“Measure not by the scale of perfection”: fixed-effects versus peers in hedonic valuation of spatial amenities [a]

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Abstract

A fine spatial scale is essential to the widely applied omitted variable bias treatment of small-area-fixed-effects in Hedonic Pricing. However, amenity valuation is subsumed into the fixed-effects when amenities vary at a coarser spatial scale. To recover amenity valuation while retaining the treatment of spatial omitted variable bias, we derive a new “Differenced-Price-Peers” specification by integrating Hedonic Pricing and the prices and attributes of spatiotemporal peers. We show that small-area-fixed-effects and Differenced-Price-Peers treat omitted variable bias and capitalize the distance to the city center equally well in a spatiotemporally dense US data context. Conversely, in a sparse Greek data context with anisotropic noise pollution where small-area fixed-effects fail, we show Differenced-Price-Peers to successfully recover aviation noise capitalization. The noise discount of house prices is -0.71% per decibel and about 70% higher than the magnitude of the non-fixed-effects Hedonic Pricing, which suggest potential undervaluation due to spatial omitted variable bias.

KEYWORDS: Omitted Variable Bias; Hedonic Pricing; Peers; Nearest Neighbors; Spatiotemporal; House Prices; Linear-in-means; Aviation Noise

JEL Code: C21, C23, D62, Q51, R21, R31, R32

[a] “Measure not by the scale of perfection the meager product of reality.” — Friedrich Schiller

1. Introduction

Hedonic Pricing (HP) applications to the housing market are key for valuing non-market goods, including spatial amenities, but suffer from endogeneities and several types of bias, many of which are caused by space varying unobservables (Kuminoff and Pope, 2014). In addressing spatial omitted variable bias (OVB), Kuminoff et al. (2010) show that in HP small-area fixed-effects (FEs) is an effective treatment, as long as their spatial scale is equal to or below the scale of variation of any omitted variables. This suggests that there is a significant advantage in spatially fine FEs. Abbott and Klaiber (2011) demonstrate the following important tradeoff: a diminishing spatial scale of FEs leads to increasing OVB treatment but decreasing capacity to recover marginal values of spatial amenities. FEs subsume marginal capitalization of amenities varying at spatial scales equal or coarser than that of the FEs. Employing large area or no FEs in HP is a common practice that does not hinder capitalization of spatial amenities, such as air quality, open space, quietness, and urban forests (Beron et al., 2001; Irwin, 2002; Thanos et al., 2015; Tyrvainen and Miettinen, 2000), but introduces a considerable OVB risk (Abbott and Klaiber, 2011). Motivated by this problem, we aim to improve the recovery of amenity capitalization in challenging data contexts while treating spatial OVB. Based on the introduction of spatiotemporal peers into HP, we develop a new 'Differenced-Price-Peers' specification that provides more transparency and flexibility in specifying the scales of amenity capitalization than a typical small-area-FEs-HP[1].

[1] In practice, FEs are often specified at political and administrative boundaries, which are not designed to correspond to the nonlinearities of spatial amenities and their gradient of capitalization into house prices (Abbott and Klaiber, 2011). The literature has moved forward to using spatial differencing techniques, such as discontinuity design and difference-in-difference (DiD), geared towards exploiting exogenous shocks/variation to capture causal effects. However, the assumption of no spatial and/or temporal OVB is incorporated within key assumptions of such techniques, as for instance in defining the bandwidth of the two way fixed effects and in the parallel trends assumption of DiD (Butts, 2023). The cases of continuous treatment and variation in treatment timing and intensity continue to pose unresolved challenges for two way fixed effects regression (Callaway et al., 2021). Given that our approach exhibits advantages precisely in challenging contexts of continuous and anisotropic (dis)amenities, such as aviation noise; we avoid confounding the focus of this paper, which is OVB treatment, with robust DiD or discontinuity design implementation.

The theoretical rationale of employing peer information and linear-in-means models in analyzing the housing market has long been established (Ioannides 2002, 2003). Recently, Szumilo (2021) extended this approaches by proposing a framework based on learning from the prices of neighborhood peers. The linear-in-means model structure is identical to spatial autoregressive models under certain conditions (Goldsmith-Pinkham and Imbens, 2013). Nevertheless, spatial error and spatial lag models do not provide substantial OVB treatment when compared to small-area FEs HP (Kuminoff et al., 2010). Furthermore, spatial econometrics have been criticized as uninformative about the economic processes at work and susceptible to identification problems (Gibbons and Overman, 2012).[2] To remove unobservables as sources of endogeneity, we extend the learning from peers in Szumilo (2021) into a generalized spatiotemporal framework, which, in turn, encompasses linear-in-means and spatial autoregressive specifications.

Differencing does not, on its own, offer a solution to Manski's (1993) "reflection" endogeneity issue in linear-in-means models and spatial econometrics. To address this, we exploit the unique properties of spatial one-time house price data.[3] For time-variant prices of assets exhibiting spatial fixity, the price-peers are literally the nearest neighbors in both geographical space and time. Spatial one-time house price data allow us to map a unique unidirectional relation between the prices of two houses sold in close spatial proximity but at different times, which disentangles the endogeneity feedback loop of price interrelationship found in linear-in-means and spatial autoregressive models. Our approach is based on the actual market practice of eliciting information from what the industry terms as "comparables" (Ratcliffe, 1972), where past information is taken as exogenous and

[2] There are well-established criticisms on applying typical spatial econometric approaches to spatially disaggregate one-time house price observations with detailed comparisons between different types of spatial and spatiotemporal models (Dubé and Legros, 2014; Thanos et al., 2016; Dubé et al., 2018; Nase et al., 2019).

[3] Spatial one-time house price observations are typically employed in HP and do not exhibit a spatial panel structure. There are approaches that transform spatial one-time house price observations in to a panel structure and may include owner demographics, but these approaches are outside the scope of this paper. Repeat sales modelling is a fundamentally different approach as well. Repeat sales are only a very small part of the total sales within a period, and the methodological focus of repeat sales models addresses related biases and the evolution of prices and time-varying attributes.

future information cannot travel backward in time. A priory selection of a limited number of comparables/"price-peers," in close spatial proximity and tight time-window before a sale, is required to enable the assumption of common observed and unobserved spatial amenities between price-peers. Next, the common spatial amenities, which include the unobservables, drop out through differencing to deliver a spatial OVB treatment. An additional step of specifying conditions for minimum variation of an observed spatial amenity between price-peers enables recovering marginal capitalization, while retaining the OVB treatment.

We evidence the effectiveness of our approach through two diverse data contexts that allow the testing of OVB treatment and marginal capitalization recovery under differing types and scales of amenity variation in space. The first context is a spatiotemporally dense US dataset, in which the focus variable of 'distance to center' varies at a finer spatial scale than the FEs. Focusing on aviation noise in Greece, the second context draws on challenging data due to the geography of the study area. We employ the data in Thanos et al. (2012; 2015), who recovered aviation noise values only when forgoing spatial FEs, which subsume any aviation noise capitalization.

Our US study area results demonstrate parity in OVB treatment between small-area FEs and 'Differenced-Price-Peers', while a statistically significant capitalization for 'distance to center' is recovered by both specifications. The application of 'Differenced-Price-Peers' to the data from Greece shows the successful recovery of aviation-noise marginal capitalization through the specification of minimum variation conditions. Conversely, the small-area-FEs-HP fails to recover any noise capitalization. Furthermore, the per decibel house price depreciation of the non-FEs HP is significantly lower than the noise value magnitude recovered by the 'Differenced-Price-Peers', which implies potential undervaluation of aviation noise due to spatial OVB.

Our contribution rests in developing an alternative specification to small-area FEs that can provide equivalent spatial OVB treatment, while further enabling marginal capitalization recovery in challenging contexts. The methodological framework provides the researcher with flexibility in adjusting price-peer selection to the spatial distribution of amenities. Our frameworks allows testing for the similarity/comparability of price-peers, as well as benchmarking for the common amenity condition between price-peers, both of which are required for spatial OVB treatment.

The structure of the paper is as follows. Section 2 provides the methodological framework for integrating HP and price-peer information. Section 3 derives the econometric 'Differenced-Price-Peers' model and specifies the conditions for spatial OVB treatment and for minimum variation of a spatial amenity that allows marginal capitalization. Section 4 presents the data and the empirical strategy. Section 5 illustrates the results of the econometric models and additional robustness checks. Section 6 draws conclusions and discusses limitations and further research.

2. The methodological framework of integrating HP and price-peer information

Signposting the development of our methodological framework, this section starts by setting up the problem of spatial OVB and the FE treatment. The rationale for employing the peer information in analyzing the housing market follows in 2.2. Section 2.3 establishes that, through the practice of employing "comparables", a common knowledge base for market participants is formed and comprises the prices, house attributes, and their weights. Section 2.4 stresses the importance of close temporal and spatial proximity in specifying price-peers and parametrizes the similarity or spatiotemporal "equivalence" between price-peers. Section 2.5 brings together the discussion and assumptions to formalize a hedonic price function that includes price-peer information.

2.1. HP, OVB and fixed effects

The price of housing y is expressed as a general parametric function in equation 1:

$$y = y(\boldsymbol{\xi}, \boldsymbol{x}; \boldsymbol{b}) \quad (1)$$

where we follow Kuminoff and Pope (2014) to deliberately distinguish between $\boldsymbol{x}$, which is a vector of individual property characteristics and ξ, which is a vector of spatial amenities varying at higher levels of spatial aggregation. For brevity, $\boldsymbol{b}$ is a vector of parameters of both $\boldsymbol{x}$ and ξ.

We set up the empirical process by rewriting equation 1 as a typical econometric model of 1st stage HP, in which we explicitly present the problem of unobserved spatial amenities. The naïve HP is presented in equation 2a.

$$y_h = \alpha + \sum_{k=1}^{K} \beta_k x_{hk} + \sum_{\lambda=1}^{\Lambda} \gamma_\lambda^{\#} \xi_{h\lambda}^{\#} + \upsilon_h \quad \text{(2a)}$$

Where β_k is the impact of the k observable structural characteristics x_k on the price of observation h. $\gamma_\lambda^{\#}$ is the estimate of the price impact of the λ observed spatial amenity $\xi_\lambda^{\#}$. The subscript for time is superfluous here due to the single observation of sale price for each house (no repeat sales) and the time-fixed effects are not shown in equation 2 for simplicity. The impact γ_z^* of the zth unobserved spatial amenity ξ_z^* would be included into the regression error term υ_h, which introduces spatial OVB when it is correlated with any of the structural or spatial characteristics of house h. Unless $\sum_{z=1}^{Z} \gamma_z^* \xi_{hz}^* = 0$, equation 2a is typically plagued by OVB. We write equation 2b by substituting: $\upsilon_h = \sum_{z=1}^{Z} \gamma_z^* \xi_{hz}^* + \varepsilon_h$, where ε_h is an independent and identically distributed (IID) random error term.

$$y_h = \alpha + \sum_{k=1}^{K} \beta_k x_{hk} + \sum_{\lambda=1}^{\Lambda} \gamma_\lambda^{\#} \xi_{h\lambda}^{\#} + \sum_{z=1}^{Z} \gamma_z^* \xi_{hz}^* + \varepsilon_h \quad \text{(2b)}$$

If the additive separability assumption holds for FEs, γD, let a ψ number of FE areas be specified at a finer spatial scale than the observed, $\xi_\lambda^{\#}$, and unobserved, ξ_z^*, amenities. It follows that: $\left(\sum_{\lambda=1}^{\Lambda} \gamma_\lambda^{\#} \xi_{h\lambda}^{\#} + \sum_{z=1}^{Z} \gamma_z^* \xi_{hz}^*\right) \in \gamma_\psi D_\psi$, which gives us equation 2c.

$$y_h = \alpha + \sum_{k=1}^{K} \beta_k x_{hk} + \gamma_\psi D_\psi + \varepsilon_h \quad \text{(2c)}$$

We can now estimate βs free of spatial OVB, provided that observed and unobserved amenities are not time varying in the data context (Abbott and Klaiber, 2011). However, it is obvious that, if the assumptions hold, we are no longer able to recover the marginal price of the observed amenity $\gamma_\lambda^{\#}$, which is subsumed to the FEs. Conversely, only if $\xi_\lambda^{\#}$ varies at a finer scale than the FEs and ξ_z^* at a coarser scale, then equation 2c becomes:

$$y_h = \alpha + \sum_{k=1}^{K} \beta_k x_{hk} + \sum_{\lambda=1}^{\Lambda} \gamma_\lambda^\# \xi_{h\lambda}^\# + \gamma_\psi D_\psi + \varepsilon_h \quad (2d)$$

2.2. Peers, endogeneity and spatial-one-time price data

The rationale of employing peer information in analyzing the housing market is motivated by Ioannides (2002, 2003), and Ioannides and Zabel (2008). The typical property transaction data neither exhibit a panel structure nor contain any information about neighboring households, as do the data in Ioannides and Zabel (2008). This constrains the peer information source to the prices and attributes of neighboring house sales, which we define as price-peers. Gibbons (2004) was one of the first papers to employ a linear-in-means (LIM) specification for housing transaction data in order to retrieve the capitalization of property crime by instrumenting to account for the endogeneity. Szumilo (2021) further develops the framework of learning and anchoring as mechanisms that can explain price-peer effects. This captures the impact of changes in characteristics to the own house price and to the prices of neighboring properties through the average price in the neighborhood.

