework is not restricted to  $2 \times 2$  systems. It is possible to apply the design procedure to systems of order  $m \times m$ . It should be noticed that in [5] a generalization of the ICAD has been demonstrated. In this case, there exist several multivariable structure functions. They relate the interaction among the different individual channels with the rest of the system. For example, in [34], the analysis, design and successful implementation of ICAD-based controllers are given for a gyroscope model with four inputs and two outputs, with highly-coupled dynamics. The system is decomposed into two SISO systems followed by one  $2 \times 2$  system. Recently, in [35], a detailed explanation of the application of the ICAD to  $3 \times 3$  and  $4 \times 4$  systems has been reported.

4. Example: the control of an induction motor

The controller design methodology proposed in Section 3 is applied to an induction motor  $\alpha$ - $\beta$  model extracted from [49,50].

The model of the typical two-phase induction motor in static coordinates is [36]:

$$\begin{aligned} \dot{i}_{\alpha s} &= -\frac{L_r^2 R_s + L_m^2 R_r}{\sigma L_s L_r^2} i_{\alpha s} + \frac{L_m R_r}{\sigma L_s L_r^2} \psi_{\alpha r} + \frac{L_m \omega_r}{\sigma L_s L_r} \psi_{\beta r} + \frac{1}{\sigma L_s} v_{\alpha s}, \\ \dot{i}_{\beta s} &= -\frac{L_r^2 R_s + L_m^2 R_r}{\sigma L_s L_r^2} i_{\beta s} - \frac{L_m \omega_r}{\sigma L_s L_r} \psi_{\alpha r} + \frac{L_m R_r}{\sigma L_s L_r^2} \psi_{\beta r} + \frac{1}{\sigma L_s} v_{\beta s}, \\ \dot{\psi}_{\alpha r} &= -\frac{R_r}{L_r} \psi_{\alpha r} - \omega_r \psi_{\beta r} + \frac{L_m R_r}{L_r} i_{\alpha s}, \\ \dot{\psi}_{\beta r} &= -\frac{R_r}{L_r} \psi_{\beta r} + \omega_r \psi_{\alpha r} + \frac{L_m R_r}{L_r} i_{\beta s}, \\ T_E &= \frac{3}{2} \left(\frac{P}{2}\right) \frac{L_m}{L_r} (\psi_{\alpha r} i_{\beta s} - \psi_{\beta r} i_{\alpha s}), \\ \dot{\omega}_r &= \left(\frac{P}{2J}\right) (T_E - T_L), \end{aligned}$$

where  $i_{\alpha s}, i_{\beta s}$  are the stator currents;  $\psi_{\alpha r}, \psi_{\beta r}$  are the rotor fluxes;  $v_{\alpha s}, v_{\beta s}$  are the stator voltages; and  $\omega_r$  is the rotor angular velocity.  $\omega_r = 2\pi f_r$ , where  $f_r$  is the rotor electrical frequency.  $T_E$  is the moment generated by the motor.

The parameters of the model are: the stator, rotor and mutual inductances  $L_s, L_r$  and  $L_m$ ; the stator and rotor resistance  $R_s$  and  $R_r$ ;  $J$  is the moment of inertia;  $T_L$  is the external load;  $P$  is the number of poles of the motor; and  $\sigma = 1 - L_m^2 / (L_r L_s)$ .

