BER Analysis of Multi-User NOMA Networks in the Presence of Interference

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Abstract—This letter considers a multi-user non-orthogonal multiple access (NOMA) system in the presence of arbitrary co-channel interference (CCI) signals. Taking into account the precise statistics of the self-interference signals of NOMA users, it derives a new unified expression for the bit error rate (BER) of an arbitrary number of users. The users may employ various modulation schemes and orders, including M-ary pulse amplitude modulation with arbitrary orders, M-ary quadrature amplitude modulation with arbitrary orders, or a combination of both with arbitrary orders. The newly derived expression is then applied to a multi-user NOMA in two different interference scenarios: (i) fixed located CCI, and (ii) Poisson point process-based randomly located CCI. The preciseness of the BER expression for different number of users and modulation schemes/orders is verified via numerical and simulation results.

Index Terms—Arbitrary number of users, bit error rate, non-orthogonal multiple access, co-channel interference.

I. INTRODUCTION

ON-ORTHOGONAL multiple access (NOMA) has been recognized as a promising candidate for future wireless networks. It has the potential to significantly enhance the spectral efficiency, provide low transmission latency, improve user fairness, and facilitate massive connectivity by enabling multiple users to simultaneously share the same frequency/time resource blocks [1]. Specifically, in the power-domain (PD) downlink NOMA, the transmitter implements the superposition coding technique to superimpose the data of multiple users with different power levels, while the receiver employs the successive interference cancellation (SIC) technique to decode and acquire the desired data [1].

In the existing literature, a very limited number of studies have analyzed the bit error rate (BER) or symbol error rate (SER) performance of NOMA systems subject to co-channel interference (CCI) [2]–[4]. In more details, the authors in [2] and [3] have considered multi-user NOMA systems under Rayleigh fading channels and CCI. By considering the binary phase shift keying and quadrature phase shift keying modulation schemes, they have investigated the approximate BER performance of the system. In [4], the authors have considered a two-user NOMA cooperative relaying system in the presence of CCI and shadowed Rician fading channels. Taking into account the M-ary phase-shift keying modulation scheme, the SER performance of the system has been analyzed. Additionally, the authors in [5] and [6] respectively proposed new beamforming and joint beamforming and NOMA power allocation schemes to mitigate the effects of CCI.

It is important to note that the aforementioned studies have approximated the information symbols of self-interference NOMA users by Gaussian distribution. However, such an assumption does not lead to a precise analysis of NOMA systems, especially for low-order modulations. Moreover, in practice, different NOMA users may employ various modulation schemes and orders to meet specific requirements. Therefore, for a precise BER analysis, it is essential to consider the exact statistics of NOMA signals rather than relying on a Gaussian approximation. Additionally, the BER expressions presented in the aforementioned studies are only applicable to specific fading channels and environments. Whereas, practical wireless communication systems may operate under different scenarios and environments, such as line-of-sight (LOS) scenarios, non-LOS scenarios, indoor and outdoor scenarios, and so forth [7]. Therefore, a unified expression is required that can be applied to different scenarios under any arbitrary types of fading channels.

Motivated by the aforementioned considerations, this letter aims to evaluate the BER performance of multi-user NOMA systems in the presence of multiple independent but not necessarily identical interferers. Therefore, it derives a new unified expression for the precise BER analysis, where the users can employ same or different modulation schemes and orders such as M-ary pulse amplitude modulation (MPAM) with arbitrary orders, M-ary quadrature amplitude modulation (MQAM) with arbitrary orders, or a combination of both with arbitrary orders. Although the newly derived expression is applicable to any arbitrary types of fading channels, for the sake of conciseness, it is applied to a multi-user NOMA under Nakagami--$m$ fading channels to assess the system’s BER in two different interference scenarios: (i) fixed located interferers scenario, and (ii) Poisson point process (PPP)-based randomly located interferers scenario. Numerical and simulation results verify the preciseness of the unified expression for an arbitrary number of users employing various modulation schemes and orders. The results also show that the system performance is more severely impacted by the PPP-based interferers compared to the fixed interferers.

