

Elasticity of Demand and Behaviour-Based Price Discrimination *

Rosa Branca Esteves

Carlo Reggiani

Universidade do Minho and NIPE

School of Social Sciences - Economics

Braga, Portugal

University of Manchester, United Kingdom

rbranca@eeg.uminho.pt.

carlo.reggiani@manchester.ac.uk.

October 2013

Abstract

Behaviour-based price discrimination (BBPD) is typically analysed in a framework characterised by perfectly inelastic demand. This paper provides a first assessment of the role of demand elasticity on the profit, consumer and welfare effects of BBPD. We show that the demand expansion effect, that is obviously overlooked by the standard framework with unit demand, plays a relevant role. In comparison to uniform pricing, we show that firms are worse off under BBPD, however, as demand elasticity increases the negative impact of BBPD on profits gets smaller. Despite a possible slight increase in the average prices charged over the two periods in comparison to uniform pricing, we show that BBPD boosts consumer surplus and that this benefit is independent of elasticity. In contrast to the welfare results derived under the unit demand assumption, where BBPD is always bad for welfare, the paper shows that BBPD can be welfare enhancing if demand elasticity is sufficiently high.

JEL: D43, L13.

Keywords: Price discrimination based on purchase history, elasticity of demand, welfare

*We are grateful to the Editor, Yongmin Chen, and to two anonymous referees for their thorough and thoughtful reports. Thanks for comments are also due to Yiqian Gu. This work is funded by FEDER funds through the Operational Program for Competitiveness Factors - COMPETE and National Funds through FCT - Foundation for Science and Technology within the project PTDC/EGE-ECO/108784/2008. Financial support from the DERS Super Pump Priming, University of York is also gratefully acknowledged. The usual disclaimer applies.

1 Introduction

The increasing diffusion of the internet as a marketplace and the unprecedented capability of firms to gather and store information on the past shopping behaviour of consumers is enhancing their ability to make use of this information and price differently their own previous customers with respect to the rivals' consumers. This form of price discrimination, termed behaviour-based price discrimination (BBPD) or price discrimination by purchase history or dynamic pricing, is now widely observed in many markets. Examples of firms that adopt BBPD include supermarkets, web retailers, telecom companies, banks, restaurants and many others.

As this business practice is becoming increasingly prevalent, a good economic understanding of its profit, consumer surplus and welfare implications needs to be founded on a good understanding of the market in which it is implemented. Although this type of price discrimination has recently received much attention in economics,¹ the literature has hitherto focused on the assumption that consumers have perfectly inelastic (or unit) demand. Because price discrimination has no role to increase aggregate output, one must be careful when interpreting the welfare results obtained in these models (Stole, 2007).

In real markets, however, the consumers' decision does not only involve choosing a firm but also the amount of good(s) purchased. Therefore, an important issue remains to be explored. What happens if the restrictive assumption of unit demand is relaxed? What are the welfare implications of price discrimination when aggregate output can vary with prices?

The main contribution of this paper is to offer a first assessment of the profit and welfare effects of BBPD when firms face a demand that can vary with the price level. With this goal in mind, we relax the perfectly inelastic demand assumption, in a BBPD model where purchase history discloses information about consumers' exogenous preferences for brands. Based on the CES (constant elasticity of substitution) representative consumer model we allow demand elasticity to vary between zero and unity, i.e., $\varepsilon \in [0, 1)$. Our basic model follows the brand preference approach proposed in Fudenberg and Tirole (2000) by considering a two-period model with two horizontally differentiated firms competing for consumers with stable exogenous preferences across periods. These preferences are specified in the Hotelling-style linear market of unit length with firms positioned at the endpoints. Firms cannot commit to future prices. As firms have no information about consumers' brand preferences in period 1, they quote a uniform price.

¹Chen (2005), Fudenberg and Villas-Boas (2007) and Esteves (2009b) present comprehensive literature surveys on BBPD.

In period 2, firms use the consumers' first period purchase history to draw inferences about consumers' preferences and price accordingly. Unlike their assumption of perfectly inelastic demand, we maintain that consumers' demand can respond to price changes.

The assumption that firms are competing in a unit demand framework *à la Hotelling* is widely adopted by the literature on BBPD, implying that the role of demand elasticity on the effects of competitive BBPD has been mostly overlooked. The assumption may be justified by the challenge posed by introducing demand elasticity in a Hotelling framework. Nero (1999) and Rath and Zhao (2001) seem to be the first to tackle the issue. They use quadratic utility preferences to show that a Hotelling game with not perfectly inelastic demand has a unique price-location equilibrium. Both papers emphasize the role of the transport cost to reservation price ratio as a determinant of the optimal location of firms. Anderson and De Palma (2000) introduce the CES representative consumer model in a spatial framework to analyse issues related to localised and global competition. In so doing they allow the elasticity of demand to vary between zero and unity. Gu and Wenzel (2009, 2011) use the same system of preferences to address the optimality of firms' entry in a spatial model.

In this paper we also adopt the CES formulation to introduce the elasticity of demand in the analysis of competitive BBPD. This approach has the advantage of tractability which allows us to provide a closed form solution to a two periods BBPD model.

The economics literature on price discrimination based on purchase history has followed two main approaches. In the switching costs approach, consumers initially view the two firms as perfect substitutes; but in the second period they face a switching cost if they change supplier. In this setting, purchase history discloses information about exogenous switching costs (e.g. Chen, 1997 and Taylor, 2003). In the brand preferences approach (e.g. Villas-Boas, 1999; Fudenberg and Tirole, 2000), purchase history discloses information about a consumer's exogenous brand preference for a firm. Although the framework of competition differs in the two approaches their predictions have some common features. First, when price discrimination is permitted, firms offer better deals to the competitor's consumers than to its previous customers. Second, because both firms have symmetric information for price discrimination purposes and the market exhibits best-response asymmetry,² industry profits fall with price discrimination (e.g., Chen, 1997; Villas-Boas, 1999; Fudenberg and Tirole, 2000, Taylor, 2003; Esteves, 2010; Gerigh et

²In the terminology of Corts (1998) there is best response asymmetry when each firm's strong market is the rival's weak market.

al. 2011,2012).³ Third, when consumers have perfectly inelastic demand, there is no welfare benefit when prices fall due to discrimination. The exclusive effect of BBPD is to give rise to a deadweight loss to society due to the mismatch of preferences caused by excessive switching (Chen, 1997; Fudenberg and Tirole, 2000; Gerigh et al., 2011, 2012). Nonetheless, important differences arise in both approaches when taking into account the effects of poaching on initial prices. While in the brand preferences' approach when BBPD is permitted initial prices are high and then decrease, in the switching costs approach the reverse happens.

Some authors have recently explored new avenues in the literature on BBPD. Chen and Percy (2010), for instance, look at BBPD under the assumption of correlated preferences across time. They show that if there is sufficiently strong dependence between preferences, BBPD reduces industry profits and increases consumer surplus. In contrast, under weak dependence they show that BBPD increases industry profits and reduces consumer surplus.⁴

This paper enriches the literature on BBPD following the avenue of relaxing the assumption of perfectly inelastic demand. As said, following the brand preference approach proposed in Fudenberg and Tirole (2000), the main goal is to investigate whether the results derived with perfectly inelastic demand hold or are rather contradicted when demand elasticity is allowed to vary between zero (perfectly inelastic) and unity. We show that the model yields the same results as in Fudenberg and Tirole when demand elasticity is low. New results are nonetheless obtained when demand elasticity is sufficiently high. When demand is perfectly inelastic BBPD reduces average prices in comparison to uniform pricing. The paper shows that this result holds for low levels of the elasticity. In contrast, the average price with BBPD can be above its non discrimination counterpart when the elasticity of demand is sufficiently high. Additionally, when demand elasticity is high both loyal and poached consumers can face a higher present value of total payment for the two periods of consumption. This reverses the results obtained under perfectly inelastic demand.

This paper highlights that a complete picture of the welfare effects of price discrimination based on purchase history should be drawn under the assumption that consumers' demand do respond to price changes. Although average prices can increase with BBPD (for high ε) in

³On the Pros and Cons of Price Discrimination, Chen (2005) argues also that firms tend to be worse off being able to recognize consumers and price discriminate. Targeted pricing is also bad for profits in Shaffer and Zhang (1995) and Bester and Petrakis (1996).

⁴Esteves (2013) extends the literature on BBPD allowing firms to employ retention strategies to avoid losing part of the old customers willing to switch. Esteves and Vasconcelos (2012) look at mergers when BBPD is permitted.

comparison to uniform pricing, we show that, in aggregate, BBPD always increases overall consumption. The model confirms that regardless of the elasticity of demand, BBPD by competing firms often intensifies competition and benefits consumers. The CES formulation implies that the benefit of BBPD on consumer surplus does not depend on the elasticity of demand.

Our results confirm that BBPD reduces overall social surplus in the special case of $\varepsilon = 0$ and when demand elasticity is low enough ($0 < \varepsilon < 0.5$). In contrast, if the elasticity of demand is high enough, the welfare result derived in models with unit demand no longer applies. Specifically, when $0.5 < \varepsilon < 1$, *price discrimination can actually boost overall welfare* in comparison to uniform pricing. The inefficiency created by sub-optimal consumption is more than compensated by the increase in the overall quantity consumed. Additionally, we show that if consumers are myopic, which tends to be the case in many markets, the positive impact of BBPD on overall welfare is expected to be positive for a wider range of the elasticity of demand ($0.125 < \varepsilon < 1$).

