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Essays In Empirical Asset Pricing

Sungjun Cho

Sponsor: Robert Hodrick

Submitted in partial fulfillment of the
requirements for the degree
of Doctor of Philosophy
in the Graduate School of Arts and Sciences

COLUMBIA UNIVERSITY

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ABSTRACT

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Sungjun Cho

This dissertation consists of two chapters, all of which attempt to shed some light on what constitutes the time-varying risk premia in financial markets. The first chapter demonstrates that monetary policy shocks identified from New-Keynesian dynamic stochastic general equilibrium (DSGE) models explain the risk premia in stock markets. Indeed, the implied ICAPMs explain the value and the industry premia for the periods of 1980 to 2004. In particular, the permanent monetary policy shocks to inflation target capture the value premium and part of industry risk premium once I account for the capital market imperfection endogenously in New-Keynesian models. The shocks to investment technology, as a main determinant of the external finance premium, are also important for understanding the value premium.

The second chapter examines determinants of stochastic relative risk aversion in conditional asset pricing models by utilizing nonlinear state space model with GARCH specification. After imposing general version of the conditional CAPM or ICAPM, I develop non-ad-hoc empirical models and search for valid specifications of relative risk aversion along with appropriate hedging components. I discover that the surplus consumption ratio implied by the external habit formation model is the most important determinant of time varying relative risk aversion. The CAY of Lettau and Ludvigson (2001a) without a look-ahead bias also captures part of relative risk aversion. The short term interest rate (RREL) has explanatory power for hedging components. I use the implied conditional asset pricing models in explaining the cross-section of average returns on either the Fama-French 25 size and book-to-market sorted portfolios alone or with 30 industry portfolios. I find that

the chosen conditional CAPM and ICAPM with time-varying relative risk aversion and a hedging component are at least comparable to or better than the Fama-French three-factor model for the sample periods 1957 to 2005.

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To ILJIN

Chapter 1

Stock Returns and Monetary Policy

1.1 Introduction

The stock market continuously watches and forms expectations about the Federal Reserve Board(Fed) decisions. It seems that investors in Wall Street take it for granted that the actions of the Fed have a considerable impact on stock market returns while there is controversy on the issue among macro-economists. Two crucial monetary transmission mechanisms have been suggested through which stock prices respond to monetary news. The first is the interest rate channel, which relates to economic activity primarily through consumption and investment since a cut in the borrowing cost should raise the quantity of funds demanded for investment and promotes the current over future consumption, which leads to an increase in economic activity. The second mechanism is the credit channel under the capital market imperfection assumption. When credit markets are tight, unanticipated monetary easing reduces the external finance premium which is a wedge between the external financing by issuing equity or debt and the internal financing by retaining earnings.

Bernanke and Gertler (1989) argue that the effect of the capital market imperfection are largest in recessions, when weak balance sheets lead to higher costs of external finance, resulting in lower investment demand and reduced economic activity.

In recent empirical asset pricing studies, several researchers have confirmed that monetary policy shocks affect the future risk premium. Notably, Bernanke and Kuttner (2005) find that monetary policy shocks are important for understanding the risk premium using Campbell and Ammer (1993) type decomposition. Specifically, they find that unanticipated changes in monetary policy affect stock prices not so much by influencing expected dividends or the risk-free real interest rate, but rather by affecting the perceived riskiness of stocks. By employing the long-horizon regression methodology, Patelis (1997) finds that some portion of the observed predictability in excess returns in US stock market can be attributed to shifts in the monetary policy stance. Patelis relates his findings to the credit channel of monetary policy transmission (Bernanke and Gertler (1995)) and to the financial propagation mechanism (Bernanke and Gertler (1989)). By estimating a vector autoregressive (VAR) system that includes monthly equity returns, output growth, inflation, and the federal funds rate, Thorbecke (1997) finds that monetary policy shocks, measured by orthogonalized innovations in the federal funds rate, have a greater impact on smaller capitalization stocks, which is in line with the hypothesis that monetary policy affects firms' access to credit. Jensen, Mercer, and Johnson (1996) find that predictable variation in stock returns depends on monetary as well as business conditions, with expected stock returns being higher in tight money periods than in easy money periods. And business conditions could predict future stock returns only in periods of expansive monetary policy.

While there seems enough time-series evidence of the effect of monetary policy on stock returns, none of the papers investigates directly its implications on the cross-section of stock returns.¹ Fama (1991) conjectures that we should relate the cross-sectional properties

¹Some empirical asset pricing studies (e.g. Hahn and Lee (2006)) using cross section of stock returns seem to interpret that significant risk price of the short term interest rate or term spread exists since it is a proxy

of expected returns to the expected returns through time. In fact, since Merton (1973)'s theoretical presentation of the ICAPM, it has been recognized that there exist state variables that capture variations in future investment opportunities, and assets' covariations with such variables should be priced in the cross-section of average returns. Campbell (1996), Brennan, Wang, and Xia (2004), and Petkova (2006) build their models based on Merton (1973) in which only factors that forecast future investment opportunities or stock returns are admitted. From the time-series evidence of return forecastability of monetary policy instruments, it seems natural to investigate the effects of monetary policy on cross-section of stock returns.

In this paper, I examine whether monetary policy shocks extracted from New-Keynesian dynamic stochastic general equilibrium(DSGE) models can explain the cross sectional variability of U.S. stock returns. I employ New-Keynesian models since we must carefully identify monetary policy shocks. In fact, New-Keynesian models utilized in this paper have become benchmarks of much of the recent monetary policy literature since they can explain many stylized facts in monetary economics.² In addition, these models provide many interesting structural shocks. For example, a series of shocks to investment technology can be identified as a major determinant of the external finance premium. Recently, the implication of the capital market imperfection on cross section of stock returns are investigated by Hahn and Lee (2006). While they find that the size and the value premia are compensation for higher exposure to the risks related to changing credit market conditions and interest rates(monetary policy), their results should be cautiously interpreted since other hypotheses might be developed consistent with their measure of risks(yield spreads). In this sense, I can investigate directly whether the identified external finance premium shock is really a

for monetary policy.

²I summarize the stylized facts in monetary economics and the failure of models before New-Keynesian models in appendix A

determinant for the size or value premium.³

I develop the whole estimation procedures in Bayesian methods. After dividing the estimation steps into three blocks based on identifying assumptions frequently used in finance literature, I sequentially estimate each block by Bayesian Markov Chain Monte Carlo method(MCMC) and repeat those steps until every parameter converges. In the first block, I estimate New-Keynesian models using DYNARE-Matlab package. First, I use three different versions of linearized Rational Expectations models consisting of AS, IS and monetary policy rule equations proposed by Cho and Moreno (2006). Their models are parsimonious yet rich enough to capture the macro dynamics of inflation, real GDP growth and the Federal funds rate.⁴ Second, I use an extended version of Smets and Wouters (2005) model proposed by Graeve (2006) to precisely uncover the importance of the external finance premium and the permanent monetary policy shock. Finally, as a robustness check for my analysis, I use the structural shocks estimated from factor-augmented vector autoregressive model(FAVAR) of Bernanke, Boivin, and Elias (2005) since results from that model could be robust to model misspecifications and small data set problems. In the second and the third blocks, I estimate the posterior distributions of risks and the risk prices with identified structural shocks. I also calculate the posterior densities of several diagnostic measures for model comparison.

Several empirical findings emerge from this analysis using New-Keynesian models and FAVAR model. First, I find that both the permanent monetary policy shock to inflation target and a proxy of shock to the external finance premium successfully capture major portions of the size and the value premia. While I use more structural methods to identify

³Lee told me, during one of private conversations, that some of researchers they met argue that the capital market imperfection should not be an asset pricing factor since it has no implication for the aggregate stock market. My results could be interesting in this venue since more direct measure of the external finance premium could be developed to test this hypothesis

⁴Cho and Moreno (2006) show that their model with serial correlation is only marginally rejected at the 5 % level against vector autoregressive(VAR) model of inflation, real GDP growth and the federal funds rate using the small-sample likelihood ratio test statistic.

proxies for the external finance premium and monetary policy shock, these results support the findings of Hahn and Lee (2006) that innovations in the default and term spreads as proxies for capturing revisions in the market's expectation about future credit market conditions and interest rates explain the size and value premium. They argue that small-sized and high-book-to-market firms would be more vulnerable to worsening credit market conditions and higher interest rates. The identified structural shocks for the external finance premium and the monetary policy shock from my models indeed indicate that their conclusions are valid with a structural investigation based on equilibrium models while additional structural shock related to inflation seem also indispensable.

Second, the permanent monetary policy shock to inflation target can explain part of industry risk premium once I correctly account for the capital market imperfection. In particular, this shock is the only statistically significant variable to determine industry risk premium after controlling for business cycle. This could reflect that while the credit channel under the capital market imperfection is important for determining the value and the size premia, interest rate channel would be more important for explaining industry premium. Peersman and Smets (2005) show that there is considerable cross-industry heterogeneity in the overall monetary policy effects. After exploring which industry characteristics can account for the cross-industry heterogeneity, they find that durability of the output produced by the sector is an important determinant of its sensitivity to monetary policy changes. They argue this fact as an evidence for interest rate/cost-of-capital channel since the demand for durable products, such as investment goods, is known to be much more affected by a rise in the interest rate through the cost-of-capital channel than the demand for non-durables such as food. Recently, Gomes, Kogan, and Yogo (2007) argue that durability of output is a risk factor since the demand for durable goods is more cyclical than that for nondurable goods and services. Consequently, the cash flow and stock returns of durable-good producers are exposed to higher systematic risk and thus investors request higher risk

premium. This study indicates that monetary policy shock is one of the fundamental shocks behind this risk premium.

Third, temporary monetary policy shock extracted from several models is not statistically significant as a determinant for any risk premium. The stock market seems to respond only to fundamental target changes, which by definition have persistent effects on the future economy.

Finally, selected ICAPMs using New-Keynesian models are capable of explaining the cross-section of the Fama-French 25 size and B/M sorted portfolios significantly ($R^2=72\%$) and a part of risk premium for 55 portfolios with their 30 industry portfolios ($R^2=30\%$). Lewellen, Nagel, and Shanken (2006) criticize most of empirical asset pricing models because they only explain the value premium but not any part of the risk premium of industry portfolios. Based on empirical results, I argue that New-Keynesian models with more appropriate firm heterogeneity could be developed to fully account for industry risk premium with fundamental economic shocks.⁵

The rest of the paper is organized as follows. Section 2 presents the structural New-Keynesian models employed in this study. Section 3 outlines the empirical methods used to extract structural shocks from the given models. Section 4 presents the data and discusses the cross-sectional results of my empirical models for 25 size and B/M portfolios alone or with 30 industry portfolios. Section 5 summarizes the main findings and concludes.

⁵Lewellen, Nagel, and Shanken (2006) criticizes most of the cross-sectional asset pricing studies for the choice of the Fama-French 25 portfolios. (Possible Data Snooping problem) Especially, they show that many empirical asset pricing models could price only the Fama-French 25 portfolios (R^2 is above 75 %) but not the 55 portfolios including 30 industry portfolios. (R^2 is typically below 10%) Therefore I check the robustness of the proposed ICAPMs for the value premium and for the capability to explain the industry portfolios. I argue that monetary policy shock is indeed important across different test assets.

1.2 Models

This section discusses the models to be estimated; the first subsection briefly explains the discrete time asset pricing model implied by new Keynesian equilibrium models and the second subsection presents three different Keynesian macro models to identify monetary policy shocks implemented in this paper.

1.2.1 The Pricing Kernel implied by New-Keynesian macro models

Without imposing any theoretical structure, the fundamental existence theorem of Harrison and Kreps (1979) states that, in the absence of arbitrage, there exists a positive stochastic discount factor, or pricing kernel, M_{t+1} , such that, for any traded asset with a gross return at time t of $R_{i,t+1}$, the following equation holds:

$$1 = E_t[M_{t+1}(R_{i,t+1})] \quad (1.2.1)$$

where E_t denotes the expectation operator conditional on information available at time t .

Standard New-Keynesian macro models employ the following external habit specification in utility function built on Fuhrer (2000).⁶

$$E_t \sum_{s=t}^{\infty} \psi^{s-t} U(C_s; F_s) = E_t \sum_{s=t}^{\infty} \psi^{s-t} \left[\frac{F_s C_s^{1-\sigma} - 1}{1-\sigma} \right]$$

where C_s is the composite index of consumption, F_s represents an aggregate demand shifting factor and usually denotes as $H_s G_s$ where H_s is an external habit level and G_s is a preference shock.; ψ denotes the subject discount factor and σ is the inverse of the intertemporal elasticity of consumption.

Bekaert, Cho, and Moreno (2005) derive the following pricing kernel implied by Fuhrer (2000) assuming standard log-normality:

⁶I closely follow the representation given in Bekaert, Cho, and Moreno (2005)

$$m_{t+1} = \ln \psi - \sigma y_{t+1} + (\sigma + \eta) y_t - (g_{t+1} - g_t) - \pi_{t+1} \quad (1.2.2)$$

where $m_{t+1} = \ln(M_{t+1})$, y_{t+1} is detrended log output, $g_{t+1} = \ln(G_{t+1})$ and π_{t+1} is the inflation rate.

They express (1.2.2) in terms of the structural shocks in the economy.

$$m_{t+1} = -i_t - \frac{1}{2} \Lambda' D \Lambda - \Lambda' \varepsilon_{t+1} \quad (1.2.3)$$

where Λ' is a vector of prices of risks entirely restricted by the structural parameters of New-Keynesian models and D is the covariance matrix of structural shocks.

1.2.2 A digestion of Risk premium and the New-Keynesian Pricing Kernel

The pricing kernel (1.2.3) is a linear combination of structural shocks to the overall economy. Following Cochrane (2001), I can interpret (1.2.3) as an example of the ICAPMs.

One major problem of (1.2.3) is that this pricing kernel assumes constant risk premium. Bekaert, Cho, and Moreno (2005) articulate that without either heteroscedasticity of structural shocks or time-varying market price of risk, their model essentially imposes that expectation hypothesis holds in bond market. In such a case, ICAPM implication would be seriously challenged since time-varying risk premium implied by ICAPM is inconsistent with this type of New-Keynesian pricing kernel.