The remainder of this letter is organized as follows. The system model and the BER expression are presented in Section II and Section III, respectively. The results and summary of the letter are presented in Section IV and Section V, respectively.
II. SYSTEM MODEL

We consider a downlink PD-NOMA system with a base station (BS) and multiple users, labeled as $U_1, \ldots, U_N$, in which each user receives the signal of interest in the presence of arbitrary CCI signals. Based on the PD-NOMA protocol, the BS multiplexes the information symbols of $N$ users simultaneously sharing the same transmission bandwidth, in which based on the channels' conditions, each user is assigned a distinct power coefficient. Without loss of generality, the channels are sorted as $|h_1|^2 > |h_2|^2 > \ldots > |h_N|^2$. Consequently, in order to ensure fairness, the power assignments are allocated to users as $\beta_1 < \beta_2 < \ldots < \beta_N$, where $\sum_{n=1}^N \beta_n = 1$. Moreover, the receiver of the $n$-th user applies SIC to eliminate the signals of users with poor channel gains and acquire its own signal. To assess the impacts of arbitrary CCI signals and maintain conciseness, we assume a perfect SIC process at the $n$-th user. However, perfect SIC may not always be feasible due to limited detection capabilities, the complexity of modulation techniques, and varying channel qualities [8], [9]. Please note that, the effects of imperfect SIC in the absence of CCI have been studied in several studies (e.g. [8], [9]). However, due to space limitations, the combined effects of imperfect SIC and CCI are left for our future work.

By assigning each user implements SIC and taking into account the independent but not necessarily identically distributed CCI signals, the received NOMA signal at the $n$-th user is written as

$$ y_n = \sqrt{E_b D_n^{-\alpha}} h_n \left( \sqrt{\beta_n d_n s_n} + \sum_{m=n+1}^N \sqrt{\beta_m d_m s_m} \right) + \sum_{k=1}^K \sqrt{P_k r_k^{-\alpha}} g_k s_k + w_n, \tag{1} $$

where $E_b$, $D_n^{-\alpha}$, $\alpha$, $D_n$, $h_n$, and $w_n \sim \mathcal{CN}(0, N_0)$ represent the bit energy, the large-scale path-loss, the path-loss exponent, the distance between the BS and the $n$-th user, the small-scale fading channel coefficient, and the additive white Gaussian noise, respectively. Moreover, $s_n$ and $2d_n$, $r \in \{n, m\}$ respectively represent the information symbols and the Euclidean distance of two adjacent information symbols for the $r$-th user. Furthermore, $P_k$, $g_k$, and $s_k$ represent the transmit power, the small-scale fading channel, and the information symbols for the $k$-th CCI interferer, respectively. Additionally, $r_k^{-\alpha}$ represents the large-scale path-loss for CCI, where $r_k$ is the distance between the $k$-th interferer and the NOMA user.

III. BIT ERROR RATE (BER) ANALYSIS

We assumed that each user can employ different modulation schemes, including MPAM and square MQAM, with an arbitrary order. In addition, Gray coding is employed to map the $\log_2 M_r$, $r \in \{n, m\}$ bits of the signal stream of the users to a two-dimensional signal constellation, where $M_r$ represents the modulation order [10]. By considering the characteristics of the symbols of both MPAM and square MQAM modulations, the real and imaginary components of the NOMA signal could be expressed as $Re/Im\{s_r\} \in \{\pm 1, \pm 3, \ldots, \pm (M_r - 1)\}$, and $Re/Im\{s_r\} \in \{\pm 1, \pm 3, \ldots, \pm (\sqrt{M_r} - 1)\}$, respectively [9]-[11]. Furthermore, $d_r$, $r \in \{n, m\}$ given in (1) can be defined in the MPAM modulation scheme as $d_r = \sqrt{\frac{3 \log_2 M_r}{(M_r - 1)}}$, while in the square MQAM modulation scheme can be expressed as $d_r = \sqrt{\frac{3 \log_2 M_r}{2(M_r - 1)}}$ [10].

In MPAM and square MQAM modulations, the in-phase and quadrature components of the equally probable data symbols are symmetric. Thus, it is sufficient to consider the in-phase component at the received signal. The in-phase component of the received signal at the $n$-th user is expressed as

$$ \chi_n = \sqrt{ \frac{E_b D_n^{-\alpha}}{2} |h_n| } \text{Re} \left\{ \left( \eta_n s_n + \sum_{m=n+1}^N \eta_m s_m \right) \right\} + \text{Re} \left\{ \sum_{k=1}^K \sqrt{P_k r_k^{-\alpha}} g_k s_k (t) + w \right\}, \tag{2} $$