The rest of the paper is structured as follows. Section 2 introduces the model. Section 3 sets the benchmark case with no discrimination. Section 4 solves the model when firms practice BBPD. Section 5 discusses the effects of BBPD on prices, quantities and profits and Section 6 provides the welfare analysis. Section 7 concludes.

2 The model

Two firms, A and B , produce at zero marginal cost⁵ a nondurable good and compete over two periods, 1 and 2. On the demand side, there is a large number of consumers whose mass is normalised to one. In each period a consumer can either decide to buy the good from firm A or from firm B , but not from both. We assume that the two firms are located at the extremes of a unit interval $[0, 1]$, and consumers are uniformly distributed along this interval. A consumer located at $x \in [0, 1]$ is at a distance $d_A(x) = x$ from firm A and at distance $d_B(x) = 1 - x$ from firm B and t is the unit transport cost. Transport cost is linear in distance and does not depend on the quantity purchased. Note that the location of a consumer x represents his relative preference for firm B over A while $t > 0$ measures how much a consumer dislikes buying a less preferred brand. A consumer's brand preference x remains fixed for both periods. In contrast to Fudenberg and Tirole (2000) consumers are not restricted to buy a single unit of

⁵The assumption of zero marginal costs can be relaxed without altering the basic nature of the results derived throughout the model.

the differentiated good. Thus, the amount bought depends on the price charged. Following Anderson and de Palma (2000) and Gu and Wenzel (2009, 2011) the (indirect) utility for a consumer located at x conditional on buying from firm i , $i = A, B$ is:

$$V_i = Y + v(p_i) - td_i(x).$$

where Y is the consumer income for the two periods of consumption⁶ and $v(p_i)$ is the consumer surplus (net of transport costs) if firm i 's product is bought at price p_i . For simplicity assume that $v(p_i) = v - \frac{p_i^{1-\varepsilon}}{1-\varepsilon}$ where the reservation value v is assumed to be high enough such that all consumers purchase in both periods. Therefore, using Roy's identity it can be shown that the demand for the differentiated product i is $q_i = -\frac{\partial v(p_i)}{\partial p_i} = p_i^{-\varepsilon}$, where

$$\varepsilon = -\frac{\partial q_i}{\partial p_i} \frac{p_i}{q_i} \in [0, 1).$$

is the constant elasticity of demand for firm i 's product. The case with $\varepsilon = 0$ corresponds to perfectly inelastic demand. Like in Fudenberg and Tirole (2000) each consumer buys one unit from the firm offering the highest surplus. For other values of ε , i.e., in the range $0 < \varepsilon < 1$, the quantity demanded responds to changes in prices although the percentage change in quantity demanded is less than the percentage change in price.

The indifferent consumer between buying from firm A or B is located at:

$$x = \frac{1}{2} + \frac{p_B^{1-\varepsilon} - p_A^{1-\varepsilon}}{2t(1-\varepsilon)}. \quad (1)$$

Total demand for firms A and B, respectively given by D_A and D_B , now depends on market share and on the quantity per consumer. Therefore:

$$D_A = xp_A^{-\varepsilon} \quad \text{and} \quad D_B = (1-x)p_B^{-\varepsilon}, \quad (2)$$

while profits are respectively given by

$$\pi_A = xp_A^{1-\varepsilon} \quad \text{and} \quad \pi_B = (1-x)p_B^{1-\varepsilon}. \quad (3)$$

In each period firms act simultaneously and non-cooperatively. In the first period, consumers are *anonymous* and firms quote the same price for all consumers. In the second period, whether or not a consumer bought from the firm in the initial period reveals that consumer's brand preference. Thus, as firms have the required information, they will set different prices to their own customers and to the rival's previous customers. If price discrimination is not adopted (for example, if it is forbidden) firms quote again a single price to all consumers. Firms and consumers have a common discount factor $\delta \in [0, 1]$.

⁶We shall assume throughout that Y is high enough such that income is never a binding constraint.

3 No discrimination benchmark

Suppose that for some reason (e.g. regulation, costs of changing prices) firms in the second period can not price discriminate. In that case, the two-period model reduces to two replications of the static equilibrium. To solve for this equilibrium, consider the one period model, and let p_A and p_B denote the prices set by firms A and B, respectively. Firm A solves the following problem:

$$\max_{p_A} \pi_A = \left\{ p_A^{1-\varepsilon} \left(\frac{1}{2} + \frac{p_B^{1-\varepsilon} - p_A^{1-\varepsilon}}{2t(1-\varepsilon)} \right) \right\}.$$

Solving for the equilibrium, the following proposition can be stated.

Proposition 1. *In the no discrimination benchmark case equilibrium prices in each period are equal to:*

$$p^{nd} = [t(1-\varepsilon)]^{\frac{1}{1-\varepsilon}},$$

and each consumer buys:

$$q^{nd} = [t(1-\varepsilon)]^{\frac{-\varepsilon}{1-\varepsilon}}.$$

Thus, each firm's equilibrium profits are

$$\pi^{nd} = \frac{t(1-\varepsilon)}{2} (1+\delta),$$

consumer surplus is

$$CS^{nd} = Y + \left(v - \frac{5}{4}t \right) (1+\delta),$$

and social welfare equals

$$W^{nd} = Y + \left(v - \frac{5}{4}t + t(1-\varepsilon) \right) (1+\delta).$$

Proof. See the Appendix.

4 Equilibrium analysis

Price discrimination is now feasible. In period 1 because firms cannot recognise customers they set a single first period price, denoted p_{1i} , $i = A, B$. Consumers' first period choices reveal information about their brand preferences, so firms can set their second period prices accordingly. In the second period, each firm can offer two prices, one to its own past customers, denoted p_{oi} , and another price to the rival's previous customers, denoted p_{ri} . To derive the subgame perfect equilibrium, the game is solved using backward induction from the second period.

4.1 Second-period pricing

As in Fudenberg and Tirole (2000) the consumers first-period decisions will lead to a cut-off rule, so that first-period sales identify two intervals of consumers, corresponding to each firm's turf. Suppose that at given first-period prices p_{1A} and p_{1B} , there is a cut-off x_1^* such that all consumers with $x < x_1^*$ bought from firm A in period 1. Thus, firm A's turf is the interval $[0, x_1^*]$, while firm B's turf is the remaining $[x_1^*, 1]$.

Look first on firm A's turf (i.e. firm A's strong market and firm B's weak market). Firm A offers price p_{oA} , while firm B offers price p_{rB} . The marginal consumer, x_{2A} who is indifferent between buying again from firm A and switching to firm B is identified by the following condition:

$$x_{2A} = \frac{1}{2} + \frac{p_{rB}^{1-\varepsilon} - p_{oA}^{1-\varepsilon}}{2t(1-\varepsilon)}. \quad (4)$$

Each consumer in the market segment $[0, x_{2A}]$ buys $q_{oA} = p_{oA}^{-\varepsilon}$ units from firm A in the second period and each consumer in the market segment $[x_{2A}, x_1^*]$ switches to firm B in period 2 and buys $q_{rB} = p_{rB}^{-\varepsilon}$ units. Thus, firm A's demand from retained customers in period 2 is given by $D_{oA} = x_{2A}p_{oA}^{-\varepsilon}$ and, similarly, firm B's demand from switching customers is: $D_{rB} = (x_1^* - x_{2A})p_{rB}^{-\varepsilon}$.

Firm A's second period profit from old customers is $\pi_{oA} = p_{oA}D_{oA}$ and firm B's second period profit from switching customers is $\pi_{rB} = p_{rB}D_{rB}$. Solving for the equilibrium yields the following proposition.