One possible remedy is to adapt the external habit specification of Fuhrer (2000) to that of Campbell and Cochrane (1999) and develop a pricing kernel with time-varying risk aversion. Since time-varying risk aversion is emphasized in the finance literature, this extension would be beneficial for explaining asset pricing facts. However, the real challenge behind this scenario is to develop IS model consistent with this new utility function in order

to explain the stylized facts in monetary economics before it is implemented in asset market research.⁷

Another suggestion would be introducing heteroscedasticity in the pricing kernel and structural shocks. While this specification naturally allows time-varying risk premium, typical log-linearization of New-Keynesian models is not valid anymore. At least second order approximation of the models should be employed to estimate the models. Unfortunately, an estimation of second-order approximated New-Keynesian models with likelihood-based methods and particle filter have not been ripen in the literature because of computational difficulties. Some steps in this direction have begun to be taken only recently.⁸ The common practice is to estimate the log-linearized economy and plug the estimates into the second-order approximation.

The easiest but ad-hoc solution of the problem is recently implemented by Rudebusch and Wu (2004) and Hordahl, Tristani, and Vestin (2006) when they jointly estimate macro models and term structure of yields. They simply ignore pricing kernel implications of their IS equations and set them exogenously.

Sometimes empirical asset pricing studies in finance seem to have somewhat convenient approximation mechanism similar to the ad-hoc approach presented above. For example, Campbell (1996)'s ICAPM is derived under constant risk premium. Strictly speaking, simple pricing kernel form implied by the homoskedastic version of the model combined with the vector autoregression(VAR) pricing model recommended by Campbell (1996) is inconsistent since it does not have any mechanism to generate time-varying risk premium. However, actual implementation of Campbell ICAPM is usually done in the homoskedastic form with the usual VAR. Recently, Petkova (2006) implements this homoskedastic VAR to extract state variables and argue that her five factor "ICAPM" model is better than

⁷I defer the explanation of terminology and implications of New-Keynesian models employed in this paper to the next section.

⁸Refer to An (2006) for bayesian estimation of this type of models

Fama-French three factor model to explain the value premium.

There seems to be trade-off between developing complex models to estimate tightly restricted models and estimating inconsistent but plausible mechanisms to extract economic state variables. Even though it seems possible to modify the pricing framework in (1.2.3) using one of the two approaches with time-varying price of risk or heteroscedasticity, I defer these attempts to future work. Instead, I follow a convenient approach, for example, taken by Rudebusch and Wu (2004) in this paper. I argue that this approximation is reasonable in terms of obtaining appropriate monetary policy shocks since current studies in monetary policy literature typically use models of the current study for explaining the stylized facts in monetary economics. While it is convenient to extract state variables in this way, pricing kernels should be interpreted as an approximation or exogenously re-specified to avoid constant risk premium. Specifically, I extract the innovations of state variables from New-Keynesian models and interpret (1.2.3) as a reasonable approximation of pricing kernel of ICAPM since these New-Keynesian models imply structural VAR of the economy.

Recent research suggests that monetary policy shocks can be identified as a determinant of market risk premium. It seems natural to check the implications of time-series relationship between monetary policy shocks and stock returns for the cross-section of portfolio returns in the spirit of ICAPM but to my knowledge, no one has investigated this issue before. Furthermore, it seems to be critical to investigate this issue closely following recent progress of the monetary policy literature.⁹ In this sense, employing New-Keynesian framework is important since most successful monetary policy analysis is done with log-linearized models considered in this paper.¹⁰ I explicitly investigate the relationship between monetary policy shocks and the cross-section of portfolio returns. Therefore, my approach seems

⁹In this sense, it is not clear that modification of (1.2.3) with heteroscedasticity or time-varying risk aversion would be better as macro models. To my knowledge, this type of research have not yet done much in monetary economics

¹⁰Refer to first nine chapters in Woodford (2003) for detailed explanations.

to capture critical but unexplored areas of empirical asset pricing.

Usual ICAPM intuition suggests that state variables should forecast the changing investment opportunity set in that economy. In this sense, reasonably identified state variables from New-Keynesian models are natural candidates since these models capture the essential feature and satisfy the intuition of various aspects of the data. Specifically, impulse response analysis implied by these models show that each shock explains the future course of the economy consistent with the stylized facts in monetary economics.¹¹

1.2.3 New-Keynesian dynamic general stochastic equilibrium models(DSGE)

1.2.3.1 A summary of three-equation New-Keynesian Macro Models

In this section, I summarize three different versions of three equation New-Keynesian models implemented by Cho and Moreno (2006)¹² These models are simple but effective since they find that their three-equation model augmented with autocorrelation is just marginally rejected against a usual vector autoregression(VAR) of observed data using small-sample likelihood ratio test statistics.

These models combine the rigor of the Real Business Cycle (RBC) approach, which is characterized by the derivation of behavioral relationships from the optimizing behavior of agents subject to technological and budget constraints and the specification of a well-defined equilibrium concept, with the tractable introduction of nominal rigidities in order to accommodate nontrivial roles for monetary policy. These models can avoid the Lucas (1976) critique by a consistent treatment of expectations formation and rigorous treatment of micro-foundations.

The first structural model(model1) proposed by Cho and Moreno (2006) contains three equations: The aggregate supply(AS) equation, the IS equation and the monetary policy

¹¹Refer to appendix A for a summary of the stylized facts in monetary economics and theoretical advances before New-Keynesian macro models.

¹²For detailed review of microfoundation of these models, see Woodford (2003)

rule.

First, the AS equation is a generalization of the Calvo (1983) pricing model:

$$\pi_t = \delta E_t \pi_{t+1} + (1 - \delta) \pi_{t-1} + \lambda y_t + \varepsilon_{AS,t} \quad (1.2.4)$$

where π_t is inflation between $t-1$ and t and y_t stands for the output gap between $t-1$ and t . $\varepsilon_{AS,t}$ can be interpreted as marginal cost push shock, which is assumed to be independently and identically distributed with homoskedastic variance σ_{AS}^2 . E_t is the rational expectations operator conditional on the information set at time t .

To account for the observed persistence in aggregate inflation, this model assumes that price-setting intermediate goods producers engage in multi-period, staggered price setting. Since price-setting responds to movements in marginal costs, its persistence arises from shocks to marginal costs if inflation is purely forward looking. In addition to that, Galí and Gertler (1999) generate additional inertia in the Phillips curve by adding rule of thumb price setters with backward-looking behavior. In their model, prices remain fixed in monetary terms during stochastic intervals of time. But unlike the Calvo model, when prices are adjusted, some prices are chosen optimally while others are adjusted according to a backward-looking rule of thumb that introduces dependence upon lagged inflation.

Second, the IS equation is derived with representative agent model with the external habit as in Fuhrer (2000):

$$y_t = \mu E_t y_{t+1} + (1 - \mu) y_{t-1} - \phi(r_t - E_t \pi_{t+1}) + \varepsilon_{IS,t} \quad (1.2.5)$$

where $\varepsilon_{IS,t}$ is the preference shock, which is assumed to be independently and identically distributed with homoskedastic variance σ_{IS}^2 . The monetary policy channel in the IS equation is captured by the contemporaneous output gap dependence on the ex ante real rate of interest ($r_t - E_t \pi_{t+1}$).

In this model, the effects of policy-induced changes on real interest rates directly affect aggregate spending decisions. For example, the central bank with an inflation target increases the short-term nominal interest rate if inflation is above the desired rate. Because of price stickiness, the central bank directly influences the real interest rate through its influence on the short-term nominal interest rate. Then, by influencing the real interest rate, aggregate demand is affected through consumption, via intertemporal substitution effects, and investment, via the cost of capital effects.

Finally, the monetary policy equation is the forward looking rule proposed by Clarida, Gali, and Gertler (2000):

$$r_t = \alpha_{MP} + \rho r_{t-1} + (1 - \rho)[\beta E_t \pi_{t+1} + r y_t] + \varepsilon_{MP,t} \quad (1.2.6)$$

where α_{MP} is a constant and $\varepsilon_{MP,t}$ is the monetary policy shock, which is assumed to be independently and identically distributed with homoskedastic variance σ_{MP}^2 .

The policy rule exhibits interest rate smoothing with a weight of ρ on the past interest rate. In this specification, the Fed reacts to high expected inflation and to deviations of output from its trend.

Cho and Moreno (2006) extend their model with ad-hoc exogenous autocorrelation(model2) or with both auto- and cross-correlation of structural shocks(model3) to accommodate the persistence of the macro-dynamics.

$$\varepsilon_{t+1} = F\varepsilon_t + \omega_{t+1} \quad (1.2.7)$$

where F is a 3x3 stationary matrix, captures the correlation of structural shocks, and is either diagonal(model2) or full matrix(model3), ω_{t+1} is independently and identically distributed with diagonal covariance matrix D .

1.2.3.2 A summary of an extended New-Keynesian DSGE

Cho and Moreno (2006) find that adding persistence to the baseline New-Keynesian model improves the fit of the model. This might indicate the need for producing more complex models to provide realistic macro-dynamics. Furthermore, their models have one critical shortcoming by maintaining the assumption of frictionless capital markets. The seminal paper by Bernanke and Gertler (1989) and a number of subsequent calibration studies document how relaxing this perfect capital market assumption can generate additional features observed in macroeconomic data. Since then, considerable interest has been placed on the role of credit rather than money in determining business cycle fluctuations.

A series of papers proposed by Smets and Wouters(e.g.Smets and Wouters (2003)) incorporate a number of additional frictions to capture this persistence in the macro-economic data and they also add an exogenous mechanism to impose capital market imperfection. Their New-Keynesian models have become an standard approach in monetary policy literature.¹³ This model contains three agents;Households consume, work, set wages, and invest; firms hire labor and capital, produce goods and set the prices of those goods; and the central bank sets the short-term interest rate in response to the deviation of inflation from the inflation target and output gap. The model accommodates both real and nominal frictions such as monopolistic competition in goods and labor markets with sticky nominal prices and wages, partial indexation of prices and wages, costs of adjustment in capital accumulation, external habit formation and variable capital utilization and fixed costs.

In order to endogenize capital market imperfection mechanism into standard New-Keynesian models, Graeve (2006) extends the role of entrepreneurs in Smets and Wouters's economy by explicitly accounting for the external finance premium equation in the sense of Bernanke, Gertler, and Gilchrist (1999). He finds that his measure of the external finance premium is closely related to readily available proxies of the premium such as the corporate

¹³Refer to Smets and Wouters (2006) in order to fully understand micro-foundations of this model.

bond spread(Baa-Aaa) and the high-yield bond spread(Bbb-Aaa).

This explicit capital market imperfection mechanism could be a significant channel for understanding the effects of monetary policy shocks on the cross-section of stock returns. Hahn and Lee (2006) investigate the role of yields spreads as proxies for this mechanism and find that the size and the value premia are compensation for higher exposure to the risks related to changing credit market conditions and interest rates(monetary policy) proxied by changes in yield spreads. However, yield spreads can be interpreted in a number of ways. Probably, other hypotheses could be developed consistent with these results from their reduced form model. For example, risk aversion can be time-varying and is proxied by two variables. In this sense, my paper can be interpreted as more structural investigation of the roles played by the capital market imperfection and monetary policy in explaining the cross-section of stock returns.

In Graeve's model, nine equations are incorporated to capture the macro dynamics of the economy. Most of the equations are just adopted from Smets and Wouters (2003) or Smets and Wouters (2005) except for the role of entrepreneurs.

First, households' maximization provides the aggregate consumption equation and wage equation. In addition to the external habit specification as in Cho and Moreno (2006), households have differentiated labor characteristics and some monopoly power over wages, which introduce sticky nominal wages in the sense of Calvo (1983). Households act as price-setters in the labor market and partial indexation of the wages is allowed. "Hat" means the steady state value.

The aggregate consumption(\hat{C}_t) in this model is determined by:¹⁴

$$\hat{C}_t = \frac{h}{1+h}\hat{C}_{t-1} + \frac{h}{1+h}E_t\hat{C}_{t+1} + \frac{\sigma_c - 1}{(1+\lambda_w)(1+h)\sigma_c}(\hat{L}_t - E_t\hat{L}_{t+1}) - \frac{(1-h)}{(1+h)\sigma_c}\hat{R}_t$$

¹⁴This is the equation (1) of Smets and Wouters (2005).

$$+ \frac{(1-h)}{(1+h)\sigma_c} (\hat{\varepsilon}_t^B - E_t \hat{\varepsilon}_{t+1}^B) \quad (1.2.8)$$

where $\hat{\varepsilon}_t^B$ is interpreted as preference shock and follows a first-order autoregressive process with an i.i.d normal error term; \hat{L}_t stands for the labor supply included as the non-separability of the utility function of labor and consumption; $\hat{R}_t(\hat{R}_t^n - E_t \hat{\pi}_{t+1})$ is the ex-ante real interest rate, where \hat{R}_t^n is the nominal interest rate and $\hat{\pi}_{t+1}$ is the inflation rate; Finally, E_t indicates conditional expectation given information up to time t.

Households set their wages with the following Calvo (1983) type staggered wage-setting scheme proposed by Christopher, Henderson, and Levin (2000). In this model, the real wage \hat{w}_t is a function of expected and past real wages and the expected, current and past inflation rates($\hat{\pi}_t$).¹⁵

$$\begin{aligned} \hat{w}_t = & \frac{\beta}{1+\beta} E_t \hat{w}_{t+1} + \frac{1}{1+\beta} \hat{w}_{t-1} + \frac{\beta}{1+\beta} (E_t \hat{\pi}_{t+1} - \bar{\pi}_t) - \frac{1+\beta\gamma_w}{1+\beta} (\hat{\pi}_t - \bar{\pi}_t) - \frac{\gamma_w}{1+\beta} (\hat{\pi}_{t-1} - \bar{\pi}_t) \\ & - \frac{1}{1+\beta} \frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\frac{(1+\lambda_w)\sigma_l}{\lambda_w})\xi_w} \left[\hat{w}_t - \sigma_l \hat{L}_t - \frac{\sigma_c}{1-h} (\hat{C}_t - h\hat{C}_{t-1}) - \hat{\varepsilon}_t^L \right] + \eta_t^W \end{aligned} \quad (1.2.9)$$

where η_t^W is interpreted as a wage-markup disturbance. And $\hat{\varepsilon}_t^L$ represents the shock to the labor supply and is assumed to follow a first-order autoregressive process with an i.i.d. normal error term.