where $\eta_n = \sqrt{\frac{d_n}{\lambda}}$ and $\eta_m = \sqrt{\frac{d_m}{\lambda}}$. Here, we suppose that the $n$-th user utilizes MPAM modulation scheme with an arbitrary order. In contrast, the remaining users, $U_m \forall m = n+1, \ldots, N$, can employ various modulation schemes with arbitrary orders, such as MPAM, square MQAM, or their combination. Hence, by leveraging [10, Eq. (9), Eq. (10)], [12, Eq. (5)] and relying on (2), the conditional BER for MPAM modulation scheme can be expressed as

$$ p_c(e|h_n, s_m, s_N, g_k, r_k) = \frac{1}{\log_2 M_n} \sum_{i=1}^{M_n-1} A_i \text{erfc} \left[ \sqrt{\sum_{k=1}^K \frac{1}{SIR_k} |g_k|^2 r_k^{-\alpha} D_n^\alpha + D_n^\alpha \text{SNR}} \right], \tag{3} $$

where erfc(.) is the complementary error function [13, Eq. (2.3-17)]. $A_i \forall i = 1, \ldots, (M_n - 1)$ are modulation dependent constants, which are specified according to the constellation size as given in [12, TABLE I], and SNR $= E_b/N_0$.

On the other hand, when the $n$-th user uses square MQAM with arbitrary orders, and the remaining users $U_m \forall m = n+1, \ldots, N$, may have MPAM, square MQAM or a combination of both with arbitrary orders. Therefore, by exploiting [10, Eq. (14), Eq. (16)] and according to (2), the conditional BER for square MQAM modulation scheme is written as

$$ p_c(e|h_n, s_m, s_N, g_k, r_k) = \sqrt{\frac{M_{n} - 2}{\log_2 M_r}} \sum_{i=0}^{\frac{M_{n} - 2}{2}} B_i \text{erfc} \left[ \sqrt{\sum_{k=1}^K \frac{1}{SIR_k} |g_k|^2 r_k^{-\alpha} D_n^\alpha + D_n^\alpha \text{SNR}} \right], \tag{4} $$

where $B_i \forall i = 0, 1, \ldots, \sqrt{M_r} - 2$ are modulation dependent coefficients, such that $\sum_{i=0}^{\sqrt{M_r} - 2} B_i = \frac{1}{2}$. For example, for 4QAM, 16QAM and 64QAM modulation schemes, the modulation dependent coefficients are respectively $B_i \in \{ \frac{1}{2} \}$, $B_i \in \{ \frac{2}{3}, \frac{4}{5}, \frac{1}{8} \}$ and $B_i \in \{ \frac{2}{3}, \frac{4}{5}, \frac{1}{8} \}$ [7], [11].
In order to derive the unconditional BER given an arbitrary number of users with arbitrary modulations and CCI, it is required to obtain the average of (3) and (4).

**Lemma 1.** The average BER for an arbitrary number of users having arbitrary modulation schemes and orders in the presence of CCI is obtained as

\[ p_c = \frac{2}{\pi} \int_0^\infty \frac{1 - e^{-\frac{\pi}{2} R^2}}{\omega} \mathcal{M}(\omega) \mathcal{C}(\omega) \, d\omega, \tag{5} \]

where \( \mathcal{M}(\omega) = \mathbb{E}\left[e^{-\sum_{k=1}^K \frac{\omega^2 D_m^2}{4m_k \Omega_k}}\right] \) is the moment generating function (MGF) of CCI, and \( \mathcal{C}(\omega) \) for both MPAM and MQAM modulation can be respectively expressed as

\[ \mathcal{C}(\omega)_{\text{MPAM}} = \frac{1}{M_m} \sum_{i=1}^{M_m-1} A_i \text{Im} \phi(\omega) = \frac{1}{2j} \sum_{m=n+1}^{N} \text{Re} \left[ \frac{1}{M_m} \left[ \Phi_{|h_n|} (\omega \Phi_{\rho_1}) - \Phi_{|h_n|} (-\omega \Phi_{\rho_2}) \right] \right], \tag{6} \]

\[ \mathcal{C}(\omega)_{\text{MQAM}} = \frac{1}{\sqrt{M_m} \sum_{i=0}^{M_m-2} B_i \text{Im} \phi(\omega) = \frac{1}{2j} \sum_{m=n+1}^{N} \text{Re} \left[ \frac{1}{\sqrt{M_m}} \left[ \Phi_{|h_n|} (\omega \Phi_{\rho_1}) - \Phi_{|h_n|} (-\omega \Phi_{\rho_2}) \right] \right], \tag{7} \]

where \( \Phi_{|h_n|} \) is the characteristic function (CHF) of the nth user’s channel amplitude, which can follow any arbitrary types of fading channels\(^{2}\). In addition, \( \Phi_{\rho_1} = ((2i - 1) \eta_m + \eta_m \text{Re}(s_m)), \Phi_{\rho_2} = ((2i - 1) \eta_m - \eta_m \text{Re}(s_m)), \Phi_{\varphi_1} = ((2i + 1) \eta_m + \eta_m \text{Re}(s_m)), \) and \( \Phi_{\varphi_2} = ((2i + 1) \eta_m - \eta_m \text{Re}(s_m)). \)