Field oriented control (FOC) is one of the most common strategies used for high performance induction motors. This scheme is based on achieving a dynamical simplification of the motor dynamics. That is, reducing the model above into:

$$\begin{aligned} \dot{\psi}_{\alpha r} &= -a_{44}\psi_{\alpha r} - \omega_r\psi_{\beta r} + a_{42}i_{\alpha s}, \\ \dot{\psi}_{\beta r} &= -a_{44}\psi_{\beta r} + \omega_r\psi_{\alpha r} + a_{42}i_{\beta s}, \\ T_E &= K_T(\psi_{\alpha r}i_{\beta s} - \psi_{\beta r}i_{\alpha s}), \\ \dot{\omega}_r &= \left(\frac{P}{2J}\right)(T_E - T_L), \end{aligned}$$

where  $a_{44} = \frac{R_r}{L_r}$ ,  $a_{42} = \frac{L_m R_r}{L_r}$ ,  $K_T = \frac{3}{2} \left(\frac{P}{2}\right) \frac{L_m}{L_r}$ . This simplification is achieved by controlling the stator currents through the stator voltages [37,38].

This inner loop is devised in order to achieve a two-time scale system. Thus, a high bandwidth and robust current controller is needed. In this case, a reduced motor model consisting only of the stator and current voltages is usually considered. In the rest of the section, the approach proposed in this paper is applied in order to control the stator currents of an induction motor at a particular operating condition. The data considered correspond to the motor reported in [49,50].

The reduced-state model of the induction motor can be written as system (3) and (4) with  $\mathbf{x} = (i_{\alpha s}, i_{\beta s}, \psi_{\alpha r}, \psi_{\beta r})^T$ ,  $\mathbf{u} = (v_{\alpha s}, v_{\beta s})^T$ ,  $\mathbf{y} = (i_{\alpha s}, i_{\beta s})^T$ , and,

$$\begin{aligned} \mathbf{A} &= \begin{pmatrix} -127.4 & 0 & 90.1 & 8.1\omega_r \\ 0 & -127.4 & -8.1\omega_r & 90.1 \\ 4.97 & 0 & -11.13 & -\omega_r \\ 0 & 4.97 & \omega_r & -11.13 \end{pmatrix}, \\ \mathbf{B} &= \begin{pmatrix} 9.15 & 0 \\ 0 & 9.15 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \end{aligned} \tag{15}$$

Notice that matrix  $\mathbf{A}$  is stated in terms of  $\omega_r$ . The elements of the matrix  $\mathbf{G}(s)$  are:

$$\begin{aligned} g_{11}(s) = g_{22}(s) &= \frac{9.15(s + 89.03)(s^2 + 60.63s + 1.392 \times 10^5)}{(s^2 + 176.2s + 7849)(s^2 + 100.9s + 1.376 \times 10^5)}, \\ g_{12}(s) = -g_{21}(s) &= \frac{138865.2628(s + 0.006543)}{(s^2 + 176.2s + 7849)(s^2 + 100.9s + 1.376 \times 10^5)}. \end{aligned} \tag{16}$$

The control goal can be recast as to design a controller for (15) which provides appropriate references to the pulse-width modulated voltage source inverter, which in turn will supply the voltage signals to the induction motor terminals. The control system is presented for the case of  $f_r = 60$  Hz.

From Fig. 2, it can be seen that the Nyquist plot of

$$\gamma(s) = \frac{g_{12}(s)g_{21}(s)}{g_{11}(s)g_{22}(s)} = \frac{-2.303 \times 10^8 (s + 0.006543)^2}{(s + 89.03)(s + 89.03)(s^2 + 60.63s + 1.392 \times 10^5)^2}$$

is to the left of (1,0), and due to the fact that it does not touch this critical point and has no RHP's,  $\gamma(s)$  is stable and the system is minimum-phase. Consequently, the passification is possible. Moreover, from the Bode plot of  $\gamma(s)$ , it is obtained that  $20\log_{10}(|\gamma(j\omega)|) < 0$  dB,  $\forall \omega$ , then the passivity of  $C_i(s)$  can be achieved by the passification of  $g_{ii}(s)$  by means of  $k_{ii}(s)$ , as it was established in Remark 5. In Fig. 2, the DYNsMAR of  $\mathbf{G}(s)$  can be also appreciated. In this case,  $DYNsMAR_p = \infty$  and  $DYNsMAR_g = 7.38$  dB.

