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Newdistns: An R Package for New Families of Distributions

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Abstract

A new R contributed package written by the authors is introduced. This package computes the probability density function, cumulative distribution function, quantile function, random numbers and some measures of inference for nineteen families of distributions. Each family is flexible enough to encompass an uncountable number of structures. The use of the package is illustrated using a real data set. Also robustness of random number generation is checked by simulation.

Keywords: Cumulative distribution function, Probability density function, Quantile function, Random numbers.

1. Introduction

Let G be *any* valid cumulative distribution function defined on the real line. The last decade or so has seen many approaches proposed for generating new distributions based on G. All of these approaches can be put in the form

$$F(x) = B\left(G(x)\right), \tag{1}$$

where $B : [0, 1] \to [0, 1]$ and F is a valid cumulative distribution function. So, for every G one can use (1) to generate a new distribution.

The first approach of the kind of (1) proposed in recent years was that due to Marshall and Olkin (1997). In Marshall and Olkin (1997), B was taken to be $B(p) = \beta p / \left\{1 - (1 - \beta)p\right\}$ for $\beta > 0$. The distributions so generated using (1) will be referred to as Marshall Olkin G distributions. Since Marshall and Olkin (1997), many other approaches have been proposed. We mention: exponentiated G distributions due to Gupta, Gupta, and Gupta (1998), beta G distributions due to Eugene, Lee, and Famoye (2002), gamma G distributions due to Zografos and Balakrishnan (2009), Kumaraswamy G distributions due to Cordeiro and Castro

(2011), generalized beta G distributions due to Alexander, Cordeiro, Ortega, and Sarabia (2012), beta extended G distributions due to Cordeiro, Ortega, and Silva (2012b), gamma G distributions due to Ristić and Balakrishnan (2012), gamma uniform G distributions due to Torabi and Montazeri (2012), beta exponential G distributions due to Alzaatreh, Lee, and Famoye (2013b), Weibull G distributions also due to Alzaatreh *et al.* (2013b), log gamma G I distributions due to Amini, MirMostafaee, and Ahmadi (2013), log gamma G II distributions also due to Amini *et al.* (2013), exponentiated generalized G distributions due to Cordeiro, Ortega, and da Cunha (2013d), exponentiated Kumaraswamy G distributions due to Lemonte, Barreto-Souza, and Cordeiro (2013), geometric exponential Poisson G distributions due to Nadarajah, Cancho, and Ortega (2013a), truncated-exponential skew-symmetric G distributions due to Nadarajah, Nassiri, and Mohammadpo (2013b), modified beta G distributions due to Nadarajah, Teimouri, and Shih (2013c), and exponentiated exponential Poisson G distributions due to Ristić and Nadarajah (2013).

The nineteen approaches and the corresponding families of G distributions are the ones that we are aware of since 1997. Each of these family can be motivated by lifetime issues, as we shall see in Section 2. The applications of these G distributions have been widespread. A list of applications for each family of G distributions is given in Section 2.

The aim of this paper is to present a new contributed package for R (R Development Core Team 2014) that computes basic properties for *any* G distribution from each of the nineteen families. The properties considered include the probability density function, cumulative distribution function, quantile function, random numbers and measures inferred based on fitting the family of distributions to some data. Calling sequences for the computation of all of these properties are given in Section 2. Also given in Section 2 are explicit expressions for the probability density, cumulative distribution and quantile functions. The computation of the measures of inference is based on the package **AdequacyModel**. Illustrations of the practical use of the new R package are given in Section 3. Finally, the robustness of the routines for random number generation is checked by simulation in Section 4.

2. Families of distributions and R programs

Here, we list the nineteen families of G distributions and the corresponding calling sequences to compute the probability density function, cumulative distribution function, quantile function, random numbers and measures of inference when the family of distributions is fitted to some data. The latter include the following:

- Maximum likelihood estimates, standard deviations and 95 percent confidence intervals based on asymptotic normality;
- Akaike Information Criterion;
- Consistent Akaikes Information Criterion;
- Bayesian Information Criterion;
- Hannan-Quinn information criterion;
- Cramer-von Misses statistic;
- Anderson Darling statistic;

- Minimum value of the negative log-likelihood function;
- Kolmogorov Smirnov test statistic and its p-value;
- Convergence status - Algorithm converged or algorithm not converged.

The following format is used for the listing: the first line gives the expression for the probability density function; the second line gives the expression for the cumulative distribution function; the third line gives the expression for the quantile function; the fourth line gives the calling sequence for the probability density function; the fifth line gives the calling sequence for the cumulative distribution function; the sixth line gives the calling sequence for the quantile function; the seventh line gives the calling sequence for random number generation; the eighth line gives the calling sequence for the measures of inference.

The notation used for the calling sequences in the last five lines can be described as follows.

The `spec` (a character string) specifies the distribution corresponding to the probability density function, $g(\cdot)$, and the cumulative distribution function, $G(\cdot)$. The distribution should be one that is recognized by R. It could be one of the distributions implemented in the R base package or one of the distributions implemented in an R contributed package or one freshly written by a user. In any case, there should be functions `dspec`, `pspec` and `qspec`, computing the probability density function, cumulative distribution function and quantile function of the G distribution.

Some examples of `spec` are: `spec = "norm"`, meaning that $g(x) = (1/\sigma)\psi\left((x-\mu)/\sigma\right)$ and $G(x) = \Phi\left((x-\mu)/\sigma\right)$, where $\psi(\cdot)$ and $\Phi(\cdot)$ denote, respectively, the probability density function and the cumulative distribution function of a standard normal random variable; `spec = "lnorm"`, meaning that $g(x) = \{1/(\sigma x)\}\,\psi\left((\log x-\mu)/\sigma\right)$ and $G(x) = \Phi\left((\log x-\mu)/\sigma\right)$; `spec = "exp"`, meaning that $g(x) = \lambda\exp(-\lambda x)$ and $G(x) = 1-\exp(-\lambda x)$.

If `log = TRUE` then log of the probability density function will be returned. If `log.p = TRUE` then log of the cumulative distribution function will be returned and the quantile function will be computed for `exp(p)`. If `lower.tail = FALSE` then one minus the cumulative distribution function will be returned and the quantile function will be computed for `1 - p`. The code `n` denotes the number of random numbers to be generated.

Additional arguments in the form of `...` can be supplied for each calling sequence. These arguments could give inputs (e.g., parameter values) for the distribution specified by `spec`. For example, if `spec = "norm"` then `...` can be replaced by `mean = 1, sd = 1` to mean that $g(x) = \psi(x-1)$ and $G(x) = \Phi(x-1)$; if `spec = "lnorm"` then `...` can be replaced by `meanlog = 1, sdlog = 1` to mean that $g(x) = \psi(\log x - 1)$ and $G(x) = \Phi(\log x - 1)$; if `spec = "exp"` then `...` can be replaced by `rate = 1` to mean that $g(x) = \exp(-x)$ and $G(x) = 1-\exp(-x)$.

In the calling sequence for the measures of inference, `spec2` is limited to be one of the following fourteen distributions:

- Chi-square (`"chisq"`) with

$$g(x) = \Gamma^{-1}\left(\frac{r}{2}\right) 2^{-\frac{r}{2}} x^{\frac{r}{2}-1} \exp\left(-\frac{x}{2}\right)$$

  for $x > 0$ and $r > 0$.

- Exponential (`"exp"`) with

$$g(x) = r \exp(-rx)$$

for $x > 0$ and $r > 0$.

- F (`"f"`) with

$$g(x) == \frac{\Gamma\left(\frac{r+s}{2}\right)}{\Gamma(r/2)\Gamma(s/2)} \left(\frac{r}{s}\right)^{\frac{r}{2}} x^{\frac{r}{2}-1} \left(1 + \frac{r}{s}x\right)^{-\left(\frac{r+s}{2}\right)}$$

for $x > 0$, $r > 0$ and $s > 0$.

- Gamma (`"gamma"`) with

$$g(x) = \left[s^r \Gamma(r)\right]^{-1} x^{r-1} \exp\left(-\frac{x}{s}\right)$$

for $x > 0$, $r > 0$ and $s > 0$.

- Lognormal (`"lognormal"`) with

$$g(x) = \left(\sqrt{2\pi} s x\right)^{-1} \exp\left[-\frac{1}{2}\left(\frac{\log(x) - r}{s}\right)^2\right]$$

for $x > 0$, $r > 0$ and $s > 0$.

- Weibull (`"weibull"`) with

$$g(x) = \left(\frac{r}{s}\right)\left(\frac{x}{s}\right)^{r-1} \exp\left[-\left(\frac{x}{s}\right)^r\right]$$

for $x > 0$, $r > 0$ and $s > 0$.

- Burr XII (`"burrxii"`) with

$$g(x) = rsx^{s-1}\left(1 + x^s\right)^{-r-1}$$

for $x > 0$, $r > 0$ and $s > 0$.

- Chen (`"chen"`) with

$$g(x) = rsx^{r-1} \exp\left(x^r\right) \exp\left\{-s\left[\exp\left(x^r\right) - 1\right]\right\}$$

for $x > 0$, $r > 0$ and $s > 0$.

- Frechet (`"frechet"`) with

$$g(x) = rs^{-1} \exp\left[-\left(\frac{x}{s}\right)^{-r}\right]\left(\frac{x}{s}\right)^{-r-1}$$

for $x > 0$, $r > 0$ and $s > 0$.

- Gompertz (`"gompertz"`) with

$$g(x) = s \exp\left\{rx - \frac{s}{r}\left[\exp(rx) - 1\right]\right\}$$

for $x > 0$, $r > 0$ and $s > 0$.

- Linear failure rate (`"lfr"`) with

$$g(x) = (r + sx) \exp\left(-rx - \frac{sx^2}{2}\right)$$

for $x > 0$, $r > 0$ and $s > 0$.

- Log-logistic (`"log-logistic"`) with

$$g(x) = rs^{-r}x^{r-1}\left[\left(\frac{x}{s}\right)^r + 1\right]^{-2}$$

for $x > 0$, $r > 0$ and $s > 0$.

- Lomax (`"lomax"`) with

$$g(x) = rs\,(1 + rx)^{-(s+1)}$$

for $x > 0$, $r > 0$ and $s > 0$.

- Rayleigh (`"rayleigh"`) with

$$g(x) = 2rx \exp\left(-rx^2\right)$$

for $x > 0$ and $r > 0$.

Each of these distributions is defined on the positive real line and has one or two parameters. These distributions in fact include the most popular distributions for lifetime modeling. As we shall see the nineteen families of distributions can be motivated by lifetime issues.

In the calling sequence for the measures of inference, `data` must be a vector of data values for which the family of distributions is to be fitted. `starts` must be a vector of initial values for the parameters of the family of distributions and those of g. The vector must contain the initial values for the parameters of the family of distributions in the order specified by the calling sequence for the probability density function, and then the initial value for `r` if g has only one parameter. The vector must contain the initial values for the parameters of the family of distributions in the order specified by the calling sequence for the probability density function, then the initial value for `r` and then the initial value for `s` if g has two parameters. `method` is the method for optimizing the log likelihood function. It can be one of `"Nelder-Mead"`, `"BFGS"`, `"CG"`, `"L-BFGS-B"` or `"SANN"`. The default is `"BFGS"`. The option `"L-BFGS-B"` can be used only if each parameter specified by `starts` takes values on the positive real line. The details of these options can be found in the manual pages for `optim`.

For each of the nineteen families of G distributions, we now list motivation, particular members of the family studied in the literature and the applications they have received.