New-Keynesian economists emphasize the role of nominal rigidities(price stickiness) based on microfoundations of imperfect competition. However, for these rigidities to have important implications, it is necessary that wages do not respond much to fluctuations in demand. The fall in output also results in a fall in labor demand which, in turn, would drive down the equilibrium wage in the labor market and the firm's marginal cost curves. This may increase the gain from price adjustment significantly. Thus, for the lack of price

¹⁵This is the equation (6) of Smets and Wouters (2005).

adjustment to be a macroeconomic equilibrium, we need real rigidity in the labor market. Staggered wage-setting equation is one of the mechanisms to generate this real rigidity in labor market. In fact, Smets and Wouters (2003) use partial or full indexation of this kind for both wages and prices, and find that this extension of the Calvo pricing model improves the empirical fit of their models.

Intermediate goods firms' optimizations in monopolistic competition markets yield the following equations. First, Cobb-Douglas production function augmented with fixed costs and variable capital utilization is given by:¹⁶

$$\hat{Y}_t = \phi \hat{\varepsilon}_t^A + \phi \alpha \hat{K}_{t-1} + \frac{\phi \alpha}{\psi} \hat{r}_t^k + \phi(1 - \alpha) \hat{L}_t \quad (1.2.10)$$

where output(\hat{Y}_t) is produced using capital (\hat{K}_{t-1}) and labor services (\hat{L}_t). Total factor productivity ($\hat{\varepsilon}_t^A$) is assumed to follow a first-order autoregressive process.

The firm's labor demand(\hat{L}_t) depends negatively on the real wage(\hat{w}_t) and positively on the rental rate of capital(\hat{r}_t^k) by equalizing marginal cost:¹⁷

$$\hat{L}_t = -\hat{w}_t + (1 + \frac{1}{\psi}) \hat{r}_t^k + \hat{K}_{t-1} \quad (1.2.11)$$

Finally, price is determined following Calvo (1983) scheme.¹⁸

$$\begin{aligned} \hat{\pi}_t - \bar{\pi}_t &= \frac{\beta}{1 + \beta} (E_t \hat{\pi}_{t+1} - \bar{\pi}_t) + \frac{\gamma_p}{1 + \beta \gamma_p} (\hat{\pi}_{t-1} - \bar{\pi}_t) \\ &+ \frac{1}{1 + \beta \gamma_p} \frac{(1 - \beta \xi_p)(1 - \xi_p)}{\xi_p} [\alpha \hat{r}_t^k + (1 - \alpha) \hat{w}_t - \hat{\varepsilon}_t^A] + \eta_t^P \end{aligned} \quad (1.2.12)$$

where the deviation of inflation($\hat{\pi}_t$) from the target inflation rate ($\bar{\pi}_t$) depends on past and

¹⁶This is embedded in the equation (8) of Smets and Wouters (2005).

¹⁷This is the equation (7) of Smets and Wouters (2005).

¹⁸This is the equation (5) of Smets and Wouters (2005).

expected future inflation deviations and on the current marginal cost $(\alpha \hat{r}_t^K + (1 - \alpha) \hat{w}_t - \hat{\varepsilon}_t^A)$. The stochastic component $\hat{\varepsilon}_t^A$ is assumed to follow a first-order autoregressive process and η_t^P is an i.i.d. normal price mark-up shock.

Capital goods producers work in a perfectly competitive environment and their investment decision can be summarized as:¹⁹

$$\hat{I}_t = \frac{1}{1 + \beta} \hat{I}_{t-1} + \frac{\beta}{1 + \beta} E_t \hat{I}_{t-1} + \frac{1/\varphi}{1 + \beta} (\hat{Q}_t + \hat{\varepsilon}_t^I) \quad (1.2.13)$$

where \hat{Q}_t is the real value of installed capital and φ is the investment adjustment cost parameter. A positive shock to the investment-specific technology, $\hat{\varepsilon}_t^I$ increases investment in the same way as an increase in the value of the existing capital stock \hat{Q}_t . This investment shock is also assumed to follow a first-order autoregressive process with an i.i.d normal error term.

And the capital stock evolves as:²⁰

$$\hat{K}_{t+1} = (1 - \tau) \hat{K}_t + \tau \hat{I}_t + \tau \hat{\varepsilon}_t^I \quad (1.2.14)$$

where τ is the depreciation rate, \hat{I}_t stands for investment and $\hat{\varepsilon}_t^I$ represents a shock to the investment technology.

Unlike the forward-looking monetary policy used in Cho and Moreno (2006), the monetary policy rule follows a generalized Taylor rule by gradually responding to deviations of lagged inflation from an inflation objective and the lagged output gap. This reaction mechanism contains two monetary policy shocks: a temporary i.i.d. normal interest rate shock (η_t^R) and a persistent shock for changes in inflation target ($\hat{\pi}_t - \bar{\pi}_t$).²¹

¹⁹This is the equation (2) of Smets and Wouters (2005).

²⁰This is the equation (4) of Smets and Wouters (2005).

²¹This is the equation (9) of Smets and Wouters (2005)

$$\begin{aligned} \hat{R}_t^n &= \rho \hat{R}_{t-1}^n + (1 - \rho) \left\{ \bar{\pi}_t + r_\pi (\hat{\pi}_t - \bar{\pi}_t) + r_Y (\hat{Y}_t - \hat{Y}_t^P) \right\} + r_{\Delta\pi} (\hat{\pi}_t - \bar{\pi}_{t-1}) \\ &\quad + r_{\Delta Y} \left(\hat{Y}_t - \hat{Y}_t^P - (\hat{Y}_{t-1} - \hat{Y}_{t-1}^P) \right) + \eta_t^R \end{aligned} \quad (1.2.15)$$

where \hat{R}_t^n is the federal funds rate, $\bar{\pi}_t$ is the inflation target set by the central bank and potential output (\hat{Y}_t^P) is defined as the level of output that would prevail under flexible price and wages in the absence of cost-push shocks and in frictionless credit market equilibrium. Finally \hat{Y}_t is the actual real GDP and $\hat{\pi}_t$ is the actual inflation rate.

The goods market equilibrium condition can be written as:²²

$$\hat{Y}_t = c_y \hat{C}_t + \tau k_y \hat{I}_t + \varepsilon_t^G + \frac{(\bar{R}^K - 1 + \tau)}{\psi k_y} \hat{r}_t^k + k_y (\bar{R}^K - \bar{R}) \left(1 - \frac{\bar{N}}{\bar{K}}\right) (\hat{R}_t^K + \hat{Q}_{t-1} + \hat{K}_t) \quad (1.2.16)$$

where c_y and k_y denotes the steady-state ratio of consumption and capital to output respectively. And ε_t^G is interpreted as government spending shock, which follows a first-order autoregressive process with an i.i.d. normal error term.

Finally, entrepreneurs buy the capital stock K_{t+1} from capital goods producers at a given price Q_t with internal funds (net worth, N_{t+1}) and bank loans. And they choose capital utilization and rent out capitals to intermediate goods firms at a rate \hat{r}_t^k .²³

The aggregate expected real return to capital is given by:

$$E_t \hat{R}_{t+1}^K = \frac{1 - \tau}{\bar{R}^K} E_t \hat{Q}_{t+1}^K + \frac{\bar{r}^k}{\bar{R}^K} E_t \hat{r}_{t+1}^k - \hat{Q}_t \quad (1.2.17)$$

²²This is the equation (8) of Smets and Wouters (2005)

²³This is modified equation (3) of Smets and Wouters (2005) without exogenous risk premium shock. From now on, I closely follows page 8 and 9 of Graeve (2006)

where \bar{R}^K denotes the steady state return to capital and \bar{r}^k stands for the steady state rental rate. The first term in the equation states the value of remaining capital ($\frac{1-\tau}{\bar{R}^K} E_t \hat{Q}_{t+1}^K$), the second term indicates the return from renting out the capital ($\frac{1-\tau}{\bar{R}^K} E_t \hat{Q}_{t+1}^K$) and the last term indicates the paid price for the purchase of capital stock (\hat{Q}_t).

While Graeve (2006) uses set of equations adopted directly from Smets and Wouters (2005) for the equations described up to now, Graeve (2006) extends the Smets-Wouters model by assuming that entrepreneurs cannot borrow at the risk-less rate because of capital market imperfection. In that case, because of the asymmetric information between the financial intermediary and entrepreneurs, the bank should pay a state verification cost for monitoring entrepreneurs. In equilibrium, entrepreneurs borrow up to the point where the expected return to capital equals the cost of external finance.

At equilibrium, Graeve (2006) argues that the external finance premium is given by:

$$E_t \hat{R}_{t+1}^K = -\varepsilon E_t \left[\hat{N}_{t+1} - \hat{Q}_t - \hat{K}_{t+1} \right] + \bar{R}_t \quad (1.2.18)$$

where ε measures the elasticity of the external finance premium to variations in entrepreneurial financial health ($E_t \left[\hat{N}_{t+1} - \hat{Q}_t - \hat{K}_{t+1} \right]$), measured by net worth relative to capital expenditures. Following Bernanke, Gertler, and Gilchrist (1999), he assumes that the premium over the risk-free rate required by the financial intermediary is a negative function of the amount of collateralized net worth. When entrepreneurs have sufficient net worth to finance the entire capital stock, Graeve (2006) explains that his model reduces to the Smets and Wouters model.

And Graeve (2006) sets the net worth equation of entrepreneurs by:

$$\hat{N}_{t+1} = \gamma \bar{R}^K \left[\frac{\bar{K}}{\bar{N}} \left(\hat{R}_t^K - E_{t-1} \hat{R}_t^K \right) + E_{t-1} \hat{R}_t^K + \hat{N}_t \right] \quad (1.2.19)$$

where γ is the entrepreneurial survival rate and $\frac{\bar{K}}{\bar{N}}$ is the steady state ratio of capital to net

worth.

Graeve (2006) concludes that his model with the financial accelerator (endogenous external finance premium) performs substantially better in matching the macro-dynamics relative to the Smets-Wouters model without that mechanism from examining the Bayes factor.

1.2.3.3 Factor-augmented vector autoregressions (FAVAR)

Central banks increasingly pay attention to a much larger number of time-series in their forecasts than is commonly assumed by academic econometricians. For example, the US Fed is thought to keep and evaluate thousands of series. Usual criticisms of the common VAR approach to monetary policy shock identification center around that a relatively small amount of information is used by usual VARs with at most eight variables. The New-Keynesian models discussed in the last section are also subject to the same criticism since they use at most eight variables. This information set is very small available to the information sets of actual central bankers who constantly watch even hundreds of data series.

Recently, Bernanke, Boivin, and Elias (2005) propose a solution to deal with this critique on limited information problem by combining the usual VAR analysis and principal component analysis to incorporate latent factors extracted with a large panel of economic data. They argue that those latent factors can be interpreted as state variables of the economy such as unobserved potential output or credit conditions.

In this paper, I use a series of monetary policy shock identified from FAVAR model in order to investigate two issues. First, since this model provides monthly estimates of structural shocks, we can examine the relationship between monetary policy shocks and stock returns further at a higher frequency than quarterly horizon. Second, the size of the data set used in estimating New-Keynesian models may be too small to extract monetary policy shocks with 8 variables and 20 years of quarterly data. Since I use almost 40 years

of monthly data to check for the effects of monetary policy on the cross-section of stock returns, empirical results using this model would provide another testing ground for the effects of monetary policy.

However, there is a criticism on using this type of models. Even though structural vector autoregressions (VARs) are widely used to show the effect of monetary policy shocks on the economy, these models typically use simple identifying restrictions consistent with various models.²⁴ Following the standard structural VAR approach, FAVAR proposed by Bernanke, Boivin, and Elias (2005) also focuses on specifications that identify the monetary policy shock while remaining agnostic about the structure of the rest of the model and the number of unobservable factors. Because this is not a true structural model based on microfoundations, it is questionable whether the results are robust and not subject to Lucas (1976) critique problems.

Bernanke, Boivin, and Elias (2005) propose the following state space model:

Transition Equation

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + v_t, v_t \sim (0, Q)$$

where Y_t is an $M \times 1$ vector of observable economic variables assumed to drive the dynamics of the economy and F_t is a $K \times 1$ vector of “unobserved” factors such as unobserved potential output or credit conditions.

Measurement equation

$$X_t = \Lambda^F F_t + \Lambda^Y Y_t + e_t$$

$$(N \times 1) \quad (N \times K) \quad (N \times M) \quad (N \times 1)$$

²⁴For example, Sims (1992) assumes that monetary policy only affects output with a lag. (money granger causes output)

where $e_t \sim N(0, R)$ and R is a diagonal matrix and e_t and v_t are independent. X_t is $N \times 1$ vector of economic variables useful to extract F_t

In matrix forms, measurement equation can be expressed as the following:

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} \Lambda^f & \Lambda^y \\ 0 & I \end{bmatrix} \begin{bmatrix} F_t \\ Y_t \end{bmatrix} + \begin{bmatrix} e_t \\ 0 \end{bmatrix}$$

Following Bernanke, Boivin, and Eliasch (2005)'s empirical specifications, I extract three latent factors from 120 monthly macro series and use the federal funds rate as the single observed factor. In this case, $Y_t = R_t$ and R_t is monetary policy instrument.

Even though I use several complex models to identify monetary policy shocks, the models presented here should be viewed only as approximations to the actual monetary transmission mechanism. A realistic quantitative model still needs to incorporate additional complexities. Nonetheless, the basic elements of the optimizing model of the monetary transmission mechanism used in this paper are interpreted as being representative of crucial elements of a realistic model; and indeed, Smets and Wouters (2003) argue that the illustrative models discussed here have many elements in common with rational-expectations models of the monetary transmission mechanism that are already being used for quantitative policy evaluation at a number of central banks.

1.3 Empirical Specification

1.3.1 Baseline Empirical Models

This section provides a simple Bayesian framework for estimating and evaluating a version of ICAPM based on the state variables estimated from several New-Keynesian models. In order to estimate the models explained in the previous section, I present a Markov Chain

Monte Carlo(MCMC) algorithm to encompass all of the estimation procedures.²⁵

1.3.1.1 Assumptions and Estimation algorithm

First, I express the joint density of all the parameters by:²⁶

$$f(\lambda, \beta, \sigma_v^2, \sigma_e^2, X|R, M)$$

where λ denotes the risk price; β is the risk(betas); σ_v^2 and σ_e^2 will be defined later; X is a vector of state variables(structural shocks); R is a vector of portfolio returns; M is a vector of macro variables.