Proposition 2. *When firms can recognise their old and the rivals' previous customers and price discriminate, second-period equilibrium prices and quantities are:*

(i) if $\frac{1}{4} \leq x_1^* \leq \frac{3}{4}$:

$$\begin{aligned} p_{oA} &= \left[\frac{t(1-\varepsilon)(2x_1^*+1)}{3} \right]^{\frac{1}{1-\varepsilon}} \quad \text{and} \quad p_{rA} = \left[\frac{t(1-\varepsilon)(3-4x_1^*)}{3} \right]^{\frac{1}{1-\varepsilon}}, \\ q_{oA} &= \left[\frac{t(1-\varepsilon)(2x_1^*+1)}{3} \right]^{\frac{-\varepsilon}{1-\varepsilon}} \quad \text{and} \quad q_{rA} = \left[\frac{t(1-\varepsilon)(3-4x_1^*)}{3} \right]^{\frac{-\varepsilon}{1-\varepsilon}}, \end{aligned}$$

$$\begin{aligned} p_{oB} &= \left[\frac{t(1-\varepsilon)(3-2x_1^*)}{3} \right]^{\frac{1}{1-\varepsilon}} \quad \text{and} \quad p_{rB} = \left[\frac{t(1-\varepsilon)(4x_1^*-1)}{3} \right]^{\frac{1}{1-\varepsilon}}, \\ q_{oB} &= \left[\frac{t(1-\varepsilon)(3-2x_1^*)}{3} \right]^{\frac{-\varepsilon}{1-\varepsilon}} \quad \text{and} \quad q_{rB} = \left[\frac{t(1-\varepsilon)(4x_1^*-1)}{3} \right]^{\frac{-\varepsilon}{1-\varepsilon}}. \end{aligned}$$

(ii) if $x_1^* < \frac{1}{4}$:

$$\begin{aligned}
p_{oA} &= [t(1-\varepsilon)(1-2x_1^*)]^{\frac{1}{1-\varepsilon}} \text{ and } p_{rA} = \left[\frac{t(1-\varepsilon)(3-4x_1^*)}{3} \right]^{\frac{1}{1-\varepsilon}}, \\
q_{oA} &= [t(1-\varepsilon)(1-2x_1^*)]^{\frac{-\varepsilon}{1-\varepsilon}} \text{ and } q_{rA} = \left[\frac{t(1-\varepsilon)(3-4x_1^*)}{3} \right]^{\frac{-\varepsilon}{1-\varepsilon}}, \\
p_{oB} &= \left[\frac{t(1-\varepsilon)(3-2x_1^*)}{3} \right]^{\frac{1}{1-\varepsilon}} \text{ and } p_{rB} = 0, \\
q_{oB} &= \left[\frac{t(1-\varepsilon)(3-2x_1^*)}{3} \right]^{\frac{-\varepsilon}{1-\varepsilon}} \text{ and } q_{rB} = 0.
\end{aligned}$$

(iii) if $x_1^* > \frac{3}{4}$:

$$\begin{aligned}
p_{oA} &= \left[\frac{t(1-\varepsilon)(2x_1^*+1)}{3} \right]^{\frac{1}{1-\varepsilon}} \text{ and } p_{rA} = 0, \\
q_{oA} &= \left[\frac{t(1-\varepsilon)(2x_1^*+1)}{3} \right]^{\frac{-\varepsilon}{1-\varepsilon}} \text{ and } q_{rA} = 0, \\
p_{oB} &= [t(1-\varepsilon)(2x_1^*-1)]^{\frac{1}{1-\varepsilon}} \text{ and } p_{rB} = \left[\frac{t(1-\varepsilon)(4x_1^*-1)}{3} \right]^{\frac{1}{1-\varepsilon}}, \\
q_{oB} &= [t(1-\varepsilon)(2x_1^*-1)]^{\frac{-\varepsilon}{1-\varepsilon}} \text{ and } q_{rB} = \left[\frac{t(1-\varepsilon)(4x_1^*-1)}{3} \right]^{\frac{-\varepsilon}{1-\varepsilon}}.
\end{aligned}$$

Proof. See the Appendix.

4.2 First-period pricing

Consider now the equilibrium first-period pricing and consumption decisions. If firms have no commitment power, their market shares in the first period will affect their second period pricing and profits. Thus, forward looking firms take this interdependence into account when setting their first period prices. As consumers are non-myopic they also anticipate the firms' second period pricing behaviour. Suppose the first-period prices lead to a cut-off x_1 that is in the interior of the interval $[0, 1]$. Then the marginal consumer must be indifferent between buying q_{1A} units in the first period at price p_{1A} , and buying q_{rB} units next period at the poaching price p_{rB} ; or buying q_{1B} units in the first period at price p_{1B} , and switching to buy q_{rA} units in the second period at the poaching price p_{rA} . Hence, at an interior solution:

$$\frac{1}{1-\varepsilon} p_{1A}^{1-\varepsilon} + t x_1 + \delta \left[\frac{1}{1-\varepsilon} p_{rB}^{1-\varepsilon} + t(1-x_1) \right] = \frac{1}{1-\varepsilon} p_{1B}^{1-\varepsilon} + t(1-x_1) + \delta \left[\frac{1}{1-\varepsilon} p_{rA}^{1-\varepsilon} + t x_1 \right]$$

yielding:

$$x_1 = \frac{1}{2} + \frac{p_{1B}^{1-\varepsilon} - p_{1A}^{1-\varepsilon}}{2t(1-\varepsilon)(1-\delta)} + \frac{\delta(p_{rA}^{1-\varepsilon} - p_{rB}^{1-\varepsilon})}{2t(1-\varepsilon)(1-\delta)},$$

in which p_{rA} and p_{rB} are given by the expressions in Proposition 2.⁷ Since we know from Proposition 2 that $p_{rB} = p_{rA}$ when $x_1 = \frac{1}{2}$, the previous equation shows that $x_1 = \frac{1}{2}$ if and only if $p_{1A} = p_{1B}$, as expected given the symmetry of the problem. First period profits for firm A and B can be written respectively as:

$$\pi_{1A} = p_{1A}D_{1A} = x_1 p_{1A}^{1-\varepsilon}, \quad (5)$$

$$\pi_{1B} = p_{1B}D_{1B} = (1 - x_1) p_{1B}^{1-\varepsilon} \quad (6)$$

We are now able to characterize the firms' first period problem. Firm A, for example, chooses p_{1A} to maximize its overall profits:

$$\max_{p_{1A}} \pi_A = \pi_{1A} + \delta \pi_{2A},$$

where $\pi_{2A}(x_1(p_{1A}, p_{1B})) = x_{2A} p_{oA}^{1-\varepsilon} + (x_{2B} - x_1) p_{rA}^{1-\varepsilon}$. Solving the problem allows us to state:

Proposition 3. *There is a symmetric Subgame Perfect Nash Equilibrium in which:*

(i) *first-period equilibrium price and quantity purchased are respectively given by*

$$p_1 = \left[t(1 - \varepsilon) \left(1 + \frac{\delta}{3} \right) \right]^{\frac{1}{1-\varepsilon}},$$

$$q_1 = \left[t(1 - \varepsilon) \left(1 + \frac{\delta}{3} \right) \right]^{\frac{-\varepsilon}{1-\varepsilon}}$$

and both firms share equally the market in period 1, thus $x_1^*(p_A^1, p_B^1) = \frac{1}{2}$;

(ii) *second-period equilibrium prices and quantities are:*

$$p_o = \left[\frac{2}{3} t(1 - \varepsilon) \right]^{\frac{1}{1-\varepsilon}}, \quad q_o = \left[\frac{2}{3} t(1 - \varepsilon) \right]^{\frac{-\varepsilon}{1-\varepsilon}},$$

$$p_r = \left[\frac{1}{3} t(1 - \varepsilon) \right]^{\frac{1}{1-\varepsilon}}, \quad q_r = \left[\frac{1}{3} t(1 - \varepsilon) \right]^{\frac{-\varepsilon}{1-\varepsilon}}.$$

and consumers in the intervals $[\frac{1}{3}, \frac{1}{2}]$ and $[\frac{1}{2}, \frac{2}{3}]$ switch from one firm to the other in equilibrium.

(iii) *Each firm's overall profit is equal to*

$$\pi^d = \frac{(8\delta + 9)(1 - \varepsilon)t}{18}.$$

⁷Note that if consumers were myopic they would not take into account the firms' second period prices and so as in the static case we would have $x_1 = \frac{1}{2} + \frac{p_{1B}^{1-\varepsilon} - p_{1A}^{1-\varepsilon}}{2t(1-\varepsilon)}$.

Proof. See the Appendix.

Notice that the properties of CES preferences are such that the level of switching (S) is independent of ε :

$$S = (x_1^* - x_{2A}^*) + (x_{2B}^* - x_1^*) = \frac{1}{3}.$$

The price effects of price discrimination offset each other due to the constant elasticity and, as the quantity demanded is independent of the transport costs, all that matters for consumers' decision to switch is the distance from firms. Hence, as long as price discrimination is employed, like in the Fudenberg and Tirole (2000) model ($\varepsilon = 0$), one third of total consumers switch to their least favorite firm in the second period regardless of ε .

5 Price, output and profit effects

The effects of BBPD *vis à vis* non discriminatory prices can now be evaluated. As previously underlined, perfectly inelastic demand is captured in our model as the limit case of $\varepsilon = 0$. We look next at the impact of demand elasticity, ε , on prices, quantities and profits. We consider the effect on prices first.

Prices The comparison of the two pricing regimes, uniform and BBPD, allows us to state Proposition 4.

Proposition 4. *(i) When $\varepsilon \in [0, 1)$ the following relationship between first period, second period and non discriminatory prices holds no matter the elasticity of demand:*

$$p_r < p_o < p^{nd} \leq p_1.$$

(ii) Provided that demand elasticity is sufficiently high and consumers are patient, $\delta \in (0, 1]$, the average price paid under BBPD can be higher than the uniform price .

Proof. See the Appendix.

The relation between the prices paid by different types of consumers ($p_r < p_o < p^{nd} \leq p_1$) is not affected by the elasticity of demand. As in Fudenberg and Tirole (2000), under BBPD with variable elasticity consumers are overcharged in the first period but then strong competition leads to reduced prices in the second period. The reduction is more pronounced for switchers that need to be encouraged to buy their less favorite good. Intuitively, the difference between

the prices tends to fade out as demand elasticity increases. These findings are illustrated in Figure 1. This figure and the ensuing ones are plotted assuming that $\delta = 1$ and $t = 1$.

