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Antinomialism reconsidered

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Abstract. Many philosophers of mathematics are attracted by nominalism – the doctrine that there are no sets, numbers, functions, or other mathematical objects. John Burgess and Gideon Rosen have put forward an intriguing argument against nominalism, based on the thought that philosophy cannot overrule internal mathematical and scientific standards of acceptability. I argue that Burgess and Rosen’s argument fails because it relies on a mistaken view of what the standards of mathematics require.

I. Burgess and Rosen’s argument

In several places,¹ John Burgess and Gideon Rosen have argued against nominalism on the grounds that philosophical arguments cannot trump internal mathematical and scientific standards of acceptability. Similar arguments have been put forward by David Lewis.² In their recent article ‘Nominalism reconsidered’,³ Burgess and Rosen present the most detailed version of this naturalistic line of thought to date. The argument they give is complex and intriguing:

(1) Standard mathematics, pure and applied, abounds in ‘existence theorems’ that appear to assert the existence of mathematical objects, and to be true only if such objects exist; which is to say, to be true only if nominalism is false. Such, for instance, are:

There are infinitely many prime numbers. ...

¹ J. Burgess, ‘Review of Field’s *Realism, Mathematics and Modality*’, *Philosophia Mathematica*, 1 (1993), pp. 180-188, at p. 187; J. Burgess and G. Rosen, *A Subject with No Object: Strategies for Nominalistic Interpretation of Mathematics* (Oxford: Clarendon Press, 1997), at p. 212; J. Burgess, ‘Mathematics and *Bleak House*’, *Philosophia Mathematica*, 12 (2004), pp. 18-36, at pp. 29-30.

² D. Lewis, *Parts of Classes* (Oxford: Blackwell, 1991), at pp. 58-9; D. Lewis ‘Mathematics is megethology’, repr. in his *Papers in Philosophical Logic* (Cambridge: Cambridge UP, 1998), pp. 203-229, at p. 218.

³ J. Burgess and G. Rosen, ‘Nominalism reconsidered’, in Stewart Shapiro (ed.) *Oxford Handbook of the Philosophy of Mathematics and Logic* (New York: Oxford UP, 2005), pp. 515-535.

(2) Well-informed scientists and mathematicians – the ‘experts’ – accept these existence theorems in the sense both that they assent verbally to them without conscious silent reservations, and that they rely on them in both theoretical and practical contexts. They use them as premises in demonstrations intended to convince other experts of novel claims, and together with other assumptions as premises in arguments intended to persuade others to some course of action.

(3) The existence theorems are not merely *accepted* by mathematicians, but are *acceptable* by mathematical standards. They, or at any rate the great majority of them, are supplied with proofs; and while the mathematical disciplines recognize a range of grounds for criticizing purported proofs, ... nonetheless the proofs of the existence theorems, or at any rate the great majority of them, are not susceptible to this kind of internal mathematical criticism. ...

(4) The existence theorems really do assert and imply just what they appear to, that there are such mathematical objects as prime numbers greater than 1000, ... and so on.

(5) To accept a claim in the sense of assenting verbally to it without conscious silent reservations, of relying on it theoretical demonstrations and practical deliberations, and so on, just *is* to believe what it says, to believe that it is true.

(6) The existence theorems are not merely acceptable by specifically *mathematical* standards, but are acceptable by more general *scientific* standards. ...

(7) There is no philosophical argument powerful enough to override or overrule mathematical and scientific standards of acceptability in the present instance.

From (1), (2), (4), and (5) there follows the intermediate conclusion:

(8) Competent mathematicians and scientists believe in prime numbers greater than 1000, ... and so on. Hence, if nominalism is true, expert opinion is systematically mistaken.