Constructing identification strategies for endogenous peer effects is particularly difficult due to unobservables, the reflection problem, and data limitations (Manski, 1993; Goldsmith-Pinkham and Imbens, 2013). Therein, though, lies the potential to partially disentangle the endogeneity feedback loop that is often created by spatial and/or temporal aggregation. Due to the unique properties of spatially disaggregate one-time (i.e. temporally disaggregate) house price observations, the interrelationship caused by spatial and/or temporal aggregation between the prices of houses *h, i*, and *j* can be disentangled into the complete map of unidirectional relations. For spatial one-time price data, the interrelationship between price-peers *j* and *h*, for instance, would be impossible, if house *h* was sold after house *j*. Price information can only flow from past *(j)* to future *(h)* and not the other way around. This breaks the “reflection” feedback loop and satisfies a key condition,[4] as noted in Szumilo (2021), for estimating linear-in-means (LIM) models on house prices: that the price of house h should not influence the average price in the neighborhood (i.e. of peers), used, in turn, to estimate its own price. In taking the relation between *j* and *h* to be, strictly, a unidirectional information flow, we make the first step in relaxing the sweeping assumption in Manski (1993) that buyers/sellers of *j* and *h* belong to the same group and tend to behave similarly.

The falsely assumed interrelationship between price-peers j and h, when house h is sold after house j, encapsulates the time-travel problem of using typical spatial econometric approaches for spatial one-time price data (Thanos et al., 2016). This also renders the concepts of "social multiplier" (Glaeser et al., 2003) and "spatial multiplier" (Anselin, 1988), together with their endogeneities, inapplicable in spatial one-time price data contexts.

2.3. Comparable sales approach and attribute amenity weights

The key source of inspiration for developing our price-peer framework is the common market practice of employing "comparables" or "comparable sales" in the real estate field. This is an explicitly nearest neighbor (NN) approach (Isakson 1986) that is implicitly spatiotemporal due to spatially disaggregate one-time house price observations. Equation 3 provides the simple case of expressing the sale price of house, h, as a function of one comparable past sale *j* (Colwell et al. 1983). Appendix A provides an exposition of the grid comparable sales approach (CSA) with more than one comparables, and the relevant real estate literature, including its gaps.

$$y_h = \alpha + \sum_{\kappa=1}^{K} \tilde{\beta}_\kappa x_{h\kappa} + \left[y_j - \alpha - \sum_{\kappa=1}^{K} \tilde{\beta}_\kappa x_{j\kappa}\right] = y_j + \sum_{\kappa=1}^{K} \tilde{\beta}_\kappa (x_{h\kappa} - x_{j\kappa}) \quad (3)$$

Where $\tilde{\beta}_\kappa$ is a scalar estimator of the impact or weight of housing characteristic κ on the price of houses *h* and j. $\tilde{\beta}$ is assumed to be equal between *h* and *j*. When houses *h* and *j* are

[4] Szumilo (2021) employs a 2SLS estimation on repeat house sales to capture the effects of renovations (changes in internal and external housing characterizes) after fire. This is fundamentally different to our approach, as we do not seek to model changes in houses or neighborhoods in this paper. The remaining 3 conditions in Szumilo (2021) address the threat to identification posed by exogenous effects, correlated to renovations. It concern changes in characteristics, and prices of house *h* and the neighborhood between sales.

in close spatial proximity, all characteristics depending on locality drop out (not shown for simplicity), as do the constants. Therefore, the only information required for a price estimate is knowledge of the weights ($\tilde{\beta}_\kappa$) of the housing characteristics that diverge between houses h and j.

Equation 3 does not have an error term because it is not estimated through statistical modelling in the simple grid CSA. $\tilde{\beta}_\kappa$ is the weight employed by the real estate professional on a grid and in that setting it is (naively) treated as strictly exogenous scalar along with the information on the price and attributes of house j. Even in cases of statistical estimation of equation 3, the resulting $\tilde{\beta}$s tend to be treated by the professionals and market participant as scalars rather than means of estimates with error and confidence intervals (Isakson 1986). This practice is in accordance with the grid CSA function of reducing the computational burden for individual households or small producers when compared to HP. The information about the weights ($\tilde{\beta}$) is routinely transmitted through real estate professionals to market participants. In a well-functioning and stable housing market, this creates a common knowledge that allows market participants to generate relatively precise cognitive shortcuts.

The HP framework (Rosen, 1974; Palmquist, 1992) assumes coefficients to be stable and equal across observations in a specific market. The stability of CSA attribute weights requires the assumption that the $\tilde{\beta}$s are equal (and time and space invariant) between comparables. Banerjee and Green (2015) indicate price signals to have more influence on subsequent transactions in markets with greater information asymmetry. Major exogenous shock in the market may create information asymmetry between comparables, which obfuscates the puzzle of attribute weight adjustments. Therefore, market participant reliance upon $\tilde{\beta}$s is expected to diminish and reliance upon house prices to increase after major shocks. This is further considered in the set-up of our empirical approach.

2.4. *Spatial and temporal considerations in specifying price-peers*

Peer-price or "comparables" selection is critically underpinned by the mechanism of learning from prices of similar assets. However, we agree with Szumilo (2021) in that both mechanisms of anchoring and learning from prices of similar assets produce empirical outcomes that are too similar to be able to provide clear evidence in support of either. As disentangling "learning" from anchoring is not the focus of this paper, we avoid exacerbating the divergence between "learning" and anchoring outcomes by employing data that do not exhibit substantial macroeconomic and technological shocks.

House sales in close spatial proximity can provide the precise reflection of how a specific combination of spatially varying amenities is valued at a specific location (Kuminoff and Pope, 2014). We avoid the a priori definition of neighborhood shape, size, and structure. Instead, we address spatial variation by selecting a set number of nearest neighbors (NNs) in space to be the price-peers. Henceforth, we employ NNs and price-peers interchangeably. This closely reflects the common practice by housing market participants of selecting and examining comparables in close spatial proximity of a prospective sale. The price-peer selection can be constrained to account for known discontinuities and boundaries. For addressing spatially varying unobservables, price-peers seem a more natural (i.e. actually employed in the market) and nuanced process than that of pre-specified spatial fixed effects in equation 2c.[5]

[5] The spatial scale of capitalization in nonmarket goods is poorly understood, as well as the scale of what can be considered neighborhood and/or housing submarket. These are highly dependent on the local context and there is little reason for the spatial scale to conform to official spatial units, such as census tracks/areas. Our divergence from Abbott and Klaiber (2011) rests in that they specify the unobserved spatial patterns as panel random effects at a pre-specified neighborhood level. Their framework conceptualizes spatiotemporal data as a panel of repeated transactions of houses in a single neighborhood and requires a two-stage estimation to instrument for the endogeneity. This estimation relies on a categorization of characteristics as neighborhood variant or invariant, and on an a priori partition of the neighborhood-varying characteristics into exogenous and endogenous.

The time-varying nature of some spatial attributes is indeed characterized as a pervasive problem in the HP literature (Bajari et al., 2012).[6] The necessary assumption for OVB treatment in small-area-FEs-HP is often unrealistic, since spatial amenities are treated as time invariant within a data timeframe typically spanning many years (Abbott and Klaiber, 2011). Rather, a unidirectional information flow of a few months between peers is a far more realistic assumption. The myopic consumer assumption in HP (Rosen, 1974) is relevant in that households may not look to prices of house sales too far in the past. Dubé et al. (2018) suggest that house price information from beyond 9 months in the past does not improve HP estimation. When information from price-peers is up to date, it accurately reflects time-varying prices and amenity levels. Price-peers, *j* and *h*, are selected to be sufficiently proximal in time for otherwise time-varying observables and unobservables to become time-invariant. Hence, by minimizing spatiotemporal distance between NNs, we can reach the condition of price-peers exhibiting common observed and unobserved amenities that otherwise could be space and/or time varying. Henceforth, we term this as the 'common amenities' condition, which is implicit in the grid CSA of equation 3.

The spatiotemporal proximity of information sources is ignored in much of the real estate valuation literature (see Appendix A), which focuses, instead, on a range of metrics such as the Minkowski, Manhattan, and Mahalanobis distances measuring the degree of attribute similarity between different houses (McCluskey and Borst 2017). The primacy of matching structural house attributes disregards spatially local unobservables; a sale price of an identical house in very distant location/time from another cannot be "comparable" with regard to spatial and/or time varying attributes. This criticism extends beyond the real estate literature when attribute matching estimators, which are fundamentally different to our approach, are applied to house price data. Robust applications of attribute matching models require not only carefully designed quasi-experimental settings, but also explicit space and time restrictions for identifying comparable neighbors (Abbott and Klaiber, 2013;

[6] Bajari et al. (2012) construct a consistent identification process based on the variation of prices and time-varying attributes between repeated sales of house *j*. We propose that otherwise space and/or time varying unobservables become common between price-peers *j* and *h* and can be differenced out.

Haninger et al., 2017). It is outside the scope of this paper to further discuss when such restrictions may forego the effects of spatially localized unobservables.

A key part of the mechanism of learning from the prices of similar assets is the degree of similarity between these assets. We conceptualize this similarity as a spatiotemporal "equivalence" index and parametrize it in ρ ($\rho \in (0, +\infty]$). $\rho = 1$ denotes equivalence, meaning that the prices of the selected peers broadly equate when adjusted for the varying (but known and broadly comparable) attributes[7]. $\rho \ncong 1$ reflects the discount of price-peer information that starts to diverge, not accurately capturing market conditions and prices at the time and location of sale. $\rho = 0$ denotes either that there are no price-peers or that the information contained therein is completely irrelevant, returning the data generating process (DGP) to a typical HP form. $\rho \gg 1$ has scant economic justification and may reflect a data or model issue. ρ is assumed to be a scalar here. It could be feasible in principle for ρ to be specified as a vector and produce model forms akin to geographically and temporally weighted regression (Huang, et al., 2010), but that requires a robust theoretical justification and is outside the scope of this paper.