[Insert Figure 1]

Finally, on average, if demand is perfectly inelastic, BBPD leads to lower prices. Interestingly, however, as the elasticity increases this feature of BBPD may no longer hold: *the average price paid under BBPD can exceed the uniform price*. In comparison to uniform pricing, this tends to be the case when the demand elasticity is sufficiently high. In that case the reduction in prices in the second period is not sufficient to compensate for the price increase in the first period.

[Insert Figure 2]

Additional results emerge if we take into account different consumers' present value payment for the two periods of consumption. Under no discrimination, each consumer pays a total discounted charge $TP_{nd} = (1 + \delta) [t(1 - \varepsilon)]^{\frac{1}{1-\varepsilon}}$ for the two units. If discrimination takes place, loyal consumers in the interval $[0, \frac{1}{3}]$ and $[\frac{2}{3}, 1]$ buy from the same firm in each period, the total discounted charge is now equal to $TP_o = [t(1 - \varepsilon)]^{\frac{1}{1-\varepsilon}} \left[\left(1 + \frac{\delta}{3}\right)^{\frac{1}{1-\varepsilon}} + \delta \left(\frac{2}{3}\right)^{\frac{1}{1-\varepsilon}} \right]$. While BBPD with perfectly inelastic demand ($\varepsilon = 0$) has no effect on loyal consumers' total payment ($\frac{TP_{nd}}{TP_o} = 1$), the same is not true when $0 < \varepsilon < 1$. If demand elasticity is high enough it can be shown that $\frac{TP_{nd}}{TP_o} < 1$, implying that loyal consumers' present value payment for the two periods is higher under BBPD than in the uniform price case.

New insights are also obtained for the poached consumers, in the interval $[\frac{1}{3}, \frac{2}{3}]$. These consumers switch from one firm to another with a present value of payments equal to $TP_r = [t(1 - \varepsilon)]^{\frac{1}{1-\varepsilon}} \left[\left(1 + \frac{\delta}{3}\right)^{\frac{1}{1-\varepsilon}} + \delta \left(\frac{1}{3}\right)^{\frac{1}{1-\varepsilon}} \right]$. In the unit demand framework, this group of consumers pays strictly less moving from no discrimination to BBPD ($\frac{TP_{nd}}{TP_r} > 1$). The same might not be true when we allow demand elasticity to vary with the price level. In fact, it can be shown that poached consumers can be charged more for the two periods under BBPD than under uniform pricing ($\frac{TP_{nd}}{TP_r} < 1$). In the example in Figure 3 this happens for $\varepsilon \geq \hat{\varepsilon} \simeq 0.6$. More generally, poached consumers are charged more in the two periods with price discrimination if $\hat{\varepsilon}$ is such that $(1 + \delta) - \left(1 + \frac{\delta}{3}\right)^{\frac{1}{1-\varepsilon}} + \delta \left(\frac{1}{3}\right)^{\frac{1}{1-\varepsilon}} < 0$ and $\hat{\varepsilon}$ is decreasing in δ . Intuitively, when demand elasticity is sufficiently high ($\varepsilon > \hat{\varepsilon}$), the reduction in prices in the second period is not sufficient to compensate for the increase in the first period. As a result of that, total discounted charge increases.

[Insert Figure 3]

Quantities Prices clearly affect the quantity demanded by each type of consumer and the overall output supplied. This leads to:

Proposition 5. (i) When $\varepsilon \in (0, 1)$, the following relationship between the quantity demanded by each individual consumer in the first period, second period and under no discrimination (each period) holds:

$$q_1 \leq q^{nd} < q_o < q_r.$$

(ii) For perfectly inelastic demand, then $q_1 = q^{nd} = q_o = q_r = 1$.

(iii) The quantity consumed by any switching consumer over the two periods Q^r exceeds the quantity consumed over the two periods by any loyal consumer Q^o ; moreover, Q^o exceeds the quantity consumed over the two periods by any consumer under non discrimination Q^{nd} .

(iv) BBPD increases the aggregate quantity exchanged on the market.

Proof. See the Appendix.

In the perfectly inelastic case ($\varepsilon = 0$), any given consumer demands one unit of the good with and without price discrimination. A higher elasticity of demand ($0 < \varepsilon < 1$), instead, implies an inverse relation between price and demand. Switching consumers are demanding a higher quantity, both individually and on aggregate through the two periods. Loyal consumers, despite consuming less than switchers, get more of the good than in case discrimination did not take place. This holds both in the second period and over the two periods. Under BBPD the effect of a lower price in the second period leads to a demand increase that more than compensates for the higher price and lower consumption in the first period. In aggregate, this implies that BBPD increases overall consumption over the two periods compared with no discrimination. The results are further illustrated in Figures 4 that plot quantities as a function of ε .

[Insert Figure 4]

Profits Finally, we consider the profit effects of BBPD when the elasticity of demand is allowed to vary in the interval $[0, 1)$. From Proposition 1 it follows that under no discrimination industry profits are equal to $\Pi^{nd} = (1 + \delta)(1 - \varepsilon)t$, while from Proposition 3 under discrimination they are equal to $\Pi^d = \frac{(8\delta + 9)(1 - \varepsilon)t}{9}$. The effect of demand elasticity on firms' profits with and without price discrimination is unambiguous. In both pricing regimes larger demand elasticity reduces profits. Hence, product market competition is tougher as consumers react more strongly to price changes. As usual, higher product differentiation (higher t) raises profits.

Corollary 1. *BBPD reduces industry profits for all $\varepsilon \in [0, 1)$. As demand elasticity increases the negative impact of BBPD on profits gets smaller in absolute terms but it is constant in relative terms.*

The difference in industry profits with uniform pricing and BBPD is:

$$\Pi^{nd} - \Pi^d = \frac{t(1-\varepsilon)\delta}{9} > 0.$$

As in the perfectly inelastic case, our results also suggest that a ban on price discrimination would act to promote industry profits. The previous expression also highlights that, in absolute terms, the difference between profits decreases as demand elasticity increases. The profits' ratio Π^{nd}/Π^d is however $\frac{9(\delta+1)}{8\delta+9}$ and hence it is independent of the elasticity. As profits decrease in both scenarios, the reduction due to price discrimination compared to no discrimination is, in absolute terms, smaller under a larger demand elasticity. The proportion between the two remains however constant: clearly, this also implies that the ranking between them is unchanged all over the support.

6 Welfare analysis

The welfare effects of BBPD when demand elasticity is allowed to vary in the interval $[0, 1)$ can now be investigated. As shown in the previous section, in comparison to uniform pricing, BBPD is always bad for industry profits but it is worst if demand elasticity is lower (i.e., closer to the perfectly inelastic case). The aforementioned properties of CES preferences are such that both switching and consumers' surplus are not affected by the elasticity of demand. Consequently as shown in Lemma 1 consumer surplus in our framework is independent of ε .

Lemma 1. *With behaviour-based price discrimination overall consumer surplus is:*

$$CS^d = Y + v(1 + \delta) - \frac{5}{4}t - \frac{43}{36}t\delta \quad (7)$$

and overall welfare is equal to:

$$W^d = Y + v(1 + \delta) + \frac{(8\delta + 9)(1 - \varepsilon)t}{9} - \frac{5}{4}t - \frac{43}{36}t\delta. \quad (8)$$

Proof. See the Appendix.

From Proposition 1 we have that consumer surplus with no discrimination equals

$CS^{nd} = Y + (v - \frac{5}{4}t)(1 + \delta)$. The difference in consumer surplus with uniform pricing and BBPD is:

$$CS^{nd} - CS^d = -\frac{t\delta}{18} < 0. \quad (9)$$

We can now state the following result.

Proposition 6. *In comparison to uniform pricing:*

- (i) *BBPD boosts consumer surplus, and the increase is independent of ε .*
- (ii) *BBPD raises the switchers' average consumer surplus and has no effect on loyal consumers' average surplus. However, under BBPD the average consumer surplus of loyal consumers exceeds the one of switchers.*

Proof.

See the Appendix.

Part (i) of Proposition 6 confirms that BBPD promotes consumer surplus compared with uniform pricing also when we relax the perfectly inelastic demand assumption. When demand is completely inelastic consumer surplus increases with BBPD because prices fall with discrimination. In our framework, no matter the price effect of BBPD, there is also a demand expansion effect that comes into play. This implies that even if the average price increases under BBPD, the demand expansion effect linked to demand elasticity prevails. We show however that consumer surplus is exactly the same for all levels of demand elasticity, i.e., for any $\varepsilon \in [0, 1)$. Therefore, our analysis highlights that the positive impact of BBPD on consumer surplus is independent of the elasticity of demand. This property is related once more with CES preferences: price and demand expansion effects exactly offset each other to leave consumer surplus unaffected by the elasticity, both under BBPD and no discrimination.