I assume that structural shocks(X) are identified only from New-Keynesian models with macro variables but not from stock market returns. However, this assumption might not be innocuous. Modelling and identifying the interaction between monetary policy and the stock market is complicated by endogeneity problem since the Federal Reserve may react to movements in asset price returns while asset price returns are sensitive to the interest rate set by the monetary authorities.(e.g.Rigobon and Sack (2003))

I argue why these assumption can still be valid with the following reasons. First, the reaction of the Fed on stock market behavior might not be too big. For example, Bernanke and Gertler (1999) augment an otherwise conventional Taylor rule with the lag of asset price returns and find that the Fed does not react to the stock market. They argue that the Fed with inflation target objective should respond only to inflation component of stock

²⁵Alternative classical approach can be specified following Brennan, Wang, and Xia (2004).In their approach, they first estimate an essential affine term structure models using maximum likelihood estimation with Kalman filter and do the usual Fama-Macbeth two-step regressions. In this setup, GMM estimation of all parameters is impossible because of Kalman filter, which is true in estimating New-Keynesian DSGEs since it also involves Kalman filter. In fact, since I use diffuse priors for almost all parameters, we might be able to interpret the estimation results similar to bootstrap estimation of all parameters in classical approach.

²⁶I suppress structural parameters in New-Keynesian models since these parameters are not primary concerns.

market volatility. Since inflation rate itself is included in estimation of New-Keynesian models, adding stock market variables would not be that much important for estimation results. Second, none of the standard New-Keynesian models include stock market data to identify structural shocks for macro economy. Probably one notable exception is FAVAR model, which includes stock market returns to extract latent factors from observed variables. Therefore, robustness checks using FAVAR models might be interesting to check whether monetary policy shock is really important for explaining the cross-section of stock returns. Finally, empirical asset pricing studies using macro-variables typically assume that state variables extracted from some macro-models are exogenous to the stock market. For example, Thorbecke (1997) include stock return as most endogenous variables in his structural VAR to investigate the time-series relationship between monetary policy shock and stock returns.

This joint density can now be decomposed as:

$$f(\lambda, \beta, \sigma_v^2, \sigma_e^2 | X, R) f(X | M) \quad (1.3.1)$$

Furthermore, I assume that prior independence of parameters in $f(X|M)$ and those in $f(\lambda, \beta, \sigma_v^2, \sigma_e^2 | X, R)$. Then I can estimate posterior distributions of all parameters in separate blocks. Based on the given assumptions, I use unified MCMC scheme to estimate all parameters by repeating estimation steps given below.²⁷ I estimate the given models in step 1 for burn-in period.²⁸ And after burn-in period, I start to repeat step 1 through 3 and save series of identified structural shocks and calculate posterior distributions of corresponding risks and risk prices. During the repetition, I calculate and save various statistics for the comparison of different models. However, for the Fama-French three factor model or CAPM, I skip the step 1 since factors are all observed.

²⁷In fact, it becomes a sequential estimation of three blocks of MCMC using assumptions.

²⁸I use Brooks and Gelman (1998)'s convergence diagnostic to make sure actual convergence of algorithm

Step 1. estimation of $f(X|M)$ I estimate New-Keynesian or FAVAR models and obtain the series of structural errors from given models after burn-in periods. In order to estimate New-Keynesian models and FAVAR model, I utilize a Bayesian method. While maximum likelihood estimation(MLE) is known to produce better results than generalized methods of moments(GMM) in estimating DSGE models, several researchers(e.g.An and Schorfheide (2005)) criticize MLE with "dilemma of absurd parameter estimates" when applying MLE to DSGE models and argue that Bayesian methods often produce more acceptable parameter estimates combining prior information from micro-economic studies.

Specifically, I use the DYNARE package to estimate the Linearized DSGE models.²⁹ The Bayesian estimation methodology in DYNARE package contains the following steps. First, the linearized rational expectations model is solved with the generalized Schur decomposition(QZ) method³⁰, which leads to results in a state equation in the predetermined state variables. Second, the model is written in the state space form by adding a measurement equation that links the observable variables to the vector of state variables. Third, the likelihood function is derived using the Kalman filter. Finally, after the posterior density is formed by combining the likelihood function with a prior distribution over the parameters, a Random-Walk Metropolis(RWM) algorithm is utilized to generate draws from the posterior distribution of parameters. The RWM algorithm belongs to the more general class of Metropolis-Hastings algorithms which generate Markov chains with stationary distributions that correspond to the posterior distributions of interest.³¹ In order to get appropriate starting values, DYNARE estimates the posterior mode with Sims' "*cminwel*" function and the inverse of the Hessian is computed at that posterior mode. Since it is well known that

²⁹To estimate New-Keynesian models I employ DYNARE version 3.5, by S. Adjemian, M. Juillard and O.Kamenik. I appreciate for their kindness to make this phenomenal toolbox available. For FAVAR, I utilize Matlab programs downloaded from Jean Boivin's website. I also thank him for sharing his programs and data

³⁰I refer to Klein (2000) for a reference of the QZ method

³¹I refer to Chib and Greenberg (1995) for an excellent introduction to Metropolis-Hastings algorithms.

the posterior distribution of each parameter is asymptotically normal, a Gaussian approximation around the posterior mode and a scaled asymptotic covariance matrix is used as the proposal distribution. And the scale parameters are chosen to obtain approximately 35 % acceptance rate. The debugging features of DYNARE are used to determine if the optimization routines have found the optimum and if enough draws have been executed for the posterior distributions to be accurate.

Before developing step 2, I state the implied ICAPM in the form of the fundamental asset pricing equation and the corresponding beta-regression.

$$E (M_{t+1}R_{t+1}^e|\psi_t) = 0 \quad (1.3.2)$$

where R_{t+1}^e is an N-vector of excess returns(test assets) to a short term risk-free from t to t+1; M_{t+1} is the asset pricing kernel(or stochastic discount factor) given in (1.2.3); ψ_t denotes the information at time t. By plugging (1.2.3) into (1.3.2), implied ICAPM model can be intuitively expressed by (expected return = risk price(λ)*risk(β)).

Since I use non-return factors implied from New-Keynesian models, I can not obtain the risk price(λ) and risk(β) simultaneously. Therefore, I assume that we can estimate λ and β in two steps with the Fama-MacBeth analogy. Now, I can express (1.3.1) by:

$$f(\lambda, \sigma_\nu^2|\beta, \sigma_e^2, \mathbf{X}, \mathbf{R})f(\beta, \sigma_e^2|\mathbf{X}, \mathbf{R})f(X|M) \quad (1.3.3)$$

where λ and σ_ν^2 are the parameters in cross-sectional regression; β and σ_e^2 are the parameters in time-series regression.

Step 2. estimation of risk(β) with $f(\beta, \sigma_e^2|\mathbf{X}, \mathbf{R})$ If I assume that the econometrician has access to both excess returns and the historical values of the true state variables(structural shocks) given from the first step, then the second step in this estimation scheme is a multivariate time-series regression of excess returns(r^i) on the state variables

or structural shocks from the first step(X^i):

$$r^i = X^i \beta^i + e^i, \text{ for } i = 1, 2, \dots, m \quad (1.3.4)$$

where r^i is a $T \times 1$ vector of the T observations of the dependent variable (portfolio returns), X^i is a $T \times p^i$ matrix of independent variables (state variables or structural shocks), β^i is a $p^i \times 1$ vector of the regression coefficients (betas), and e^i is the vector of errors for the T observations of the i th regression.

In matrix forms,

$$\tilde{r} = \tilde{X} \tilde{\beta} + \tilde{e} \quad (1.3.5)$$

where

$$\tilde{r} = \begin{pmatrix} r^1 \\ r^2 \\ \dots \\ r^m \end{pmatrix}, \tilde{X} = \begin{pmatrix} X^1 & 0 & \dots & 0 \\ 0 & X^2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & X^m \end{pmatrix}, \tilde{\beta} = \begin{pmatrix} \beta^1 \\ \beta^2 \\ \dots \\ \beta^m \end{pmatrix}, \tilde{e} = \begin{pmatrix} e^1 \\ e^2 \\ \dots \\ e^m \end{pmatrix}, \tilde{e} \sim N\left(0, \sum \otimes I_T\right)$$

As the above remarks indicate, \tilde{r} are asset returns and the independent variables, \tilde{X} are factors (structural shocks) in asset pricing models, $\tilde{\beta}$ are risks of factors. Since the set of factors is the same across all equations, i.e., ($X \equiv X^1 = X^2 = \dots, X^m$), we can interpret (1.3.5) as a seemingly unrelated regressions (SUR) model with the same independent variables. Typically, we assume, in linear factor models, that \sum is diagonal, i.e., $e^i = N(0, \sigma_i^2)$ and $\text{cov}(e^i, e^j) = 0$ for $i \neq j$. Since generalized least square (GLS) estimation is numerically equal to OLS estimation in this case, we can estimate each equation in the above system with Bayesian estimation of ordinary least squares regression. ($r^i = X \beta^i + e^i$ for all i)

Based on the given models, I can utilize simple Bayesian regression method with non-informative priors. If $r^i | \beta^i, \sigma_e^2, X^i \sim N(X^i \beta^i, \sigma_e^2 I)$ and $p(\beta^i, \log \sigma_e^2) \propto \sigma_e^{-2}$ then it follows

that the conditional posterior distribution $p(\beta^i | \sigma_e^2; \mathbf{X}, \mathbf{R})$ can be written as:

$$p(\beta^i | \sigma_e^2; \mathbf{X}, \mathbf{R}) \sim N_{\text{MVN}} \left(\left(X^{i'} X^i \right)^{-1} X^{i'} r^i, \sigma_e^2 \left(X^{i'} X^i \right)^{-1} \right) \quad (1.3.6)$$

The posterior distribution of σ_e^2 can be written as: $p(\sigma_e^2 | data) \sim \text{Inv } \chi^2(n - k, s^2)$. Using Gibbs-sampling technique, I can estimate β with previous returns data with state variables(\mathbf{X}) from the step 1. However, it is well known that in this case, the following marginal distribution of β holds: $p(\beta | data) \sim \text{MVT}_{n-k}(\beta^{i'}, s^2 X^{i'} X^i)^{-1}$. Therefore, I just use modes of multivariate t distribution as estimates of β .

Step 3. estimation of risk price(λ) with $\mathbf{f}(\lambda, \sigma_\nu^2 | \beta, \sigma_e^2, \mathbf{X}, \mathbf{R})$ Finally, I estimate risk prices(λ) using β as independent variables for each time. In classical approach, Fama-MacBeth approach suggests that I should run this cross-sectional regressions each quarter, generating time-series of estimates for risk prices(λ). Means, standard errors, and t-statistics are then computed from these time series and inference proceeds in the usual manner, as if the time series are independently and identically distributed. It is well known that security returns are cross-sectionally correlated, due to common market and industry factors, and also are heteroscedastic. As a result, the usual formulas for standard errors are not appropriate for the OLS cross-sectional regressions(CSR). Fama-Macbeth approach is interpreted as a remedy for this phenomenon. Since the true variance of each quarterly estimator depends on the covariance matrix of returns, cross-sectional correlation and heteroskedasticity are reflected in the time series of quarterly estimates. Following this argument, I run Bayesian cross-sectional regression every time using (1.3.6). However, its independent variables become (β) instead of X^i , and λ become a vector of coefficients from this regression

$$p(\lambda^i | \sigma_\nu^2; data) \sim N_{\text{MVN}} \left(\left(\beta^{i'} \beta^i \right)^{-1} \beta^{i'} r^i, \sigma_\nu^2 \left(\beta^{i'} \beta^i \right)^{-1} \right) \quad (1.3.7)$$

where β is obtained from the previous step.

However, estimates given in step 2 provide some information about covariance matrix for error terms in cross-sectional regressions since intercept terms in time-series regressions are related to error terms in cross-sectional regression. In fact, Cochrane (2001) argues, in classical sense, that while GLS is asymptotically more efficient, estimation results may not be trustworthy for large portfolios since GLS meaning OLS on transformed portfolios and it pays attention to uninteresting but low residual variance or near riskless portfolios. Therefore, consistent with previous studies using OLS, I use either Bayesian Gibbs sampling using (1.3.7) or use modes of multivariate t distribution as before.

$$p(\lambda|X, R) \sim \text{MVT}_{n-k} \left(\lambda^{i'}, s^2 \beta^{i'} \beta^{i-1} \right). \quad (1.3.8)$$

The whole procedure can be interpreted as usual integrating-out of nuisance parameters. After simulating the posterior distribution of parameters, simulated parameters can then be used to construct the posterior distribution of risk prices by the following form:

$$f[\lambda] = \iiint f(\lambda, \beta, \sigma_e^2, \sigma_\nu^2, X | M, R) d\sigma_\nu^2 d\beta d\sigma_e^2 dX$$

Any function of estimated parameters can also be constructed with same logic. For example, I construct Jagannathan and Wang (1996)'s adjusted R^2 to judge the goodness of fit of the suggested empirical models.

$$R^2 = \frac{\sigma_C^2(\bar{R}) - \sigma_C^2(\bar{e})}{\sigma_C^2(\bar{R})}$$

where σ_C^2 represents the in-sample cross-sectional variance, \bar{R} is a vector of average excess returns, and \bar{e} stands for the vector of average residuals in cross-sectional regression.

1.4 Data and Empirical Results

1.4.1 Data

In this study, I use three different macro data sets. First, in order to estimate three versions of Cho and Moreno model, I download their data set from the website of Journal of Money, Credit, and Banking.³² They use U.S. quarterly time series for three variables: output gap, inflation rate using GDP deflator and the quarterly average of Federal funds rate from 1980:Q4 to 2004:Q4.³³ They choose this sample period after several parameter stability tests and report their results are robust across several output gap measures with the Consumer Price Index(CPI) and the 3 Month T-Bill rate. I estimate their models using linearly-detrended output gap and inflation rate using GDP deflator and the Federal funds rate. Perhaps, this sample period explicitly shows the relationship between monetary policy and stock returns because Fed has responded more vigorously to inflation variations since 1979.

Second, I estimate Graeve (2006)'s model using his data and DYNARE program.³⁴ His data set consists of real GDP, consumption, investment, real wages, hours worked, price(GDP deflator) and the short-term interest rate of Smets and Wouters (2006) from 1947:Q1 to 2004:Q4.³⁵ Nominal variables are deflated by the GDP-deflator and aggregate real variables are expressed in per capita terms. All variables except for hours, inflation and the interest rate are linearly detrended. Following Smets and Wouters (2006) and Graeve (2006), I estimate Graeve's model using full sample data. But for comparison, I use extracted structural shocks only from 1980:Q4 to 2004:Q4 for explaining the cross-

³²I thank Cho and Moreno for sharing their data set and JMCB for its data policy.