2.5. Formalizing price functions to include price-peer information

We bring together the discussion and assumptions in this section to formalize a price function that includes price-peer information. The pricing of a house is a complicated matter and a potential cognitive or computational burden to individual households or small producers, to which the widely employed CSA in equation 3 provides a cognitive and practical shortcut for market participants. It is stressed that price-peers (house sales and their comparables) share a location/neighborhood that is known to market participants. The location in itself provides a slew of information, especially since it is routinely visited for a viewing by prospective buyers or real estate professionals. Even though the totality of

[7] This implicitly raises an issue not directly addressed in this paper, since a small one-bedroom flat sale may not serve well as a comparable for a mansion in spatial and temporal proximity, which also implies different sub-markets and discontinuities. Explicitly combining spatial, temporal, and attribute distance in a single weight matrix is not feasible, but, adding "similarity" constraints to the selection of spatiotemporal NNs is straightforward (Yousfi et al., 2020).

such local information may not be explicitly quantified by the participant, it is at least implicitly included in their decision process. This includes information on the prices, attributes and implicit or explicit weights, and a perception of the similarity of the comparables.

We rewrite equation 1 to acknowledge its dependence on model primitives (Kuminoff and Pope, 2014) and the role of peer information in the price formation process. The primitives of the model include parameters for the unobserved consumer types, $\boldsymbol{\delta}$**,** and the unobserved producer types, $\boldsymbol{\theta}$ (Kuminoff and Pope, 2014), but the house attributes and location are assumed to be exogenous and known by both $\boldsymbol{\delta}$ and $\boldsymbol{\theta}$. The information on *C* price-peers is provided by a vector of prices and the corresponding vectors of individual property characteristics, spatial amenities, and parameters $[\boldsymbol{y}_C, \boldsymbol{\xi}_C, \boldsymbol{x}_C; \boldsymbol{b}_C)] \equiv \boldsymbol{\eta}$. $\boldsymbol{\eta}$ is a shorthand that does not contain any utility or profit bearing attributes beyond exogenous information about price-peers based on CSA. $\boldsymbol{\eta}$ provides the prices $\boldsymbol{y}_C$ and attribute information $(\boldsymbol{\xi}_C, \boldsymbol{x}_C)$ house buyers $\boldsymbol{\delta}$ and producers $\boldsymbol{\theta}$ typically rely on to make decisions. $\boldsymbol{\eta}$ also includes in $\boldsymbol{b}_C$ the information on CSA weighting as scalar proxies of *β*s that is available to $\boldsymbol{\delta}$ and $\boldsymbol{\theta}$. A price function that acknowledges its dependence on model primitives and price-peer information equivalence is provided in Equation 4a.

$$y(\boldsymbol{\xi}, \boldsymbol{x}; \boldsymbol{b}) \equiv y\{\boldsymbol{\xi}, \boldsymbol{x}; \boldsymbol{b}[\boldsymbol{\delta}(\boldsymbol{\eta}; \rho), \boldsymbol{\theta}(\boldsymbol{\eta}; \rho)]\} \Rightarrow y(\boldsymbol{y}_C, \boldsymbol{\xi}_C, \boldsymbol{x}_C, \boldsymbol{b}_C, \boldsymbol{\xi}, \boldsymbol{x}; \boldsymbol{b}, \rho) \quad (4a)$$

The reduced form parameters, $\boldsymbol{b}$, describing the shape of the price function, are determined by $\boldsymbol{\delta}$ and $\boldsymbol{\theta}$ (Kuminoff and Pope, 2014). The $\boldsymbol{\delta}$ and $\boldsymbol{\theta}$**,** in turn, learn from and/or anchor on the exogenous information $(\boldsymbol{y}_C, \boldsymbol{\xi}_C, \boldsymbol{x}_C, \boldsymbol{b}_C)$. If this learning/anchoring leads to generating cognitive shortcuts that operate in terms of attribute differences and CSA weights described in equation 3, the common exogenous information $(\boldsymbol{y}_C, \boldsymbol{\xi}_C, \boldsymbol{x}_C, \boldsymbol{b}_C)$ can be included in the reduced form price function.

A $\boldsymbol{b}_C$ generated by local real estate practitioners is not typically available to the researcher, which complicates estimation. In a competitive and stable housing market, it may be plausible to assume that the parameters for a house sale, $\boldsymbol{b}$**,** will converge under certain conditions to the CSA weighting of the *C* price-peers, $\boldsymbol{b}_C$. It is stressed that these peers are in close spatial and temporal proximity to the sale. This admittedly strong assumption ($\boldsymbol{b} = \boldsymbol{b}_C$) is found in the two early influential papers that specifically scrutinize the inclusion of "comparables" or "neighbouring observations" in the HP function. Pace and Gilley (1998) explicitly mention $\boldsymbol{b} = \boldsymbol{b}_C$ as a necessary assumption for grid CSA. They move on to construct a model based on the differences/errors between the CSA ($\boldsymbol{b}_C$) and HP ($\boldsymbol{b}$) parameters implicitly assuming that these parameters are estimated simultaneously (see Appendix A for details on that strand of the literature). Gibbons (2004) defines the "neighbouring observations" for each house sale through a spatial distance kernel, but has to implicitly assume $\boldsymbol{b} = \boldsymbol{b}_C$ for the exogenous housing characteristics in equation 3 of that paper (Gibbons, 2004; pp: F445).

If the assumption ($\boldsymbol{b} = \boldsymbol{b}_C$) becomes valid, equation 4a can be simplified to the reduced form price function in 4b.

$$y(\boldsymbol{y}_C, \boldsymbol{\xi}_C, \boldsymbol{x}_C, \boldsymbol{b}_C, \boldsymbol{\xi}, \boldsymbol{x}; \boldsymbol{b}, \rho) \overset{(\boldsymbol{b}=\boldsymbol{b}_C)}{\Longrightarrow} y(\boldsymbol{y}_C, \boldsymbol{\xi}_C, \boldsymbol{x}_C, \boldsymbol{\xi}, \boldsymbol{x}; \boldsymbol{b}, \rho) \quad \text{(4b)}$$

Equation 4b produces two types of parameters to be estimated: $\boldsymbol{b}$ capturing the marginal capitalization of housing attributes; and ρ capturing the equivalence of the price-peers. We specify in section 3.2 and empirically test in section 5.1 the conditions under which the ($\boldsymbol{b} = \boldsymbol{b}_C$) assumption may hold. This is not found elsewhere in the literature.

3. Introducing price-peers into econometric HP models

This section begins by specifying the price-peer selection process. This is followed by the derivation of the econometric 'Differenced-Price-Peers' model and the conditions for treating OVB. The section closes by enabling amenity valuation in the 'Differenced-Price-Peers' model through the specification of 'minimum variation conditions' for a spatial amenity.

3.1. *Defining price-peers*

To define price-peers, let matrix **W** map the NNs of each observation. Let spatiotemporal distance from h to j $(j \neq h)$ be defined as $f(s_{hj}) \times m(\tau_{hj})$, where $f(s_{hj})$ is a function of spatial distance s_{hj} between h and j, multiplied by a function $m(\tau_{hj})$ of temporal distance τ_{hj}. The spatiotemporal distances from h to all remaining observations (n - 1) in the data are then ranked from closest to furthest.[8] The set $G_C(h) = \{g(h1), \ \ldots, \ g(hC)\}$ contains the C closest spatiotemporal NNs or price-peers of h. C is the number of NNs selected by the researcher. g_{hj} in equation 5 evaluates whether j $(j \in 1, \ldots, n-1)$ is a price-peer to h.

$$g_{hj} = \begin{cases} 1 & \forall\, j \in G_C(h) \wedge \tau_{hj} \geq 0 \\ 0 & \forall\, j \notin G_C(h) \wedge \tau_{hj} < 0 \end{cases} \quad (5)$$

Temporal distance between price-peers is constrained to be non-negative in order to include only information flow from past to future and avoid time-travel. Further constraints can account for known discontinuities and boundaries; for instance, price-peers can be selected only within a local government area that provides the same level of public services. τ can also be conceptualized as a well-behaved time decay function when combined to spatial distance (Dubé et al., 2018). Therefore, we can explicitly include the spatiotemporal distance, w_{hj}, between price-peers in equation 6.

$$\mathrm{w}_{hj} = \begin{cases} f(s_{hj}) \times m(\tau_{hj}) & \forall\, j \in G_C(h) \wedge \tau_{hj} \geq 0 \\ 0 & \forall\, j \notin G_C(h) \wedge \tau_{hj} < 0 \end{cases} \quad (6)$$

A minor advantage of equation 6 compared to 5 is that it can further smooth out any minor linear temporal and spatial variation between price-peers and potentially increase the number of price-peers that can exhibit equivalence (ρ=1). Section 4 provides the empirical specifications of $f(s)$ and $m(\tau)$.

Equations 5 is extended to all other pairs of observations to provide the 0 or 1 elements of **W** in equation 7a, which is subsequently row-standardized to become the algebraic

[8] This is not done simultaneously for all observations, but in loops for each line in **W** in order to facilitate estimation.

equivalent to the average price-peer terms in the equations of Section 3.2. Equation 6 results in the **W** of equation 7b, which is equivalent to a row-standardized Hadamard product of spatial and temporal distance matrices (Thanos et al., 2016).

$$\mathbf{W} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ g_{21} & 0 & 0 & \cdots & 0 \\ g_{31} & g_{32} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{n1} & g_{n2} & g_{n3} & \cdots & 0 \end{bmatrix} \text{(7a)}; \mathbf{W} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ w_{21} & 0 & 0 & \cdots & 0 \\ \mathrm{w}_{31} & \mathrm{w}_{32} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathrm{w}_{n1} & \mathrm{w}_{n2} & \mathrm{w}_{n3} & \cdots & 0 \end{bmatrix} \text{(7b)}$$

When the transactions are chronologically ordered, both **W** matrices return a lower triangular format, which provides a respite from endogeneities caused by the interrelationship between y_h and y_i.

3.2. Deriving the 'Differenced-Price-Peers' model

To operationalize the equation 4b into the 'Differenced-Price-Peers' (DPP) model, we introduce to equation 2b[9] the price-peer information multiplied by the equivalence index ρ. We deliberately include spatial observable and unobservable amenities in order to demonstrate when these drop out between price-peers. Only when the unobservables drop out, the assumptions of equation 4b may hold. To make the derivation clear, information for the price-peer 1 (c=1) multiplied by ρ is introduced to equation 2b: $\rho(y_1 - \alpha - \sum_{k=1}^{K} \beta_k x_{1k} - \sum_{\lambda=1}^{\Lambda} \gamma_\lambda^{\#} \xi_{1\lambda}^{\#} - \sum_{z=1}^{Z} \gamma_z^{*} \xi_{1z}^{*})$. This is repeated for C price peers to produce the equation system in 8a, whose terms are, in turn, added vertically, divided by C and rearranged in equation 8b. We also provide 8b in matrix format to illustrate all N observations (not just of house h) and their interactions defined by the weight matrix **W**.

$$y_h = \alpha + \sum_{k=1}^{K} \beta_k x_{hk} + \sum_{\lambda=1}^{\Lambda} \gamma_\lambda^{\#} \xi_{h\lambda}^{\#} + \sum_{z=1}^{Z} \gamma_z^{*} \xi_{hz}^{*} - \rho y_1 - \rho\alpha - \rho \sum_{k=1}^{K} \beta_k x_{1k} - \rho \sum_{\lambda=1}^{\Lambda} \gamma_\lambda^{\#} \xi_{1\lambda}^{\#} - \rho \sum_{z=1}^{Z} \gamma_z^{*} \xi_{1z}^{*} + \varepsilon_{h1}$$

$+$

$$y_h = \alpha + \sum_{k=1}^{K} \beta_k x_{hk} + \sum_{\lambda=1}^{\Lambda} \gamma_\lambda^{\#} \xi_{h\lambda}^{\#} + \sum_{z=1}^{Z} \gamma_z^{*} \xi_{hz}^{*} - \rho y_2 - \rho\alpha - \rho \sum_{k=1}^{K} \beta_k x_{1k} - \rho \sum_{\lambda=1}^{\Lambda} \gamma_\lambda^{\#} \xi_{2\lambda}^{\#} - \rho \sum_{z=1}^{Z} \gamma_z^{*} \xi_{2z}^{*} + \varepsilon_{h2}$$