Beta exponential G distributions due to Alzaatreh *et al.* (2013b):

$$f(x) = \frac{\lambda}{B(a,b)} g(x) \left[1 - G(x)\right]^{\lambda b - 1} \left\{1 - \left[1 - G(x)\right]^{\lambda}\right\}^{a-1},$$
$$F(x) = 1 - I_{[1-G(x)]^{\lambda}} \left(\lambda(b-1) + 1, a\right),$$
$$F^{-1}(p) = G^{-1}\left(1 - \left\{I_{1-p}^{-1}\left(\lambda(b-1) + 1, a\right)\right\}^{1/\lambda}\right),$$

```
dbetaexpg(x, spec, lambda = 1, a = 1, b = 1, log = FALSE, ...),
pbetaexpg(x, spec, lambda = 1, a = 1, b = 1, log.p = FALSE, lower.tail = TRUE, ...),
qbetaexpg(p, spec, lambda = 1, a = 1, b = 1, log.p = FALSE, lower.tail = TRUE, ...),
rbetaexpg(n, spec, lambda = 1, a = 1, b = 1, ...),
mbetaexpg(spec2, data, starts, method = "BFGS")
```

for x in the range of g, $0 \leq p \leq 1$, $\lambda > 0$, the first shape parameter, $a > 0$, the second shape parameter, and $b > 0$, the third shape parameter, where $I_x(a,b) = \int_0^x t^{a-1}(1-t)^{b-1} dt / B(a,b)$ denotes the incomplete beta function ratio, $B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1} dt$ denotes the beta function, and $I_x^{-1}(a,b)$ denotes the inverse function of $I_x(a,b)$. The default values for λ, a and b are 1.

Beta extended G distributions due to Cordeiro *et al.* (2012b):

$$f(x) = \frac{\alpha g(x)}{B(a,b)} \left\{1 - \exp\left[-\alpha G(x)\right]\right\}^{a-1} \exp\left[-\alpha b G(x)\right],$$
$$F(x) = I_{1-\exp[-\alpha G(x)]}(a,b),$$
$$F^{-1}(p) = G^{-1}\left(-\frac{1}{\alpha} \log\left[1 - I_p^{-1}(a,b)\right]\right),$$

```
dbeg(x, spec, alpha = 1, a = 1, b = 1, log = FALSE, ...),
pbeg(x, spec, alpha = 1, a = 1, b = 1, log.p = FALSE, lower.tail = TRUE, ...),
qbeg(p, spec, alpha = 1, a = 1, b = 1, log.p = FALSE, lower.tail = TRUE, ...),
rbeg(n, spec, alpha = 1, a = 1, b = 1, ...),
mbeg(spec2, data, starts, method = "BFGS")
```

for x in the range of g, $0 \leq p \leq 1 - \exp(-\alpha)$, $\alpha > 0$, the scale parameter, $a > 0$, the first shape parameter, and $b > 0$, the second shape parameter. The default values for α, a and b are 1.

Beta extended G distributions have been used to model lifetimes of mechanical components (Cordeiro *et al.* 2012b).

Beta G distributions due to Eugene *et al.* (2002):

$$f(x) = \frac{1}{B(a,b)} g(x) \left[G(x)\right]^{a-1} \left[1 - G(x)\right]^{b-1},$$
$$F(x) = I_{G(x)}(a,b),$$
$$F^{-1}(p) = G^{-1}\left(I_p^{-1}(a,b)\right),$$

```
dbetag(x, spec, a = 1, b = 1, log = FALSE, ...),
pbetag(x, spec, a = 1, b = 1, log.p = FALSE, lower.tail = TRUE, ...),
```

```
qbetag(p, spec, a = 1, b = 1, log.p = FALSE, lower.tail = TRUE, ...),
rbetag(n, spec, a = 1, b = 1, ...),
mbetag(spec2, data, starts, method = "BFGS")
```

for x in the range of g, $0 \leq p \leq 1$, $a > 0$, the first shape parameter, and $b > 0$, the second shape parameter. The default values for a and b are 1.

These distributions were motivated to model the failure time of a a-out-of-$a + b - 1$ system when the failure times of the components are independent and identical random variables with cumulative distribution function G.

Particular beta G distributions studied in the literature include the beta Birnbaum-Saunders distribution (Cordeiro and Lemonte 2011a), the beta Burr III distribution (Gomes, da Silva, Cordeiro, and O 2013), the beta Burr XII distribution (Paranaba, Ortega, Cordeiro, and Pescim 2011), the beta Cauchy distribution (Alshawarbeh, Famoye, and Lee 2014), the beta Dagum distribution (Domma and Condino 2013), the beta exponential distribution (Nadarajah and Kotz 2006), the beta exponential geometric distribution (Bidram 2012; Nassar and Nada 2012), the beta exponentiated Pareto distribution (Zea, Silva, Bourguignon, Santos, and Cordeiro 2012), the beta exponentiated Weibull distribution (Cordeiro, Gomes, da Silva, and Ortega 2013c), the beta Frechet distribution (Barreto-Souza, Cordeiro, and Simas 2011), the beta gamma distribution (Kong, Carl, and Sepanski 2007), the beta generalized exponential distribution (Barreto-Souza, Santos, and Cordeiro 2010), the beta generalized gamma distribution (Cordeiro, Castellares, Montenegro, and de Castro 2013a), the beta generalized half normal geometric distribution (Ramires, Ortega, Cordeiro, and Hamedani 2013), the beta generalized Lindley distribution (Oluyede and Yang 2014), the beta generalized logistic distribution (Morais, Cordeiro, and Cysneiros 2013), the beta generalized normal distribution (Cintra, Rego, Cordeiro, and Nascimento 2014), the beta generalized Pareto distribution (Mahmoudi 2011; Nassar and Nada 2011), the beta generalized Rayleigh distribution (Cordeiro, Cristino, Hashimoto, and 2013b), the beta generalized Weibull distribution (Singla, Jain, and Sharma 2012), the beta Gompertz distribution (Jafari, Tahmasebi, and Alizadeh 2014), the beta Gumbel distribution (Nadarajah and Kotz 2004), the beta half-Cauchy distribution (Cordeiro and Lemonte 2011b), the beta inverse Rayleigh distribution (Leao, Saulo, Bourguignon, Cintra, Rego, and Cordeiro 2014), the beta inverse Weibull distribution (Hanook, Shahbaz, Mohsin, and Golam Kibria 2013), the beta linear failure rate distribution (Jafari and Mahmoudi 2014), the beta Laplace distribution (Kozubowski and Nadarajah 2008; Cordeiro and Lemonte 2011c), the beta Lindley distribution (Merovci and Sharma 2014), the beta lognormal distribution (Montenegro and Cordeiro 2013), the beta Lomax distribution (Rajab, Aleem, Nawaz, and Daniya 2013), the beta modified Weibull distribution (Silva, Ortega, and Cordeiro 2010), the beta Moyal distribution (Cordeiro, Nobre, Pescim, and Ortega 2014a), the beta Nakagami distribution (Shittu and Adepoju 2013), the beta normal distribution (Eugene *et al.* 2002), the beta Pareto distribution (Akinsete, Famoye, and 2008), the beta power distribution (Cordeiro and Brito 2012), the beta power exponential distribution (Adepoju, Chukwu, and Wang 2014), the beta skew normal distribution (Mameli and Musio 2013), the beta transmuted Weibull distribution (Pal and Tiensuwan 2014), the beta truncated Pareto distribution (Lourenzutti, Duarte, and Azevedo 2014), the beta Weibull geometric distribution (Cordeiro, Silva, and Ortega 2013e; Bidram, Behboodian, and Towhidi 2013), the beta Weibull Poisson distribution (Percontini, Blas, and Cordeiro 2013) and the beta weighted Weibull distribution (Idowu and Ikegwu 2013; Badmus and Bamiduro 2014).

Beta G distributions have been used to model: adult numbers for *Tribolium Castaneum* and *Tribolium Confusum* (Eugene *et al.* 2002; Kong *et al.* 2007); breaking strength of glass fibers (Barreto-Souza *et al.* 2010, 2011; Cordeiro and Lemonte 2011a; Cordeiro *et al.* 2013a; Domma and Condino 2013; Adepoju *et al.* 2014; Alshawarbeh *et al.* 2014); breaking stress of carbon fibers (Barreto-Souza *et al.* 2011; Cordeiro and Lemonte 2011a; Alshawarbeh *et al.* 2014; Leao *et al.* 2014; Oluyede and Yang 2014); carbon monoxide measurements in several brands of cigarettes (Cordeiro *et al.* 2013c); daily ozone level measurements in New York (Cordeiro *et al.* 2013e); exceedances of flood peaks of the Wheaton river in Yukon Territory, Canada (Akinsete *et al.* 2008; Mahmoudi 2011; Alshawarbeh *et al.* 2014; Cordeiro *et al.* 2014a); failure times of a polyster/viscose yarn in a textile experiment (Pal and Tiensuwan 2014); failure times of motorettes with a new insulation (Cordeiro *et al.* 2013c; Pal and Tiensuwan 2014); failure times of turbocharger of one type of engine (Singla *et al.* 2012); fatigue life of 6061-T6 aluminum coupons cut parallel with the direction of rolling (Mahmoudi 2011; Bidram 2012; Bidram *et al.* 2013); fatigue life of bearings of a certain type (Montenegro and Cordeiro 2013); flood data for the Floyd river located in James, Iowa, USA (Akinsete *et al.* 2008); household income and consumption in Italy (Domma and Condino 2013); lifetimes of mechanical components (Silva *et al.* 2010; Badmus and Bamiduro 2014; Jafari *et al.* 2014); maximum values of monthly flood rates of the Castelo river, Brazil (Lourenzutti *et al.* 2014); monthly actual taxes revenue in Egypt (Nassar and Nada 2011); national index of consumer prices of Brazil corresponding to health and personal care (Cordeiro and Lemonte 2011c); number of successive failures of the air-conditioning system of each number of a fleet of Boeing 720 jet airplanes (Nassar and Nada 2012; Bidram *et al.* 2013); remission times of a random sample of bladder cancer patients (Zea *et al.* 2012; Merovci and Sharma 2014; Oluyede and Yang 2014); repair times for an airborne communication transceiver (Cordeiro *et al.* 2013b; Percontini *et al.* 2013; Cordeiro *et al.* 2014a); SAR image processing (Cintra *et al.* 2014); short-term and long-term outcomes of constraint induced movement therapy after stroke (Nassar and Nada 2012); strength of ball bearings (Nassar and Nada 2012); stress-rupture life of kevlar epoxy strands subjected to constant sustained pressure (Cordeiro *et al.* 2013b); survival times of cutaneous melanoma (a type of malignant cancer) patients (Paranaba *et al.* 2011); survival times of guinea pigs injected with different doses of tubercle bacilli (Cordeiro and Lemonte 2011b; Merovci and Sharma 2014); survival times of myelogenous leukemia patients (Mahmoudi 2011); times to first failure of devices (Jafari and Mahmoudi 2014).

Exponentiated exponential Poisson G distributions due to Ristić and Nadarajah (2013):

$$f(x) = a\lambda \left\{1-\exp(-\lambda)\right\}^{-1} g(x) G^{a-1}(x) \exp\left[-\lambda G^a(x)\right],$$
$$F(x) = \left\{1-\exp(-\lambda)\right\}^{-1} \left\{1-\exp\left[-\lambda G^a(x)\right]\right\},$$
$$F^{-1}(p) = G^{-1}\left(\left[-\frac{1}{\lambda}\log\left\{1-p\left[1-\exp(-\lambda)\right]\right\}\right]^{1/a}\right),$$

```
deepg(x, spec, lambda = 1, a = 1, log = FALSE, ...),
peepg(x, spec, lambda = 1, a = 1, log.p = FALSE, lower.tail = TRUE, ...),
qeepg(p, spec, lambda = 1, a = 1, log.p = FALSE, lower.tail = TRUE, ...),
reepg(n, spec, lambda = 1, a = 1, ...),
meepg(spec2, data, starts, method = "BFGS")
```

for x in the range of g, $0 \leq p \leq 1$, $\lambda > 0$, the scale parameter, and $a > 0$, the shape parameter. The default values for λ and a are 1.

These distributions were motivated to model the time to failure of the first out of a Poisson number of systems functioning independently where each system has a fixed number of parallel units and their failure times are independent and identical random variables with cumulative distribution function G. These distributions have been used to model the daily average air temperature (F) in Cairo (Ristić and Nadarajah 2013).