³³They use data from 1980:Q1 to 2000:Q1 in their main estimation but extend their data set for robustness check. Their data is annualized and in percentages. Federal funds rate data was collected from the Board of Governors of the Federal Reserve website. Real GDP and the GDP deflator were obtained from the National Income and Product Accounts (NIPA). I refer to Cho and Moreno (2006) for the details.

³⁴I greatly appreciate for his sharing of his program and data

³⁵For data description, I refer to data appendix of Smets and Wouters (2006).

section of stock returns. Even though sample period used in estimation is different, identified structural shocks are similar across different models. For example, any measure of monetary policy shocks from models 1,2 and 3 of Cho and Moreno (2006) has a correlation above 80 % of temporary monetary shocks from Graeve (2006)'s model. This might indicate the robustness of this Smets-Wouters model over simple three equations of New-Keynesian models since parameter instability of simple models would be endogenously incorporated in this complex model.

Finally, I download data from the website of Jean Boivin to estimate FAVAR model.³⁶ They use Federal funds rate and other monthly macro-economic data from January 1959 through August 2001 and extract latent factors from a balanced panel of 120 monthly macroeconomic time series, which is transformed to induce stationarity. Probably, one characteristic of their data is noteworthy to be mentioned. Unlike New-Keynesian models using only macro variables, Bernanke, Boivin, and Elias (2005) include stock prices data(NYSE and S&P stock index, dividend yield, price-earning ratio) to extract their latent factors. I use structural shocks from this model to check the relationship between monetary policy and stock returns for full sample period.

In cross-sectional analysis, I use ,as test assets, the returns on Fama-French 25 portfolios sorted by size and book-to-market and 30 industry portfolios. Even though the 25 portfolios have become the benchmark in testing competing asset pricing models, Lewellen, Nagel, and Shanken (2006) show that the 55 portfolios are the more appropriate to rigorously compare the models. All the portfolio returns and the Fama-French three-factors -the returns of the market portfolio(Rmrf), HML, and SMB are downloaded from French's website.

³⁶I thank him for sharing his data and program. I refer to the data appendix of Bernanke, Boivin, and Elias (2005) for the description of the data set and their detrending methods.

1.4.2 Priors and estimation results for New-Keynesian models

The Bayesian approach facilitates the incorporation of prior information from other macro as well as micro studies. As is well known from Bayes rule, the posterior distribution of a parameter is proportional to the product of its prior distribution and the likelihood function of the data. This prior distribution describes the available information prior to observing the data used in the estimation. The observed data is then used to update the prior, via Bayes theorem, to the posterior distribution of the parameters. However, Bayesian analysis is often criticized for its subjectivity bias from prior selections. In this study, I employ non-informative priors whenever possible. Especially, for the second and the third blocks of estimation, readily available non-informative Jeffrey priors are used. $(p(\beta^i, \log \sigma_e^2) \propto \sigma_e^{-2})$ In specifying the prior density, I assume that all parameters in each block are independently distributed of other parameters in the other blocks. This assumption simplifies my estimation since all blocks of estimation can be now separately done along with identifying assumptions given in the previous section.

For the first block estimation of New-Keynesian models, however, informative priors seem to be indispensable. While maximum likelihood estimation(MLE) or Bayesian estimation with non-informative priors is known to produce better results than the generalized methods of moments(GMM) in estimating DSGE models, several researchers(e.g.An and Schorfheide (2005)) criticize MLE with "dilemma of absurd parameter estimates" when applying the MLE to DSGE models and argue that Bayesian methods often produce more acceptable parameter estimates. For the estimation of Graeve (2006) and FAVAR, I exactly follow their selections of prior distributions.³⁷ However, I need to select prior distributions of the parameters in Cho and Moreno (2006)'s models since they use MLE in their paper. First, I choose distributional assumptions following Graeve (2006) in the sense that if some parameters have positivity restrictions(e.g.Calvo parameter), I impose beta distribution

³⁷I refer to their papers for the details on their prior selection.

and I use usual inverted gamma distribution for variance parameters. However, in order to minimize biases caused by the selection of prior distribution and obtain consistent results with Cho and Moreno (2006), I choose almost non-informative priors. For example, with DYNARE, I can check whether posterior modes are uniquely identifiable with given prior density and likelihood function. I set the variance of prior density as large as possible if unique mode is identified.

In Bayesian analysis, monitoring the convergence of parameters is critical since without it, we are not sure whether estimated parameters can be considered as a valid sample from the posterior distribution.³⁸ Therefore, in order to ensure convergence, I do several checks. First, I simulate samples from each New-Keynesian model at least 200,000 draws from five different chains and after discarding 50 % of them in each chain as burn-in replications, I calculate the convergence diagnostics of Brooks and Gelman (1998) offered in DYNARE package. I find every parameter converged with this statistics. When I also draw one long chain of 1,000,000 draws from each model with 500,000 as burn-in periods, I obtain similar results.

While all of the parameter estimates are similar to those presented in Cho and Moreno (2006) and Graeve (2006),³⁹ I report the estimated structural shocks omitted from the tables of Cho and Moreno (2006) and Graeve (2006). Several points deserve to be mentioned.⁴⁰ First, in Table 1.1, model 2(M2) and model 3(M3) are extended versions of model 1(M1) with autocorrelation alone or cross-correlation together. We expect that even though estimated shocks are similar across the models, autocorrelations of model 2 and of model 3 will be much lower than that of model 1. These facts are confirmed in the table. Correla-

³⁸For FAVAR model, I exactly follow the suggestions given in Bernanke, Boivin, and Eliasch (2005) and obtain similar estimates using their original programs.

³⁹I omit results of parameter estimation and refer to the tables in each paper for the details since there is no additional information provided from reporting the same estimates.

⁴⁰I also omit results of FAVAR and just show Figure 1.3 for overall picture since these results are not primary concerns in this paper.

tions of estimated shocks are above 90% in every case, but autocorrelations of shocks are significantly lower in model 2 and model 3 compared with that of model 1. This pattern is also confirmed in Figure 1.1. Table 1.2 and Figure 1.2 reports the sample statistics and patterns of estimated structural shocks from Graeve's model.

Two facts deserve special attention for understanding the risk premium reported in the next section. In Graeve's model, there are two monetary policy shocks; permanent shock to inflation target(GE_PIE_BAR) and temporary monetary policy shock(GETA_R). Table 1.3 reports correlation between these two shocks and monetary policy shocks identified from Cho and Moreno's models. Interestingly, monetary shocks of Cho and Moreno's models seem to capture mostly temporary monetary shock. Correlation between Cho and Moreno's monetary policy shocks and Graeve's temporary monetary policy shocks are around 75% but with permanent shocks, correlation is just 30%. Therefore, I interpret Cho and Moreno's monetary policy shocks as temporary shocks. Finally, I do not report here but confirm an important result from the table 4 of Graeve (2006). He calculates which structural shock is important for explaining the external finance premium for several different horizons using variance decomposition. In this estimates, the shock to investment technology corresponds to almost 85 % of the external finance premium especially for horizons over two years.

1.4.3 Cross-sectional implications of New-Keynesian models

In this section, I examine the pricing performance of the full set of state variables from New-Keynesian models over the period from 1980:Q1 to 2004:Q4. The full set of shocks of the state variables consists of the price mark-up shock(AS), the preference shock(IS) and the temporary monetary shock for models 1,2 and 3 proposed by Cho and Moreno (2006). For Graeve's model, I choose 4 structural shocks in order to investigate the source of risk premium more precisely. For consistent comparison with Cho and Moreno's models, I choose price mark-up shock(Price-Markup), permanent shock to inflation target(Monetary1) and

temporary monetary shock(Monetary2). In addition, I select a shock to investment technology(Investment) as a main determinant for the external finance premium.⁴¹ These state variables derived from New-Keynesian models are risk factors in the ICAPM model. The objective is to test whether assets' loadings with respect to these risk factors are important determinants of its average returns.

I repeat the following estimation steps for at least 200,000 for each model. After burn-in periods, estimated structural shocks from the first step estimation of New-Keynesian models are saved. In the second block of Bayesian estimation, I regress the portfolio returns on these structural shocks saved from the first block to obtain the risk prices(betas). As in Lettau and Ludvigson (2001b), the full-sample loadings, which are the independent variables in the second stage regressions, are computed in this multiple time-series regression using marginal posterior density of parameters. With diffuse priors, I just can use modes of parameters in multivariate t-distribution instead of using Gibbs sampling.⁴² The following regression is specifically used.

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,1}X_{1,t} + \beta_{i,2}X_{2,t} + \beta_{i,3}X_{3,t} + \varepsilon_{i,t}$$

where $R_{i,t}$ is portfolio returns; $R_{f,t}$ is treasury bill returns; $R_{M,t}$ is market returns; $X_{i,t}$ for $i=1,2$ and 3 are structural shocks extracted from the first block of estimation. For unconditional CAPM and the Fama-French three factor model, usual Rmrf, HML, SMB are used as state variables.

Finally, in the third block of estimation, the posterior means of betas from the previous regressions are subsequently used as independent variables in this cross-sectional regression for all time period. Following Fama-Macbeth analogy, I estimate means and standard

⁴¹This shock corresponds to 85% of the external finance premium using variance decomposition reported in the table 4 of Graeve (2006)

⁴²Details of this Bayesian estimation methods are given in the previous section.

deviation of risk prices(λ) for each time using risks(betas) as independent variables from the previous step. The following forms are used in cross sectional regressions.

1. CAPM: $R_{i,t} - R_{f,t} = \alpha_i + \lambda_i \text{Rmrf}$
2. FF3: $R_{i,t} - R_{f,t} = \alpha_i + \lambda_1 \text{Rmrf} + \lambda_2 \text{SMB} + \lambda_3 \text{HML}$
3. Model1,2 and 3: $R_{i,t} - R_{f,t} = \alpha_i + \lambda_1 \text{AS} + \lambda_2 \text{IS} + \lambda_3 \text{Monetary}$
4. GRAEVE: $R_{i,t} - R_{f,t} = \alpha_i + \lambda_1 \text{Investment} + \lambda_2 \text{Monetary1} + \lambda_3 \text{price mark-up} + \lambda_3 \text{Monetary2}$
5. FAVAR: $R_{i,t} - R_{f,t} = \alpha_i + \lambda_1 \text{latent1} + \lambda_2 \text{latent2} + \lambda_3 \text{latent3} + \lambda_4 \text{Monetary2}$

where definition of models and variables are reported in the tables of 4,5 and 6.

After burn-in periods in the first and the second steps of MCMC estimation of models, I save samples of the risk prices and ,using only the last 1,000 samples, I calculate means and standard deviations of them.⁴³ To judge the goodness of fit of the suggested empirical models, I use the posterior modes of cross-sectional R^2 measure employed first by Jagannathan and Wang (1996). This R^2 represents the fraction of cross-sectional variation in average returns explained by the model. This measure is calculated as:⁴⁴

$$R^2 = \frac{\sigma_C^2(\bar{R}) - \sigma_C^2(\bar{e})}{\sigma_C^2(\bar{R})}$$

where σ_C^2 represents the in-sample cross-sectional variance, \bar{R} is a vector of average excess returns, and \bar{e} stands for the vector of average residuals in cross-sectional regression.

I also report the root mean square of pricing errors(α) in cross-sectional regression(RMSE) as another intuitive diagnostic to compare the models. I use $\sqrt{\frac{1}{25} \sum_{i=1}^{25} \alpha_i^2} = \sqrt{\frac{1}{N} \alpha' \alpha}$ for

⁴³Following Bernanke, Boivin, and Elias (2005), I also use medians of parameters to obtain qualitatively the same results in cross-sectional implication of models.

⁴⁴Usual model comparison using Bayes factors are not employed for the following reasons. First, I use non-informative or diffuse priors on many parameters in order to minimize possible biases from selections of prior distribution. However, with these diffuse priors, closed form of marginal densities are hard to obtain. Furthermore, New-Keynesian models involve one more step to estimate structural shocks than unconditional CAPM and the Fama-French three factor model. It is unclear whether usual comparison is still valid in this case.

all the models. Cochrane advocates this measure by arguing that even though Hansen-Jagannathan(HJ) distance measure is invariant to portfolio formation, this simple RMSE could be more informative if the original portfolios were primary concerns and the second moment matrix of the test assets is quite close to singular⁴⁵ since the HJ distance places too much weight on pricing near-riskless portfolios rather than pricing the original assets.

Table 1.4 reports the posterior modes of coefficients, standard errors and the degrees of freedom-adjusted R^2 of Jagannathan and Wang (1996) and the RMSE for the cross-sectional regressions using the excess returns on 25 portfolios sorted by book-to-market and size. In this table, structural shocks are identified from New-Keynesian models. First, in most cases, the market factor(Rmrf) receives a negative and statistically insignificant risk premium consistent with the findings of Fama and French (1992).⁴⁶ Even though it appears to be a severe problem for the CAPM, the negative market risk premium has not been understood yet. Since the issue is beyond the scope of this paper, I defer it to the future studies. Second, there is a mixed evidence for the value premium in the data. Even though the HML in the Fama-French three factor itself is not priced in the cross section of average returns, inclusion of the HML tends to increase the cross-section R^2 (R^2 is above 80%). Third, model 1 of Cho and Moreno is even worse than CAPM in terms of R^2 and RMSE. Their models 2 and 3 are comparable to or slightly better than CAPM. Among other things, monetary shock(Monetary) is not statistically significant while aggregate supply shock,i.e., price mark-up shock(AS) and IS shock are statistically significant.

Parameter estimates of Graeve's model will shed some lights on why Cho and Moreno models fail to fully explain risk premium. This is mainly because Graeve's model is a significant extension of Cho and Moreno's models and provides more precise description

⁴⁵Lewellen, Nagel, and Shanken (2006) suggest that Fama-French 25 size and B/M sorted portfolios have essentially three degree of freedom.