[9] Equation 2b is employed instead of 2a because in the derivation below we need to make clear when the unobservables ($\gamma_z^{*} \xi_z^{*}$) drop out between price-peers and when they remain a problem. The temporal ordering and proximity of peer selection in equations 5 and 6 account for the typical time and spatial fixed effects in HP (eq. 2c). This is further demonstrated in the empirical results.

$$\vdots \tag{8a}$$

$$y_h = \alpha + \sum_{k=1}^{K} \beta_k x_{hk} + \sum_{\lambda=1}^{\Lambda} \gamma_\lambda^{\#} \xi_{h\lambda}^{\#} + \sum_{z=1}^{Z} \gamma_z^{*} \xi_{hz}^{*} - \rho y_c - \rho\alpha - \rho \sum_{k=1}^{K} \beta_k x_{ck} - \rho \sum_{\lambda=1}^{\Lambda} \gamma_\lambda^{\#} \xi_{c\lambda}^{\#} - \rho \sum_{z=1}^{Z} \gamma_z^{*} \xi_{cz}^{*} + \varepsilon_{hc}$$

$$=$$

$$y_h = (1-\rho)\alpha + \rho C^{-1} \sum_{c=1}^{C} y_c + \sum_{k=1}^{K} \beta_k (x_{hk} - \rho C^{-1} \sum_{c=1}^{C} x_{ck}) + \sum_{\lambda=1}^{\Lambda} \gamma_\lambda^{\#} (\xi_{h\lambda}^{\#} - \rho C^{-1} \sum_{c=1}^{C} \xi_{c\lambda}^{\#}) + \sum_{z=1}^{Z} \gamma_z^{*} (\xi_{hz}^{*} - \rho C^{-1} \sum_{c=1}^{C} \xi_{cz}^{*}) + \varepsilon_h \tag{8b}$$

$$\mathbf{y} = \boldsymbol{\iota}(1-\rho)\alpha + \rho\mathbf{W}\mathbf{y} + (\mathbf{X} - \rho\mathbf{W}\mathbf{X})\boldsymbol{\beta} + (\boldsymbol{\Xi}^{\#} - \rho\mathbf{W}\boldsymbol{\Xi}^{\#})\boldsymbol{\gamma}^{\#} + (\boldsymbol{\Xi}^{*} - \rho\mathbf{W}\boldsymbol{\Xi}^{*})\boldsymbol{\gamma}^{*} + \boldsymbol{\varepsilon}$$

where the sale prices are in vector $\mathbf{y}$, matrix $\mathbf{X}$ is the house varying attributes multiplied by the vector of parameters $\boldsymbol{\beta}$. $\boldsymbol{\Xi}^{\#}$ and $\boldsymbol{\Xi}^{*}$ are matrices of observed and unobserved spatially varying attributes multiplied by the respective vectors of parameters $\boldsymbol{\gamma}^{\#}$ and $\boldsymbol{\gamma}^{*}$. The equivalence index ρ typically multiplies the weight matrix $\boldsymbol{W}$ to provide the price-peers information for the respective terms. $\boldsymbol{\iota}$ is the identity vector, and $\boldsymbol{\varepsilon}$ is the error term vector. Equation 8b is an illustration of the general DGP which we do not estimate in this form but helps with the derivation and exposition of the models below. It also highlights the equivalence to the LIM specification when $\rho = 1$[10] and to the spatial Durbin model when $\rho \neq 1$. In both cases, the unobserved spatial amenities $\boldsymbol{\Xi}^{*}\boldsymbol{\gamma}^{*}$ do not drop out, explicitly demonstrating the cause of spatial OVB.

By minimizing spatiotemporal distance between NNs, we reach the condition of price-peers exhibiting common observed ($\xi_{h\lambda}^{\#} = \xi_{c\lambda}^{\#}$) and unobserved ($\xi_{hz}^{*} = \xi_{cz}^{*}$) amenities that. The terms $\sum_{\lambda=1}^{\Lambda} \gamma_\lambda^{\#} (\xi_{h\lambda}^{\#} - \rho C^{-1} \sum_{c=1}^{C} \xi_{c\lambda}^{\#})$ and $\sum_{z=1}^{Z} \gamma_z^{*} (\xi_{hz}^{*} - \rho C^{-1} \sum_{c=1}^{C} \xi_{cz}^{*})$ in equation 8b are simplified to $\sum_{\lambda=1}^{\Lambda} \gamma_\lambda^{\#} (1-\rho) \xi_{h\lambda}^{\#}$ and $\sum_{z=1}^{Z} \gamma_z^{*} (1-\rho) \xi_{hz}^{*}$. When the 'equivalence' ($\rho = 1$) holds in combination with the 'common amenities' condition, both

[10] When NNs tend to the sample size ($C \to n-1$), Eq. 8 becomes the LIM: $(\mathbf{y} - \mathbf{W}\mathbf{y}) = (\mathbf{X} - \mathbf{W}\mathbf{X})\boldsymbol{\beta} + (\boldsymbol{\Xi}^{\#} - \mathbf{W}\boldsymbol{\Xi}^{\#})\boldsymbol{\gamma}^{\#} + (\boldsymbol{\Xi}^{*} - \mathbf{W}\boldsymbol{\Xi}^{*})\boldsymbol{\gamma}^{*} + \boldsymbol{\varepsilon}$. This is equivalent to equation 3 of Gibbons (2004) with the exception of the selection process for "neighbouring observations" based on a spatial distance kernel. The choice of distance decay in the kernel "is such that the spatially weighted means explain around one third of the variation in property prices, as measured by the R^2" (ibid, p: F460) of regressing the spatially weighted mean prices to observed house prices. Further evidencing is required as to how this neighbor selection process can effectively remove spatial unobservables. In this paper we improve Gibbons' (2004) approach in the following ways: a) we provide a robust justification for including information from neighbouring observation into HP; b) we explicitly account for unidirectional temporal distance and similarity/equivalence (ρ) between peers in estimating neighbour averages; c) we compare the LIM results to small area FEs in order to benchmark spatial OVB treatment. The UK local authorities employed in Gibbons (2004) as FEs are quite coarse.

$\sum_{\lambda=1}^{\Lambda}\gamma_{\lambda}^{\#}(1-\rho)\xi_{h\lambda}^{\#}$ and $\sum_{z=1}^{Z}\gamma_{z}^{*}(1-\rho)\xi_{hz}^{*}$ become equal to zero. This causes observed and unobserved amenities, along with the constant, to completely drop out. As a result, equation 8b is simplified to the mean price of *C* price-peers adjusted for K differences between each variable *x* and its *C* price-peers mean. This produces the DPP of equation 9, which satisfies the assumptions of equation 4b:

$$y_h = C^{-1}\sum_{c=1}^{C} y_c + \sum_{k=1}^{K}\beta_k(x_{hk} - C^{-1}\sum_{c=1}^{C} x_{ck}) + \varepsilon_h$$

$$\mathbf{y} = \mathbf{Wy} + (\mathbf{X} - \mathbf{WX})\boldsymbol{\beta} + \boldsymbol{\varepsilon} \qquad (9)$$

The additional assumption of no systematic unobserved differences between the price-peers is required here. Any unobservables varying at house level (*x*) in equation 9 can potentially still cause OVB. However, identical unobserved *x* terms between price-peers drop out in DPP providing limited OVB treatment even from unobserved house varying characteristics. This is not the case for small-area FEs-HP. It also is possible for an unobserved spatial amenity $\xi_{un}^{*}\,(un \in Z)$ not to drop out due to extended discontinuities/nonlinearities causing spatial OVB in equation 9. Nevertheless, there is no inherent advantage of small-area FEs-HP over DPP in this case.

The challenge for empirical estimation is selecting a high enough number of NNs that contain sufficient comparable information, while preserving the 'equivalence' and 'common amenities' conditions and assumption ($\boldsymbol{b} = \boldsymbol{b}_C$). As the number of price-peers increases, so does the probability of OVB due to amenity divergence between peers:

$$E(\sum_{c=1}^{C}\sum_{\lambda=1}^{\Lambda}(\xi_{h\lambda}^{\#} - \xi_{c\lambda}^{\#})) \sim 0\,, (c \rightarrow 1, \zeta_{Low} \leq c \leq \zeta_{High}) \qquad (10)$$

where ζ is the range of NNs, in which the common amenities between price-peers condition ($\xi_{h\lambda}^{\#} = \xi_{c\lambda}^{\#}\ and\ \xi_{hz}^{*} = \xi_{cz}^{*}$) holds. This issue is exemplified in the common real estate practice of selecting as a low number of NNs (Ratcliffe 1972), around 10, as opposed to computer assisted automated valuation models (AVMs), where a tendency of introducing hundreds or thousands of price-peers would invalidate the common amenities condition. Furthermore, a very high number of price-peers ($C \rightarrow n-1$) means that their attributes values asymptotically equal the sample/population mean. For instance:

$$\sum_{k=1}^{K}\mu_k C^{-1}\sum_{c=1}^{C} x_{ck} \sim \sum_{k=1}^{K}\mu_k \bar{x}_k\,, (C \rightarrow n-1) \qquad (11)$$

Where $\bar{x}_k$ is the attribute mean over the whole population/sample, which (when $\rho = 1$) turns equation 8b into a Manski (1993) type of LIM model, requiring additional identification strategies.

A Monte Carlo (MC) simulation in Appendix B verifies that the simplifications proposed in equation 9 are numerically consistent and provides the price-peers range for which these simplifications hold. The simulation demonstrates the substantial bias stemming from an omitted spatial amenity, when it is correlated with observed housing attributes, and the effectiveness of DPP in nullifying this problem. The hypothesized spatial OVB treatment holds for DPP when employing a low number of price-peers (ζ_{High}< 20) by recovering the original unbiased coefficients. This is further tested through our empirical estimation.

In Appendix C, we derive the typical spatiotemporal HP models from equation 8b: the Spatiotemporal Durbin model (STDM) (Pace et al. 1998, 2000), the Spatiotemporal Autoregressive (STAR) (Dubé et al., 2018; Hyun and Milcheva 2018), and the Spatiotemporal Error model (STEM) (Thanos et al., 2012, 2015). The "equivalence" and "common spatial amenities" conditions necessary for OVB treatment simplify STDM into DPP but cannot be fully satisfied by either STAR or STEM. This further supports the Kuminoff et al. (2010) findings of spatial error and lag models (the spatial equivalents of STEM and STAR) not delivering any significant spatial OVB reductions.

3.3. DPP and valuation of spatial amenities

There can be special circumstances where 'equivalence' ($\rho = 1$) holds but, due to spatial nonlinearities/discontinuities or significant anisotropy, a specific amenity $\xi_o^{\#}$ ($o \in \Lambda$) is not common ($\xi_{ho}^{\#} \neq \xi_{co}^{\#}$) between the C price-peer pairs. This is equivalent to a quasi-experimental setting of substantial amenity variation within close spatial proximity, which the researcher needs to a priory identify. Such quasi-experimental data are uncommon and difficult to apply in many contexts of non-market valuation through the housing market. Even where an amenity is anisotropically and/or nonlinearly distributed, resulting to variation between some of the price-peers, this may not be sufficient to provide a statistically significant estimate.