Exponentiated generalized G distributions due to Cordeiro *et al.* (2013d):

$$f(x) = abg(x)\left[1 - G(x)\right]^{a-1}\left\{1 - \left[1 - G(x)\right]^{a}\right\}^{b-1},$$
$$F(x) = \left\{1 - \left[1 - G(x)\right]^{a}\right\}^{b},$$
$$F^{-1}(p) = G^{-1}\left(1 - \left(1 - p^{1/b}\right)^{1/a}\right),$$

```
deg(x, spec, a = 1, b = 1, log = FALSE, ...),
peg(x, spec, a = 1, b = 1, log.p = FALSE, lower.tail = TRUE, ...),
qeg(p, spec, a = 1, b = 1, log.p = FALSE, lower.tail = TRUE, ...),
reg(n, spec, a = 1, b = 1, ...),
meg(spec2, data, starts, method = "BFGS")
```

for x in the range of g, $0 \leq p \leq 1$, $a > 0$, the first shape parameter, and $b > 0$, the second shape parameter. The default values for a and b are 1.

These distributions were motivated to model the failure of time of a system having b units functioning in parallel and each of these units have a subunits functioning in series. The failure times of the subunits are assumed to be independent and identical with cumulative distribution function G.

Particular exponentiated generalized G distributions studied in the literature include the exponentiated generalized Birnbaum-Saunders distribution (Cordeiro and Lemonte 2014).

Exponentiated generalized G distributions have been used to model: breaking stress of carbon fibers (Cordeiro *et al.* 2013d); effects of mechanical damage on banana fruits (Cordeiro *et al.* 2013d); exceedances of flood peaks of the Wheaton river near Carcross in Yukon Territory, Canada (Cordeiro *et al.* 2013d; Cordeiro and Lemonte 2014); lifetimes for industrial devices put on life test at time zero (Cordeiro and Lemonte 2014); stress-rupture life of kevlar epoxy strands subjected to constant sustained pressure (Cordeiro *et al.* 2013d).

Exponentiated G distributions due to Gupta *et al.* (1998):

$$f(x) = ag(x)G^{a-1}(x),$$
$$F(x) = G^{a}(x),$$
$$F^{-1}(p) = G^{-1}\left(p^{1/a}\right),$$

```
dexpg(x, spec, a = 1, log = FALSE, ...),
pexpg(x, spec, a = 1, log.p = FALSE, lower.tail = TRUE, ...),
qexpg(p, spec, a = 1, log.p = FALSE, lower.tail = TRUE, ...),
rexpg(n, spec, a = 1, ...),
mexpg(spec2, data, starts, method = "BFGS")
```

for x in the range of g, $0 \leq p \leq 1$ and $a > 0$, the shape parameter. The default value for a is 1.

These distributions were motivated to model the failure of time of a system having a units functioning in parallel the failure times of which are assumed to be independent and identical with cumulative distribution function G.

Particular exponentiated G distributions studied in the literature include the exponentiated Frechet distribution (Nadarajah and Kotz 2003), the exponentiated gamma distribution (Nadarajah and Gupta 2007), the exponentiated generalized inverse Weibull distribution (Elbatal and Muhammed 2014), the exponentiated Gumbel distribution (Nadarajah 2006), the exponentiated Lomax distribution (Abdul-Moniem and Abdel-Hameed 2012; Salem 2014), the exponentiated Pareto distribution (Shawky and Abu-Zinadah 2009) and the exponentiated transmuted Weibull distribution (Hady and Ebraheim 2014).

Exponentiated G distributions have been used to model: annual maximum daily rainfall from Orlando, Florida (Nadarajah 2006); drought data from Nebraska (Nadarajah and Gupta 2007); remission times of a random sample of bladder cancer patients (Elbatal and Muhammed 2014).

Exponentiated Kumaraswamy G distributions due to Lemonte *et al.* (2013):

$$f(x) = abcg(x)G^{a-1}(x)\left[1-G^a(x)\right]^{b-1}\left\{1-\left[1-G^a(x)\right]^b\right\}^{c-1},$$

$$F(x) = \left\{1-\left[1-G^a(x)\right]^b\right\}^c,$$

$$F^{-1}(p) = G^{-1}\left(\left\{1-\left[1-p^{1/c}\right]^{1/b}\right\}^{1/a}\right),$$

```
dexpkumg(x, spec, a = 1, b = 1, c = 1, log = FALSE, ...),
pexpkumg(x, spec, a = 1, b = 1, c = 1, log.p = FALSE, lower.tail = TRUE, ...),
qexpkumg(p, spec, a = 1, b = 1, c = 1, log.p = FALSE, lower.tail = TRUE, ...),
rexpkumg(n, spec, a = 1, b = 1, c = 1, ...),
mexpkumg(spec2, data, starts, method = "BFGS")
```

for x in the range of g, $0 \leq p \leq 1$, $a > 0$, the first shape parameter, $b > 0$, the second shape parameter, and $c > 0$, the third shape parameter. The default values for a, b and c are 1.

These distributions were motivated to model the failure of time of a system having c units functioning in parallel and each of these units have b subunits functioning in series and each of these subunits have a subsubunits functioning in parallel. The failure times of the subsubunits are assumed to be independent and identical with cumulative distribution function G. These distributions have been applied model lifetimes (Lemonte *et al.* 2013).

Gamma G I distributions due to Zografos and Balakrishnan (2009):

$$f(x) = \frac{1}{\Gamma(a)}g(x)\left\{-\log\left[1-G(x)\right]\right\}^{a-1},$$

$$F(x) = Q\left(a, -\log\left[1-G(x)\right]\right),$$

$$F^{-1}(p) = G^{-1}\left(1-\exp\left[-Q^{-1}(a,p)\right]\right),$$

```
dgammag1(x, spec, a = 1, log = FALSE, ...),
pgammag1(x, spec, a = 1, log.p = FALSE, lower.tail = TRUE, ...),
qgammag1(p, spec, a = 1, log.p = FALSE, lower.tail = TRUE, ...),
```

```
rgammag1(n, spec, a = 1, ...),
mgammag1(spec2, data, starts, method = "BFGS")
```

for x in the range of g, $0 \leq p \leq 1$, and $a > 0$, the shape parameter, where $Q(a,x) = \int_0^x t^{a-1} \exp(-t)dt/\Gamma(a)$ denotes the regularized incomplete gamma function, $\Gamma(a) = \int_0^\infty t^{a-1} \exp(-t)dt$ denotes the gamma function, and $Q^{-1}(a,x)$ denotes the inverse function of $Q(a,x)$. The default value for a is 1.

These distributions were constructed as the distribution of the ath upper record value for a random sample from the cumulative distribution function G.

Particular gamma G I distributions studied in the literature include the gamma Dagum distribution (Oluyede, Huang, and Pararai 2014), the gamma exponentiated Weibull distribution (Castellares and Lemonte 2014), the gamma extended Frechet distribution (da Silva, de Andrade, Maciel, Ca 2013), the gamma half normal distribution (Alzaatreh and Knight 2013), the gamma inverse Weibull distribution (Pararai, Warahena-Liyanage, and Oluyede 2014), the gamma linear failure rate distribution (Cordeiro, Ortega, and Popovic 2014b), the gamma log-logistic distribution (Ramos, Cordeiro, Marinho, Dias, and Hamedani 2013), the gamma logistic distribution (Castellares, Santos, Montenegro, and Cordeiro 2015), the gamma Lomax distribution (Cordeiro, Ortega, and Popovic 2015) and the gamma normal distribution (Alzaatreh, Famoye, and Lee 2014).

Gamma G distributions have been used to model: breaking stress of carbon fibers (Alzaatreh *et al.* 2014; Cordeiro *et al.* 2014b); flood levels for the Susquehanna river at Harrisburg, PA (Alzaatreh and Knight 2013); gene expression levels on human cancer cells (Castellares *et al.* 2015); number of million of revolutions before failure of ball bearings in a life testing experiment (Pararai *et al.* 2014); number of successive failures for the air conditioning system of each member in a fleet of Boeing 720 jet airplanes (Oluyede *et al.* 2014); remission times of a random sample of bladder cancer patients (Cordeiro *et al.* 2015; Oluyede *et al.* 2014; Castellares and Lemonte 2014); salaries of professional baseball players (Oluyede *et al.* 2014); strengths of glass fibers (Alzaatreh *et al.* 2014); survival times of breast cancer patients (Ramos *et al.* 2013); survival times of cutaneous melanoma (a type of malignant cancer) patients (Cordeiro *et al.* 2014b); survival times of guinea pigs injected with different doses of tubercle bacilli (Pararai *et al.* 2014); tensile strength for single-carbon fibers (Alzaatreh and Knight 2013); the cDNA microarray data of the NC160 cancer cell lines (Castellares *et al.* 2015); waiting times between consecutive eruptions of the Kiama Blowhole (da Silva *et al.* 2013).

Gamma G II distributions due to Ristić and Balakrishnan (2012):

$$f(x) = \frac{1}{\Gamma(a)} g(x) \left\{ -\log G(x) \right\}^{a-1},$$
$$F(x) = 1 - Q\left(a, -\log G(x)\right),$$
$$F^{-1}(p) = G^{-1}\left(\exp\left[-Q^{-1}(a, 1-p)\right]\right),$$

```
dgammag2(x, spec, a = 1, log = FALSE, ...),
pgammag2(x, spec, a = 1, log.p = FALSE, lower.tail = TRUE, ...),
qgammag2(p, spec, a = 1, log.p = FALSE, lower.tail = TRUE, ...),
rgammag2(n, spec, a = 1, ...),
mgammag2(spec2, data, starts, method = "BFGS")
```

for x in the range of g, $0 \leq p \leq 1$, and $a > 0$, the shape parameter. The default value for a

is 1.

These distributions were constructed as the distribution of the ath lower record value for a random sample from the cumulative distribution function G.

Particular gamma G II distributions studied in the literature include the gamma exponential distribution (Ristić and Balakrishnan 2012) and the gamma exponentiated Weibull distribution (Pinho, Cordeiro, and Nobre 2012).

Gamma G distributions have been used to model: annual maximum precipitation for one rain gauge in Fort Collins, Colorado (Ristić and Balakrishnan 2012); daily minimum wind speed at the Midwest ISO area in the USA (Pinho *et al.* 2012); survival times of guinea pigs which received a dose of tubercle bacilli (Ristić and Balakrishnan 2012); the number of successive failures of the air conditioning system of a fleet of Boeing 720 jet airplanes (Ristić and Balakrishnan 2012).

Gamma uniform G distributions due to Torabi and Montazeri (2012):

$$f(x) = \frac{1}{\Gamma(a)} \frac{g(x)}{[1-G(x)]^2} \left[\frac{G(x)}{1-G(x)}\right]^{a-1} \exp\left[-\frac{G(x)}{1-G(x)}\right],$$

$$F(x) = Q\left(a, \frac{G(x)}{1-G(x)}\right),$$

$$F^{-1}(p) = G^{-1}\left(\frac{Q^{-1}(a,p)}{1+Q^{-1}(a,p)}\right),$$

```
dgammag(x, spec, a = 1, log = FALSE, ...),
pgammag(x, spec, a = 1, log.p = FALSE, lower.tail = TRUE, ...),
qgammag(p, spec, a = 1, log.p = FALSE, lower.tail = TRUE, ...),
rgammag(n, spec, a = 1, ...),
mgammag(spec2, data, starts, method = "BFGS")
```

for x in the range of g, $0 \le p \le 1$, and $a > 0$, the shape parameter. The default value for a is 1.

These distributions were constructed by considering the distribution of $G^{-1}\left(W/(1+W)\right)$, where W is a gamma random variable. These distributions have been used to model survival times of leukemia patients (Torabi and Montazeri 2012).

Generalized beta G distributions due to Alexander *et al.* (2012):

$$f(x) = \frac{c}{B(a,b)} g(x) G^{ac-1}(x) \left[1-G^c(x)\right]^{b-1},$$

$$F(x) = I_{G^c(x)}(a,b),$$

$$F^{-1}(p) = G^{-1}\left(\left[I_p^{-1}(a,b)\right]^{1/c}\right),$$

```
dgbg(x, spec, a = 1, b = 1, c = 1, log = FALSE, ...),
pgbg(x, spec, a = 1, b = 1, c = 1, log.p = FALSE, lower.tail = TRUE, ...),
qgbg(p, spec, a = 1, b = 1, c = 1, log.p = FALSE, lower.tail = TRUE, ...),
rgbg(n, spec, a = 1, b = 1, c = 1, ...),
mgbg(spec2, data, starts, method = "BFGS")
```

for x in the range of g, $0 \leq p \leq 1$, $a > 0$, the first shape parameter, $b > 0$, the second shape parameter, and $c > 0$, the third shape parameter. The default values for a, b and c are 1.