In general, the maximum bandwidth that can be achieved is limited to technology aspects. In our case, the aim is to obtain a settling time of around 0.1 s, which correspond to a bandwidth of around 50 rad/s.

The four steps of the design methodology are followed:

**Step 1** The conditions for the passification of  $C'_1(s)$  and  $C'_2(s)$  established in Section 3.1 are met. From Fig. 3,  $DYNsMAR_p = \infty$  and  $DYNsMAR_g = 45$  dB for both channels. This gives an indicator of how far the channels are from being non-minimum-phase, and as it was established in Section 3.1,  $DYNsMAR_g$  reflects how robust the passification conditions are despite uncertainties. The value of  $DYNsMAR_g$  is high enough to ensure that the non-passive channels can be robustly passifiable.

**Step 2** Design of  $k_{11}(s)$  and  $k_{22}(s)$ . For the motor,  $k_{11}(s) = k_{22}(s)$  is considered. Following the design procedure for  $k_{11}(s)$  established in Section 3.2, it is obtained that:

$$k_{11}^t(s) = k_{22}^t(s) = \frac{7(s + 100)}{s}, \tag{17}$$

$$k_{11}^p(s) = k_{22}^p(s) = \frac{(s + 250)(s^2 + 100s + 135000)}{(s + 370)(s^2 + 60s + 136300)}. \tag{18}$$



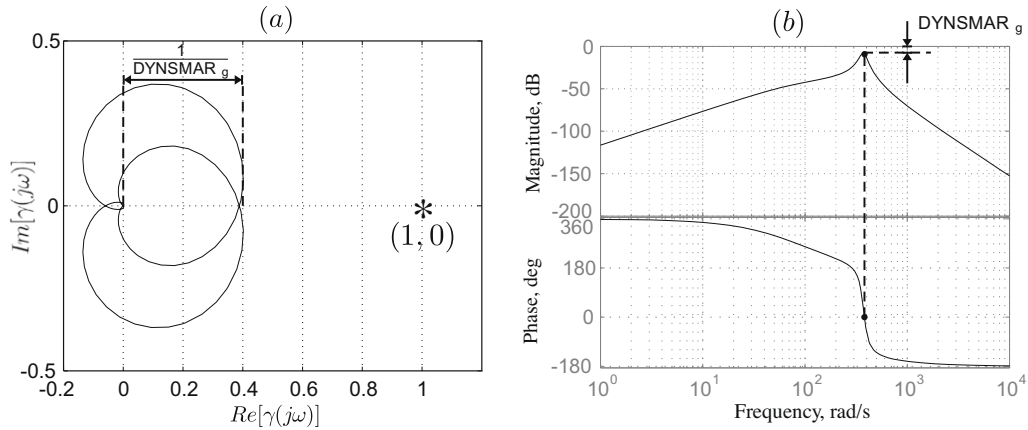


Fig. 2. Evaluation of the MSF  $\gamma(s)$  for the induction motor with  $f_r = 60$  Hz: (a) Nyquist plot; (b) Bode plot.

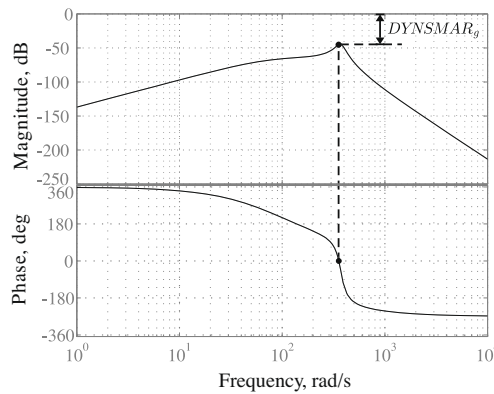


Fig. 3. Bode plots of  $\gamma(s)h'_1(s)$  and  $\gamma(s)h'_2(s)$ , for the induction motor with  $f_r = 60$  Hz.