⁴⁶Jagannathan and Wang (1996) and Lettau and Ludvigson (2001b) also report negative estimates for the market risk premium, using monthly or quarterly data.

of the structural shocks. Specifically, Graeve's model uses two monetary policy shocks. Only the loading with respect to permanent monetary policy shock to inflation target is an important determinant of average returns. Since Cho and Moreno's monetary shock can mostly be interpreted as temporary monetary shock, we can conclude that investors in the stock market primarily respond to the persistent shock from the Fed decision. This seems natural since target changes, by definition, have persistent effects on the future economy. Second, the loading on a shock to investment as the main proxy of the external finance premium is also an important cross-sectional determinant of average returns. The value premium has been an important yet controversial subject in the asset pricing literature. In fact, Fama and French (1993) argue that HML and SMB represent compensations for risk consistent with Merton (1973)'s ICAPM. While several models have been developed to explain these premiums, Hahn and Lee (2006) find that the size and value premiums are compensation for higher exposure to the risks related to changing credit market conditions and interest rates(monetary policy). They argue that small-sized and high book-to-market firms would be more vulnerable to worsening credit market conditions and higher interest rates since small firms tend to be young, poorly collateralized, and have limited access to external capital markets (Gertler and Gilchrist (1994)) and high book-to-market firms tend to have high financial leverage and cash flow problems. However, there is some controversy over the interpretation of Hahn and Lee (2006)'s findings since they use term spreads and default spreads as proxies for the external finance premium and monetary policy. Other hypotheses might well be developed to explain their results. My empirical results suggest that their hypotheses are indeed valid for explaining part of the size and value premium since my empirical proxies are obtained from equilibrium models more precisely and the loadings on both components price Fama-French 25 portfolios significantly.

Finally, the loading on price mark-up shock is statistically significant from models of either Cho and Moreno or Graeve. This shock determines the major component of the

inflation rate. Even though stocks are claims on real asset, in the short run, they are not good hedges against inflation.⁴⁷ One possible hypothesis is that volatile inflation tends to accompany with volatile levels of real output, which induces inflation risk premium in asset markets. Using reduced form models, Brennan, Wang, and Xia (2004) also find that portion of risk premium is related to the inflation rate.

Figure 1.4 plots the realized versus predicted returns of the models examined. The closer a portfolio lies on the 45-degree line, the better the model can explain the returns of the portfolio. It can be seen from the graph that the ICAPM implied from Graeve's model explains the value effect comparable to Fama-French three-factor model: In general, the fitted expected returns on value portfolios (larger second digit) are higher than the fitted expected returns on growth portfolios (smaller second digit).

Recently, Lewellen, Nagel, and Shanken (2006) argue that the proposed models for the value premium do not seem to explain premium of industry portfolios. Typically, they find that most of the models are even worse than the Fama-French three factor model in explaining industry premium. Therefore, Lewellen, Nagel, and Shanken (2006) recommend that when three factors explain nearly all of the time-series variation in returns of size-B/M portfolios, we should augment them with 30 industry portfolios which don't correlate with SMB and HML as much for correct comparison of the models. Furthermore, since there are essentially three degrees of freedom in Fama-French 25 portfolios, Cochrane (2006) suggests that asset pricing models with more than three factors, should be carefully investigated even though those models tend to explain Fama-French 25 portfolios.⁴⁸ Following the suggestions of Lewellen, Nagel, and Shanken (2006), I test the robustness of the proposed empirical models by examining the ability of the competing models to price industrial portfolios. I expect that if we have meaningful asset pricing models, my proposed models should describe

⁴⁷I refer to the chapter 11 of Siegel (2002) for the details

⁴⁸Brennan, Wang, and Xia (2004) also tests their model with this 55 portfolios and finds that their model is statistically rejected. However, they don't report any intuitive statistics.

these asset returns better than the Fama-French model does.

Table 1.5 reports the cross-sectional regression results on the 55 portfolios returns. Now ICAPMs implied from New-Keynesian models perform clearly better than unconditional CAPM or the Fama-French three factor model, in explaining the test assets in terms of the intuitive measures(both R^2 and RMSE). Therefore, suggested models clearly satisfy the robustness criteria of Lewellen, Nagel, and Shanken (2006) since they have a higher explanatory power in terms of the 55 portfolios than all the other models. The plots of Figure 1.5 also confirm these facts. However, while permanent monetary policy shock to inflation target prices industry portfolios significantly, all other structural shocks fail to explain industry risk premium. This could reflect that while the credit channel under the capital market imperfection is important for determining the value and the size premia, interest rate channel would be more important for explaining industry premium. Peersman and Smets (2005) show that there is considerable cross-industry heterogeneity in monetary policy effects and find that durability of the output produced by the sector is an important determinant of its sensitivity to monetary policy changes. They argue these facts as evidence for interest rate/cost-of-capital channel since the demand for durable products, such as investment goods, is known to be much more affected by a rise in the interest rate through the cost-of-capital channel than the demand for non-durables such as food. Recently, Gomes, Kogan, and Yogo (2007) argue that durability of output is a risk factor since the demand for durable goods is more cyclical than that for nondurable goods and services. Consequently, the cash flow and stock returns of durable-good producers are exposed to higher systematic risk and investors request higher risk premium for that. This study might indicate that monetary policy shock is one of fundamental shocks behind this risk premium.

New-Keynesian models employed in this paper use at most eight variables in the estimation. However, this information set is very small compared to the information sets which the Fed uses for its conducts monetary policy. Since FAVAR model uses principal com-

ponents of 120 monthly macro data including stock market variables for extracting state variables of the economy, the result would be robust to possible omitted variable problems in New-Keynesian models. I report the cross-sectional regression results on the 25 and 55 portfolios returns using structural shocks from FAVAR model in Table 1.6 and Figure 1.6.

In summary, identified monetary policy shock is a significant risk factor with same negative sign for 25 and 55 portfolios of monthly returns. This indicate that monetary policy shocks are indeed important for determining risk premium and this result is quite robust to model misspecification. While other structural shocks can be risk factors for explaining part of the value and the size premia, once confronted with industry portfolios, all the factors except for monetary policy shock lose their statistical significance.

1.5 Conclusion

This study contributes to the asset pricing literature in several respects. While there seems to be ample time-series evidence of the effects of monetary policy on stock returns, it has not been clear whether monetary policy shocks affect the cross section of stock returns. Even though some research(e.g.Hahn and Lee (2006)) find that connection between the policy shocks and stock returns using indirect measure of the capital market imperfection and monetary policy, many researchers seem to be reluctant to accept the results since they are established from reduced form empirical models.

In this paper, I employ a more direct measure of the capital market imperfection and monetary policy utilizing the recent advance in monetary economics literature. In fact, the present study is the first application of New-Keynesian models to explain the cross-section of stock returns with two monetary transmission mechanisms. I show that empirical models suggested in this study provide more structural interpretation using direct measure of structural shocks.

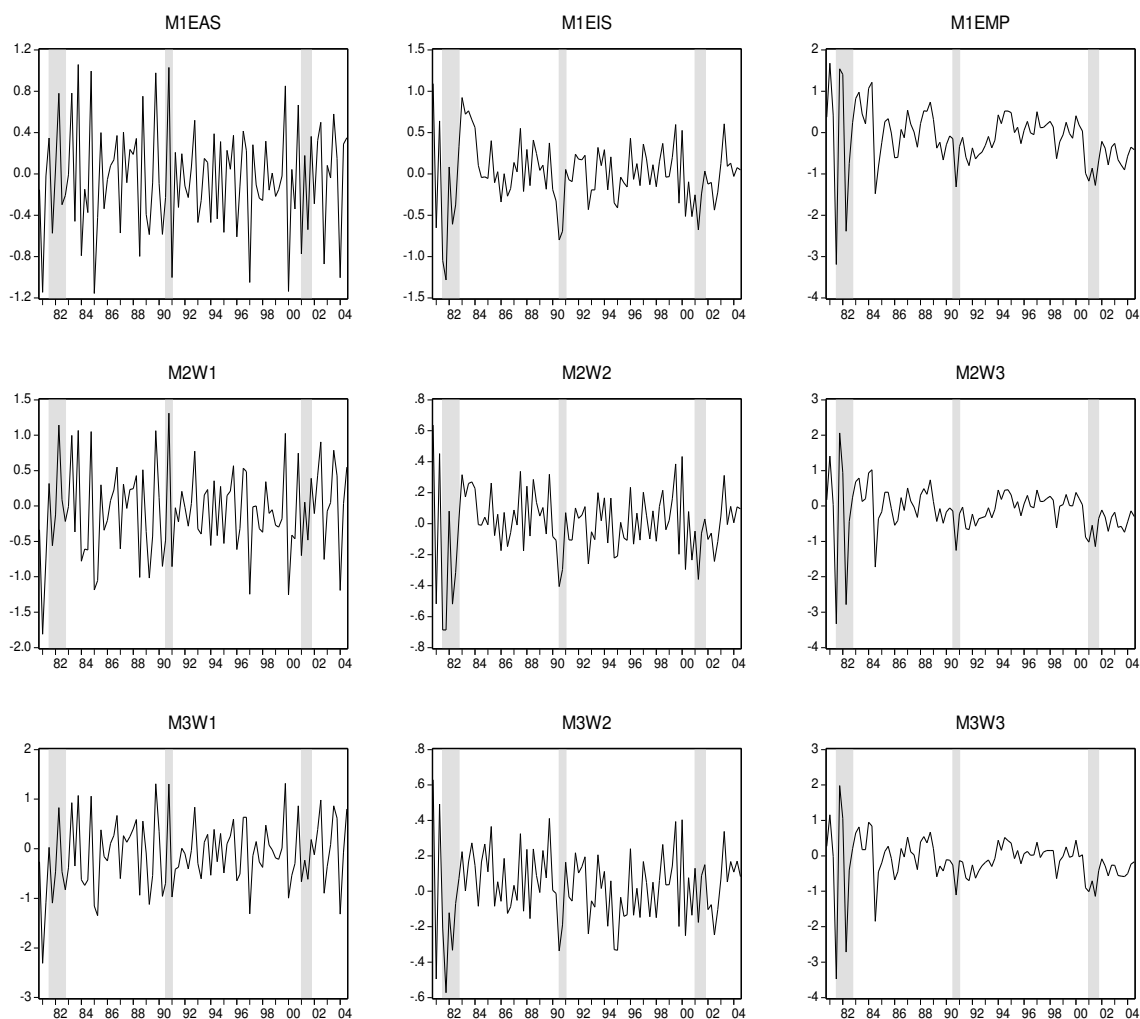
Two empirical findings emerge from this analysis using quarterly data from 1980 to 2004

and monthly data from 1960 to 2001. First, I find that both the permanent monetary policy shock to inflation target and the shock to the external finance premium successfully capture major portions of the size and the value premia. These results support the findings of Hahn and Lee (2006) that state variables reflecting revisions in the market's expectation about future credit market conditions and interest rates explain the size and the value premia. They argue that small-sized and high-book-to-market firms would be more vulnerable to worsening credit market conditions and higher interest rates.

Second, the permanent monetary policy shock to inflation target explains part of industry risk premium once I correctly account for the capital market imperfection. This may well reflect that while the credit channel with the capital market imperfection is important for determining the value and the size premia, interest rate channel would be more important for explaining industry premium. Peersman and Smets (2005) show that there is a considerable amount of cross-industry heterogeneity in the overall monetary policy effects. Specifically, they find that the durability of the outputs produced by industry sector is an important determinant of its sensitivity to monetary policy changes. Recently, Gomes, Kogan, and Yogo (2007) argue that the demand for durable goods is more cyclical than that for nondurable goods and services. Consequently, the cash flow and stock returns of durable-good producers are exposed to higher systematic risk and thus investors request higher risk premium. This study shows that monetary policy shock is one of crucial fundamental shocks behind this risk premium.

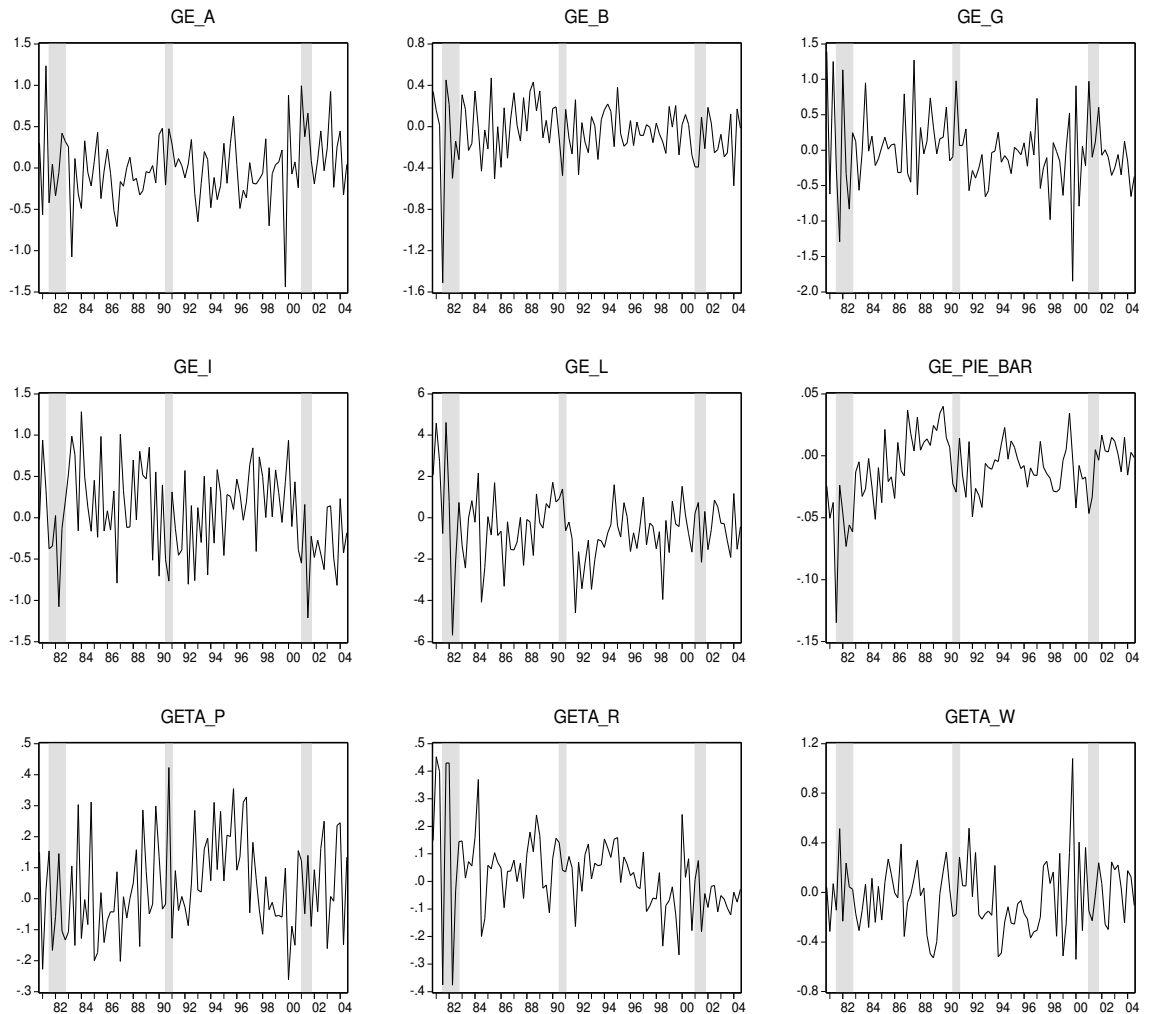
Finally, selected ICAPMs using New-Keynesian models are capable of explaining the cross-section of the Fama-French 25 size and B/M sorted portfolios significantly ($R^2=72\%$) and part of risk premia for 55 portfolios with their 30 industry portfolios ($R^2=30\%$). This result satisfy the robustness criterion of Lewellen, Nagel, and Shanken (2006) that criticize most of the empirical asset pricing models because they only explain the value premium but not any part of the risk premia of industry portfolios.