Particular generalized beta G distributions studied in the literature include the generalized beta exponentiated Pareto distribution (Mead 2014), the generalized beta gamma distribution (Marciano, Nascimento, Santos-Neto, and Cordeiro 2012) and the generalized beta log-logistic distribution (Tahir, Mansoor, Zubair, and Hamedani 2014).

Generalized beta G distributions have been used to model: effects of mechanical damage on banana fruits (Alexander *et al.* 2012); exceedances of flood peaks of the Wheaton river near Carcross in Yukon Territory, Canada (Mead 2014); monthly actual taxes revenue in Egypt (Mead 2014); survival times of breast cancer patients (Tahir *et al.* 2014); times of failure and running times for a sample of devices from a field-tracking study of a larger system (Alexander *et al.* 2012); times of unscheduled maintenance actions for the USS Halfbeak number 4 main propulsion diesel engine (Marciano *et al.* 2012).

Geometric exponential Poisson G distributions due to Nadarajah *et al.* (2013a):

$$f(x) = \frac{\theta(1-\eta)\left[1-\exp(-\theta)\right] g(x) \exp\left[-\theta+\theta G(x)\right]}{\left\{1-\exp(-\theta)-\eta+\eta\exp\left[-\theta+\theta G(x)\right]\right\}^2},$$

$$F(x) = \frac{\exp\left[-\theta+\theta G(x)\right]-\exp(-\theta)}{1-\exp(-\theta)-\eta+\eta\exp\left[-\theta+\theta G(x)\right]},$$

$$F^{-1}(p) = G^{-1}\left[\frac{1}{\theta}\log\left\{\frac{\left[1-\exp(-\theta)-\eta\right]p+\exp(-\theta)}{(1-\eta p)\exp(-\theta)}\right\}\right],$$

```
dgepg(x, spec, theta = 1, eta = 0.5, log = FALSE, ...),
pgepg(x, spec, theta = 1, eta = 0.5, log.p = FALSE, lower.tail = TRUE, ...),
qgepg(p, spec, theta = 1, eta = 0.5, log.p = FALSE, lower.tail = TRUE, ...),
rgepg(n, spec, theta = 1, eta = 0.5, ...),
mgepg(spec2, data, starts, method = "BFGS")
```

for x in the range of g, $0 \leq p \leq 1$, $\theta > 0$, the first scale parameter, and $0 < \eta < 1$, the second scale parameter. The default values for θ and η are 1 and 0.5, respectively.

These distributions were motivated to model the time to failure of the first out of a geometric number of systems functioning independently where each system has a Poisson number of parallel units and their failure times are independent and identical random variables with cumulative distribution function G. These distributions have been used to model adult numbers of *Tribolium Confusum* and failure times for epoxy insulation specimens in an accelerated voltage life test (Nadarajah *et al.* 2013a).

Kumaraswamy G distributions due to Cordeiro and Castro (2011):

$$f(x) = abg(x)G^{a-1}(x)\left[1-G^a(x)\right]^{b-1},$$

$$F(x) = 1-\left[1-G^a(x)\right]^b,$$

$$F^{-1}(p) = G^{-1}\left(1-\left[1-(1-p)^{1/b}\right]^{1/a}\right),$$

```
dkumg(x, spec, a = 1, b = 1, log = FALSE, ...),
pkumg(x, spec, a = 1, b = 1, log.p = FALSE, lower.tail = TRUE, ...),
qkumg(p, spec, a = 1, b = 1, log.p = FALSE, lower.tail = TRUE, ...),
```

```
rkumg(n, spec, a = 1, b = 1, ...),
mkumg(spec2, data, starts, method = "BFGS")
```

for x in the range of g, $0 \leq p \leq 1$, $a > 0$, the first shape parameter, and $b > 0$, the second shape parameter. The default values for a and b are 1.

These distributions were motivated to model the failure of time of a system having b units functioning in series and each of these units have a subunits functioning in parallel. The failure times of the subunits are assumed to be independent and identical with cumulative distribution function G.

Particular Kumaraswamy G distributions studied in the literature include the Kumaraswamy Birnbaum-Saunders distribution (Saulo, Leao, and Bourguignon 2012), the Kumaraswamy Burr XII distribution (Paranaiba, Ortega, Cordeiro, and de Pascoa 2013), the Kumaraswamy exponentiated Pareto distribution (Elbatal 2013b), the Kumaraswamy generalized exponentiated Pareto distribution (Shams 2013a), the Kumaraswamy generalized gamma distribution (de Pascoa, Ortega, and Cordeiro 2011), the Kumaraswamy generalized half normal distribution (Cordeiro, Pescim, and Ortega 2012c), the Kumaraswamy generalized linear failure rate distribution (Elbatal 2013a), the Kumaraswamy generalized Lomax distribution (Shams 2013b), the Kumaraswamy generalized Pareto Distribution (Nadarajah and Eljabri 2013), the Kumaraswamy generalized Rayleigh distribution (Gomes, da Silva, Cordeiro, and Ortega 2014), the Kumaraswamy geometric distribution (Akinsete, Famoye, and Lee 2014), the Kumaraswamy Gumbel distribution (Cordeiro, Nadarajah, and Ortega 2012a), the Kumaraswamy half-Cauchy distribution (Ghosh 2014), the Kumaraswamy inverse exponential distribution (Oguntunde, Babatunde, and Ogunmola 2014), the Kumaraswamy inverse Rayleigh distribution (Roges, de Gusmao, and Diniz 2014), the Kumaraswamy inverse Weibull distribution (Shahbaz, Shahbaz, and Butt 2012), the Kumaraswamy Kumaraswamy distribution (El-Sherpieny and Ahme 2014), the Kumaraswamy Lindley distribution (Cakmakyapan and Kadilar 2014), the Kumaraswamy log-logistic distribution (de Santana, Ortega, Cordeiro, and Silva 2012), the Kumaraswamy modified inverse Weibull distribution (Aryal and Elbata 2015), the Kumaraswamy modified Weibull distribution (Cordeiro, Ortega, and Silva 2014c), the Kumaraswamy Pareto distribution (Bourguignon, Silva, Zea, and Cordeiro 2013), the Kumaraswamy quasi Lindley distribution (Elbatal and Elgarhy 2013) and the Kumaraswamy Weibull distribution (Cordeiro, Ortega, and N 2010).

Kumaraswamy G distributions have been used to model: breaking strengths of glass fibers (Paranaiba *et al.* 2013); breaking strengths of polyster/viscose yarns (Aryal and Elbata 2015); breaking stress of carbon fibers (Shams 2013b,a); carbon monoxide levels from several cigarette brands (Gomes *et al.* 2014); exceedances by the river Nidd at Hunsingore Weir (Nadarajah and Eljabri 2013); exceedances of flood peaks of the Wheaton river near Carcross in Yukon Territory, Canada (Bourguignon *et al.* 2013); failure times for epoxy insulation specimens (Gomes *et al.* 2014); failure times of mechanical components (Cordeiro *et al.* 2012c); flood data for the Floyd river located in James, Iowa, USA (Cordeiro *et al.* 2012c); flood discharge of at least seven consecutive days and return period of 10 years in the Brazilian Pantanal (Cordeiro *et al.* 2012a); frequencies of the purchases of a brand X breakfast cereals (Akinsete *et al.* 2014); lifetimes of industrial devices put on life test at time zero (de Pascoa *et al.* 2011; Cordeiro *et al.* 2014c); number of absences among shift-workers in a steel industry (Akinsete *et al.* 2014); stress-rupture life of kevlar epoxy strands subjected to constant sustained pressure (Paranaiba *et al.* 2013); survival times of cutaneous melanoma (a type of malignant cancer) patients (de Santana *et al.*

2012); survival times of guinea pigs injected with different doses of tubercle bacilli (Cordeiro *et al.* 2012c); survival times of patients given radiation therapy and radiation plus chemotherapy (Cordeiro *et al.* 2014c); the number of millions revolutions reached by ball bearings before fatigue failure (Ghosh 2014); times of failure and running times of devices from a field-tracking study of a larger system (Cordeiro *et al.* 2010); times to serum reversal of children exposed to HIV by vertical transmission (de Pascoa *et al.* 2011; de Santana *et al.* 2012; Paranaiba *et al.* 2013); times until bulls reach the weight of 160kg since birth (Roges *et al.* 2014).

Log gamma G I distributions due to Amini *et al.* (2013):

$$f(x) = \frac{b^a}{\Gamma(a)} g(x) \left\{ -\log \left[1 - G(x) \right] \right\}^{a-1} \left[1 - G(x) \right]^{b-1},$$

$$F(x) = 1 - Q\left(a, -b \log \left[1 - G(x) \right] \right),$$

$$F^{-1}(p) = G^{-1}\left(1 - \exp\left[-\frac{1}{b} Q^{-1}(a, 1-p) \right] \right),$$

```
dloggammag1(x, spec, a = 1, b = 1, log = FALSE, ...),
ploggammag1(x, spec, a = 1, b = 1, log.p = FALSE, lower.tail = TRUE, ...),
qloggammag1(p, spec, a = 1, b = 1, log.p = FALSE, lower.tail = TRUE, ...),
rloggammag1(n, spec, a = 1, b = 1, ...),
mloggammag1(spec2, data, starts, method = "BFGS")
```

for x in the range of g, $0 \leq p \leq 1$, $a > 0$, the first shape parameter, and $b > 0$, the second shape parameter. The default values for a and b are 1.

These distributions were motivated as the distribution of the ath upper b-record value for a random sample from the cumulative distribution function G. They have been applied to model weekly earnings of full-time wage and salary workers from the US Bureau of Labor Statistics (Amini *et al.* 2013).

Log gamma G II distributions also due to Amini *et al.* (2013):

$$f(x) = \frac{b^a}{\Gamma(a)} g(x) \left\{ -\log G(x) \right\}^{a-1} G^{b-1}(x),$$

$$F(x) = Q\left(a, -b \log G(x) \right),$$

$$F^{-1}(p) = G^{-1}\left(\exp\left[-\frac{1}{b} Q^{-1}(a, p) \right] \right),$$

```
dloggammag2(x, spec, a = 1, b = 1, log = FALSE, ...),
ploggammag2(x, spec, a = 1, b = 1, log.p = FALSE, lower.tail = TRUE, ...),
qloggammag2(p, spec, a = 1, b = 1, log.p = FALSE, lower.tail = TRUE, ...),
rloggammag2(n, spec, a = 1, b = 1, ...),
mloggammag2(spec2, data, starts, method = "BFGS")
```

for x in the range of g, $0 \leq p \leq 1$, $a > 0$, the first shape parameter, and $b > 0$, the second shape parameter. The default values for a and b are 1.

These distributions were motivated as the distribution of the ath lower b-record value for a random sample from the cumulative distribution function G. They have been applied to model weekly earnings of full-time wage and salary workers from the US Bureau of Labor Statistics (Amini *et al.* 2013).

Marshall Olkin G distributions due to Marshall and Olkin (1997):

$$f(x) = \frac{\beta g(x)}{\left[\beta + (1-\beta)G(x)\right]^2},$$
$$F(x) = \frac{G(x)}{\beta + (1-\beta)G(x)},$$
$$F^{-1}(p) = G^{-1}\left(\frac{\beta p}{1-(1-\beta)p}\right),$$

```
dmog(x, spec, beta = 1, log = FALSE, ...),
pmog(x, spec, beta = 1, log.p = FALSE, lower.tail = TRUE, ...),
qmog(p, spec, beta = 1, log.p = FALSE, lower.tail = TRUE, ...),
rmog(n, spec, beta = 1, ...),
mmog(spec2, data, starts, method = "BFGS")
```

for x in the range of g, $0 \leq p \leq 1$ and $\beta > 0$, the scale parameter. The default value for β is 1.

These distributions were motivated to model the time to failure of the first out of a geometric number of systems. The failure times of the systems are independent and identical random variables with cumulative distribution function G.