Controller  $k_{11}^l(s)$  has been designed so that the steady-state error is zero and the closed-loop system bandwidth is 10 rad/s. Controller  $k_{11}^p(s)$  has been designed by Bode-plot shaping so that  $C_1(s)$  is passive, indeed,  $k_{11}^p(s)$  passifies  $g_{11}(s)$  and consequently, it passifies  $h_1(s)$ . The results can be appreciated in Fig. 6. No resonant response (bumps) in the frequency responses are present and the Nyquist plot of the open-loop ( $C_i(s)$ ) and the closed-loop ( $T_i(s)$ ) channels are located in the right-hand side of the complex plane. Due to the system configuration, the design for channel 1 is valid for channel 2.

**Step 3** By means of the analysis of the sensitivity and complementary sensitivity functions of each channel, the practical passivity of the closed-loop system is ensured. This can be checked in Figs. 4 and 5. Here, it is appreciated that the  $S_i$ -shaping conditions (13) and (14) are accomplished (Fig. 4). Furthermore, the magnitude of the cross-coupling term  $S_i(s)Q_i(s)$  is small in comparison with  $T_i(s)$  (Fig. 5(b)), above all, for  $\omega \geq \omega_s$  (Fig. 5(a)). In addition,  $T_i(s)$  is passive as well as  $C_i(s)$  and  $S_i(s)$ ,  $i = 1, 2$ . This fact can be appreciated from the Nyquist and Bode plots depicted in Fig. 6(a) and (b).

**Step 4** From the properties of the MSF-based design, the passivity of each open-loop channel ensures the stability of the closed-loop system. The step-response for each closed-loop output is given in Fig. 7(a). From the figure, it can be appreciated that the step-response is remarkably good with no overshooting and adequate settling time. Furthermore, in the same manner that the structural stability of the channels  $C'_i(s)$  was defined in terms of the characteristics of  $\gamma(s)h'_j(s)$  in Section 3.1, for the induction motor, the structural stability of the channels  $C_i(s)$ , and consequently, of the closed-loop system, can be quantified by obtaining the  $DYNSMAR$  of  $\gamma(s)h_j(s)$  for  $i, j = 1, 2, i \neq j$ . From Fig. 7(b),  $DYNSMAR_p = \infty$  and  $DYNSMAR_g = 26$  dB for both channels. This  $DYNSMAR_g$  gives the index of structural stability and the indicator of the robustness of the passivity achieved under structured and unstructured uncertainties. The value of 26 dB can be considered as a good gain  $DYNSMAR$  for the closed-loop system obtained.

There are several methods for controlling the stator currents in the class of induction motors treated here. Among the most successful one:

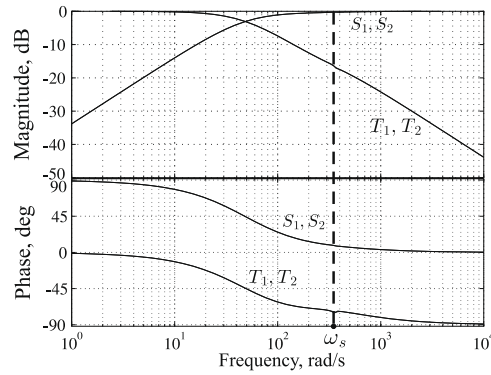


Fig. 4. Sensitivity and complementary sensitivity functions for system (15) with controllers (17)–(18) with  $f_r = 60$  Hz.