Part (ii) shows that the impact of BBPD on consumer surplus is different for specific groups of consumers, namely loyal consumers and switchers (poached consumers). In comparison to no discrimination, BBPD boosts the average consumer surplus of poached consumers, while it has no effect on loyal consumers' one. Regarding the group of switchers in the interval $[\frac{1}{3}, \frac{2}{3}]$, BBPD raises average transport costs (ATC) due to inefficient switching: more precisely, $ATC_r^{nd} - ATC_r^d = \frac{5}{12}t(\delta + 1) - (\frac{5}{12}t + \delta\frac{7}{12}t) = -\frac{1}{6}t\delta$. However BBPD leads to lower second-period prices and higher second-period consumption for the switchers that more than compensates the increase in first period prices and the decrease in first-period consumption. The net gain for each consumer in this group is given by $\frac{1}{3}t\delta$: consequently the group of switchers face an increase in consumer surplus equal to $\frac{1}{6}t\delta$. The impact of BBPD for loyal consumers, in the

intervals $[0, \frac{1}{3}]$ and $[\frac{2}{3}, 1]$, is different. Since these consumers buy from the closer firm with and without price discrimination, average transport costs are exactly the same in both price regimes. Additionally it is straightforward to see that the benefit of lower second-period prices due to BBPD are exactly offset by the losses associated with higher first period prices.⁸ As a result, consumer surplus for these intervals is unaffected by price discrimination.

As a whole, the positive impact of BBPD on overall consumer surplus is entirely explained by the increase in consumer surplus for the group of poached consumers, i.e., $\frac{1}{3} (\frac{1}{6}t\delta)$.

Finally, we look at the impact of BBPD on social welfare with variable demand elasticity. From Proposition 1 we have that: $W^{nd} = Y + [v - \frac{5}{4}t + t(1 - \varepsilon)](1 + \delta)$. Therefore, the difference in overall welfare under uniform pricing and BBPD is:

$$W^{nd} - W^d = \frac{1}{18}t\delta(1 - 2\varepsilon). \quad (10)$$

Proposition 7. *BBPD is welfare enhancing if $0.5 < \varepsilon < 1$; it reduces social welfare if $0 \leq \varepsilon < 0.5$. Finally, if $\varepsilon = 0.5$, BBPD has no effect on overall welfare.*

Proof. The result follows directly from (10). ■

A relevant contribution of this paper is to highlight that a complete picture of the welfare effects of price discrimination based on purchase history should be drawn under the assumption that consumers' demand do react to price changes, which means that total output is no longer fixed. In comparison to uniform pricing, when ε is low ($0 < \varepsilon < 0.5$), the ability of firms to engage in behavior-based pricing reduces social welfare, as in Fudenberg and Tirole (2000) for the special case where $\varepsilon = 0$. When elasticity is low the increase in output is not enough to overcome the preference mismatch caused by excessive switching. This familiar welfare result in the literature no longer applies if elasticity is sufficiently high ($0.5 < \varepsilon < 1$); in this case *BBPD actually boosts overall social surplus*. When the elasticity of demand is sufficiently high, the inefficiency created by increased transportation costs (i.e. sub-optimal consumption) is more than compensated by the increase in the overall quantity consumed induced by the reduced profit margins that firms can charge.

When $\varepsilon = 0.5$ the raise in consumer surplus due to BBPD is exactly offset by the decrease in industry profits. When demand elasticity is low ($0 \leq \varepsilon < 0.5$) the negative impact of BBPD on industry profits is higher than the positive impact on consumer surplus. BBPD reduces overall

⁸Using the equilibrium prices notice that: $\frac{(1+\delta)}{1-\varepsilon} (p_1^{nd})^{1-\varepsilon} = \frac{1}{1-\varepsilon} (p_1^{1-\varepsilon} + \delta p_o^{1-\varepsilon})$.

welfare. As demand elasticity is high enough ($0.5 < \varepsilon < 1$) the negative impact of BBPD on industry profits is smaller and lower than the positive impact on consumer surplus. Thus, BBPD enhances overall welfare.

The next table summarizes the main effects of BBPD in comparison to uniform pricing in the brand preference approach proposed by Fudenberg and Tirole (2000) when demand elasticity is allowed to vary in the interval $[0, 1)$.

Elasticity of demand	Average Prices	Aggregate Consumption	Profit	Consumers	Welfare
$\varepsilon = 0$	–	no impact	–	+	–
$0 < \varepsilon < 1$	–/+	+	–	+	–/+

Table 1: Effects of BBPD in comparison to uniform pricing

Myopic consumers The analysis so far has assumed that consumers are strategic. Myopia relates to the consumer’s perception of the future. Myopic consumers may not take into account how their actions today affect the prices they will be charged tomorrow. That is, consumers may not understand that their decision to purchase good i today reveals information to firms about them that may be used to price discriminate in the future. Consumers may not fully understand the strategies and incentives that they and the firm(s) face. The previous analysis has assumed that consumers understand the incentives and strategies of the firm. Next we assume that consumers are myopic in the sense that they do not foresee price discrimination in period 2. In this case it is straightforward to show that first-period prices would be equal to the static non-discrimination counterparts, that is $[(1 - \varepsilon)t]^{\frac{1}{1-\varepsilon}}$ while second-period prices would be the same as in Proposition 3. Consumer surplus with BBPD is now $CS^{d,m} = 2v - \frac{19}{9}t$. The difference in consumer surplus with uniform pricing and BBPD is now $CS^{nd} - CS^{d,m} = -\frac{7}{18}t < 0$. It is important to stress that in contrast to the non-myopic case, BBPD now boosts the average consumer surplus of loyal consumers, in the intervals $[0, \frac{1}{3}]$ and $[\frac{2}{3}, 1]$. Specifically, if consumers in these intervals are myopic in comparison to uniform pricing each of them faces an increase in consumer surplus equal to $\frac{t}{3}$. Regarding the group of switchers we find that in comparison to non-discrimination BBPD increases each consumer surplus by $\frac{t}{2}$.

Industry profits are in this case equal to $\Pi^{d,m} = \frac{t(1-\varepsilon)(5\delta+9)}{9}$. If firms use a discount factor equal to $\delta = 1$, then the difference in industry profits with uniform pricing and BBPD is $\Pi^{nd} - \Pi^{d,m} = \frac{4}{9}t(1 - \varepsilon)$. It follows that the difference in overall welfare under uniform pricing and BBPD is: $W^{nd} - W^{d,m} = \frac{1}{18}t(8(1 - \varepsilon) - 7)$. We can therefore conclude that *if consumers are*

myopic BBPD is welfare enhancing if $0.125 < \varepsilon < 1$; it reduces social welfare if $0 < \varepsilon < 0.125$. Finally, if $\varepsilon = 0.125$, BBPD has no effect on overall welfare. Consequently, under myopic consumers it is even more likely that price discrimination enhances social welfare.

7 Conclusion

The economics literature on price discrimination by purchase history is relatively new and, to our knowledge, has focused exclusively on markets where firms face consumers who do not react to price changes, i.e., with perfectly inelastic demands. In the brand preferences approach (e.g. Fudenberg and Tirole, 2000) where purchase history discloses information about a consumer's exogenous brand preference for a firm, the use of BBPD by competing firms often results in intensified competition. In comparison to uniform price average prices and profits fall while consumer surplus increases. Under the assumption of perfectly inelastic demand, welfare falls with BBPD exclusively due to excessive inefficient switching.

To our knowledge this paper constitutes the first assessment of the economic and welfare effects of BBPD when consumers' demand reacts to price changes. With this goal in mind the Fudenberg and Tirole (2000) model (with uniform distribution of preferences) was extended using the CES approach to allow demand elasticity to vary in the interval $[0, 1)$. The aim was to investigate in which circumstances the results derived under perfectly inelastic demand hold or are rather contradicted as demand elasticity increases.

As expected, our model yields the same results as Fudenberg and Tirole (2000) when demand is perfectly inelastic. The same happens when the elasticity of demand is sufficiently low. New results are nonetheless obtained when demand elasticity is high enough. We show that the average price with BBPD can be above its non discrimination counterpart when elasticity of demand is sufficiently high. Additionally, when demand elasticity is high both loyal and poached consumers can face a higher present value of total payment for the two periods of consumption. This reverses the results obtained under perfectly inelastic demand. In comparison to uniform pricing, although average prices can increase with BBPD, we show that, in aggregate, BBPD always increases overall consumption across the two periods.

A relevant contribution of this paper is to highlight that a complete picture of the welfare effects of price discrimination based on purchase history should be drawn under the assumption that consumers' demand responds to price changes. Our analysis confirms that BBPD is bad for welfare in the special case of perfectly inelastic demand ($\varepsilon = 0$) and when demand elasticity

is low ($0 < \varepsilon < 0.5$). In contrast, we show that if the elasticity of demand is high enough, the welfare result derived in models with fixed output no longer applies. Specifically, if $0.5 < \varepsilon < 1$, price discrimination actually enhances overall social surplus in comparison to uniform pricing.

Although our analysis is based on a stylized model, it may have some relevant policy implications. The model confirms that regardless of the elasticity of demand, BBPD by competing firms often intensifies competition and benefits consumers. It also shows that the benefit of BBPD on consumer surplus is higher if consumers are naive. If consumer surplus is the competition authority's standard, as it is the case in most antitrust jurisdictions, then BBPD should not be blocked. In contrast, if total welfare is the criterion adopted by competition agencies to evaluate the effects of this pricing practices, then the policy indication is not as clear cut. If demand elasticity is low a ban on BBPD is likely to boost industry profit and overall welfare at the expense of consumer welfare. If instead, demand elasticity is sufficiently high, in comparison to uniform pricing, BBPD promotes not only consumer surplus but also overall welfare. Additionally, our model suggests that if consumers are myopic, as evidence suggests is the case in many markets, the positive impact of BBPD on welfare is expected to be positive for a wider range of the elasticity of demand ($0.125 < \varepsilon < 1$).