Particular Marshall Olkin G distributions studied in the literature include the Marshall-Olkin asymmetric Laplace distribution (Krishna and Jose 2011), the Marshall-Olkin beta distribution (Jose, Joseph, and Ristic 2009), the Marshall-Olkin Birnbaum-Saunders distribution (Lemonte 2013), the Marshall-Olkin Burr type XII distribution (Al-Saiari, Baharith, and Mousa 2014), the Marshall-Olkin discrete uniform distribution (Sandhya and Prasanth 2014), the Marshall-Olkin Frechet distribution (Krishna, Jose, Alice, and Ristic 2013), the Marshall-Olkin gamma distribution (Ristic, Jose, and Ancy 2007), the Marshall-Olkin Laplace distribution (George and George 2013), the Marshall-Olkin Lindley distribution (Zakerzadeh and Mahmoudi 2012), the Marshall-Olkin log-logistic distribution (Gui 2013b), the Marshall-Olkin Lomax distribution (Ghitany, Al-Awadhi, and Alkhalfan 2007), the Marshall-Olkin Morgenstern Weibull distribution (Jose and Sebastian 2013), the Marshall-Olkin q-Weibull distribution (Jose, Naik, and Ristic 2010), the Marshall-Olkin Weibull distribution (Ghitany, Al-Hussaini, and Al-Jarallah 2005), the Marshall-Olkin uniform distribution (Jose and Krishna 2011) and the Marshall-Olkin Zipf distribution (Perez-Casany and Casellas 2014).

Marshall Olkin G distributions have been used to model: daily ozone measurements in New York (Jose *et al.* 2009); daily weighted discharge of Neyyar river in Kerala (Jose *et al.* 2010); frequency of occurrence of words in the novel Moby Dick by Herman Melville (Perez-Casany and Casellas 2014); length of time until a breakdown is recorded in electrical insulating (Al-Saiari *et al.* 2014); number of connections of a total of 225409 electronic mail addresses (Perez-Casany and Casellas 2014); number of days students attended a class for the whole year (Sandhya and Prasanth 2014); number of miles to first and succeeding major motor failures of buses operated by a large city bus company (Gui 2013a); number of times that a given paper is cited in a given database (Perez-Casany and Casellas 2014); permeability values from horizons of the Dominquez field of Southern California (Jose *et al.* 2009); remission times of a random sample of bladder cancer patients (Ghitany *et al.* 2005, 2007); survival times of guinea pigs injected with different doses of tubercle bacilli (Krishna *et al.* 2013); vinyl chloride data obtained from clean up gradient monitoring wells (Zakerzadeh and Mahmoudi 2012); waiting times before service of bank customers (Zakerzadeh and Mahmoudi 2012).

Modified beta G distributions due to Nadarajah *et al.* (2013c):

$$f(x) = \frac{\beta^a}{B(a,b)} \frac{g(x)\left[G(x)\right]^{a-1}\left[1-G(x)\right]^{b-1}}{\left[1-(1-\beta)G(x)\right]^{a+b}},$$

$$F(x) = I_{\beta G(x)/\{1+(\beta-1)G(x)\}}(a,b),$$

$$F^{-1}(p) = G^{-1}\left(\frac{I_p^{-1}(a,b)}{\beta-(\beta-1)I_p^{-1}(a,b)}\right),$$

```
dmbetag(x, spec, beta = 1, a = 1, b = 1, log = FALSE, ...),
pmbetag(x, spec, beta = 1, a = 1, b = 1, log.p = FALSE, lower.tail = TRUE, ...),
qmbetag(p, spec, beta = 1, a = 1, b = 1, log.p = FALSE, lower.tail = TRUE, ...),
rmbetag(n, spec, beta = 1, a = 1, b = 1, ...),
mmbetag(spec2, data, starts, method = "BFGS")
```

for x in the range of g, $0 \leq p \leq 1$, $\beta > 0$, the scale parameter, $a > 0$, the first shape parameter, and $b > 0$, the second shape parameter. The default values for β, a and b are 1.

These distributions were constructed by combining beta G distributions due to Eugene *et al.* (2002) with Marshall Olkin G distributions due to (Marshall and Olkin 1997). They have been applied to model S & P / IFC (Standard & Poor's / International Finance Corporation) global daily price indices in United States dollars for South Africa (Nadarajah *et al.* 2013c).

Truncated-exponential skew-symmetric G distributions due to Nadarajah *et al.* (2013b):

$$f(x) = \frac{\lambda}{1-\exp(-\lambda)} g(x) \exp\left\{-\lambda G(x)\right\},$$

$$F(x) = \frac{1-\exp\left\{-\lambda G(x)\right\}}{1-\exp(-\lambda)},$$

$$F^{-1}(p) = G^{-1}\left(-\frac{1}{\lambda}\log\left\{1-p\left(1-\exp(-\lambda)\right)\right\}\right),$$

```
dtessg(x, spec, lambda = 1, log = FALSE, ...),
ptessg(x, spec, lambda = 1, log.p = FALSE, lower.tail = TRUE, ...),
qtessg(p, spec, lambda = 1, log.p = FALSE, lower.tail = TRUE, ...),
rtessg(n, spec, lambda = 1, ...),
mtessg(spec2, data, starts, method = "BFGS")
```

for x in the range of g, $0 \leq p \leq 1$, and $-\infty < \lambda < \infty$, the skewness parameter. The default value for λ is 1.

These distributions were constructed as modifications of Azzalini (Azzalini 1985)'s skew-symmetric distributions. They have been used to model annual maximum daily rainfall data for 14 locations in west central Florida: Clermont, Brooksville, Orlando, Bartow, Avon Park, Arcadia, Kissimmee, Inverness, Plant City, Tarpon Springs, Tampa International Airport, St Leo, Gainesville and Ocala (Nadarajah *et al.* 2013b).

Weibull G distributions also due to Alzaatreh *et al.* (2013b):

$$f(x) = \frac{c}{\beta^c} \frac{g(x)}{1 - G(x)} \left\{ -\frac{\log\left[1 - G(x)\right]}{\beta} \right\}^{c-1} \exp\left\{ - \left[-\frac{\log\left[1 - G(x)\right]}{\beta} \right]^c \right\},$$

$$F(x) = 1 - \exp\left\{ \left[-\frac{\log\left[1 - G(x)\right]}{\beta} \right]^c \right\},$$

$$F^{-1}(p) = G^{-1}\left(1 - \exp\left\{ -\beta \left[-\log(1-p) \right]^{1/c} \right\} \right),$$

```
dweibullg(x, spec, beta = 1, c = 1, log = FALSE, ...),
pweibullg(x, spec, beta = 1, c = 1, log.p = FALSE, lower.tail = TRUE, ...),
qweibullg(p, spec, beta = 1, c = 1, log.p = FALSE, lower.tail = TRUE, ...),
rweibullg(n, spec, beta = 1, c = 1, ...),
mweibullg(spec2, data, starts, method = "BFGS")
```

for x in the range of g, $0 \le p \le 1$, $\beta > 0$, the scale parameter, and $c > 0$, the shape parameter. The default values for β and c are 1.

Particular Weibull G distributions studied in the literature include the Weibull exponentiated exponential distribution (Salem and Selim 2014) and the Weibull Pareto distribution (Alzaatreh, Famoye, and Lee 2013a).

Weibull G distributions have been used to model: adult numbers for *Tribolium Confusum* and *Tribolium Castaneum* cultured at 24C and *Tribolium Confusum* strain (Alzaatreh *et al.* 2013a); breaking stress of carbon fibers (Salem and Selim 2014).

The new package **Newdistns** (Nadarajah 2013) computes the probability density function, $f(x)$, the cumulative distribution function, $F(x)$, and the quantile function, $F^{-1}(p)$, for each of the nineteen families of G distributions. It also generates random numbers from each of the nineteen families of G distributions. It also computes the measures of inference for each of the nineteen families of G distributions. The package has been uploaded to **CRAN**, see

`http://cran.r-project.org/web/packages/Newdistns/index.html.`

3. Illustrations

Here, we provide three illustrations of the practical use of the package **Newdistns**.

The first illustration plots the probability density function of the beta-gamma distribution for varying parameter values. We have taken the shape and scale parameters of the gamma distribution to be one of $(1, 1)$, $(2, 2)$, $(3, 1)$ or $(5, 2)$. The shape parameters of the beta distribution are taken to be 2 and 3.

The following codes will produce Figure 1, the plot of the probability density functions of the beta-gamma distribution.

```
R> x <- seq(0.1, 10, 0.1)
R> y1 <- dbetag(x, "gamma", a = 2, b = 3, shape = 1, scale = 1)
R> y2 <- dbetag(x, "gamma", a = 2, b = 3, shape = 2, scale = 2)
R> y3 <- dbetag(x, "gamma", a = 2, b = 3, shape = 3, scale = 1)
R> y4 <- dbetag(x, "gamma", a = 2, b = 3, shape = 5, scale = 2)
```

```
R> xrange <- range(x)
R> yrange <- range(y1, y2, y3, y4)
R> plot(x, y1, type = "l", xlab = "x", ylab = "PDF",
+ col = "black", xlim = xrange, ylim = yrange)
R> par(new = TRUE)
R> plot(x, y2, type = "l", xlab = "", ylab = "",
+ col = "red", xlim = xrange, ylim = yrange)
R> par(new = TRUE)
R> plot(x, y3, type = "l", xlab = "", ylab = "",
+ col = "blue", xlim = xrange, ylim = yrange)
R> par(new = TRUE)
R> plot(x, y4, type = "l", xlab = "", ylab = "",
+ col = "brown", xlim = xrange, ylim = yrange)
R> legend(3, 1, legend = c("shape=1,scale=1",
+ "shape=2,scale=2",
+ "shape=3,sclae=1", "shape=5,scale=2"),
+ col = c("black", "red", "blue", "brown"), lty = 1)
```

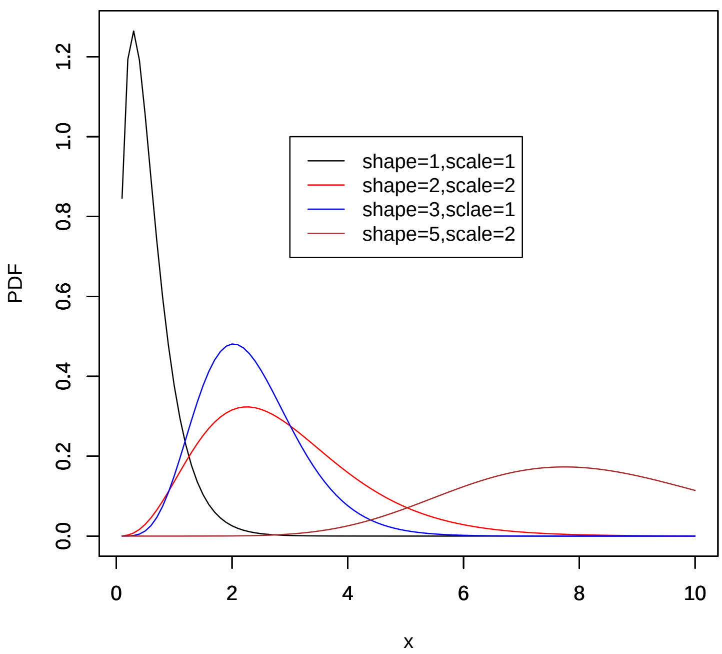

Figure 1: Probability density functions of the beta-gamma distribution.

The second illustration plots the cumulative distribution function of the beta-Student's t distribution for varying parameter values. We have taken the degree of freedom parameter of the Student's t distribution to be one of 1, 2, 5 or 10. The shape parameters of the beta distribution are taken to be 20 and 3.