To enable amenity valuation in more widespread contexts than bespoke experimental settings, the researcher needs to uncover and exploit amenity variation between price-peers. We achieve this through a price-peer selection process that ensures that amenity $\xi_o^{\#}$ levels between price-peers are not identical: $\xi_{ho}^{\#} \neq \xi_{co}^{\#} \Leftrightarrow \xi_{ho}^{\#} - \xi_{co}^{\#} \neq 0,\ (o \in \Lambda)$. Hence, the following condition for minimum variation of spatial amenity $\xi_o^{\#}$ is introduced to equation 5 or 6 for each pair of C price-peers:

$$(\xi_{ho}^{\#} - \xi_{co}^{\#}) \geq |\ d_o^{\#}|,\ \ (|\ d_o^{\#}| > 0\ \forall\ o \in \Lambda) \tag{12}$$

In this case, the term $\xi_o^{\#}$ does not drop out and the marginal capitalization of this spatial amenity can be estimated in the $\gamma_o^{\#}$ of equation 13.

$$y_h = C^{-1} \sum_{c=1}^{C} y_c + \sum_{k=1}^{K} \beta_k (x_{hk} - C^{-1} \sum_{c=1}^{C} x_{ck}) + \gamma_o^{\#}(\xi_{ho}^{\#} - C^{-1} \sum_{c=1}^{C} \xi_{co}^{\#})\ + \varepsilon_h$$

$$\mathbf{y} = \mathbf{W}\mathbf{y} + (\mathbf{X} - \mathbf{W}\mathbf{X})\boldsymbol{\beta} + (\xi_{ho}^{\#} - \mathbf{W}\boldsymbol{\xi}_{co}^{\#})\gamma_o^{\#} + \boldsymbol{\varepsilon} \tag{13}$$

Where $\gamma_o^{\#}$ is the implicit marginal price of the spatial amenity $\xi_o^{\#}$.

The necessary and sufficient assumptions for estimating equation 13, while ensuring OVB treatment, are: a) that all unobserved amenities remain common between price-peers; b) the condition of minimum amenity variation $d_o^{\#}$ does not introduce selection bias to NNs and causes a divergence of $\boldsymbol{\beta}$ magnitudes between equations 9 and 13 (the assumption $\boldsymbol{b} = \boldsymbol{b}_C$ holds). $d_o^{\#}$ can be adjusted to specific empirical settings that may work better with a nominal value close to 0 or be one sided ($\xi_{ho}^{\#} - \xi_{co}^{\#} > 0\ and/$or $\xi_{ho}^{\#} - \xi_{co}^{\#} < 0$) in cases of censored or skewed variables. However, an unreasonably high $d_o^{\#}$, which depends on the nature of the amenity, will restrict the sample size and violate the “equivalence” and/or “common spatial amenities” conditions of DPP. We test this by using two case studies: one of linear and isotropic distance-based amenity in a spatiotemporally dense data context; and another of an anisotropic and inherent nonlinear aviation noise in a sparser data context.

4. Data and Empirical Strategy

We use two data contexts to empirically test the hypotheses in section 3 and the effectiveness of DPP in markedly different settings. Given the methodological nature of the paper, we employ widely used publicly available data to accommodate verification and any extensions. The first context is of a linear and isotropic amenity, distance to city center, which is well-rooted in urban economic theory and a potential source of spatial OVB when not included in estimation. We use a well-documented and high observation dataset for single-family housing transactions from Lucas County, Ohio, US, 1993-1997 (LeSage, 1999). This dataset allows for a number of robustness checks. The second data context, in the south part of Athens, Greece, is of an anisotropic and inherently nonlinear dis-amenity, aviation noise. This is a challenging data context, which reflects the typical data constraints in many published HP studies. We employ the data in Thanos et al. (2012, 2015) to compare spatial-FEs-HP with DPP, and we re-estimate their HP base model, which is an example of employing no spatial FEs because of the hindrance to amenity capitalization.

For consistency between case studies, we use a inverse function for the spatial (s_{hj}) and temporal (τ_{hj}) distances in equation 6. Dubé et al. (2018) suggest a maximum of 8-9 months, but we employ the tighter constrain of 4 months for τ_{hj} to ensure temporal stability, especially in the Greek data.[11] Selecting a limited number of comparables is a common practice in real estate. For instance, Ratcliffe (1972) argues that less than 5 comparables is unsafe and far more than 10 comparables is probably unnecessary. The MC simulation results in Appendix B also show 10-20NNs to perform best for DPP in reducing OVB. Therefore, we specify DPP models for both case studies at 10 NNs and provide robustness checks for increasing NNs. In both study areas, we estimate $\rho\mathbf{Wy}$ in the DPP models and

[11] The temporal treatment that works best for each study area differs in the minimum temporal distance between price-peer pairs, which is one week for the Greek data and three weeks for the US data. We speculate that this is due to local market mechanisms of information dispersal. In Greece, only the real estate agents provided this information to buyers and sellers informally, whereas in the US there were more formal channels for publicizing this information, which may take longer to reach prospective market participants. Anecdotally, the real estate agents in the Greek study area, when asked, typically provide CSA derived values of housing attributes or local area characteristics uplift or discount in €/sqm or percentages.

employ $\rho = 1$ as hypothesis testing for the 'equivalence' condition and equation 9 being the correct DGP. For each DPP, we drop a few of the earliest observations to be employed as NNs for subsequent observations, which is the spatiotemporal equivalent of the Cochrane-Orcutt process (Dubé et al., 2014).

The tight temporal constrain of peer selection in combination with the small number of NNs in space and time account in DPP for the equivalent of time and small-area fixed-effects. The OLS estimator in the absence of spatial OVB is in principle unbiased and efficient, as spatiotemporal effects in lower triangular systems are exogenous (Pace et al., 1998)[12] and avoid the LIM model issues with identification, as discussed in section 2.2.

4.1. Distance type amenity in the US

Table 1 provides the variable description and descriptive statistics for the Lucas County data. We simplify the exposition by employing seven key structural variables as well as one spatial amenity, distance to center, for subsequent tests and checks. A key reason for selecting this dataset is consistency with the assumptions (sections 2.3-2.4) of no substantial financial or technological shocks. This price stability is evidenced by not being rejecting at the 99% level the following two null hypotheses:[13] a) the annual samples (1993-97) were drawn from populations with the same median price, b) there is no house price trend when the data are ordered annually (1993-97). Figure 1 tentatively suggests that house prices gradually decrease in proximity to the city center, ceteris paribus.

[12] Maximum likelihood estimation is not feasible for models with lower triangular matrices. The IV/GMM estimator of Kelejian and Prucha (1998) produced unfeasibly high values for ρ, which is a known issue (Bell and Bockstael, 2000). This is possibly exacerbated by the lower triangular nature of the matrices. The OLS and IV/GMM coefficient magnitudes, except ρ, exhibit close consistency that reinforces our view that the OLS estimator can be in principle unbiased and efficient for DPP. We do not employ a matching estimator, which precludes the use of Abadie and Imbens (2011) bias correction.

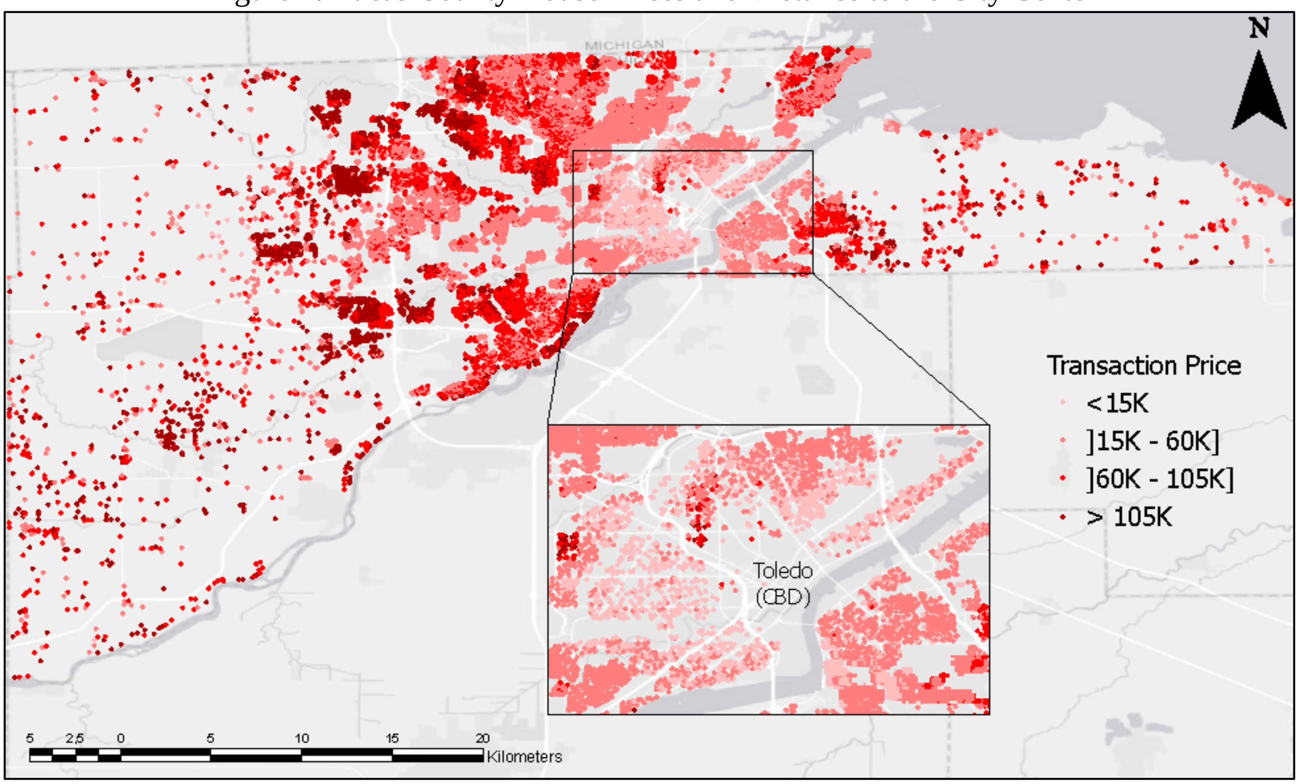
[13] a) Nonparametric K-sample test on the equality of medians: Pearson Chi2(4) = 4.8915 sig.= 0.299. b) Nonparametric test for trend across ordered groups: z = 0.81 Prob > |z| = 0.418 (Conover, 1999).

Table 1: Lucas County data description and statistics

Variable	Notation	Description	Mean	Std.Dev.	Min	Max
sale price		Sale price (USD)	79018	59655	2000	875000
ln_price	y	Ln of Sale price	11.020	0.763	7.601	13.682
age	x_1	Age of the house (years)	50.367	27.931	0.000	161.000
ln_age^	x_2	Ln of age	3.672	0.940	0.000	5.088
lotsize	x_3	Lot size (sqft*1000)	0.009	0.001	0.007	0.013
live-area	x_4	Living area (sqft*100)	0.073	0.004	0.048	0.089
bathrooms	x_5	Bathrooms (count)	0.788	0.185	0.000	2.079
Stories2+	x_6	Over 2 stories (binary)	0.317	0.465	0.000	1.000
Garage	x_7	Garage (binary)	0.862	0.344	0.000	1.000
Dist_Center	ξ_1	Distance to Toledo city center (km)	8.802	5.102	0.384	38.581
Time FEs		Quarterly dummy variables 1993Q2-1997Q4 (1993q1: base category)				
Spatial (Small Area) FEs		124 US census track areas encompassing a population between 2,500 to 8,000 people				

Note: The data from Lucas County Ohio is available in the MatLab software library (LeSage, 1999). The gamma transformation ($b_1x_1+b_2\ln x_1$) captures nonlinearities instead of the quadratic specification ($b_1x_1+b_2x_1^2$), in order to address the multicollinearity concerns for the latter (Dubé and Legros, 2013). The prices are CPI adjusted for 1997. N = 20,671

Figure 1: Lucas County House Prices and Distance to the City Center



Note: The data from Lucas County Ohio is available in the MatLab software library (LeSage, 1999). N = 20,671. The intervals are selected to illustrate the very low prices in the center.