A further contribution of this paper was to provide a closed form solution to a model of competitive BBPD where demand varies with the price level. The assumption of CES preferences was crucial to the goal. Although this functional form makes the analysis tractable and elegant, the assumption can also be seen as a limitation of the present work. Extending the analysis to an alternative setup of elastic demands, as for instance, the one suggested in Armstrong and Vickers (2010), remains as an important avenue for future research. Further directions of research might be to relax the assumption that transport costs do not depend on quantity. By so doing, the model could explore the possibility of two-stop shopping. Given the importance of this pricing practices in the digital economy this is an area of research where plenty of interesting work remains to be done.

Appendix

This appendix collects the proofs that were omitted from the text.

Proof of Proposition 1. The standard technique to derive the best response functions is adopted. From these it is immediate to find $p^{nd} = [t(1 - \varepsilon)]^{\frac{1}{1-\varepsilon}}$ and, consequently, q^{nd} . The second derivative evaluated at the candidate equilibrium prices is $-2t(1 - \varepsilon)^2 \leq 0$ for all the possible values of ε , guaranteeing that the candidate equilibrium is a maximum. As the equilibrium is symmetric, the firms have equal market shares hence their profits are: $\pi^{nd} = \frac{p^{nd}q^{nd}}{2}(1 + \delta)$; using the indirect utility function, the consumer surplus is computed as: $CS^{nd} = Y + 2(1 + \delta) \int_0^{\frac{1}{2}} \left(v - \frac{1}{1-\varepsilon} (p^{nd})^{1-\varepsilon} - tx \right) dx$. Social welfare follows immediately. ■

Proof of Proposition 2. In A's turf, firm A's second period profit from old customers is:

$$\pi_{oA} = \left(\frac{1}{2} + \frac{p_{rB}^{1-\varepsilon} - p_{oA}^{1-\varepsilon}}{2t(1 - \varepsilon)} \right) p_{oA}^{1-\varepsilon}, \quad (11)$$

and firm B's second period profit from switching customers is:

$$\pi_{rB} = \left(x_1^* - \frac{1}{2} - \frac{p_{rB}^{1-\varepsilon} - p_{oA}^{1-\varepsilon}}{2t(1 - \varepsilon)} \right) p_{rB}^{1-\varepsilon}. \quad (12)$$

On its turf, firm A chooses p_{oA} to maximise π_{oA} for any given p_{rB} . The FOC for a maximum yields:

$$\frac{1}{2t} p_{oA}^{-\varepsilon} (t(1 - \varepsilon) - 2p_{oA}^{1-\varepsilon} + p_{rB}^{1-\varepsilon}) = 0.$$

As $p_{oA}^{-\varepsilon} > 0$, firm A's best response function is:

$$p_{oA} = \left(\frac{t(1 - \varepsilon) + p_{rB}^{1-\varepsilon}}{2} \right)^{\frac{1}{1-\varepsilon}},$$

On firm A's turf, firm B chooses p_{rB} to maximize π_{rB} given p_{oA} . The FOC for a maximum yields:

$$\frac{1}{2t} p_{rB}^{-\varepsilon} (-t(1 - \varepsilon) + p_{oA}^{1-\varepsilon} - 2p_{rB}^{1-\varepsilon} + 2x_1^* t(1 - \varepsilon)) = 0.$$

As $p_{rB}^{-\varepsilon} > 0$ firm B's best response function on firm A's turf is instead:

$$p_{rB} = \left(\frac{t(1 - \varepsilon)(2x_1^* - 1) + p_{oA}^{1-\varepsilon}}{2} \right)^{\frac{1}{1-\varepsilon}}.$$

From the two best response functions it is straightforward to find that at an interior solution:

$$p_{oA} = \left[\frac{t(1 - \varepsilon)(2x_1^* + 1)}{3} \right]^{\frac{1}{1-\varepsilon}} \quad \text{and} \quad p_{rB} = \left[\frac{t(1 - \varepsilon)(4x_1^* - 1)}{3} \right]^{\frac{1}{1-\varepsilon}}. \quad (13)$$

Given that $q_i = p_i^{-\varepsilon}$ it follows that:

$$q_{oA} = \left[\frac{t(1-\varepsilon)(2x_1^* + 1)}{3} \right]^{\frac{-\varepsilon}{1-\varepsilon}} \quad \text{and} \quad q_{rB} = \left[\frac{t(1-\varepsilon)(4x_1^* - 1)}{3} \right]^{\frac{-\varepsilon}{1-\varepsilon}}. \quad (14)$$

Similar derivations in firm B's turf allow us to obtain p_{oB} , p_{rA} , q_{oB} and q_{rA} .

Finally, note that it is a dominated strategy for each firm to quote a poaching price, p_{ri} below the marginal cost, which in this case is equal to zero. In firm A's turf, from $p_{rB} > 0$ it must be true that $x_1^* > \frac{1}{4}$. Otherwise, i.e., when $x_1^* \leq \frac{1}{4}$ it follows that $p_{rB} = 0$, and so firm A's best response in order not to lose the marginal consumer located at x_1^* is to quote p_{oA} such that $\frac{1}{1-\varepsilon}p_{oA}^{1-\varepsilon} + tx_1^* = t(1-x_1^*)$. Thus, when $x_1^* \leq \frac{1}{4}$ it follows that

$$p_{oA}^{1-\varepsilon} = [(1-\varepsilon)t(1-2x_1^*)]^{\frac{1}{1-\varepsilon}} \quad \text{and} \quad p_{rB} = 0, \quad (15)$$

$$q_{oA} = [(1-\varepsilon)t(1-2x_1^*)]^{\frac{-\varepsilon}{1-\varepsilon}} \quad \text{and} \quad q_{rB} = 0. \quad (16)$$

The equilibrium prices in firm B's turf are the same reported above. Similarly it is straightforward to find that if $x_1^* \geq \frac{3}{4}$

$$p_{oB} = [t(1-\varepsilon)(2x_1^* - 1)]^{\frac{1}{1-\varepsilon}} \quad \text{and} \quad p_{rA} = 0, \quad (17)$$

$$q_{oB} = [t(1-\varepsilon)(2x_1^* - 1)]^{\frac{-\varepsilon}{1-\varepsilon}} \quad \text{and} \quad q_{rA} = 0.$$

This completes the proof. ■

Proof of Proposition 3. With no loss of generality consider the case of firm A. It's overall profit is equal to:

$$\pi_A = x_1 p_{1A}^{1-\varepsilon} + \delta [\pi_{oA} + \pi_{rA}]$$

with $\pi_{oA} = x_{2A} p_{oA}^{1-\varepsilon}$ and $\pi_{rA} = (x_{2B} - x_1) p_{rA}^{1-\varepsilon}$. Given the equilibrium solutions derived in Proposition 2 it follows that:

$$\pi_{oA} = \frac{1}{3} (1-\varepsilon) t (2x_1 + 1) \left(\frac{1}{3} x_1 + \frac{1}{6} \right) \quad \text{and,}$$

$$\pi_{rA} = \frac{1}{3} (1-\varepsilon) t (4x_1 - 3) \left(\frac{2}{3} x_1 - \frac{1}{2} \right).$$

Firm A overall profit can be rewritten as:

$$\pi_A = x_1 p_{1A}^{1-\varepsilon} + \frac{5\delta(1-\varepsilon)t(2x_1^2 - 2x_1 + 1)}{9} \quad (18)$$

with x_1 given by

$$x_1 = \frac{1}{2} + \frac{3(p_{1B}^{1-\varepsilon} - p_{1A}^{1-\varepsilon})}{2t(1-\varepsilon)(3+\delta)}. \quad (19)$$

Firm A's first-order condition is:

$$\frac{\partial \pi_{1A}}{\partial p_{1A}} = (1-\varepsilon)x_1 p_{1A}^{-\varepsilon} + \frac{\partial x_1}{\partial p_{1A}} \left(p_{1A}^{1-\varepsilon} + \frac{5}{9}t\delta(1-\varepsilon)(4x_1-2) \right) = 0$$

with

$$\frac{\partial x_1}{\partial p_{1A}} = -\frac{3}{2t} \frac{p_{1A}^{-\varepsilon}}{(\delta+3)}.$$

Thus from $\frac{\partial \pi_{1A}}{\partial p_{1A}} = 0$ it follows that:

$$p_{1A}^{-\varepsilon} \left[(1-\varepsilon)x_1 - \frac{3}{2t(\delta+3)} \left(p_{1A}^{1-\varepsilon} + \frac{5}{9}(1-\varepsilon)t\delta(4x_1-2) \right) \right] = 0.$$

As $p_{1A}^{-\varepsilon} > 0$ it must be the case that:

$$(1-\varepsilon)x_1 - \frac{3}{2t(\delta+3)} \left(p_{1A}^{1-\varepsilon} + \frac{5}{9}(1-\varepsilon)t\delta(4x_1-2) \right) = 0.$$