The following codes will produce Figure 2, the plot of the cumulative distribution functions of the beta-Student's t distribution.

```
R> x <- seq(-1, 5, 0.1)
R> y1 <- pbetag(x, "t", a = 20, b = 3, df = 1)
R> y2 <- pbetag(x, "t", a = 20, b = 3, df = 2)
R> y3 <- pbetag(x, "t", a = 20, b = 3, df = 5)
R> y4 <- pbetag(x, "t", a = 20, b = 3, df = 10)
R> xrange <- range(x)
R> yrange <- c(0, 1)
R> plot(x, y1, type = "l", xlab = "x", ylab = "CDF",
+ col = "black", xlim = xrange, ylim = yrange)
R> par(new = TRUE)
R> plot(x, y2, type = "l", xlab = "", ylab = "",
+ col = "red", xlim = xrange, ylim = yrange)
R> par(new = TRUE)
R> plot(x, y3, type = "l", xlab = "", ylab = "",
+ col = "blue", xlim = xrange, ylim = yrange)
R> par(new = TRUE)
R> plot(x, y4, type = "l", xlab = "", ylab = "",
+ col = "brown", xlim = xrange, ylim = yrange)
R> legend(3, 0.4, legend = c(expression(nu==1),
+ expression(nu==2),
+ expression(nu==5), expression(nu==10)),
+ col = c("black", "red", "blue", "brown"), lty = 1)
```

The third illustration fits the beta-exponential distribution to a strength data reported by Badar and Priest (1982) and reproduced in the code. The data represent the strength measured in GPA for single carbon fibers, and impregnated 1000-carbon fiber tows. Single fibers were tested under tension at gauge lengths of twenty millimeters.

The following code fits the beta-exponential distribution to the strength data.

```
R> install.packages("AdequacyModel")
R> library(AdequacyModel)
R> x <- c(1.312, 1.314, 1.479, 1.552, 1.700, 1.803, 1.861,
+ 1.865, 1.944, 1.958, 1.966,
+ 1.997, 2.006, 2.021, 2.027, 2.055, 2.063, 2.098, 2.14, 2.179, 2.224,
+ 2.240, 2.253, 2.270, 2.272, 2.274, 2.301, 2.301, 2.359, 2.382, 2.382,
+ 2.426, 2.434, 2.435, 2.478, 2.490, 2.511, 2.514, 2.535, 2.554, 2.566,
+ 2.57, 2.586, 2.629, 2.633, 2.642, 2.648, 2.684, 2.697, 2.726, 2.770,
+ 2.773, 2.800, 2.809, 2.818, 2.821, 2.848, 2.88, 2.954, 3.012, 3.067,
+ 3.084, 3.090, 3.096, 3.128, 3.233, 3.433, 3.585, 3.585)
R> mbetag("exp",x,starts=c(1,1,1))
```

The output will be

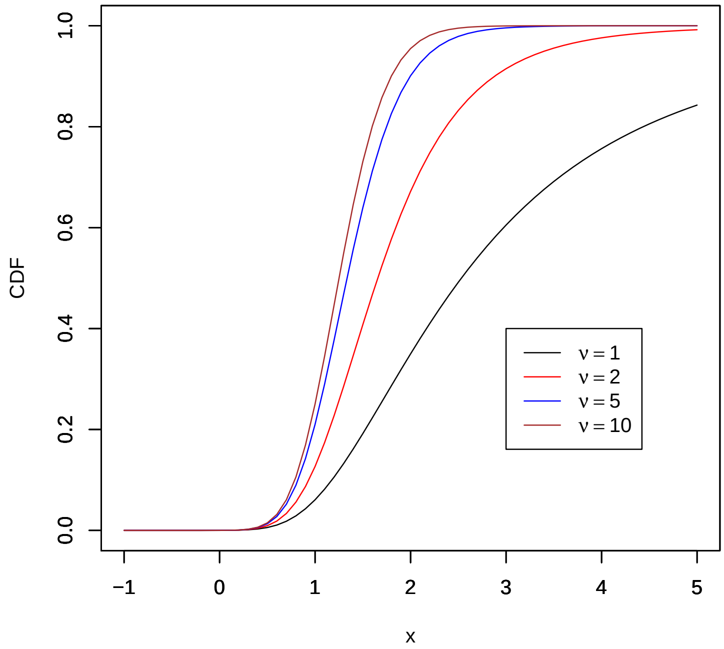

Figure 2: Cumulative distribution functions of the beta-Student's t distribution.

```
$Estimates
            MLE  Std. Dev. Inf. 95% CI Sup. 95% CI
[1,] 25.4776988  6.1826328  13.3599612   37.595436
[2,] 14.9295430 22.8137357 -29.7845573   59.643643
[3,]  0.4148489  0.4541651  -0.4752984    1.304996

$Measures
      AIC     CAIC      BIC     HQIC          W         A Min(-log(Likelihood))
 106.4235 106.7928 113.1259 109.0826 0.05017047 0.3683626              50.21177

$`Kolmogorov-Smirnov Test`
 KS Statistic KS p-value
   0.06003458  0.9647803

$`Convergence Status`

 "Algorithm Converged"
```

This output shows that the maximized log-likelihood is -50.21177, the estimate of the first shape parameter of the beta distribution is 25.4776988, the estimate of the second shape parameter of the beta distribution is 14.9295430 and the estimate of the rate parameter of the exponential distribution is 0.4148489. The output also gives the values of standard deviations, 95 percent confidence intervals based on asymptotic normality, Akaike Information Criterion, Consistent Akaikes Information Criterion, Bayesian Information Criterion, Hannan-Quinn information criterion, Cramer-von Misses statistic, Anderson Darling statistic, Kolmogorov Smirnov test statistic and its p-value, and the convergence status. In particular, the Kolmogorov-Smirnov test shows that the strength data can be adequately described by the beta exponential distribution. We see that some of the lower confidence limits are negative, this is a disadvantage of using asymptotic normality for constructing confidence intervals.

4. Checking of random number generation

In Section 2, we presented routines for random number generation from the nineteen families of distributions. It is reasonable to check if these routines generate numbers that actually follow the intended distribution. We performed this check by means of the following simulation study:

1. generate a random sample of size n from the intended distribution using the routines provided in Section 2;
2. compute the p-value of the one-sample Kolmogorov-Smirnov test that the sample comes from the intended distribution;
3. repeat steps 1 and 2 one hundred times, giving p-values $p_1, p_2, \ldots, p_{100}$ say;
4. draw a boxplot of $p_1, p_2, \ldots, p_{100}$;
5. repeat steps 1 to 4 for $n = 1, 2, \ldots, 100$.

We executed this simulation for each of the nineteen families and for a wide range of parameter values encompassing each family.

The general conclusion from the boxplots was that the p-values were significantly above 0.05 for all $n = 1, 2, \ldots, 100$ for all of the nineteen families and for all parameter values. Illustrations are given in Figures 3 to 6: Figure 3 shows the boxplots for the beta Student's t distribution with $a = 2$, $b = 3$ and one degree of freedom; Figure 4 shows the boxplots for the beta exponential gamma distribution with $\lambda = 1$, $a = 3$, $b = 3$, the gamma shape parameter equal to 2 and gamma scale parameter equal to 1; Figure 5 shows the boxplots for the exponentiated normal distribution with $a = 3$, normal mean equal to 1 and normal standard deviation equal to 3; Figure 6 shows the boxplots for the generalized beta Weibull distribution with $a = 3$, $b = 2$, $c = 3$, Weibull shape parameter equal to 4 and Weibull scale parameter equal to 1.

5. Conclusions

We have developed a new R package for computing quantities of interest for nineteen families of G distributions. Each family is flexible enough to encompass a wide range of uncountable distributions. The quantities computed for each family include the probability density function, the cumulative distribution function, the quantile function, random numbers and several measures of inference.

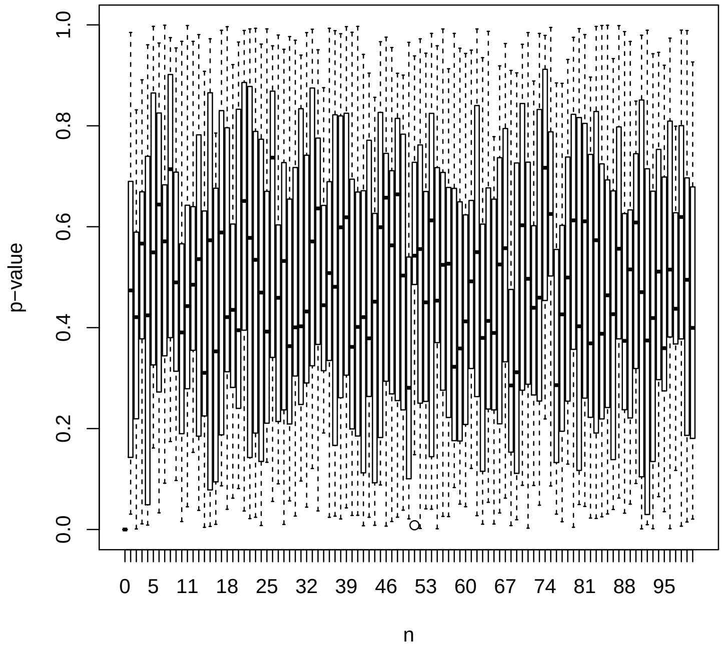

Figure 3: p-values of the Kolmogorov-Smirnov test versus n for the beta Student's t distribution.

As stated, the nineteen families are the recent ones that we are aware of. The package can be updated if other families of the kind (1) are developed in the future.

References

Abdul-Moniem I, Abdel-Hameed H (2012). "On Exponentiated Lomax Distribution." *International Journal of Mathematical Archive*, **3**, 2144–2150.

Adepoju K, Chukwu A, Wang M (2014). "The Beta Power Exponential Distribution." *Journal of Statistical Science and Application*, **2**, 37–46.

Akinsete A, Famoye F, Lee C (2008). "The Beta Pareto Distribution." *Statistics*, **42**, 547–563.

Akinsete A, Famoye F, Lee C (2014). "The Kumaraswamy-Geometric Distribution."

Al-Saiari A, Baharith L, Mousa S (2014). "Marshall-Olkin Extended Burr Type XII Distribution." *International Journal of Statistics and Probability*, **3**.

Alexander C, Cordeiro G, Ortega E, Sarabia J (2012). "Generalized Beta-Generated Distributions." *Computational Statistics and Data Analysis*, **56**, 1880–1897.

Alshawarbeh E, Famoye F, Lee C (2014). "Beta-Cauchy Distribution: Some Properties and Applications."

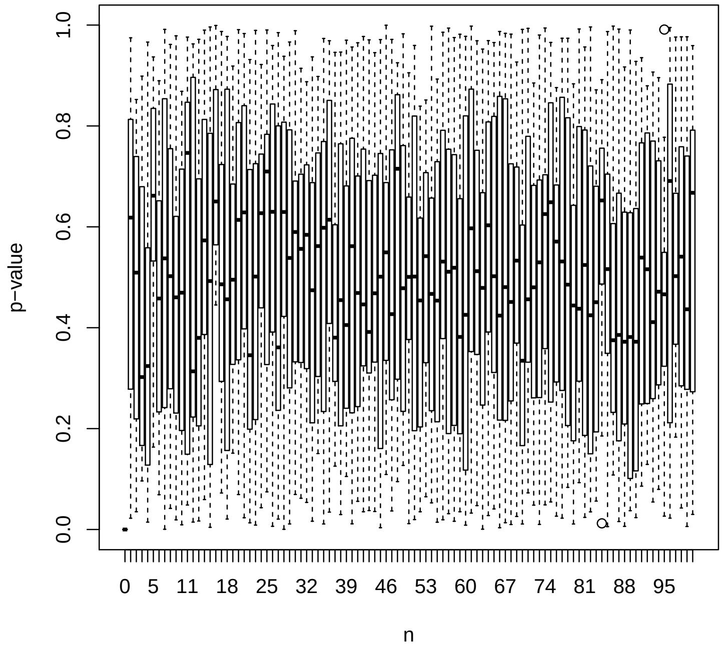

Figure 4: p-values of the Kolmogorov-Smirnov test versus n for the beta exponential gamma distribution.

Alzaatreh A, Famoye F, Lee C (2013a). "Weibull-Pareto Distribution and Its Applications." *Communications in Statistics—Theory and Methods*, **42**, 1673–1691.

Alzaatreh A, Famoye F, Lee C (2014). "The Gamma-Normal Distribution: Properties and Applications." *Computational Statistics and Data Analysis*, **69**, 67–80.

Alzaatreh A, Knight K (2013). "On the Gamma-Half Normal Distribution and Its Applications." *Journal of Modern Applied Statistical Methods*, **12**, 103–119.

Alzaatreh A, Lee C, Famoye F (2013b). "A New Method for Generating Families of Continuous Distributions." *Metron*, **71**, 63–79.