From (8) together with (3), (6), and (7) there follows the ultimate antinomialist conclusion:

(9) We are justified in believing (to some high degree) in prime numbers greater than 1000, ... and so on, which is to say we are justified in disbelieving (to the same high degree) in nominalism.⁴

Burgess and Rosen buttress their argument by defending premises (4) to (7). They do nothing to defend (1) to (3), since they find these ‘scarcely deniable’.⁵ In the next section, I will argue that (3) is far from being undeniable, and that their argument fails for that reason. But before I criticize the argument, I will spend the rest of this section clarifying its structure – for it is not at all clear how to move from the premises to the conclusion. Understanding the structure of the argument will help us to see what is wrong with it.

Burgess and Rosen’s argument revolves around the idea of *acceptance*. They explain what they mean by this term in premiss (2): to accept a mathematical claim is to assent to it without ‘conscious silent reservations’ and to employ it in one’s reasoning. There are questions as to whether this would be an adequate account of our pre-theoretical notion of acceptance: for instance, is it entirely clear that a mathematician who assents to a mathematical claim should not be classed as ‘accepting’ it if she has ‘conscious silent reservations’? However, Burgess and Rosen have no need to consider this matter, since they use ‘acceptance’ as a technical term, stipulating its meaning through (2) and (5). Whatever the word usually means, Burgess and Rosen have explained what *they* mean by it. In this note, I will follow Burgess and Rosen’s usage.

Burgess and Rosen’s conclusion, (9), is about what we are well justified in believing.⁶ Premises (3) and (6) tell us that there many existence theorems which are acceptable by mathematical and scientific standards; and (7) tells us that where existence theorems are concerned, mathematical and scientific standards of acceptability cannot be gainsaid by

⁴ J. Burgess and G. Rosen, ‘Nominalism reconsidered’, pp. 516–7.

⁵ J. Burgess and G. Rosen, ‘Nominalism reconsidered’, p. 516.

⁶ An alternative reading of (9) would take ‘(to a high degree)’ to modify ‘believing’ rather than ‘justified’. This does not seem to be the intended reading, though, since there is no mention of degrees of belief earlier in the argument (or, indeed, anywhere else in Burgess and Rosen’s article). In any case, the criticism made in the text applies equally to this reading of the argument.

philosophy. How can we put together these claims about acceptance to reach a conclusion about belief? Premiss (5) – which states that acceptance *is* belief – is intended to bridge this gap. Note that the premiss does not merely assert that everyone who accepts a claim also believes it; that would be entirely compatible with the idea that we ought to accept mathematical claims without believing them at all. Burgess and Rosen’s view is that accepting a claim and believing it are the same thing. If this is right then it is incoherent to think we ought to do one but not do the other, just as it is incoherent to command someone to draw a triangle that is not a three-sided shape.

Premises (3) and (6) speak of *mathematical* and *scientific standards*. What are these? From Burgess and Rosen’s defence of premiss (6),⁷ we can gather that scientific standards are those of explanatory power, elegance, fruitfulness, and suchlike. From what Burgess and Rosen say in premiss (3), we can gather that a claim is acceptable by mathematical standards if there is a proof of it which can stand up to the sorts of criticism offered by mathematicians.

Premiss (7) claims that mathematical and scientific standards of acceptability cannot be trumped by philosophical arguments – at least ‘in the present instance’. What is the force of this qualification? It seems likely that Burgess and Rosen think that *mathematical* claims which are acceptable by mathematical and scientific standards cannot be overridden by philosophy, where a mathematical claim is one that can be formulated using only mathematical and logical vocabulary. All that the argument requires, however, is the weaker principle that philosophy cannot override *existence theorems* if they are acceptable by mathematical and scientific standards, and so I will read (7) in this way.

Glancing at (9), which is about what we are justified in believing, we see that we must read (7) as a claim about justification as well – justification for acceptance, rather than belief (though Burgess and Rosen will soon be deploying (5) to argue that these are just the same). (7) has to mean: if an existence theorem is acceptable by mathematical and scientific standards, then philosophers are justified to a high degree in accepting it, and philosophical reasoning cannot defeat this justification.