While the present study uses a reasonable approximation to the economy, several refinements can be done in the future studies. First, the current study uses exogenous pricing kernel to investigate risk premium since it mainly focuses on obtaining reasonable structural shocks frequently used in monetary economics literature. It would be interesting to see how more consistent pricing kernels using either Campbell and Cochrane (1999) type conditional models or heteroskedasticity based models could explain both the stylized facts in monetary economics and in finance. Second, New-Keynesian models with more extensive form of firm heterogeneity can be developed to explain the industry risk premium since current models seem to capture only part of it. Third, Bekaert, Cho, and Moreno (2005) extend the models of Cho and Moreno (2006) with term structure information. This extension could also be valuable for correct inferences since term structure information links the long-term and short-term interest rates and that link is regarded as a crucial channel for gauging the real effects of monetary policy on aggregate demand equation. Finally, Dedola and Lippi (2005) find sizable and significant cross-industry differences in the effects of monetary policy, using disaggregated data on twenty-one manufacturing sectors, from five industrialized countries. This fact indicates that the international New-Keynesian models could be developed to explain the risk premia in international stock markets.



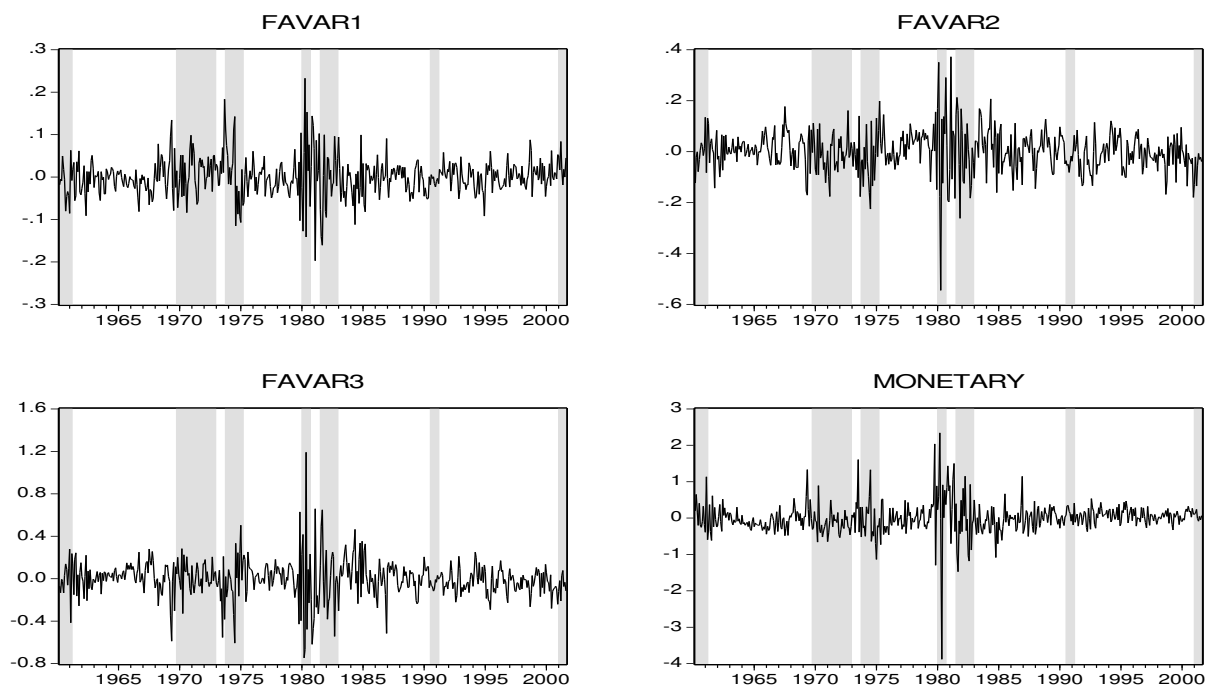
This figure plots the quarterly time series of smoothed structural shocks implied by three-equation New-Keynesian DSGE of Cho and Moreno(2006) Note: EAS or W1 is the estimated aggregate shocks(AS);EIS or W2 is the estimated IS shocks;EMP or W3 is the estimated monetary policy shocks;M1 stands for their baseline New-Keynesian models;M2 stands for M1 augmented with autocorrelation in structural shocks;M3 stands for M2 augmented with cross-correlation in structural shocks. Shared areas indicate NBER business recessions.

Figure 1.1: Estimated modes of Smoothed Structural Shocks from Cho and Moreno(2006,1980:4-2004:4)



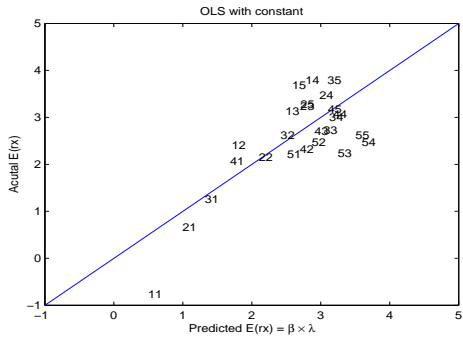
This figure plots the quarterly time series of smoothed structural shocks implied by the extended New-Keynesian DSGE of Graeve(2006) Note: GE_A is the estimated technology shocks;GE_B is the estimated preference shocks;GE_G is the estimated government spending shocks;GE_I is the estimated shocks to investment technology; GE_L is the estimated labor demand shocks;GE_PIE_BAR is the estimated shocks to inflation target set by the Federal reserve(permanent monetary policy shocks);GETA_P is the estimated price mark-up shocks;GETA_R is the estimated temporary monetary policy shocks;GETA_W is the estimated wage mark-up shocks. Shared areas indicate NBER business recessions.

Figure 1.2: Estimated modes of Smoothed Structural Shocks from Graeve(2006)(1980:4-2004:4)

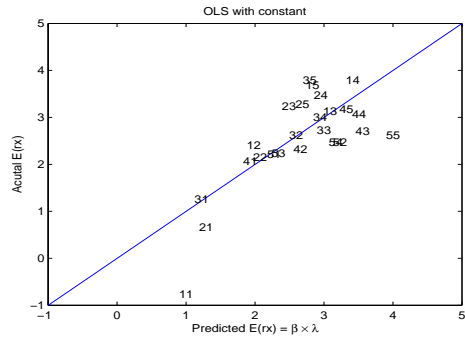


This figure plots the monthly time series of smoothed structural shocks implied by Factor-augmented VAR of Bernanke, Boivin and Elias (2005). Note: FAVAR1 is the estimated shocks from the first latent factors; FAVAR2 is the estimated shocks from the first latent factors; FAVAR3 is the estimated shocks from the first latent factors; MONETARY is the estimated monetary policy shocks. Shaded areas indicate NBER business recessions.

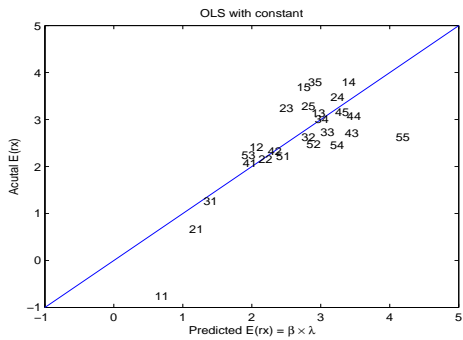
Figure 1.3: Estimated modes of Smoothed Structural Shocks from FAVAR(1960:3-2001:8)



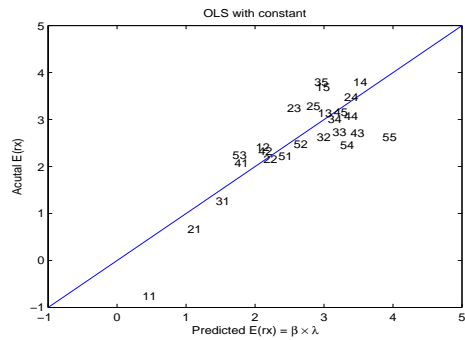
(a) Unconditional CAPM



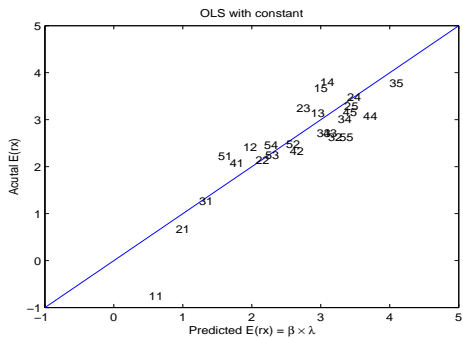
(b) Model1



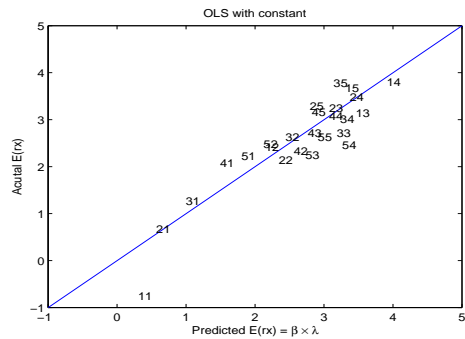
(c) Model2



(d) Model3



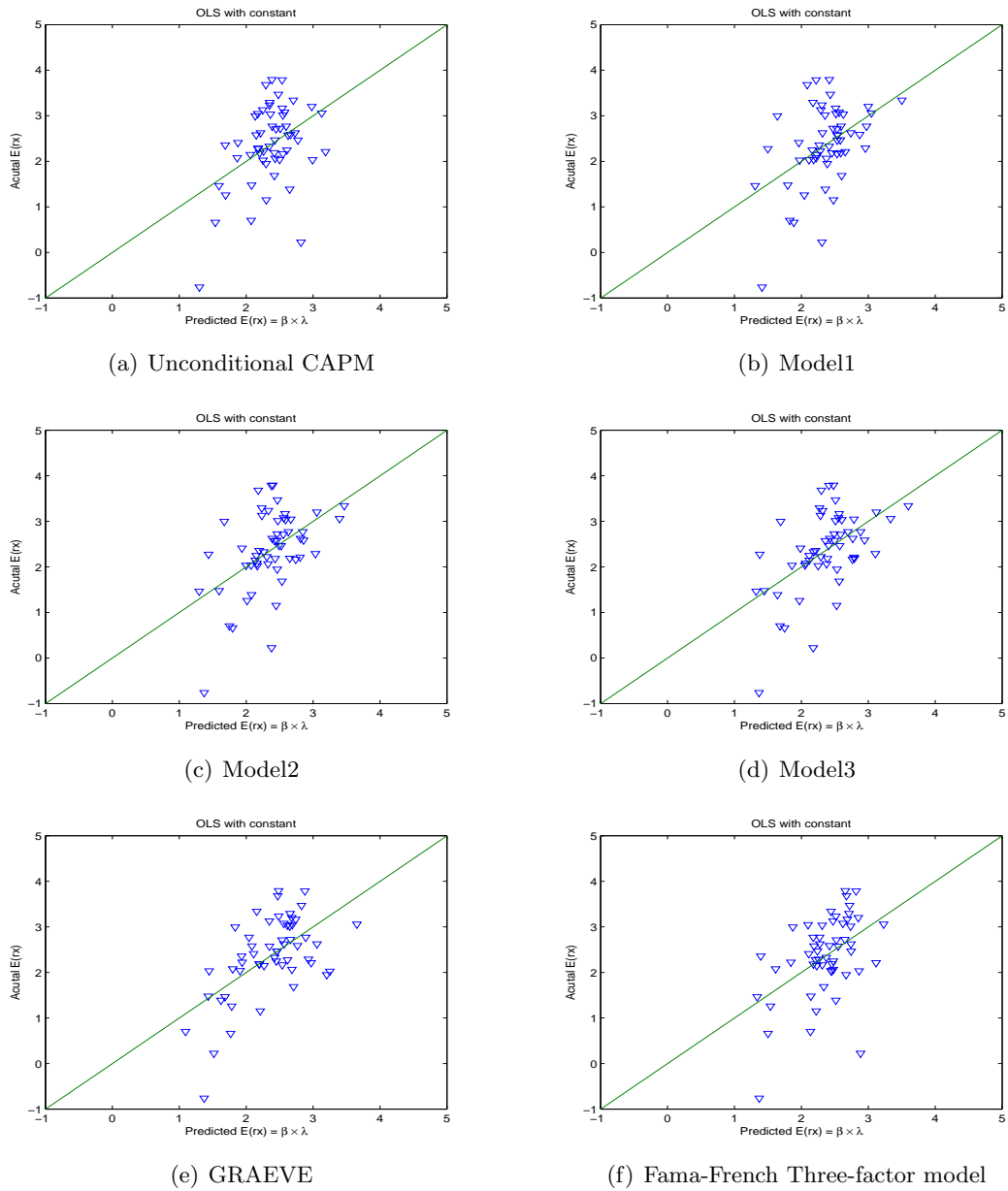
(e) GRAEVE



(f) Fama-French Three-factor model

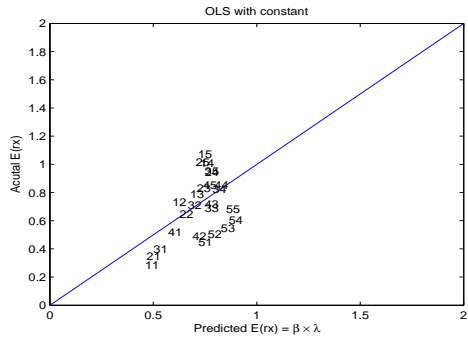
The plot shows realized average returns (in percent) on the vertical axis and fitted expected returns (in percent) on the horizontal axis for 25 size and book-to-market sorted portfolios. The first digit refers to the size quintile (1 being the smallest and 5 the largest), while the second digit refers to the book-to-market quintile (1 being the lowest and 5 the highest). For each portfolio, the realized average return is the time-series average of the portfolio return and the fitted expected return is the fitted value for the expected return from the corresponding model. The straight line is the 45-degree line from the origin. All models are defined in section 1.4.3.