4.2. Aviation noise in Greece

The Athens data from 1995-2001 include 1649 house sales, which is an order of magnitude lower and spread over a longer period than the number of observations in the US data. This, along with the challenging data situation in Greece,[14] provides a unique context to test the DPP application in non-ideal settings and compare to small-area-FE-HP. Table 2 describes the data and provides descriptive statistics. Municipality is an official spatial delineation that includes a population of tenths of thousands. It is a key local government level that operates a number of public services determining key facets of everyday life including, but not limited to, school catchment, street cleaning, garbage disposal services, policing, and local taxes. Therefore, we introduce the additional constraint of price-peers to be located in the same municipality. Otherwise, key amenities will diverge between price-peers, given the lower number of observations, and violate the conditions of equations 9 and 13.

We follow Thanos et al. (2015) to define a total 'A' weighted decibel (dBA) noise level (γ_t), given by the combination of aviation noise (γ_{av}) and the general noise level (γ_{tr}), as:

$$y_t(y_{av}, y_{tr}) = 10\ln(10^{y_{av}/10} + 10^{y_{tr}/10}) \quad (14)$$

Equation 14 captures the effect of aviation noise on the housing market above the general noise level in the area, which is not possible with typical cut-off points. If γ_{av} is dominant, γ_{tr} has an almost negligible effect on the total noise level and vice versa. As no data were available on other sources of noise pollution, the 55 dBA for background noise γ_{tr} is based on evidence from previous studies of the soundscape in Athens (Nicol and Wilson, 2004; Yang and Kang, 2005). Figure 2 displays the difference between equal-loudness contours and loudness at individual points in space. Thanos et al. (2012, 2015) included 166 observations (northeast in Figure 2) in non-coastal municipalities exposed to aviation noise much below the background level. We keep these observations in order to ensure adequate replication of the original findings and perform an additional robustness check.

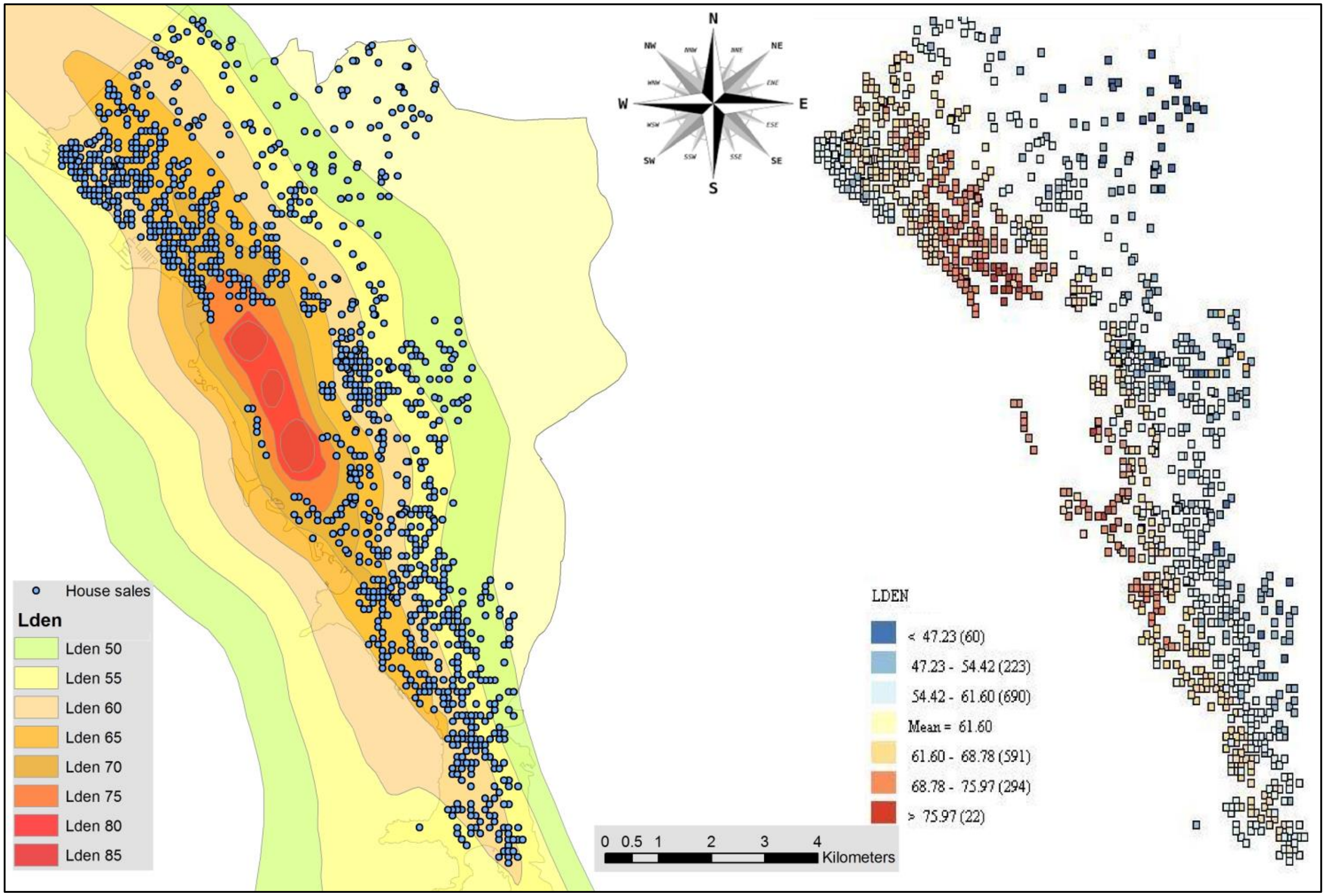
[14] Due to the legal environment, there are no official (or otherwise dependable) data of observed house sale prices available in Greece. We employ a unique dataset that includes the actual sale price acquired by local area real estate agents through the data collection exercise (Thanos et al., 2012).

Table 2: Athens data description and statistics

Variable name	Description	Mean	S.D.	Min	Max
price	House sale price (2001 €s)	176605	111798	26412	1027146
Ln_price	Ln of the house price	12.09	0.61	9.9	14
Aviation_noise (γ_{av})	'A' weighted decibel(dBA) of day–evening–night noise	63.63	5.26	55	82
Noise (γ_t)	The log-sum of aircraft(dBA L_{den}) and background (55dBA) noise	63.59	5.26	55	82
Live-area	Living floor area (square meters)	119	74	22	500
Garage	Garage or private parking (binary)	0.35	0.48	0.00	1.00
House_age	House age (years)	5.80	6.99	0.00	25.00
Ln_age	Ln of House age	1.48	0.92	0.00	3.26
Coastline	House located within 300m from the coastline (binary)	0.13	0.34	0.00	1.00
Park-sports	Distance (km) to the nearest sport facility or park	0.56	0.33	0.02	1.90
Church	Distance to the nearest church (km)	0.43	0.23	0.01	1.63
public_serv	Distance to the public service (km)	0.57	0.37	0.02	2.00
Health_cntr	Distance to the nearest hospital/ health center (km)	0.97	0.53	0.02	2.64
School	Distance to the nearest school (km)	0.40	0.29	0.01	1.98
Plaza	Distance to the nearest plaza (km)	0.35	0.21	0.01	1.36
Airport	Drive distance in meters to the airport entrance (km)	4.40	2.21	0.20	10.42
Major_road	Distance to the nearest major road with heavy traffic (km)	0.22	0.21	0.00	1.48
Dist_Center	Distance to Athens center (km)	9.87	4.13	2.87	19.51
Non-coastal	Non-coastal municipality with low aviation noise (binary)	0.10	0.30	0	1
Municipality FEs	10 Municipalities (local government area) in the south part of Athens				
Floors Num FEs	Floor number fixed effects (ground and 1st floor: base category)				
House type FEs	Detached, semi-detached/terraced, and flat/apartment (base category)				
Room Num FEs	Number of rooms fixed effects (1-2 rooms: base category)				
Time FEs	Quarterly fixed effects 1995Q2-2001Q1 (1995q1: base category)				
Small area FEs	Small area fixed effects: 107 polygons with 500m diameter				

Note: The aircraft noise data were supplied by the European Organization for the Safety of Air Navigation (EUROCONTROL), who have complete data on flight paths and aircraft movements. The consistency of the modelled data was validated with noise measurements by EUROCONTROL. Prices are CPI (2001) adjusted and converted to €s. We employ the gamma transformation for age (Dubé and Legros, 2013). N=1,649.

Figure 2: Aviation noise spatial distribution: equal-loudness contours vs individual points in space



Note: The left panel displays a typical oversimplification of aviation noise level (γ_{av}) in contours (Thanos et al., 2012). The light-yellow background shows the administrative area outline, not a noise contour. The right panel presents each data point's noise (γ_{av}) as ±3 stdev from the mean (N of each stdev category in the brackets) to illustrate the continuous spatial distribution of aviation noise.

5. Econometric modelling results

This section first discusses the US study area results and presents a number of robustness checks through this spatiotemporally dense dataset. This is followed by recovering marginal capitalization for aviation noise in a data-wise challenging Greek context.

5.1. US study area: 'Distance to center' in Lucas County

In the presence of spatial OVB, we expect more sensitive housing characteristics to spatial patterns to markedly diverge between naïve HP and Spatial-FEs-HP. The initial results of our empirical approach are illustrated in Table 3, where we first run the naïve HP model (col. 1), which is expected to suffer from OVB by missing the distance to center variable and any spatial FEs. We then introduce distance to center in HP2 (col. 2), which affects the lot size coefficient to such a degree that its sign turns negative. This is not consistent with economic theory, implying substantial spatial OVB that is not treated by including 'distance to center'. Introducing the small-area-FEs (col. 2) not only brings back the 'lot size' to a reasonable sign/size range and halves the magnitude of 'distance to center' coefficient, but also affects variables that vary across space and in proximity to city center, such as building height (Stories2+), age, and garage. These changes of coefficient magnitudes suggest a reduction, if not the full treatment, of spatial OVB. For instance, garage comes down from causing an unreasonable 40% uplift in price to a more reasonable 24%. The stability of the models is highlighted by "live-area" coefficient remaining consistent across Table 3, which, in accordance with sections 2 and 3, shows key structural characteristics varying at house level not to be sensitive to spatial OVB.

Table 3: HP and DPP estimation results for Lucas County

	HP1 No spatial FEs/Dist_Center (1)	HP2 No spatial FEs (2)	HP3 Spatial FEs (3)	DPP1 Base model: 10 NNs (4)	DPP2 $d_o^{\#} = 0.15$km btw NNs (5)
ρ				0.9998 (0.0002) ***	0.9997 (0.0002) ***
β_1: age	-0.0215 (0.0002) ***	-0.0195 (0.0002) ***	-0.0113 (0.0002) ***	-0.0108 (0.0003) ***	-0.0111 (0.0003) ***
β_2: ln_age	0.3116 (0.0061) ***	0.3322 (0.0059) ***	0.1845 (0.0057) ***	0.1845 (0.0127) ***	0.1739 (0.0111) ***
β_3: lot_size	0.0020 (0.0001) ***	-0.0002 (0.0001) ***	0.0010 (0.0001) ***	0.0007 (0.0002) ***	0.0007 (0.0002) ***
β_4: live-area	0.0462 (0.0007) ***	0.0445 (0.0007) ***	0.0414 (0.0006) ***	0.0417 (0.0009) ***	0.0432 (0.0009) ***
β_5: Garage	0.3978 (0.0090) ***	0.3943 (0.0086) ***	0.2362 (0.0069) ***	0.2232 (0.0078) ***	0.2432 (0.0081) ***
β_6: bathrooms	0.0450 (0.0089) ***	0.0374 (0.0085) ***	0.0335 (0.0068) ***	0.0356 (0.0073) ***	0.0382 (0.0075) ***
β_7: Stories2+	0.1687 (0.0070) ***	0.1746 (0.0067) ***	0.1032 (0.0056) ***	0.0993 (0.0058) ***	0.1050 (0.0059) ***
γ_1: Dist_Center		0.0311 (0.0008) ***	0.0167 (0.0028) ***	0.0039 (0.0049)	0.0124 (0.0042) **
constant	9.6424 (0.0378) ***	9.2504 (0.0377) ***	9.8009 (0.0391) ***		
Time FEs	Yes	Yes	Yes	No	No
Small area FEs	No	No	Yes	No	No
H0: ρ=1				F(1,20662)=1.58	F(1,20662)=2.71
Adj-R^2	0.6953	0.7174	0.8276		
AIC	22547	20990	10904	11453	13201
BIC	22761	21213	12102	11525	13272

Note: ***p< 0.001; **p<0.01; *p<0.05. Robust standard errors (in parentheses). The data from Lucas County Ohio is available in the MatLab software library (LeSage, 1999). As part of the spatiotemporal equivalent of the Cochrane-Orcutt process (Dubé et al., 2014), we drop the first 60 days or 308 observations resulting to the final N = 20,671 for all models. Col. 1 and 2 are the naïve HP of equation 2a, with col. 1 missing the "distance to center". Col. 3 is the small area FEs HP of equation 2c. Col. 4 is the DPP of equation 9 with10 NNs. Col. 5 is the DPP of equation 13 with10 NNs and $d_o^{\#}$ of 0.15km.