Hence as we are looking for a symmetric equilibrium the FOC evaluated at $x_1 = \frac{1}{2}$ it is straightforward to see that $p_{1A} = p_{1B} = p_1$ which is equal to:

$$p_1 = \left[t(1-\varepsilon) \left(1 + \frac{\delta}{3} \right) \right]^{\frac{1}{1-\varepsilon}}. \quad (20)$$

As $q_i = p_i^{-\varepsilon}$ we have that $q_1 = [t(1-\varepsilon)(1+\frac{\delta}{3})]^{\frac{-\varepsilon}{1-\varepsilon}}$. This completes the proof of (i). Using the fact of $x_1 = \frac{1}{2}$ is straightforward to prove part (ii). From the equilibrium prices derived for both periods it is also straightforward to show that $\pi^d = \frac{(8\delta+9)(1-\varepsilon)t}{18}$. ■

Proof of Proposition 4. (i) Consider first p_1 and p^{nd} . The two prices are identical if and only if $\delta = 0$. If $\delta \in (0, 1]$, the argument that p_1 dominates the one of p^{nd} as $t(1-\varepsilon)(1+\delta/3) > t(1-\varepsilon)$; applying a monotonically increasing transformation to both arguments does not change the relationship so $\forall \varepsilon \in [0, 1)$, $p_1 > p^{nd}$. The difference between p^{nd} and p^o is $\left(1 - \left(\frac{2}{3}\right)^{\frac{1}{1-\varepsilon}}\right) [t(1-\varepsilon)]^{\frac{1}{1-\varepsilon}} > 0$, $\forall \varepsilon \in [0, 1)$ implying $p^{nd} > p^o$. A similar argument applies to p^o and p^r , whose difference is $\left(\left(\frac{2}{3}\right)^{\frac{1}{1-\varepsilon}} - \left(\frac{1}{3}\right)^{\frac{1}{1-\varepsilon}}\right) [t(1-\varepsilon)]^{\frac{1}{1-\varepsilon}} > 0$, $\forall \varepsilon \in [0, 1)$ implying $p^o > p^r$.

(ii) As there is no change between the two periods, the present value of the average non discriminatory price coincides with $\widehat{p}^{nd} = \frac{(1+\delta)}{2} [t(1-\varepsilon)]^{\frac{1}{1-\varepsilon}}$. The average price paid by consumers under BBPD is:

$$\begin{aligned} \widehat{p}^d &= \frac{1}{2}p_1 + \frac{\delta}{2} \left(\frac{1}{3}p_r + \frac{2}{3}p_o \right) \\ &= \frac{1}{2} \left(t(1-\varepsilon) \left(1 + \frac{\delta}{3} \right) \right)^{\frac{1}{1-\varepsilon}} + \frac{\delta}{2} \left(\frac{1}{3} \left(\frac{1}{3}t(1-\varepsilon) \right)^{\frac{1}{1-\varepsilon}} + \frac{2}{3} \left(\frac{2}{3}t(1-\varepsilon) \right)^{\frac{1}{1-\varepsilon}} \right). \end{aligned}$$

Both \widehat{p}^d and \widehat{p}^{nd} are decreasing functions with respect to ε over the domain. The two prices are clearly identical as $\varepsilon \rightarrow 1$; moreover, $\widehat{p}^{nd} > \widehat{p}^d$ for $\varepsilon = 0$. Provided that $\delta > 0$:

$$\lim_{\varepsilon \rightarrow 1} \frac{\widehat{p}^{nd}}{\widehat{p}^d} = \frac{\frac{(1+\delta)}{2} (t(1-\varepsilon))^{\frac{1}{1-\varepsilon}}}{\frac{1}{2} \left(\frac{t(1-\varepsilon)(\delta+3)}{3} \right)^{\frac{1}{1-\varepsilon}} + \frac{\delta}{6} \left(\frac{1}{3} t(1-\varepsilon) \right)^{\frac{1}{1-\varepsilon}} + \frac{\delta}{3} \left(\frac{2}{3} t(1-\varepsilon) \right)^{\frac{1}{1-\varepsilon}}} = 0^+,$$

implying that $\widehat{p}^d > p^{nd}$ as $\varepsilon \rightarrow 1$. Hence, we can conclude that the two functions intersect for at least one value of $\varepsilon \in (0, 1)$. ■

Proof of Proposition 5. (i) As demand is inversely related to prices, the results follow from Proposition 4 (i). In particular, as $\varepsilon \in (0, 1)$, the function $X^{\frac{-\varepsilon}{1-\varepsilon}}$ is decreasing for any value of the argument X ; hence, for any given value of ε , the smaller the argument, the larger $X^{\frac{-\varepsilon}{1-\varepsilon}}$. But then $t(1-\varepsilon) \left(1 + \frac{\delta}{3}\right) \geq t(1-\varepsilon) > \frac{2}{3}t(1-\varepsilon) > \frac{1}{3}t(1-\varepsilon)$ implies $q_1 \leq q^{nd} < q_o < q_r$.

(ii) If demand is perfectly inelastic, it can be verified that:

$$\lim_{\varepsilon \rightarrow 0} \left(\frac{1}{3} t(1-\varepsilon) \right)^{\frac{-\varepsilon}{1-\varepsilon}} = \lim_{\varepsilon \rightarrow 0} \left(\frac{2}{3} t(1-\varepsilon) \right)^{\frac{-\varepsilon}{1-\varepsilon}} = \lim_{\varepsilon \rightarrow 0} (t(1-\varepsilon))^{\frac{-\varepsilon}{1-\varepsilon}} = \lim_{\varepsilon \rightarrow 0} \left[t(1-\varepsilon) \left(1 + \frac{\delta}{3} \right) \right]^{\frac{-\varepsilon}{1-\varepsilon}} = 1.$$

(iii) The quantity consumed over two periods by a given switching consumer is:

$$Q^r = \left[t(1-\varepsilon) \left(1 + \frac{\delta}{3} \right) \right]^{\frac{-\varepsilon}{1-\varepsilon}} + \left(\frac{1}{3} t(1-\varepsilon) \right)^{\frac{-\varepsilon}{1-\varepsilon}}. \quad (21)$$

The corresponding quantity consumed by a loyal consumer is:

$$Q^o = \left[t(1-\varepsilon) \left(1 + \frac{\delta}{3} \right) \right]^{\frac{-\varepsilon}{1-\varepsilon}} + \left(\frac{2}{3} t(1-\varepsilon) \right)^{\frac{-\varepsilon}{1-\varepsilon}}, \quad (22)$$

while any given consumer under no discrimination consumes:

$$Q^{nd} = 2 [t(1-\varepsilon)]^{\frac{-\varepsilon}{1-\varepsilon}}. \quad (23)$$

Focus first on (21) and (22). The first term is identical in both but from point (i) we know that $q_o < q_r$, $\varepsilon \in (0, 1)$. Hence, $Q^r > Q^o$. Turning to (22) and (23), we can write the difference of the two as:

$$\Delta Q = Q^o - Q^{nd} = \left[\underbrace{\left(t(1-\varepsilon) \left(1 + \frac{\delta}{3} \right) \right)^{\frac{-\varepsilon}{1-\varepsilon}} - (t(1-\varepsilon))^{\frac{-\varepsilon}{1-\varepsilon}}}_A \right] + \left[\underbrace{\left(\frac{2}{3} t(1-\varepsilon) \right)^{\frac{-\varepsilon}{1-\varepsilon}} - (t(1-\varepsilon))^{\frac{-\varepsilon}{1-\varepsilon}}}_B \right],$$

from part (i) we know that $A \leq 0$ while $B > 0$. In case $\delta = 0$ then the result is obvious. If, instead, $\delta \in (0, 1]$, we consider once again the function $X^{\frac{-\varepsilon}{1-\varepsilon}}$; as the function is decreasing in X then $|A| - B = \left(\frac{2}{3} t(1-\varepsilon) \right)^{\frac{-\varepsilon}{1-\varepsilon}} - \left[t(1-\varepsilon) \left(1 + \frac{\delta}{3} \right) \right]^{\frac{-\varepsilon}{1-\varepsilon}} < 0$ implying $\Delta Q > 0$.