**Proof:** Please refer to Appendix A.

Furthermore, by exploiting [15, Eq. (25.4.45)], the expression given in (5) can be obtained in closed-form as

\[ p_c \approx \sum_{\ell=1}^{L} \frac{2 \delta \mathcal{C}(\xi_\ell)}{\xi_\ell e^{-\xi_\ell}} \left[ 1 - e^{-\frac{4}{2\pi} \mathcal{M}(\xi_\ell)} \right], \tag{8} \]

where \( \delta_\ell \) and \( \xi_\ell \) are the weights factors and sample points of the Laguerre orthogonal polynomial, given in [15, TABLE 25.9].

To gain further performance insights, the BER in the high-SNR regime is analyzed. Thus, using (5), it is written as

\[ p_c^{\infty} = \lim_{\text{SNR} \to \infty} p_c = \frac{2}{\pi} \int_0^\infty \frac{1 - \mathcal{M}(\omega)}{\omega} \mathcal{C}(\omega) \, d\omega \approx \frac{2}{\pi} \sum_{\ell=1}^{L} \frac{\delta \mathcal{C}(\xi_\ell)}{\xi_\ell e^{-\xi_\ell}} \left[ 1 - \mathcal{M}(\xi_\ell) \right]. \tag{9} \]

**Remark.** It is evident from (9) that the BER in the high-SNR regime becomes independent of SNR, and due to CCI, it reaches a limit which is scaled by different factors such as power allocation, modulation schemes, distance, and the number of interferers and users.

The newly derived BER expression is then applied to different interference scenarios as follows.

A. **Fixed Interferers Scenario**

In this scenario, we assume that NOMA users are affected by multiple CCIs that are positioned in fixed locations, and thus the distances of the NOMA users and CCIs are fixed. Moreover, the fading channels of CCIs are assumed to be independent and non-identically distributed (i.n.i.d) Nakagami-m RVs, where the CCIs channel gains follow i.n.i.d gamma distribution. Therefore, the MGF expression required for (5) can be written as

\[ \mathcal{M}(\omega) = \prod_{k=1}^{K} \left( 1 + \frac{\Omega_k \omega^2 D_m^2}{4m_k \Omega_k} \right)^{-m_k}. \tag{10} \]

B. **Poisson point process (PPP)-based Interferers Scenario**

To consider a more practical interference scenario, we assume that potential interfering signals are distributed arbitrarily surrounding NOMA users in the space domain. To this end, the distribution of interfering signals is modeled according to PPP with average density \( \lambda \) (average number of interferers per unit area) \( \lambda = K/\pi R^2 \) (interferers/m²). Let’s consider the interference is generated in a disc of radius \( R \) around the nth NOMA user and then considering the limit as \( R \to +\infty \). Without loss of generality, based on Mecke’s theorem, the analysis is performed for an arbitrary user considered to be located at the center [16]. In such a scenario, by taking into account the practical interference model in [16] and considering homogeneous interfering signals, the MGF expression required for (5) can be written as

\[ \mathcal{M}(\omega) = \exp \left( -\lambda \pi \left( \frac{\omega^2 D_m^2}{4\Omega_k} \right)^\frac{2}{\alpha} \Gamma \left( 1 - \frac{2}{\alpha} \right) I \right), \tag{11} \]

where \( I = \left( \frac{\Omega_k}{m_k} \right)^\frac{2}{\alpha} \Gamma(m + \frac{2}{\alpha}) \). \( \frac{m_k}{\Omega_k} \Gamma(m) \)

However, in the presence of heterogeneous interferers, where the receiver suffers from \( L \) different interferers with distinct characteristics. The interferers are precisely characterized by their densities, \{\lambda_1, \lambda_2, \ldots, \lambda_L\}, SIRs, \{\text{SIR}_1, \text{SIR}_2, \ldots, \text{SIR}_L\}, shape parameters, \{m_1, m_2, \ldots, m_L\} and scale parameters \{\Omega_1, \Omega_2, \ldots, \Omega_L\}, respectively. In such a case, by virtue of [16], the MGF expression required for (5) can be written as

\[ \mathcal{M}(\omega) = \exp \left( -\pi \left( \frac{\omega^2 D_m^2}{4\Omega_k} \right)^\frac{2}{\alpha} \Gamma \left( 1 - \frac{2}{\alpha} \right) I \right), \tag{12} \]

where \( I = \sum_{l=1}^{L} \frac{\lambda_l}{4\Omega_k^{\frac{2}{\alpha}}} \left( \frac{\Omega_m}{m_l} \right)^\frac{2}{\alpha} \Gamma(m + \frac{2}{\alpha}) \).