Burgess and Rosen use the phrase ‘acceptable by mathematical standards’. This admits of at least two readings. To say that a claim is acceptable by mathematical standards might mean that mathematical standards *permit* mathematicians to accept it, or that mathematical standards *require* them to accept it. And perhaps there are further alternatives.

Different understandings of the phrase ‘acceptable by mathematical and scientific standards’ yield different readings of (7). For instance, we might read (7) as:

⁷ J. Burgess and G. Rosen, ‘Nominalism reconsidered’, pp. 518-520.

- (7a) When mathematical and scientific standards permit mathematicians and scientists to accept an existence theorem t , philosophers are well justified in accepting t regardless of what philosophical arguments are offered against it.

However, (7a) could not be the basis of a compelling argument for antinomialism, because it is implausibly strong. The guiding idea of Burgess and Rosen's argument is that philosophers should respect the internal standards of mathematics and natural science. But (7a) embodies something more extreme. Suppose that T is an existence theorem which the internal standards of mathematics and science permit us – but do not require us – to accept. (T might follow from a controversial axiom, for instance.) According to (7a), philosophers are well justified in accepting T ; since acceptance is belief, this means that they are well justified in believing T and disbelieving its negation. So philosophers are required to believe T even though mathematicians are not. But that is implausible: philosophical respect for mathematical standards ought to require no more of philosophers than mathematical standards require of mathematicians.

This point enables us to see how (7) must be read. To argue against nominalism, Burgess and Rosen aim to show that philosophers are *required* to believe those existence theorems which are acceptable by mathematical and scientific standards. So we should take a theorem to be 'acceptable by mathematical and scientific standards' only if mathematical and scientific standards *require* us to accept it. Putting anything weaker in the place of 'require' here will lead to a principle which imposes more stringent requirements on philosophers than the internal standards of mathematics and science require of mathematicians and scientists themselves. And it is hard to see why such principles should be believed. They certainly go beyond the thought that philosophical arguments cannot trump internal mathematical and scientific standards of acceptability.

Let us therefore read (7) as:

- (7b) When mathematical and scientific standards require mathematicians and scientists to accept an existence theorem t , philosophers are well justified in accepting t regardless of what philosophical arguments are offered against it.

To ensure that the argument is valid, we must read the other premises in ways which harmonize with (7b). In particular, when the word 'acceptable' appears in (3) and (6), we must understand it as referring to claims which mathematical and scientific standards *require* us to

accept. Premiss (3) then becomes the claim that mathematics requires us to accept the existence theorems, where ‘acceptance’ is understood as in (2) and (5). According to (3), then, the internal standards of mathematics require us to accept the existence theorems without any ‘conscious silent reservations’.

We have seen that we ought to read (7) as: if an existence theorem is acceptable by mathematical and scientific standards, then philosophers are justified to a high degree in accepting it, and philosophical reasoning cannot defeat this justification. Combined with (3) and (6), which say that there are many existence theorems which are acceptable by these standards, we reach the result that that there are many existence theorems we are well justified in accepting. Then Burgess and Rosen invoke (5) to reach the conclusion that we are well justified in *believing* these theorems. Since the theorems entail that there are numbers and other mathematical objects (premiss (4)), it follows that we are well justified in *disbelieving* the denial that there are any such objects, that is, in disbelieving nominalism.

This seems the only way of combining Burgess and Rosen’s premises into an argument against nominalism. But it is not quite how Burgess and Rosen state it. They themselves explain how to reach (9) by using the intermediate conclusion, (8), which says that ‘competent mathematicians and scientists’ believe that there are numbers and other mathematical objects. Burgess and Rosen claim that (9) follows from (8), in conjunction with (3), (6), and (7). But I cannot see how this works. Of (8), (3), (6), and (7), only (8) mentions belief: how can we get from that claim, which is about what mathematicians and scientists *actually* believe, to the conclusion, which is about what ‘we’ (in this context, philosophers) are *justified* in believing? Burgess and Rosen’s argument cannot validly proceed as they indicate. The argument must have a different structure; above, I indicated what seems like the only sensible way it could go.