Amini M, MirMostafaee S, Ahmadi J (2013). "Log-Gamma-Generated Families of Distributions." *Statistics*. Doi: 10.1080/02331888.2012.748775.

Aryal G, Elbata I (2015). "Kumaraswamy Modified Inverse Weibull Distribution: Theory and Application." *Applied Mathematics and Information Sciences*, **9**, 651–660.

Azzalini A (1985). "A Class of Distributions which Includes the Normal Ones." *Scandinavian Journal of Statistics*, **12**, 171–178.

Badar M, Priest A (1982). "Statistical Aspects of Fiber and Bundle Strength in Hybrid Composites." In T Hayashi, K Kawata, S Umekawa (eds.), *Progress in Science and Engineering Composites*, pp. 1129–1136.

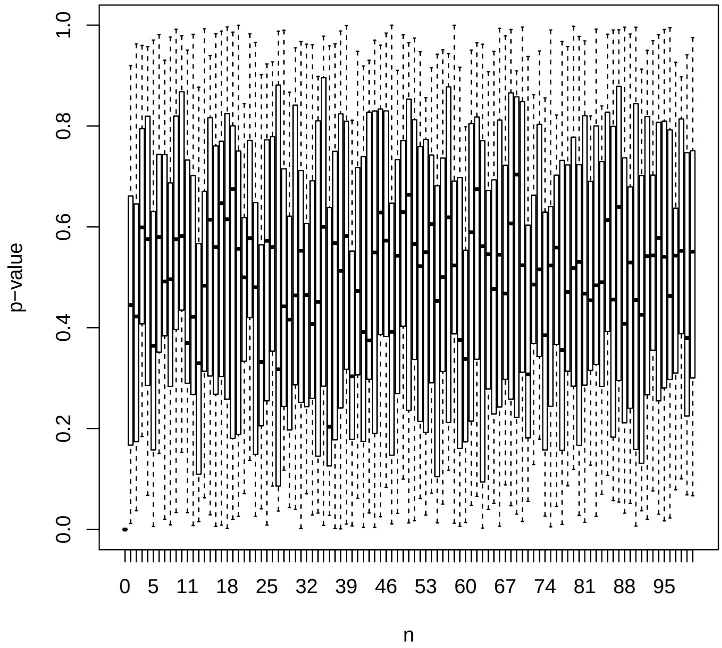

Figure 5: p-values of the Kolmogorov-Smirnov test versus n for the exponentiated normal distribution.

Badmus N, Bamiduro T (2014). "Life Length of Components Estimates with Beta-Weighted Weibull Distribution." *Journal of Statistics: Advances in Theory and Applications*, **11**, 91–107.

Barreto-Souza W, Cordeiro G, Simas A (2011). "Some Results for Beta Frechet Distribution." *Communications in Statistics—Theory and Methods*, **40**, 798–811.

Barreto-Souza W, Santos A, Cordeiro G (2010). "The Beta Generalized Exponential Distribution." *Journal of Statistical Computation and Simulation*, **80**, 159–172.

Bidram H (2012). "The Beta Exponential-Geometric Distribution." *Communications in Statistics—Theory and Methods*, **41**, 1606–1622.

Bidram H, Behboodian J, Towhidi M (2013). "The Beta Weibull Geometric Distribution." *Journal of Statistical Computation and Simulation*, **83**, 52–67.

Bourguignon M, Silva R, Zea L, Cordeiro G (2013). "The Kumaraswamy Pareto Distribution." *Journal of Statistical Theory and Applications*, **12**, 129–144.

Cakmakyapan S, Kadilar G (2014). "A New Customer Lifetime Duration Distribution: The Kumaraswamy Lindley Distribution." *International Journal of Trade, Economics and Finance*, **5**.

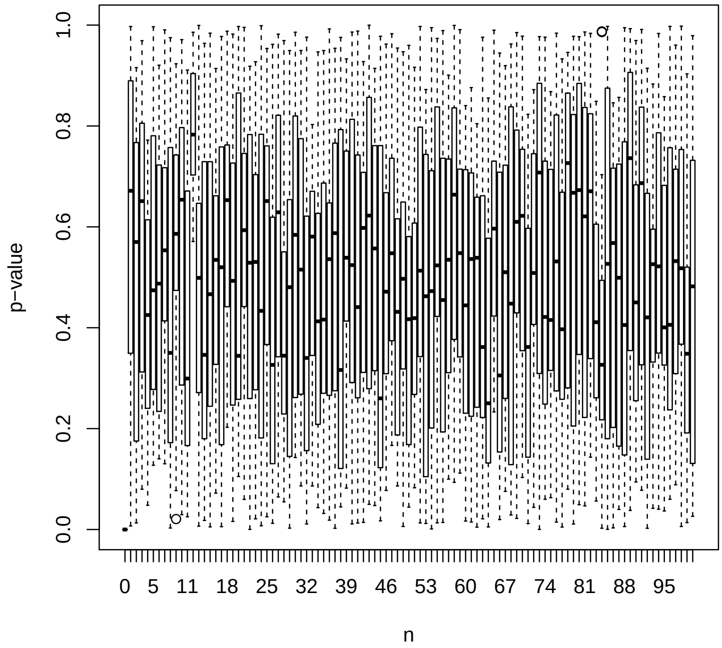

Figure 6: p-values of the Kolmogorov-Smirnov test versus n for the generalized beta Weibull distribution.

Castellares F, Lemonte A (2014). "A New Generalized Weibull Distribution Generated by Gamma Random Variables." *Journal of the Egyptian Mathematical Society*.

Castellares F, Santos M, Montenegro L, Cordeiro G (2015). "A Gamma-Generated Logistic Distribution: Properties and Inference." *American Journal of Mathematical and Management Sciences*, **34**, 14–39.

Cintra R, Rego L, Cordeiro G, Nascimento A (2014). "Beta Generalized Normal Distribution with An Application for SAR Image Processing." *Statistics*, **48**, 279–294.

Cordeiro G, Brito R (2012). "The Beta Power Distribution." *Brazilian Journal of Probability and Statistics*, **26**, 88–112.

Cordeiro G, Castellares F, Montenegro L, de Castro M (2013a). "The Beta Generalized Gamma Distribution." *Statistics*, **47**, 888–900.

Cordeiro G, Castro M (2011). "A New Family of Generalized Distributions." *Journal of Statistical Computation and Simulation*, **81**, 883–898.

Cordeiro G, Cristino C, Hashimoto E, Ortega E (2013b). "The Beta Generalized Rayleigh Distribution." *Statistical Papers*, **54**, 133–161.

Cordeiro G, Gomes A, da Silva C, Ortega E (2013c). "The Beta Exponentiated Weibull Distribution." *Journal of Statistical Computation and Simulation*, **83**, 114–138.

Cordeiro G, Lemonte A (2011a). "The Beta Birnbaum-Saunders Distribution: An Improved Distribution for Fatigue Life Modeling." *Computational Statistics and Data Analysis*, **55**, 1445–1461.

Cordeiro G, Lemonte A (2011b). "The Beta-Half-Cauchy Distribution." *Journal of Probability and Statistics*, **2011**.

Cordeiro G, Lemonte A (2011c). "The Beta Laplace Distribution." *Statistics and Probability Letters*, **81**, 973–982.

Cordeiro G, Lemonte A (2014). "The Exponentiated Generalized Birnbaum-Saunders Distribution." *Applied Mathematics and Computation*, **247**, 762–779.

Cordeiro G, Nadarajah S, Ortega E (2012a). "The Kumaraswamy Gumbel Distribution." *Statistical Methods and Applications*, **21**, 139–168.

Cordeiro G, Nobre J, Pescim R, Ortega E (2014a). "The Beta Moyal: A Useful Skew Distribution."

Cordeiro G, Ortega E, da Cunha D (2013d). "The Exponentiated Generalized Class of Distributions." *Journal of Data Science*, **11**, 1–27.

Cordeiro G, Ortega E, Nadarajah S (2010). "The Kumaraswamy Weibull Distribution with Application to Failure Data." *Journal of the Franklin Institute*, **347**, 1399–1429.

Cordeiro G, Ortega E, Popovic B (2014b). "The Gamma-Linear Failure Rate Distribution: Theory and Applications." *Journal of Statistical Computation and Simulation*, **84**, 2408–2426.

Cordeiro G, Ortega E, Popovic B (2015). "The Gamma-Lomax Distribution." *Journal of Statistical Computation and Simulation*, **85**, 305–319.

Cordeiro G, Ortega E, Silva G (2012b). "The Beta Extended Weibull Family." *Journal of Probability and Statistical Science*, **10**, 15–40.

Cordeiro G, Ortega E, Silva G (2014c). "The Kumaraswamy Modified Weibull Distribution: Theory and Applications." *Journal of Statistical Computation and Simulation*, **84**, 1387–1411.

Cordeiro G, Pescim R, Ortega E (2012c). "The Kumaraswamy Generalized Half-Normal Distribution for Skewed Positive Data." *Journal of Data Science*, **10**, 195–224.

Cordeiro G, Silva G, Ortega E (2013e). "The Beta Weibull Geometric Distribution." *Statistics*, **47**, 817–834.

da Silva R, de Andrade T, Maciel D, Campos R, Cordeiro G (2013). "A New Lifetime Model: The Gamma Extended Frechet Distribution." *Journal of Statistical Theory and Applications*, **12**, 39–54.

de Pascoa M, Ortega E, Cordeiro G (2011). "The Kumaraswamy Generalized Gamma Distribution with Application in Survival Analysis." *Statistical Methodology*, **8**, 411–433.

de Santana T, Ortega E, Cordeiro G, Silva G (2012). "The Kumaraswamy-Log-Logistic Distribution." *Journal of Statistical Theory and Applications*, **11**, 265–291.

Domma F, Condino F (2013). "The Beta-Dagum Distribution: Definition and Properties." *Communications in Statistics—Theory and Methods*, **42**, 4070–4090.

El-Sherpieny E, Ahmed M (2014). "On the Kumaraswamy Kumaraswamy Distribution." *International Journal of Basic and Applied Sciences*, **3**, 372–381.

Elbatal I (2013a). "Kumaraswamy Generalized Linear Failure Rate Distribution." *Indian Journal of Computational and Applied Mathematics*, **1**, 61–78.

Elbatal I (2013b). "The Kumaraswamy Exponentiated Pareto Distribution." *Economic Quality Control*, **28**, 1–8.

Elbatal I, Elgarhy M (2013). "Statistical Properties of Kumaraswamy Quasi Lindley Distribution." *International Journal of Mathematics Trends and Technology*, **4**, 237–246.

Elbatal I, Muhammed H (2014). "Exponentiated Generalized Inverse Weibull Distribution." *Applied Mathematical Sciences*, **8**, 3997–4012.

Eugene N, Lee C, Famoye F (2002). "Beta-Normal Distribution and Its Applications." *Communications in Statistics—Theory and Methods*, **31**, 497–512.

George D, George S (2013). "Marshall-Olkin Esscher Transformed Laplace Distribution and Processes." *Brazilian Journal of Probability and Statistics*, **27**, 162–184.

Ghitany M, Al-Awadhi F, Alkhalfan L (2007). "Marshall-Olkin Extended Lomax Distribution and Its Application to Censored Data." *Communications in Statistics—Theory and Methods*, **36**, 1855–1866.

Ghitany M, Al-Hussaini E, Al-Jarallah R (2005). "Marshall-Olkin Extended Weibull Distribution and Its Application to Censored Data." *Journal of Applied Statistics*, **32**, 1025–1034.

Ghosh I (2014). "The Kumaraswamy Half-Cauchy Distribution: Properties and Applications." *Journal of Statistical Theory and Applications*, **13**, 122–134.

Gomes A, da Silva C, Cordeiro G, Ortega E (2013). "The Beta Burr III Model for Lifetime Data." *Brazilian Journal of Probability and Statistics*, **27**, 502–543.

Gomes A, da Silva C, Cordeiro G, Ortega E (2014). "A New Lifetime Model: The Kumaraswamy Generalized Rayleigh Distribution." *Journal of Statistical Computation and Simulation*, **84**, 290–309.

Gui W (2013a). "A Marshall-Olkin Power Log-Normal Distribution and Its Applications to Survival Data." *International Journal of Statistics and Probability*, **2**.