**IV. NUMERICAL AND SIMULATION RESULTS**

To evaluate the BER performance of a multi-user NOMA with CCI, and validate the preciseness of the newly derived expression, we present numerical and simulation results by setting the required parameters as \( N = \{2, 3, 4\}, \Omega = 1, m = \{1, 2, 3, 4,\} \), \( m_k = \{1\}, \Omega_k = 1, \alpha = 2 \) for the fixed located CCI scenario, where 4 is for PPP-based randomly located CCI scenario, as the free space propagation model (\( \alpha = 2 \)) is not
Fig. 1 demonstrates the BER of the NOMA system with CCI versus $E_b/N_0$ for a three-user NOMA network employing the same modulation with different orders, and a four-user NOMA network utilizing different modulation schemes and orders. In Fig. 1 (a), the modulations employed by U1, U2, and U3 are respectively 4QAM, 4QAM and 64QAM, where the power assignments are set as $\{\beta_1, \beta_2, \beta_3\} = \{0.0001, 0.0154, 0.0845\}$. In addition, in Fig. 1 (b), the modulation used by U1, U2, U3, and U4 are 2PAM, 4QAM, 2PAM and 4QAM, respectively, and the power assignments are set as $\{\beta_1, \beta_2, \beta_3, \beta_4\} = \{0.0001, 0.0431, 0.0568, 0.9000\}$ [17]. It is evident from the figures that, the BER performance of far user U3 in Fig. 1(a) and U4 in Fig. 1(b) consistently shown to be inferior to that of other users. The main reason behind this is that U3 and U4 experiences more interference from other users, i.e., treating U1, U2 or U3 as interference when detecting its own information, adversely affecting its performance. In contrast, U1, U2, or U3 implement the SIC process, which results in better performance, as shown in both figures. It is evident from the results that the BER performance is significantly impacted by the modulation design; an increase in the modulation order, deteriorates the BER performance of NOMA networks. Also, Fig. 1 (a) compares OMA and NOMA schemes, which reveals that the BER performance of OMA outperforms the NOMA scheme, as OMA users do not experience inter-user interference. The figures also show a perfect match between the theoretical and simulation results, verifying Lemma 1.

This letter presented a new unified expression for the BER evaluation in multi-user NOMA networks with arbitrary co-

Fig. 2 BER versus $E_b/N_0$ for $N = 2$ with different modulation and orders, $K = 3$, and $m = \{1, 2\}$.

Fig. 3 BER versus $E_b/N_0$ for $N = 2$ under PPP-based randomly located CCI scenario, $m = 3$, and $m_k = 1$.

Fig. 3 illustrates the BER performance of a two-user NOMA network subject to arbitrary PPP-based interference, where U1 employs 4QAM, and U2 employs 2PAM modulation, with power assignments of $\{\beta_1, \beta_2\} = \{0.1181, 0.8819\}$. It is to note that Fig. 3 (a) shows the impact of interference in the presence of heterogeneous interferers with different densities, $\{\lambda_1, \lambda_2\} = \{10^{-3}, 10^{-4}\}$, and Fig. 3 (b) shows the impact of interference in the presence of homogeneous interferers with a density of $\{\lambda = 10^{-3}\}$. The figure demonstrates that BER performance considerably deteriorates as density $\lambda$ is increased. This phenomenon is expected to emerge as the density $\lambda$ or transmitted energy of the interfering signals increases, thus the interference power at the victim user evolves stronger. The BER performance of both users in Fig. 3 is saturated as the SNR increases (i.e., SNR $\to \infty$) due to the impact of interference. Therefore, in high-SNR regime, the presence of CCI lead to a floor in the BER performance. The theoretical and simulation results are in perfect agreement, verifying the validity of the expressions in (5) and (9) for the PPP-based randomly located CCI scenario.