II. Against Burgess and Rosen’s argument

Imagine a pure mathematician, Ada, who begins to be troubled by epistemological worries about her work. Ada has no worries about her knowledge of logical truths, and she has no problem with the idea that deducing B from A makes it reasonable to believe that A entails B. For instance, if she deduces from the axioms of arithmetic that there are infinitely many primes, then she can see how she is justified in believing that the axioms of arithmetic entail that there are infinitely many primes. But Ada is puzzled as to how her proof justifies her in believing the categorical claim that there are infinitely many primes. It seems to her as though all the proof licenses is belief in the entailment. Pursuing such reflections, Ada becomes agnostic as to whether there are any such things as numbers. She continues to practice as a pure mathematician, but without believing in

categorical mathematical claims. Suppose that an arithmetical theorem – T, say – has been supplied with a completely uncontroversial proof. Then, while Ada’s colleagues believe T, Ada merely believes that the axioms of arithmetic entail T. When she is explaining proofs to her colleagues or students, Ada utters categorical mathematical claims – but she ceases to believe them.

We have noted that if a result has a proof which can stand up to the sorts of criticism offered by mathematicians, then it is acceptable by mathematical standards. So T is acceptable by mathematical standards. In Burgess and Rosen’s terms, Ada ‘assents verbally’ to T but her assent is at least sometimes accompanied by ‘conscious silent reservations’. As we saw above, premiss (3) must be read as claiming that the internal standards of mathematics require us to assent to the existence theorems without any ‘conscious silent reservations’. Understood in this way, the premiss implies that Ada’s attitude to her work is forbidden by the standards of mathematics, which demand a less qualified acceptance. However, this seems highly unlikely. Ada could be a highly competent mathematician at the top of her profession. It would thus be very surprising if she were transgressing the standards of her discipline. Read as it has to be for Burgess and Rosen’s argument to be valid, premiss (3) is false.

If her conversion to agnosticism led Ada to abandon the standards of mathematics, then we would expect her peers to detect this. But Ada’s agnosticism would be invisible to those who read her work. It’s illuminating to contrast Ada’s case with that of a mathematician who begins to treat affirming the consequent as a valid rule. This deviation would soon be noticed: the proofs put forward would not be taken seriously, and the person who produced them would no longer be regarded as a competent mathematician. In the same way, a mathematician whose proposed proof of a result invokes non-trivial but unproven assumptions would not be treated as having established that result. Ada’s work, on the other hand, would not be susceptible to ‘internal mathematical criticism’ of this sort.

In reply, Burgess and Rosen might argue that the standards of mathematics *do* forbid Ada’s attitude to her work. They might compare Ada to a priest who loses their faith but continues to conduct worship, minister to the sick, and so on. We can imagine their loss of faith going undetected, perhaps for ever. In the same way, Burgess and Rosen might argue that the fact that Ada’s attitude to her work could go undetected does not show that it conforms to the internal standards of mathematics. Since we can only tell what beliefs people have on the basis of how they act, we should not expect mathematicians to be able to detect whether Ada has departed from the standards of their discipline.

We can deal with this criticism by refining the thought-experiment so that Ada makes her attitude perfectly explicit. Suppose then that Ada makes no secret of her reservations: she prefaces each piece of her mathematical speaking or writing with a brief account of her agnosticism about categorical mathematical claims. It seems wrong to think that Ada would be regarded as an incompetent mathematician, or as contravening mathematical standards. Unlike mathematicians who affirm the consequent, Ada's work could still be of great mathematical interest and value. Ada's colleagues will pay attention to proofs of important theorems, irrespective of any philosophical preambles.

The argument I have given uses an imaginary case to test what the standards of mathematics require. It does not rest on the claim that mathematicians like Ada actually exist. However, seems probable that there have been, and continue to be, actual mathematicians who assent to mathematical theorems only with silent reservations. For instance, some mathematicians have consciously adopted formalist philosophies of mathematics, which say that existence theorems are false or lack truth-value.⁸ We would expect these mathematicians to be prime examples.