Gui W (2013b). "Marshall-Olkin Extended Log-Logistic Distribution and Its Application in Minification Processes." *Applied Mathematical Sciences*, **7**, 3947–3961.

Gupta R, Gupta P, Gupta R (1998). "Modeling Failure Time Data by Lehman Alternatives." *Communications in Statistics—Theory and Methods*, **27**, 887–904.

Hady A, Ebraheim N (2014). "Exponentiated Transmuted Weibull Distribution: A Generalization of the Weibull Distribution." *International Journal of Mathematical, Computational, Physical and Quantum Engineering*, **8**.

Hanook S, Shahbaz M, Mohsin M, Golam Kibria B (2013). "A Note on Beta Inverse-Weibull Distribution." *Communications in Statistics—Theory and Methods*, **42**, 320–335.

Idowu B, Ikegwu E (2013). "The Beta Weighted Weibull Distribution: Some Properties and Application to Bladder Cancer Data." *Journal of Applied and Computational Mathematics*, **2**.

Jafari A, Mahmoudi E (2014). "Beta Linear Failure Rate Distribution and Its Applications."

Jafari A, Tahmasebi S, Alizadeh M (2014). "The Beta Gompertz Distribution." *Revista Colombiana de Estadística*, **37**, 139–156.

Jose K, Joseph A, Ristic M (2009). "A Marshall-Olkin Beta Distribution and Its Applications." *Journal of Probability and Statistical Science*, **7**, 173–186.

Jose K, Krishna E (2011). "Marshall-Olkin Extended Uniform Distribution." *ProbStat Forum*, **4**, 78–88.

Jose K, Naik S, Ristic M (2010). "Marshall-Olkin q-Weibull Distribution and Max/Min Processes." *Statistical Papers*, **51**, 837–851.

Jose K, Sebastian R (2013). "Marshall-Olkin Morgenstern Weibull Distribution: Generalisations and Applications." *Economic Quality Control*, **28**, 105–116.

Kong L, Carl L, Sepanski J (2007). "On the Properties of Beta Gamma Distribution." *Journal of Modern Applied Statistical Methods*, **6**.

Kozubowski T, Nadarajah S (2008). "The Beta Laplace Distribution." *Journal of Computational Analysis and Applications*, **10**, 305–318.

Krishna E, Jose K (2011). "Marshall-Olkin Generalized Asymmetric Laplace Distributions and Processes." *Statistica*, **71**, 453–467.

Krishna E, Jose K, Alice T, Ristic M (2013). "Marshall-Olkin Frechet Distribution." *Communications in Statistics—Theory and Methods*, **42**, 4091–4107.

Leao J, Saulo H, Bourguignon M, Cintra R, Rego L, Cordeiro G (2014). "On Some Properties of the Beta Inverse Rayleigh Distribution."

Lemonte A (2013). "A New Extension of the Birnbaum-Saunders Distribution." *Brazilian Journal of Probability and Statistics*, **27**, 133–149.

Lemonte A, Barreto-Souza W, Cordeiro G (2013). "The Exponentiated Kumaraswamy Distribution and Its Log-Transform." *Brazilian Journal of Probability and Statistics*, **27**, 31–53.

Lourenzutti R, Duarte D, Azevedo M (2014). "The Beta Truncated Pareto Distribution."

Mahmoudi E (2011). "The Beta Generalized Pareto Distribution with Application to Lifetime Data." *Mathematics and Computers in Simulation*, **81**, 2414–2430.

Mameli V, Musio M (2013). "A Generalization of the Skew-Normal Distribution: The Beta Skew-Normal." *Communications in Statistics—Theory and Methods*, **42**, 2229–2244.

Marciano F, Nascimento A, Santos-Neto M, Cordeiro G (2012). "The Mc-Gamma Distribution and Its Statistical Properties: An Application to Reliability Data." *International Journal of Statistics and Probability*, **1**.

Marshall A, Olkin I (1997). "A New Method for Adding A Parameter to A Family of Distributions with Application to the Exponential and Weibull Families." *Biometrika*, **84**, 641–652.

Mead M (2014). "An Extended Pareto Distribution." *Pakistan Journal of Statistics and Operation Research*.

Merovci F, Sharma V (2014). "The Beta Lindley Distribution: Properties and Applications." *Journal of Applied Mathematics*, **2014**.

Montenegro L, Cordeiro G (2013). "The Beta Lognormal Distribution." *Journal of Statistical Computation and Simulation*, **83**, 203–228.

Morais A, Cordeiro G, Cysneiros A (2013). "The Beta Generalized Logistic Distribution." *Brazilian Journal of Probability and Statistics*, **27**, 185–200.

Nadarajah S (2006). "The Exponentiated Gumbel Distribution with Climate Application." *Environmetrics*, **17**, 13–23.

Nadarajah S (2013). *Newdistns: Computes Pdf, Cdf, Quantile and Random Numbers for 19 General Families of Distributions.* R package version 1.0, URL http://cran.r-project.org/web/packages/Newdistns/index.html.

Nadarajah S, Cancho V, Ortega E (2013a). "The Geometric Exponential Poisson Distribution." *Statistical Methods and Applications*, **22**, 355–380.

Nadarajah S, Eljabri S (2013). "The Kumaraswamy GP Distribution." *Journal of Data Science*, **11**, 739–766.

Nadarajah S, Gupta A (2007). "The Exponentiated Gamma Distribution with Application to Drought Data." *Calcutta Statistical Association Bulletin*, **59**, 29–54.

Nadarajah S, Kotz S (2003). "The Exponentiated Frechet Distribution." *InterStat*.

Nadarajah S, Kotz S (2004). "The Beta Gumbel Distribution." *Mathematical Problems in Engineering*, **2004**, 323–332.

Nadarajah S, Kotz S (2006). "The Beta Exponential Distribution." *Reliability Engineering and System Safety*, **91**, 689–697.

Nadarajah S, Nassiri V, Mohammadpour A (2013b). "Truncated-Exponential Skew-Symmetric Distributions." *Statistics*. To appear.

Nadarajah S, Teimouri M, Shih S (2013c). "Modified Beta Distributions." *Sankhyā B*. To appear.

Nassar M, Nada N (2011). "The Beta Generalized Pareto Distribution." *Journal of Statistics: Advances in Theory and Applications*, **6**, 1–17.

Nassar M, Nada N (2012). "A New Generalization of the Exponential-Geometric Distribution." *Journal of Statistics: Advances in Theory and Applications*, **7**, 25–48.

Oguntunde P, Babatunde O, Ogunmola A (2014). "Theoretical Analysis of the Kumaraswamy-Inverse Exponential Distribution." *International Journal of Statistics and Applications*, **4**, 113–116.

Oluyede B, Huang S, Pararai M (2014). "A New Class of Generalized Dagum Distribution with Applications to Income and Lifetime Data." *Journal of Statistical and Econometric Methods*, **3**, 125–151.

Oluyede B, Yang T (2014). "A New Class of Generalized Lindley Distributions with Applications." *Journal of Statistical Computation and Simulation*.

Pal M, Tiensuwan M (2014). "The Beta Transmuted Weibull Distribution." *Austrian Journal of Statistics*, **43**, 133–149.

Paranaba P, Ortega E, Cordeiro G, Pescim R (2011). "The Beta Burr XII Distribution with Application to Lifetime Data." *Computational Statistics and Data Analysis*, **55**, 1118–1136.

Paranaiba P, Ortega E, Cordeiro G, de Pascoa M (2013). "The Kumaraswamy Burr XII Distribution: Theory and Practice." *Journal of Statistical Computation and Simulation*, **83**, 2117–2143.

Pararai M, Warahena-Liyanage G, Oluyede B (2014). "A New Class of Generalized Inverse Weibull Distribution with Applications." *Journal of Applied Mathematics and Bioinformatics*, **4**, 17–35.

Percontini A, Blas B, Cordeiro G (2013). "The Beta Weibull Poisson Distribution." *Chilean Journal of Statistics*, **4**, 3–26.

Perez-Casany M, Casellas A (2014). "Marshall-Olkin Extended Zipf Distribution."

Pinho L, Cordeiro G, Nobre J (2012). "The Gamma-Exponentiated Weibull Distribution." *Journal of Statistical Theory and Applications*, **11**, 379–395.

Rajab M, Aleem M, Nawaz T, Daniya M (2013). "On Five Parameter Beta Lomax Distribution." *Journal of Statistics*, **20**, 102–118.

Ramires T, Ortega E, Cordeiro G, Hamedani G (2013). "The Beta Generalized Half-Normal Geometric Distribution." *Studia Scientiarum Mathematicarum Hungarica*, **50**, 523–554.

Ramos M, Cordeiro G, Marinho P, Dias C, Hamedani G (2013). "The Zografos-Balakrishnan Log-Logistic Distribution: Properties and Applications." *Journal of Statistical Theory and Applications*, **12**, 225–244.

R Development Core Team (2014). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0, URL http://www.R-project.org.

Ristić M, Balakrishnan N (2012). "The Gamma Exponentiated Exponential Distribution." *Journal of Statistical Computation and Simulation*, **82**, 1191–1206.

Ristic M, Jose K, Ancy J (2007). "A Marshall-Olkin Gamma Distribution and Minification Process." *STARS: International Journal (Sciences)*, **1**, 11–21.

Ristić M, Nadarajah S (2013). "A New Lifetime Distribution." *Journal of Statistical Computation and Simulation*. Doi: 10.1080/00949655.2012.697163.

Roges D, de Gusmao F, Diniz C (2014). "The Kumaraswamy Inverse Rayleigh Distribution."

Salem H (2014). "The Exponentiated Lomax Distribution: Different Estimation Methods." *American Journal of Applied Mathematics and Statistics*, **2**, 364–368.

Salem H, Selim M (2014). "The Generalized Weibull-Exponential Distribution: Properties and Applications." *International Journal of Statistics and Applications*, **4**, 102–112.

Sandhya E, Prasanth C (2014). "Marshall-Olkin Discrete Uniform Distribution." *Journal of Probability*, **2014**.

Saulo H, Leao J, Bourguignon M (2012). "The Kumaraswamy Birnbaum-Saunders Distribution." *Journal of Statistical Theory and Practice*, **6**, 745–759.

Shahbaz M, Shahbaz S, Butt N (2012). "The Kumaraswamy Inverse Weibull Distribution." *Pakistan Journal of Statistics and Operation Research*, **8**, 479–489.

Shams T (2013a). "The Kumaraswamy Generalized Exponentiated Pareto Distribution." *International Journal of Statistics and Applications*, **5**, 92–99.

Shams T (2013b). "The Kumaraswamy Generalized Lomax Distribution." *Middle-East Journal of Scientific Research*, **17**, 641–646.

Shawky A, Abu-Zinadah H (2009). "Exponentiated Pareto Distribution: Different Methods of Estimations." *International Journal of Contemporary Mathematical Sciences*, **4**, 677–693.

Shittu O, Adepoju K (2013). "On the Beta-Nakagami Distribution." *Progress in Applied Mathematics*, **5**, 49–58.

Silva G, Ortega E, Cordeiro G (2010). "The Beta Modified Weibull Distribution." *Lifetime Data Analysis*, **16**, 409–430.

Singla N, Jain K, Sharma S (2012). "The Beta Generalized Weibull Distribution: Properties and Applications." *Reliability Engineering and System Safety*, **102**, 5–15.

Tahir M, Mansoor M, Zubair M, Hamedani G (2014). "McDonald Log-Logistic Distribution with An Application to Breast Cancer Data." *Journal of Statistical Theory and Applications*, **13**, 65–82.

Torabi H, Montazeri N (2012). "The Gamma-Uniform Distribution and Its Applications." *Kybernetika*, **48**, 16–30.

Zakerzadeh H, Mahmoudi E (2012). "A New Two Parameter Lifetime Distribution: Model and Properties."

Zea L, Silva R, Bourguignon M, Santos A, Cordeiro G (2012). "The Beta Exponentiated Pareto Distribution with Application to Bladder Cancer Susceptibility." *International Journal of Statistics and Probability*, **1**.

Zografos K, Balakrishnan N (2009). "On Families of Beta- and Generalized Gamma-Generated Distributions and Associated Inference." *Statistical Methodology*, **6**, 344–362.

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