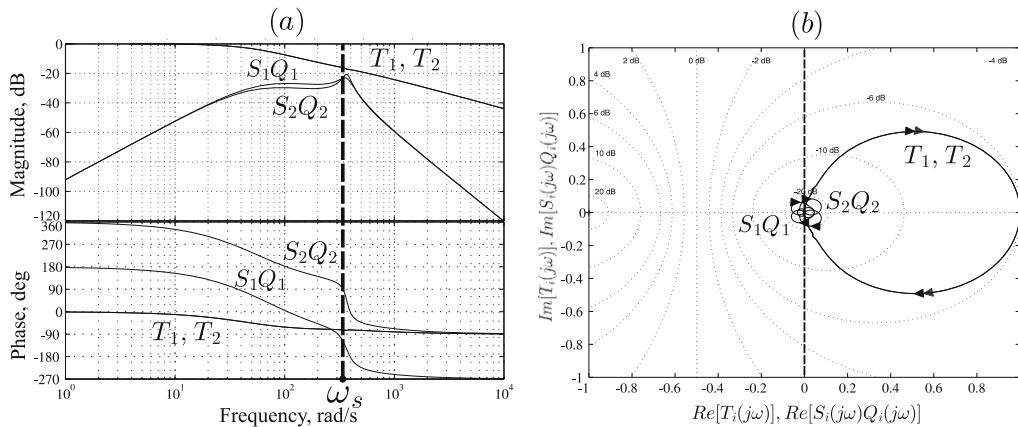


Fig. 5. Bode and Nyquist plots for  $T_i(s)$  and the cross-coupling terms  $S_i(s)Q_i(s)$  for system (15) with controllers (17)–(18) with  $f_r = 60$  Hz: (a) Bode plots; (b) Nyquist plots.

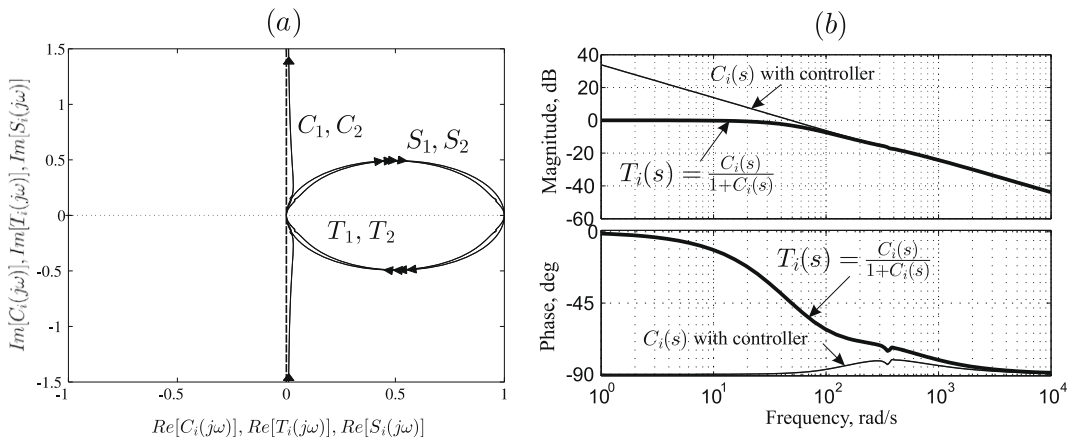
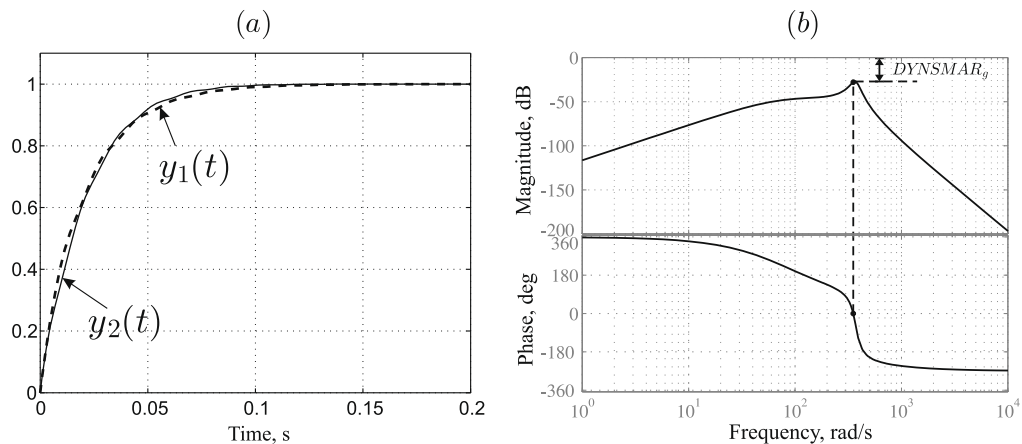