(iv) The result follows immediately from point (ii). As the market is covered under both no discrimination and BBPD and as both switchers and loyal consumers consume over the two period more than any consumer under no discrimination, then surely BBPD increases the overall quantity consumed, or $Q^d = \frac{1}{3}Q^r + \frac{2}{3}Q^o > Q^{nd}$. ■

Proof of Lemma 1. Overall consumer surplus under BBPD is obtained as:

$$\begin{aligned}
CS^d &= Y + 2 \left[\int_0^{\frac{1}{2}} \left(v - \frac{1}{1-\varepsilon} p_1^{1-\varepsilon} - tx \right) dx \right] \\
&\quad + 2\delta \left[\int_0^{\frac{1}{3}} \left(v - \frac{1}{1-\varepsilon} p_o^{1-\varepsilon} - tx \right) dx + \int_{\frac{1}{3}}^{\frac{1}{2}} \left(v - \frac{1}{1-\varepsilon} p_r^{1-\varepsilon} - t(1-x) \right) dx \right] \\
&= Y + 2 \left[\int_0^{\frac{1}{2}} \left(v - \left(1 + \frac{\delta}{3} \right) t - tx \right) dx \right] \\
&\quad + 2\delta \left[\int_0^{\frac{1}{3}} \left(v - \frac{2}{3}t - tx \right) dx + \int_{\frac{1}{3}}^{\frac{1}{2}} \left(v - \frac{1}{3}t - t(1-x) \right) dx \right] \\
&= Y + v(1 + \delta) - \frac{5}{4}t - \frac{43}{36}t\delta
\end{aligned}$$

As industry profits with discrimination are:

$$\Pi^d = \frac{(8\delta + 9)}{9}t(1 - \varepsilon)$$

total welfare with discrimination is:

$$\begin{aligned}
W^d &= \Pi^d + CS^d \\
&= Y + v(1 + \delta) + \frac{(8\delta + 9)}{9}t(1 - \varepsilon) - \frac{5}{4}t - \frac{43}{36}t\delta. \blacksquare
\end{aligned}$$

Proof of Proposition 6. The proof of (i) follows directly from (9). To prove (ii) look next at consumer surplus for old consumers and switchers respectively denoted by CS_o and CS_r :

$$\begin{aligned}
CS_o &= 2Y \int_0^{\frac{1}{3}} dx + 2 \left(\int_0^{\frac{1}{3}} \left(v - \frac{1}{1-\varepsilon} p_1^{1-\varepsilon} - tx \right) dx \right) + 2\delta \left(\int_0^{\frac{1}{3}} \left(v - \frac{1}{1-\varepsilon} p_o^{1-\varepsilon} - tx \right) dx \right) \\
&= \frac{2}{3}Y + \frac{2}{3}v(1 + \delta) - \frac{7}{9}t(1 + \delta)
\end{aligned}$$

$$\begin{aligned}
CS_r &= Y \int_{\frac{1}{3}}^{\frac{1}{2}} dx + 2 \left(\int_{\frac{1}{3}}^{\frac{1}{2}} \left(v - \frac{1}{1-\varepsilon} p_1^{1-\varepsilon} - tx \right) dx \right) + 2\delta \left(\int_{\frac{1}{3}}^{\frac{1}{2}} \left(v - \frac{1}{1-\varepsilon} p_r^{1-\varepsilon} - t(1-x) \right) dx \right) \\
&= \frac{1}{3}Y + \frac{1}{3}v(1+\delta) - \frac{t}{36}(17+15\delta).
\end{aligned}$$

Average consumer surplus in each of these two groups is:

$$\begin{aligned}
ACS_o &= \frac{CS_o}{\frac{2}{3}} = Y + v(1+\delta) - \frac{7}{6}t(1+\delta), \\
ACS_r &= \frac{CS_r}{\frac{1}{3}} = Y + v(1+\delta) - \frac{17}{12}t - \frac{5}{4}t\delta.
\end{aligned}$$

With no discrimination average consumer surplus per group is

$$\begin{aligned}
ACS_o^{nd} &= Y + v(\delta+1) - \frac{7t}{6}(1+\delta), \\
ACS_r^{nd} &= Y + v(\delta+1) - \frac{17t}{12} - \frac{17t}{12}\delta.
\end{aligned}$$

To prove (ii) note that moving from non-discrimination to BBPD leads to a differential in the average consumer surplus given by: $ACS^{nd} - ACS_o = 0$ and $ACS^{nd} - ACS_r = -\frac{t\delta}{6} < 0$. Thus switchers are better off under BBPD. The differential in the average consumer surplus is: $ACS_o - ACS_r = \frac{t(\delta+3)}{12} > 0$. ■

References

- ANDERSON, S. AND A. DE PALMA (2000), “From local to global competition,” *European Economic Review*, 44, 423-448.
- ARMSTRONG, M., AND VICKERS, J. (2010), “Competitive non-linear pricing and bundling,” *The Review of Economic Studies*, 77, 30-60.
- ARMSTRONG, M. (2006), “Recent Developments in the Economics of Price Discrimination,” in *Advances in Economics and Econometrics: Theory and Applications: Ninth World Congress of the Econometric Society*, ed. by R. Blundell, W. Newey, and T. Persson. Cambridge University Press, Cambridge, UK.
- BERNHARDT, D., LIU, Q. AND SERFES, K. (2007), “Product Customization,” *European Economic Review*, 51, 1396-1422.
- BESTER, H. AND PETRAKIS, E. (1996), “Coupons and Oligopolistic Price Discrimination.” *International Journal of Industrial Organization*, 14, 227-242.

CORTS, K. (1998), "Third-Degree Price Discrimination in Oligopoly: All-Out Competition and Strategic Commitment," *RAND Journal of Economics*, 29, 306-323.

CHEN, Y. (1997), "Paying Customers to Switch," *Journal of Economics and Management Strategy*, 6, 877-897.

CHEN, Y., NARASIMHAN, C. AND ZHANG, Z. (2001), "Individual Marketing with Imperfect Targetability," *Marketing Science*, 20, 23-41.

CHEN, Y. (2005), "Oligopoly Price Discrimination by Purchase History," In: *The Pros and Cons of Price Discrimination*, The Swedish Competition Authority, Stockholm, 101-130.

CHEN, Y., AND ZHANG, Z. (2009), "Dynamic Targeted Pricing with Strategic Consumers," *International Journal of Industrial Organization*, 27, 43-50.

CHEN, Y. AND PEARCY, J. (2010), "Dynamic Pricing: when to Entice Brand Switching and when to Reward Consumer Loyalty," *Rand Journal of Economics*, 41, 674-685.

ESTEVEZ, R.B. (2009a), "Customer Poaching and Advertising," *Journal of Industrial Economics*, vol 57 (1), 112-146.

ESTEVEZ, R.B. (2009b), "A Survey on the Economics of Behaviour-Based Price Discrimination," Working paper, WP-NIPE 5/2009.

ESTEVEZ, R.B. (2010), "Pricing with Customer Recognition," *International Journal of Industrial Organization*, 28, 669-681.

ESTEVEZ, R.B. AND VASCONCELOS, H. (2012), "Price Discrimination under Customer Recognition and Mergers," NIPE WP.

ESTEVEZ, R.B. (2013), "Customer Poaching with Retention Strategies," NIPE - WP 02/2013, RepositoriUM.

FUDENBERG, D. AND TIROLE, J. (2000), "Customer Poaching and Brand Switching," *RAND Journal of Economics*, 31, 634-657.

FUDENBERG, D. AND VILLAS-BOAS, M. (2007), "Behavior-Based Price Discrimination and Customer Recognition," In *Handbook on Economics and Information Systems*, ed. by T. Henderson. North-Holland, Amsterdam.

GEHRIG, T. O. SHY AND STENBACKA, R. (2011), "History-Based Price Discrimination and Entry in Markets with Switching Costs," *European Economic Review*, 55, 732-739.

GEHRIG, T. O. SHY AND STENBACKA, R. (2012), "A Welfare Evaluation of History-Based Price Discrimination," *Journal of Industry, Competition and Trade*, 12, 373-393.

GHOSE, A. AND K-W. HUANG (2006), "Personalized Pricing and Quality Design," Unpublished manuscript.

GU, Y. AND WENZEL, T. (2009), "A Note on the Excess Entry Theorem in Spatial Models with Elastic Demand," *International Journal of Industrial Organization*, 27, 567-571.

GU, Y. AND WENZEL, T. (2011), "Transparency, Price-dependent Demand and Product Variety," *Economics Letters*, 110, 216-219.

LIU, Q. AND SERFES, K. (2004), "Quality of Information and Oligopolistic Price Discrimination," *Journal of Economics and Management Strategy*, 13, 671-702.

LIU, Q. AND SERFES, K. (2006), "Customer Information Sharing among Rival Firms," *European Economic Review*, 50, 1571-1600.

NERO, G. (1999), "Full or partial market coverage? A note on spatial competition with elastic demand," *Managerial and Decision Economics*, 20, 107-111.

RATH, K.P. AND ZHAO, G.(2001), "Two Stage Equilibrium and Product Choice with Elastic Demand," *International Journal of Industrial Organization*, 19, 1441-1455.

SHAFFER, G. AND ZHANG, Z. (1995), "Competitive Coupon Targeting." *Marketing Science*, 14, 395-416.

SHAFFER, G. AND ZHANG, Z. (2000), "Pay to Switch or Pay to Stay: Preference-Based Price Discrimination in Markets with Switching Costs," *Journal of Economics and Management Strategy*, 9, 397-424.

SHAFFER, G. AND ZHANG, Z. (2002), "Competitive One-to-One Promotions," *Management Science*, 48, 1143-1160.

SHY, O. AND STENBACKA, R. (2012), Investment in Customer Recognition and Information Exchange, Unpublished manuscript.

STOLE, L. (2007), "Price Discrimination in Competitive Environments," In Armstrong, M. and R. Porter, eds., *The Handbook of Industrial Organization*, Vol.3, Amsterdam: North-Holland.

TAYLOR, C. (2003) "Supplier Surfing: Competition and Consumer Behaviour in Subscription Markets," *RAND Journal of Economics*, 34, 223-246.

VILLAS-BOAS, M. (1999), "Dynamic Competition with Customer Recognition," *RAND Journal of Economics*, 30, 604-631.