These mathematicians cause an additional problem for Burgess and Rosen, as I will now explain. It is not entirely clear how Burgess and Rosen's premiss (2) is meant to be read, but it is natural to read it as:

- (2a) All well-informed scientists and mathematicians accept existence theorems (in Burgess and Rosen's full-blooded sense of 'accept').

These mathematicians are counterexamples to (2a). By itself, this is not a serious problem, as premiss (2) is not required for the argument to go through: we have seen that it relies on the claim that the existence theorems are acceptable (premiss (3)), not that they are actually accepted. But if there actually have been competent mathematicians who did not accept the existence theorems, that tends to undermine the idea that accepting them is required by the standards of mathematics. So the falsity of (2a) is problematic because it makes (3) hard to believe.

⁸ See e.g. H. G. Dales, 'The mathematician as a formalist', in H. G. Dales and G. Oliveri (eds) *Truth in Mathematics*. (Oxford: Clarendon Press 1998), pp. 181-202.

Against Burgess and Rosen, then, we *do* have reason to deny (3). Burgess and Rosen's attempted refutation of nominalism does not succeed.⁹

Burgess and Rosen's argument adopts an attitude of unconditional deference to the standards of mathematics and science. (7b) tells us that what is acceptable by those standards cannot be defeated by any philosophical argument, no matter how strong it is. Many will find this sort of deference too much to take. They will think that philosophical arguments are capable of overturning the deliverances of mathematics and science, provided they are sufficiently strong. This less deferential attitude is captured by (7c):

(7c) When mathematical and scientific standards require mathematicians and scientists to accept an existence theorem *t*, philosophers are well justified in accepting *t* unless there is a sufficiently strong philosophical argument against it.

Different choices of what counts as 'sufficiently strong' produce stronger and weaker versions of this principle. Nothing in the rest of my discussion depends on which version we choose. Regardless of where the line is drawn, (7c) suggests a variant on Burgess and Rosen's argument, purged of the unconditional deference. This variant attempts to refute nominalism by arguing that no philosophical argument for nominalism is strong enough to defeat the scientific and mathematical warrant for antinomialism. Unlike Burgess and Rosen's argument, this line of thought allows that a sufficiently powerful argument for nominalism could, in principle, justify us in believing its conclusion. But it adds that no such argument actually exists, and concludes that we are justified in believing that there are mathematical objects.

The criticism I have made of Burgess and Rosen's argument also indicts this less deferential variant. Using the example of Ada, I have already argued that the internal standards of mathematics do not require us to accept any existence theorems. Consequently, for no existence theorem is there any prospect of using (7c) to show that philosophers are well justified in accepting it. So (7c) cannot contribute to a successful argument against nominalism.

Earlier on, we discussed whether Burgess and Rosen's (7) should be cashed out in terms of what we are *required* to accept, or in terms of some weaker notion. Opponents of nominalism

⁹ In my view, there is a further problem with the argument: Burgess and Rosen have not given us adequate grounds for their premiss (7). See my 'Is there a good epistemological argument against platonism?', *Analysis* 66.2 (April 2006).

might try to modify (7c) by replacing ‘require’ with something weaker. This would result in something like:

- (7d) When mathematical and scientific standards permit mathematicians and scientists to accept an existence theorem t , philosophers are well justified in accepting t unless there is a sufficiently strong philosophical argument against it.

But nothing of this sort can help the antinominalist. Such principles suffer from the same problem as (7a) did: they demand more of philosophers than mathematical standards demand of mathematicians. For instance, if the internal standards of their discipline permit mathematicians to believe an existence theorem T , then in the absence of a sufficiently strong philosophical argument to the contrary, philosophers are *required* to believe it – even if the internal standards of mathematics do not require mathematicians to believe it. This implausibly strong principle goes beyond mere respect for the internal standards of mathematics. The less deferential variant of Burgess and Rosen’s argument fares no better than the original.¹⁰

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