Despite their substantial differences in specification, the coefficients of HP3 (col. 3) and DPP1 (col. 4) have almost identical magnitudes excepting 'distance to center'. This is tentative evidence that the assumption $\boldsymbol{b} = \boldsymbol{b}_C$ in equation 4b holds under these conditions. This is also a first step to evidencing that spatial OVB treatment is equivalent between DPP and Spatial-FEs-HP. Given the high number of observations in the dataset, there is sufficient variation within the FEs for the distance to center coefficient in HP3 to achieve significance. Conversely, the spatiotemporally dense dataset means for DPP1 that the 10 NNs are too close to allow sufficient 'distance to center' variation between price-peers. The distance to center coefficient is no different to zero (col. 4), which is in accordance with the common spatial amenities dropping out between price-peers in equation 9.

The shortcoming of DPP not capturing 'distance to center' is addressed by introducing (col. 5) the condition of a minimum 'distance to center' difference between price-peers ($d_o^{\#}$) in equation 12. A $d_o^{\#}$ below 0.1km does not provide sufficient variation for estimation and $d_o^{\#}$ over 0.2km risks violating "equivalence" and "common spatial amenities" conditions. We take a $d_o^{\#}$ of 0.15km (164 yards) ($|\, d_o^{\#}| \geq 0.15$), the middle of 0.1-0.2 range, so that price-peers are located in the same neighborhood and have common spatial amenities while allowing the minimum sufficient variation for estimation. As a result, the coefficients of col. 5 are almost identical to columns 3 and 4, suggesting that the additional conditions for estimating equation 13 are met.

Most importantly, we can, in principle, capture the marginal capitalization of spatial amenities while treating spatial OVB by employing DPP in equation 13. The 1.24% (95% CI: 0.42% – 2.06%) price uplift per kilometer of 'distance to center' in DPP2 is consistent with the 1.67% (95% CI: 1.13% – 2.11%) uplift in HP3. This signifies that DPP and Spatial-FEs-HP are essentially on par regarding spatial OVB treatment and recovery of amenity marginal capitalization.

The HP3, DPP1, and DPP2 (col. 3-5) provide significant improvement in goodness of fit compared to naïve HP (col. 1-2) , even though DPP2 exhibits lowest overall goodness of fit of the three due to the extra data constraint. AIC shows HP3 to fit the data best while BIC shows DPP1 to fit best. This is not surprising as BIC favors parsimonious specifications that allow more degrees of freedom. This difference in degrees of freedom is not a big problem for the US data but can affect datasets with lower number of observations, such as the Greek data, which marks a clear advantage of DPP over Spatial-FEs-HP.

Robustness checks for the US study area

Key robustness checks are presented in Table 4. We start by deliberately introducing spatial OVB to small-area-FEs-HP and DPP specifications. In col. 1, we remove from small-area-FEs-HP 'distance to center'. If the FEs cannot treat spatial OVB, we expect a noticeable change in the spatially sensitive coefficients (age, lot_size, Garage, Stories2+ and Dist_Center), which is not the case. We replicate this for DPP by removing 'distance to center' and by adding a constant (col. 2). Not only the removal of 'distance to center' does not alter coefficients between DPP3 and DPP1, but also the constant does not pick up any of the variation and remains no different to zero. This clearly demonstrates that the unobservables and the constant drop out in accordance with equation 9.

Additional checks involve the assumption $\boldsymbol{b} = \boldsymbol{b}_C$ and the conditions of 'common amenities' and 'equivalence' between price-peers. In a falsification test, instead of 0.15km in DPP2, we increase $d_o^{\#}$ by an order of magnitude to 1.5km in DPP4 (col. 3). This causes the following: a) extensive magnitude divergence of all spatially sensitive coefficients compared to DPP2 and HP3, indicating severe bias and violation of the assumption: $\boldsymbol{b} = \boldsymbol{b}_C$; b) the violation of the "equivalence" condition, as the hypothesis of ρ = 1 is soundly rejected; and c) extensive deterioration of the overall goodness of fit, rendering DPP4 worse than the naïve HP1.

Table 4: Robustness checks for US data

Variables	HP4 No Dist_Center (1)		DPP3 No Dist_Center (2)		DPP4 $d_o^{\#}$ =1.5km (3)		DPP5 100NNs (4)	
P			1.0076 (0.0045)	***	0.9987 (0.0003)	***	1.0002 (0.0002)	***
β_1: age	-0.0114 (0.0002)	***	-0.0108 (0.0003)	***	-0.0175 (0.0003)	***	-0.0137 (0.0003)	***
β_2: ln_age	0.1863 (0.0057)	***	0.1845 (0.0127)	***	0.2778 (0.0101)	***	0.2261 (0.0123)	***
β_3: lot_size	0.0011 (0.0001)	***	0.0007 (0.0002)	***	0.0003 (0.0002)		0.0005 (0.0002)	***
β_4: live-area	0.0413 (0.0006)	***	0.0417 (0.0009)	***	0.0413 (0.0008)	***	0.0443 (0.0009)	***
β_5: Garage	0.2368 (0.0070)	***	0.2231 (0.0078)	***	0.3753 (0.0099)	***	0.2787 (0.0090)	***
β_6: bathrooms	0.0338 (0.0068)	***	0.0355 (0.0073)	***	0.0312 (0.0088)	***	0.0396 (0.0080)	***
β_7: Stories2+	0.1030 (0.0056)	***	0.0995 (0.0058)	***	0.2252 (0.0066)	***	0.1246 (0.0063)	***
γ_1: Dist_Center					0.0541 (0.0024)	***	0.0207 (0.0019)	***
Constant	9.9043 (0.0353)	***	-0.0861 (0.0504)					
Time Fes	Yes		No		No		No	
Small area FEs	Yes		No		No		No	
H0: ρ=1			F(1,20662)=2.80		F(1,20662)= 24.68	***	F(1,20662)=0.63	
Adj-R^2	0.8273		0.8217					
AIC	10939		11451		21504		14001	
BIC	12129		11522		21575		14073	

Note: ***p< 0.001; **p<0.01; *p<0.05. Robust standard errors (in parentheses). The data from Lucas County Ohio is available in the MatLab software library (LeSage, 1999). As part of the spatiotemporal equivalent of the Cochrane-Orcutt process (Dubé et al., 2014), we drop the first 60 days or 308 observations resulting to the final N = 20,671 for all models. Col. 1 is the small area FEs HP of equation 2c that is missing the “distance to center”. Col. 2 is the DPP of equation 9 with10 NNs that is missing the “distance to center”. Col. 3 is the DPP of equation 13 with10 NNs and $d_o^{\#}$ of 1.5km. Col. 4 is the DPP of equation 9 with 100 NNs instead of 10.

We increase NNs to 100 (col. 4) and the resulting DPP5 exhibits substantial deterioration of the overall goodness of fit. The spatially sensitive coefficients of DPP5, also, noticeably diverge from DPP1 and HP3 to the direction of the naive HP models (col. 1-2, Table 3), which suggests violation of $\boldsymbol{b} = \boldsymbol{b}_C$ and the 'common amenity' condition. When employing many hundreds or thousands NNs,[1] equation 9 converges to a Manski (1993) type of LIM model. Contrary to the grid CSA practice, the practice of computer assisted massive AVMs employing hundreds or thousands "NNs" is not supported by our framework or findings.

5.2. *Greek study area: aviation noise in Athens*

Even though it is challenging data-wise, the south part of metropolitan Athens, containing much of the "Athens Riviera", offered a unique context with instances of very high long-term exposure and significant variation of aviation noise. Table 5 illustrates the estimation results, starting with reproducing the base HP model (col.1) in Thanos et al (2012, 2015).[2] HP1 recovers an aviation noise marginal value of 0.42% price discount per decibel but does not include any spatial FEs. We employ in HP2 (col. 2) small-area FEs, which subsume the aviation noise and render most other spatial amenity coefficients insignificant. From the use of small-area FEs, BIC shows the overall goodness of fit to markedly deteriorate in HP2, severely impacted by the loss of 106 degrees of freedom[3] in contrast to AIC that shows improvement.

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[1] The Table D1 in the Appendix D provides the DPP estimation results when employing 30 and 1000 NNs. Overall goodness of fit starts deteriorating and spatial OVB increases DPP over 20NNs in consistence with the simulation in Appendix B. The equivalence condition is where the equations 5 and 6 diverge in statistical performance: at low NNs the difference is negligible but at higher NNs equation 6 keeps the specification from violating the 'equivalence' condition. We include relevant checks in Table D1: a) DPP1 (10NNs eq.13) with weights derived by equation 5 instead of 6 displays negligible differences in coefficients and also recovers 1.2% price uplift per kilometer of 'distance to center'; b) conversely, the equivalence condition is violated at 100 NNs when employing equation 5. Nevertheless, this divergence is moot, as above 20NNs in both cases the overall goodness of fit deteriorates, and the 'common amenities' condition is gradually violated.

[2] For consistency, we corrected some minor oversights in the original Thanos et al (2012, 2015) base model that affect none of the findings or arguments here. We are happy to present the original Thanos et al (2012, 2015) model upon request and replicate all our additional analysis on the originals.

[3] Previously, an anonymous referee noted that the equivalent to DPP and to other spatiotemporal models (Appendix C) would be spatiotemporal FEs (quarterly FEs×small-area-FEs) instead of separate spatial and temporal FEs. Beyond the deteriorating BIC, the number of FEs reduces the degrees of freedom prohibitively for estimating datasets not in the many tenths of thousands (e.g. Athens data: N=1649, 24 quarterly FEs ×107 small-area-FEs = 2,568 spatiotemporal FEs). We can also show upon request that spatiotemporal FEs provide no estimation advantage even in the US dataset of N=20,671.