V. CONCLUSION

This letter presented a new unified expression for the BER evaluation in multi-user NOMA networks with arbitrary co-
channel interference. The unified expression facilitates an exact BER computation of an arbitrary number of users employing arbitrary modulation schemes such as MPAM with arbitrary orders, square MQAM with arbitrary orders, or a combination of both with arbitrary orders. By applying the unified expression, the BER performance of a NOMA network under two different practical interference scenarios was assessed. Subsequently, numerical and simulation results validated the preciseness of the newly derived expression.

**APPENDIX A**

By applying the Fourier sine transform’s convolution theorem given in [18, Eq. (4)], and initially considering that modulation scheme for the $n$-th user, $U_n$, is MQAM with an arbitrary order. In contrast, the modulation scheme employed for the remaining NOMA users, $U_m \forall m = n + 1, \ldots, N$, can be either MPAM, MQAM, or a combination of both with arbitrary orders. Therefore, calculating the average of (4) is fundamental in order to derive Lemma 1. To this end, we first define $\mathcal{X} \triangleq |h_n|^2 \left( \sum_{m=n+1}^{N} \eta_m \text{Re} s_m \right)$ and $\mathcal{Y} \triangleq \sqrt{\sum_{k=1}^{K} \text{SIR}_k^{-1}} |d_k|^2 r^\alpha D_n^2 + \frac{N_0}{2}$. Next, by invoking the convolution theorem [18, Eq. (4)] and [19, Eq. (4.5.5)] , it can be concluded that

$$p_e = \sum_{i=0}^{\sqrt{M_n}-2} B_i E \left[ \text{erfc}\left( \frac{\mathcal{X}}{\sqrt{2}} \right) \right]$$

$$= \sum_{i=0}^{\sqrt{M_n}-2} B_i E \left[ \frac{2}{\pi} \int_0^{\infty} G_s (\omega) \text{Im} \Phi (\omega) d\omega \right].$$

(13)

$$G_s (\omega) = \int_0^{\infty} \sin (\omega \mathcal{X}) \text{erfc}\left( \frac{\mathcal{Y}}{\sqrt{2}} \right) d\mathcal{X}$$

$$= 1 - e^{-\frac{\omega^2}{2}} \text{erfc}\left( \frac{\omega \mathcal{Y}}{\sqrt{2}} \right)$$

$$= 1 - e^{-\frac{\omega^2 D_n^2}{N_0}} e^{-\sum_{k=1}^{K} \frac{\omega^2 \eta_k^2 |d_k|^2}{45 \text{SNR}}}.$$

(14)

Moreover, the second term, $\text{Im} \Phi (\omega) = \frac{\Phi (\omega) - \Phi (-\omega)}{2j}$, can be written as

$$\text{Im} \Phi (\omega) = \frac{1}{2j} \times$$

$$\left[ e^{j\omega |h_n| |\eta_n| (2i+1)} E \left[ e^{-j\omega |h_n| \sum_{m=n+1}^{N} \eta_m \text{Re} s_m} \right] |h_n| - e^{-j\omega |h_n| |\eta_n| (2i+1)} E \left[ e^{j\omega |h_n| \sum_{m=n+1}^{N} \eta_m \text{Re} s_m} \right] |h_n| \right].$$

(15)

Given $s_m \forall m = n + 1, \ldots, N$ are independent RVs, it can be deduced that

$$E \left[ e^{j\omega |h_n| \sum_{m=n+1}^{N} \eta_m \text{Re} s_m} |h_n| \right] = \prod_{m=n+1}^{N} E \left[ e^{j\omega |h_n| |\eta_m| \text{Re} s_m} |h_n| \right].$$

In case that $U_m$ employs MPAM or square MQAM modulation with arbitrary orders, it can be demonstrated that

$$\mathbb{E} \left[ e^{j\omega \eta_m \text{Re} s_m} \right] = \sum_{ \text{Re} s_m \in \{-1,1,2,\ldots,v_m-1\} } e^{j\omega \eta_m \text{Re} s_m} \frac{1}{v_m}.$$

(16)

where $v_m = M_m$ in the case of $U_m$ with MPAM, and $v_m = \sqrt{M_m}$ in the case of $U_m$ with square MQAM modulation.

By substituting (16) into (15), the term $C(\omega) \triangleq \text{Im} \Phi (\omega)$ is derived for both MPAM scheme, as outlined in (6), and for MQAM scheme, as defined in (7). Finally, by substituting (14) and (6)/(7) into (13), the average BER given in (5) is obtained, which concludes the proof.

**REFERENCES**