Fig. 6. (a) Nyquist plots of the passified  $C_i(s)$ ,  $T_i(s)$  and  $S_i(s)$  for system (15) with controllers (17)–(18) with  $f_r = 60$  Hz; (b) Bode plots of the passified  $C_i(s)$  (open-loop channels),  $T_i(s)$  (closed-loop channels) for system (15) with controllers (17)–(18) with  $f_r = 60$  Hz.

- Designs based on the comparison of several levels of hysteresis and a data switching table. These schemes generate additional harmonics and large control errors [38–41].
- Predictive control techniques. They require the measurement or estimation of the state and are not robust with respect to parameter variations [42–45].



**Fig. 7.** (a) Closed-loop outputs step-response of system (15) with  $f_r = 60$  Hz: (---)  $y_1(t)$ , (—)  $y_2(t)$ ; (b) Bode plots of  $\gamma(s)h_1(s)$  and  $\gamma(s)h_2(s)$ , for the passified channels for the induction motor with  $f_r = 60$  Hz.

- Proportional-integral-type (PI) controllers in stationary coordinates. In general, these types of controllers present large errors due to the fact that the references are sinusoidal signals. Their performance is limited due to the fact that their design is based on over-simplified models, and the multivariable aspect of the system is not considered [36–39,41,42,46–48].

A meaningful comparison of the different methods is out of the scope of this paper. Nevertheless, the approach proposed here allows:

- To consider the multivariable nature of the system.
- To apply well-known engineering methods and to assess robustness.
- To avoid unnecessary sophisticated controllers.
- To consider effective alternatives to low-performance PI designs.

Experimental results on the application of the framework here presented are currently being carried out.

## 5. Conclusions

A passification and stabilization method for MIMO systems which is based on the analysis and the design of SISO transfers functions has been proposed. This method is carried out under the ICAD framework and takes advantage of the appealing properties of the MSF-based design. The procedure presented is based on the passification of the input-output pairs in the open-loop, which is carried out by means of classical control methods of Nyquist and Bode plots shaping. The *practical passivity* of the closed-loop system is achieved by means of the shaping of the channels sensitivity functions.

The steps of the procedure are clearly established in order to be applied to a wide variety of MIMO systems. The conditions are not restrictive and the MSF-passivity-based diagonal controllers design gives rise to robust stability and good system performance, as well as easy-to-implement controllers. The passification and stabilization method has potential to be applied to systems with non-minimum-phase open-loop channels, by changing the controller and MSF structure, which is completely feasible due to the ICAD characteristics.

The passification and stabilization framework proposed can be applied to a wide range of engineering applications. A key aspect of the methodology proposed is that the channels are robustly passified and stabilised irrespective of the channels cross-coupling. For systems with weak cross-coupling between the channels, the MIMO system would be reduced to two SISO systems and the procedure would be simplified. Moreover, with minor variations in the control design, systems with delay could be considered, by being based on a frequency-domain representation and integrating the delay as a decrease of phase either in the Bode or the Nyquist plot.

Finally, although  $2 \times 2$  systems are dealt with in this paper, the analysis and control procedures under the ICAD framework are not restricted to  $2 \times 2$  systems. Its application to  $m \times m$  systems reduces to a nesting design procedure. The general case of  $m \times m$  systems has already been proposed and successfully applied to a wide variety of real-world applications.

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