Table 5: Athens Aviation Noise

	HP1 No spatial FEs (1)	HP2 Spatial FEs (2)	HP3 FEs coastal (3)	DPP1 10 NNs (4)	DPP2 $d_o^{\#}$ =.25dBA (5)	DPP3 $d_o^{\#}$ =2.5dBA (6)
ρ				1.0002 (0.0003)***	0.9988 (0.0008)***	0.9973 (0.0017)***
Noise (γ_t)	-0.0042 (0.0014)**	0.0029 (0.0043)	0.0017 (0.0044)	-0.0028 (0.0017)	-0.0071 (0.0016)**	-0.0089 (0.0038)*
live-area	0.0052 (0.0003)***	0.0051 (0.0003)***	0.0049 (0.0003)***	0.0052 (0.0006)***	0.0050 (0.0007)***	0.0047 (0.0006)***
Garage	0.0878 (0.0133)***	0.0927 (0.0135)***	0.0953 (0.0146)***	0.0928 (0.0171)***	0.0854 (0.0209)**	0.0992 (0.0211)**
House_age	-0.0111 (0.0057)	-0.0098 (0.0061)	-0.0118 (0.0066)	-0.0137 (0.0062)	-0.0150 (0.0055)*	-0.0107 (0.0057)
Ln_age	0.0508 (0.0318)	0.0427 (0.0335)	0.0530 (0.0382)	0.0554 (0.0293)	0.0736 (0.0267)*	0.0305 (0.0328)
Coastline	0.0464 (0.0223)*	-0.0095 (0.0314)	-0.0103 (0.0317)	0.0224 (0.0165)	0.0288 (0.0155)	0.0056 (0.0187)
Park-sports	-0.0647 (0.0194)***	-0.0942 (0.0400)*	-0.0849 (0.0425)*	0.0003 (0.0341)	-0.0204 (0.0238)	-0.0156 (0.0223)
Church	0.1037 (0.0295)***	-0.0284 (0.0440)	-0.0184 (0.0463)	0.0364 (0.0199)	0.0233 (0.0292)	0.0412 (0.0166)*
public_serv	0.0365 (0.0243)	-0.0400 (0.0436)	-0.0567 (0.0468)	-0.0392 (0.0197)	-0.0113 (0.0172)	-0.0299 (0.0191)
Health_cntr	-0.0113 (0.0150)	0.0412 (0.0340)	0.0418 (0.0370)	0.0086 (0.0180)	-0.0167 (0.0128)	-0.0007 (0.0213)
School	0.0955 (0.0249)***	0.0874 (0.0534)	0.0894 (0.0555)	0.0073 (0.0334)	0.0030 (0.0196)	0.0216 (0.0194)
Plaza	-0.1532 (0.0332)***	0.0012 (0.0472)	0.0024 (0.0506)	-0.0155 (0.0283)	-0.0409 (0.0409)	-0.0529 (0.0414)
Airport	0.0212 (0.0031)***	0.0011 (0.0285)	-0.0108 (0.0318)	-0.0047 (0.0081)	-0.0140 (0.0083)	-0.0132 (0.0123)
Major_road	0.0893 (0.0333)**	0.0361 (0.0563)	0.0636 (0.0598)	0.0638 (0.0377)	0.0403 (0.0413)	0.0329 (0.0239)
Dist_Center		0.0357 (0.0284)	0.0518 (0.0312)	0.0134 (0.0101)	0.0258 (0.0093)*	0.0292 (0.0108)*
Non-coastal	-0.3193 (0.0294)***	-0.1730 (0.0736)*				
Constant	10.6415 (0.1175)***	9.9334 (0.6854)***	9.8053 (0.7061)***			
Floor, House type, & Room Num FEs	Yes	Yes	Yes	Yes	Yes	Yes
Time FEs	Yes	Yes	Yes	No	No	No
Small area FEs	No	Yes	Yes	No	No	No
H0: ρ=1				F(1, 1588)=2.20	F(1, 1372)= 1.11	F(1,1108)= 2.67
Adj-R^2/ AIC/ BIC	0.85/20/323	0.86/-81/757	0.85/-44/639	N/A /-87/-38	N/A /144/192	N/A /215/255
N	1649	1649	1483	1621	1405	1140

Note: ***p< 0.001; **p<0.01; *p<0.05. Data from south Athens, 1995-2001, N=1649. We do not drop a uniform number of observations for the equivalent of the Cochrane-Orcutt process to allow replication of Thanos et al. (2012, 2015) base HP. Cols.1-3 display to robust standard errors (in parentheses). Cols.4-6, which display robust standard errors (in parentheses) clustered at the municipality level as NNs are set only within the same municipality. Col. 1 is the naïve HP of equation 2a. Col. 2 is the small area FEs HP of equation 2c. Col. 3 is the small area FEs HP of equation 2c only employing the coastal municipality data that were affected by aviation noise. Col. 4 is the DPP of equation 9 with10 NNs. Col. 5 is the DPP of equation 13 with10 NNs and a $d_o^{\#} > 0.25$ dBA. Col. 6 is the DPP of equation 13 with100 NNs and a $d_o^{\#} > 2.5$ dBA. The non-coastal dummy drops in DPP because price peers are from the same municipality.

DPP1 (col. 4) has best overall fit in this challenging data setting (BIC and AIC), marking a clear advantage to the loss of degrees of freedom by the FEs in HP. The statistically significant attributes varying at house level display almost identical coefficients between DPP1 and HP2. The only significant spatial amenity in HP2, distance to parks, drops out in DPP1 in keeping with the common amenity condition. DPP1 estimation drops 28 observations, less than 2%, to the spatiotemporal equivalent of the Cochrane-Orcutt process (Dubé et al., 2014). These initial 28 observations do not have sufficient price-peers for estimation and are, in turn, employed as the price-peers of subsequent house sales.

To recover marginal capitalization of aviation noise we introduce the condition: $d_o^{\#} > 0.25$ dBA. This a nominal noise level difference between price-peers given that sound changes below 3 dBA are not auditorily perceptible. The condition is one sided since γ_t has an asymptotical lower limit of 55 dBA due to noise masking (equation 14). DPP2 (col. 5) recovers noise capitalization in house prices of -0.71% per dBA, which is much higher to the -0.42% per dBA of the HP1[18]. Due to the geography of the study area, the aviation noise spatial distribution is correlated with distance to center, the latter just about reaches statistical significance along with age.[19] DPP2 exhibits otherwise negligible differences to DPP1 except lower overall fit to the data, due to the extra constraint.

Robustness checks for the Greek study area

The main shortcoming of DPP2 is dropping 241 observations, substantially more than the 28 observations dropped in DPP1. This is due to the tighter combined window of time, space, and noise-difference. Most of these observations are dropped due to the inclusion of non-coastal municipalities that suffered negligible aviation noise exposure with very low

[18] In spite of the data sparseness and "noisiness", the one-tailed z-score (1.36) rejects the "H_0: (-0.71%) - (-0.42%)=0" at 90% level and "Ha: $|-0.71\%| > |-0.42\%|$" is tentatively accepted.

[19] The inhabited area is a strip of land at most a few kilometers wide situated between the sea and a steep hill/mountain. The center of Athens is towards the north-northwest in Figure 2, which is also the direction of the flight paths (noise source) and the direction of the inhabited strip of land. The distance to center is highly correlated with and forces out of significance the noise coefficient. This is further addressed robustness checks (see footnote 20). Most of the housing stock was built in the '80s and late '70s. Building activity spread mostly outwards along the main roads from the center of Athens, hence the correlation with distance to center and aviation noise.

variation of γ_t. As a robustness check, we exclude the non-coastal municipalities in HP3 (col. 3) The results show that dropping non-coastal municipalities is not the reason for not recovering noise marginal values, as HP3 exhibits negligible differences to HP2. DPP2 has an acceptable deficit of 78 observations, or circa 5% sample size divergence compared to HP3. It is a feature and potential shortcoming of equation 13 to be constrained to employing data that exhibit some variation of the amenity in question. Col. 6 provides an additional robustness check of increasing $d_o^{\#}$ in DPP by an order of magnitude to 2.5 dBA. The aviation noise coefficient increases only slightly in magnitude but its standard error doubles, thus reducing statistical significance level. The overall goodness of fit deteriorates and the sample size for estimation is reduced to 1140 observations, which violates the conditions of equation 13 by substantially limiting the sample size.

We summarize a range of additional robustness checks[20] that cannot all be fully illustrated here and are found in Table D2, Appendix D. We show that aviation noise is subsumed even when we employ coarser FEs[21] in HP. We also run the HP3 in (col. 2) with a restricted sample of the exact 1,405 observations in DPP 2 (col. 5). The results do not substantially change and the aviation noise coefficient remains no different to zero. When we increase the DPP2 (col. 5) from 10 to 100 NNs the results are: a) the hypothesis of $\rho = 1$ is rejected; b) some spatial amenities are becoming statistically significant as they are no longer common between price-peers; and c) BIC deteriorates.

Predicated on the methodological framework, we observe in the empirical findings a key similarity between the US and Greek data contexts. When $d_o^{\#}$ and/or the NNs are increased, the following results are gradual and consistently monotonic: overall goodness of fit deterioration, and coefficient divergence from the base DPP.

[20] The Greek data sparseness and "noisiness" limits us from fully replicating all the robustness checks (e.g. 1000 NNs) conducted to the US data.

[21] We provide the results when we increase the size of the FEs to polygons of 1km diameter instead of the current 0.5km. We set coarser FEs at the level of municipality. We also provide the regression results of HP1 including 'distance to center' and HP3 without the distance to center.

6. Conclusions

A key shortcoming for Hedonic Pricing (HP) in valuing spatial amenities is spatial omitted variable bias (OVB). The common OBV treatment is small-area fixed-effects (FEs), which require assumptions about their spatial structure and size (Kuminoff et al. 2010) and subsume marginal values of spatial amenities varying at a coarser spatial scale. OVB treatments based on neighborhood peers, such as linear-in-means (Szumilo, 2021) and spatial autoregressive models (Gibbons and Overman, 2012), pose several identification and endogeneity challenges including Manski's (1993) "reflection" issue. Motivated by these problems and inspired by actual market practice, we develop a new 'Differenced-Price-Peers' specification. By exploiting the unique properties of spatial one-time house price data, we achieve a limited solution to the "reflection" issue. By differencing over price-peers with common amenities, we provide spatial OVB treatment. By specifying conditions for minimum variation of an observed spatial amenity between price-peers, we recover marginal capitalization while retaining the OVB treatment.

A US context with a spatiotemporally dense dataset successfully demonstrates that 'Differenced-Price-Peers' delivers spatial OVB treatment and capitalization for distance to city center equivalent to small-area-FEs. We employ a Greek study area with sparse housing data and anisotropic noise pollution to reflect typical challenges of amenity valuation in the literature. The 'Differenced-Price-Peers' model recovers marginal capitalization of aviation noise while treating spatial OVB, where capitalization recovery through small-area-FEs-HP fails. The -0.71% per decibel noise discount of house prices in 'Differenced-Price-Peers' model is much higher than the -0.42% in the non-FEs HP, which suggests potential undervaluation of aviation noise due to spatial OVB.

The empirical exercise demonstrates that 'Differenced-Price-Peers' can be robustly estimated in widely different markets through consistent empirical settings for time and space, necessitating rudimentary context knowledge. It requires a priory selection of a limited number of price-peers in spatial proximity and a tight time-window before the sale.

Not only this is a widespread practice in housing market and potentially more transparent than FEs, it also provides the researcher with additional flexibility and control in adjusting price-peer selection to the complexities of the spatial distribution of amenities and their capitalization gradient into house prices. The additional flexibility and control are checked by testing the selection of price-peers for their similarity/equivalence, and by benchmarking for the common amenity condition between price-peers.

We acknowledge the following limitations and consider further research avenues. Even though, in recovering a capitalization effect, the 'Differenced-Price-Peers' specification has the advantage of time varying unobservables becoming common between price-peers and dropping out, capitalization cannot be interpreted as a non-marginal welfare estimate. It cannot capture equilibrium effects for the whole urban housing market and marginal capitalization is valid only for small "localized" changes (Palmquist, 1992). In the Greek data context, Thanos et al. (2015) provided some evidence of sorting due to long term noise pollution, which indicates the need for equilibrium sorting models to fully capture non-marginal values. Extending 'Differenced-Price-Peers' in combination with tools such as discontinuity design and/or difference-in-difference models to robustly recover non-marginal effects is the next step. Future research can also include the combination with machine learning (Shen and Ross, 2021) to identify peers and minimize unobservables.

^The Appendices referenced in the text are available in the SSRN version of the paper: http://dx.doi.org/10.2139/ssrn.4600289

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