Figure 1.4: Fitted Expected Returns Versus Average Realized Returns for the Fama French 25 portfolios(1980:4-2004:4)

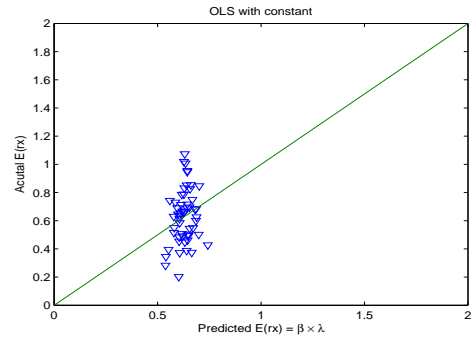


The plot shows realized average returns (in percent) on the vertical axis and fitted expected returns (in percent) on the horizontal axis for 25 size and book-to-market sorted portfolios and 30 industry portfolios. For each portfolio, the realized average return is the time-series average of the portfolio return and the fitted expected return is the fitted value for the expected return from the corresponding model. The straight line is the 45-degree line from the origin. All models are defined in section 1.4.3.

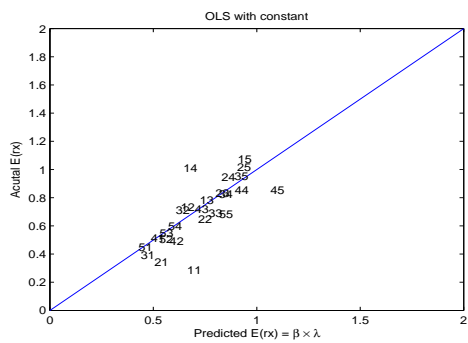
Figure 1.5: Fitted Expected Returns Versus Average Realized Returns for the Fama French 25 portfolios and 30 Industry portfolios(1980:4-2004:4)



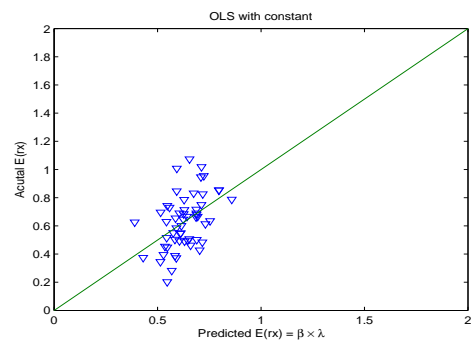
(a) Unconditional CAPM(25)



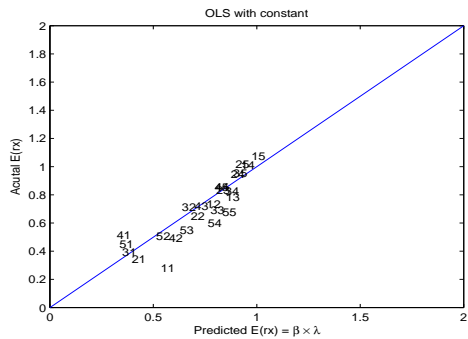
(b) Unconditional CAPM(55)



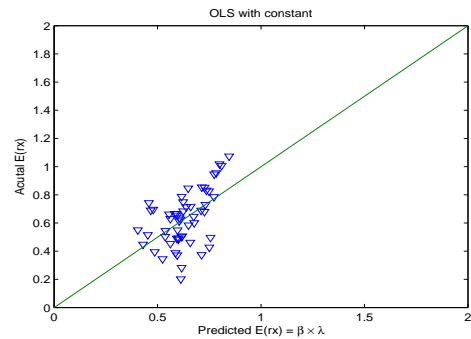
(c) FAVAR(25)



(d) FAVAR(55)



(e) Fama-French Three-factor model(25)



(f) Fama-French Three-factor model(55)

The plot shows realized average returns (in percent) on the vertical axis and fitted expected returns (in percent) on the horizontal axis for 25 size and book-to-market sorted portfolios and 30 industry portfolios. For each portfolio, the realized average return is the time-series average of the portfolio return and the fitted expected return is the fitted value for the expected return from the corresponding model. The straight line is the 45-degree line from the origin. All models are defined in section 1.4.3.

Figure 1.6: Fitted Expected Returns Versus Average Realized Returns for the Fama French 25 portfolios and the 55 portfolios with 30 Industry portfolios(1960:3-2001:8)

Table 1.1: Summary Statistics for Cho and Moreno(2006)

Summary statistics for structural shocks from Cho and Moreno(2006) from 1980:4 to 2004:4. The Auto(1) give the first autocorrelation. Note: EAS or W1 is the estimated aggregate shocks(AS);EIS or W2 is the estimated IS shocks;EMP or W3 is the estimated monetary policy shocks;M1 stands for their baseline New-Keynesian models;M2 stands for M1 augmented with autocorrelation in structural shocks;M3 stands for M2 augmented with cross-correlation in structural shocks.

	M1EAS	M1EIS	M1EMP	M2EAS	M2EIS	M2EMP	M3EAS	M3EIS	M3EMP
Panel A: Correlation Matrix									
M1EAS	1.000								
M1EIS	-0.074	1.000							
M1EMP	-0.271	0.322	1.000						
M2EAS	0.945	-0.108	-0.354	1.000					
M2EIS	-0.113	0.954	0.286	-0.153	1.000				
M2EMP	-0.245	0.267	0.975	-0.316	0.251	1.000			
M3EAS	0.889	-0.030	-0.322	0.963	-0.036	-0.288	1.000		
M3EIS	-0.109	0.840	-0.042	-0.156	0.907	-0.046	-0.072	1.000	
M3EMP	-0.232	0.292	0.970	-0.272	0.283	0.991	-0.232	-0.032	1.000
Panel B:Univariate Summary Statistics									
Mean	-0.045	0.004	-0.122	-0.071	0.001	-0.095	-0.103	0.028	-0.118
Skewness	-0.088	-0.181	-0.805	-0.059	-0.409	-1.360	-0.134	-0.032	-1.516
Std. Dev.	0.507	0.396	0.715	0.609	0.226	0.692	0.681	0.208	0.686
Kurtosis	2.798	4.003	6.498	2.947	4.180	9.617	3.132	3.452	10.276
Auto(1)	-0.438	0.179	0.189	-0.192	-0.095	0.028	-0.097	-0.128	0.034

Table 1.2: Summary Statistics for Graeve(2006)

Summary statistics for structural shocks from Graeve(2006) from 1980:4 to 2004:4. The Auto(1) give the first autocorrelation. Note: GE_A is the estimated technology shocks;GE_B is the estimated preference shocks;GE_G is the estimated government spending shocks;GE_I is the estimated shocks to investment technology; GE_L is the estimated labor demand shocks;GE_PIE_BAR is the estimated shocks to inflation target set by the Federal reserve(permanent monetary policy shocks);GETA_P is the estimated price mark-up shocks;GETA_R is the estimated temporary monetary policy shocks;GETA_W is the estimated wage mark-up shocks.

	GE_A	GE_B	GE_G	GE_I	GE_L	GE_PIE_BAR	GETA_P	GETA_R	GETA_W
Panel A: Correlation Matrix									
GE_A	1.000								
GE_B	-0.019	1.000							
GE_G	0.389	0.000	1.000						
GE_I	-0.209	0.100	-0.055	1.000					
GE_L	0.242	0.006	0.301	0.196	1.000				
GE_PIE_BAR	-0.041	0.644	-0.062	0.082	-0.031	1.000			
GETA_P	-0.067	-0.230	0.148	-0.317	-0.013	0.077	1.000		
GETA_R	0.185	0.409	0.327	0.307	0.658	-0.017	-0.200	1.000	
GETA_W	0.002	-0.109	-0.316	-0.269	0.030	0.063	-0.081	-0.273	1.000
Panel B: Univariate Summary Statistics									
Mean	-0.008	-0.045	-0.022	0.073	-0.529	-0.011	0.041	0.029	-0.038
Std. Dev.	0.405	0.282	0.524	0.522	1.666	0.026	0.152	0.148	0.276
Skewness	-0.033	-1.402	0.168	-0.084	0.025	-1.138	0.364	0.302	0.684
Kurtosis	4.826	8.976	4.653	2.461	4.594	6.768	2.405	4.528	4.444
Auto(1)	-0.127	-0.183	-0.305	-0.020	0.202	0.454	-0.023	0.084	0.018

Table 1.3: Correlation Matrix of Identified Monetary Policy Shocks

This Table reports correlation matrix of identified monetary policy shocks from Cho and Moreno(2006) and Graeve(2006) from 1980:4 to 2004:4. Note: M1EMP is the estimated monetary policy shocks from model 1; M2EMP is the estimated monetary policy shocks from model 2;M3EMP is the estimated monetary policy shocks from model 3;GE_PIE_BAR is the estimated shocks to inflation target set by the Federal reserve(permanent monetary policy shocks);GETA_R is the estimated temporary monetary policy shocks.

	M1EMP	M2EMP	M3EMP	GE_PIE_BAR	GETA_R
M1EMP	1.000				
M2EMP	0.975	1.000			
M3EMP	0.970	0.991	1.000		
GE_PIE_BAR	0.302	0.357	0.376	1.000	
GETA_R	0.770	0.737	0.740	-0.017	1.000

Table 1.4: Cross-Sectional Tests of Asset Pricing Models on Fama French 25 size-B/M portfolios(1980:4-2004:4)

The table presents the estimated results of the second step cross-sectional regression using the excess returns on 25 portfolios sorted by book-to-market and size. The full-sample factor loadings, which are the independent variables in the regressions, are computed in one multiple time-series regression following the suggestions of Lettau and Ludvigson (2001b). The Adjusted R^2 follows the specification of Jagannathan and Wang (1996). The standard errors are obtained from posterior distribution of estimated parameters. The last column reports the root mean squared pricing errors of the model. Note: Rmrf, SMB, and HML are the Fama and French (1993)'s market and size and B/M factors; Model 1, 2 and 3 are defined in section 1.2.3.1, which use aggregated supply shock(AS), IS shock and monetary policy shock from Cho and Moreno (2006); GRAEVE use investment shock as a proxy for the external finance premium, price mark-up shock, permanent monetary policy shock or shock to the inflation target(monetary1) and temporary monetary policy shock(monetary2).

Models	Variables						Statistics		
CAPM	constant	Rmrf				Adj.R2	RMSE		
	0.058	-3.252				0.574	0.0063		
S.E.	0.011	1.405							
Fama-French	constant	Rmrf	SMB	HML			Adj.R2	RMSE	
	0.089	-6.845	-0.029	1.233			0.795	0.0042	
S.E.	0.015	1.745	0.607	0.838					
Model1	constant	AS	IS	Monetary			Adj.R2	RMSE	
	0.035	-0.577	-0.473	-0.382			0.508	0.0065	
S.E.	0.008	0.205	0.157	0.357					
Model2	constant	AS	IS	Monetary			Adj.R2	RMSE	
	0.038	-0.691	-0.231	-0.246			0.579	0.006	
S.E.	0.008	0.255	0.059	0.314					
Model3	constant	AS	IS	Monetary			Adj.R2	RMSE	
	0.038	-0.829	-0.201	-0.266			0.644	0.0055	
S.E.	0.008	0.293	0.053	0.318					
Graeve	constant	Investment	Monetary1	price-markup	Monetary2			Adj.R2	RMSE
	0.042	-0.619	-0.029	-0.094	-0.035			0.721	0.0048
S.E.	0.008	0.205	0.011	0.048	0.051				

Table 1.5: Cross-Sectional Tests of Asset Pricing Models on Fama French 25 size-B/M and 30 industry portfolios(1980:4-2004:4)

The table presents the estimated results of the second step cross-sectional regression using the excess returns on 25 portfolios sorted by book-to-market and size together with 30 industry portfolios. The full-sample factor loadings, which are the independent variables in the regressions, are computed in one multiple time-series regression following the suggestions of Lettau and Ludvigson (2001b). The Adjusted R^2 follows the specification of Jagannathan and Wang (1996). The standard errors are obtained from posterior distribution of estimated parameters. The last column reports the root mean squared pricing errors of the model. Note: Rmrf, SMB, and HML are the Fama and French (1993)'s market and size and B/M factors; Model 1, 2 and 3 are defined in section 1.2.3.1, which use aggregated supply shock(AS), IS shock and monetary policy shock from Cho and Moreno (2006); GRAEVE use investment shock as a proxy for the external finance premium, price mark-up shock, permanent monetary policy shock or shock to the inflation target(monetary1) and temporary monetary policy shock(monetary2).

Models	Variables						Statistics		
CAPM	constant	Rmrf				Adj.R2	RMSE		
	0.038	-1.546				0.167	0.0079		
S.E.	0.009	1.298							
Fama-French	constant	Rmrf	SMB	HML			Adj.R2	RMSE	
	0.037	-1.628	-0.105	1.03			0.184	0.0076	
S.E.	0.009	1.257	0.595	0.808					
Model1	constant	AS	IS	Monetary			Adj.R2	RMSE	
	0.028	-0.119	-0.202	-0.063			0.165	0.0077	
S.E.	0.008	0.109	0.121	0.241					
Model2	constant	AS	IS	Monetary			Adj.R2	RMSE	
	0.029	-0.118	-0.094	-0.139			0.206	0.0075	
S.E.	0.008	0.144	0.063	0.237					
Model3	constant	AS	IS	Monetary			Adj.R2	RMSE	
	0.029	-0.198	-0.069	-0.154			0.253	0.0073	
S.E.	0.008	0.177	0.057	0.237					
Graeve	constant	Investment	Monetary1	price-markup	Monetary2			Adj.R2	RMSE
	0.031	-0.173	-0.014	0.016	-0.003			0.3	0.007
S.E.	0.008	0.182	0.005	0.037	0.042				

Table 1.6: Cross-Sectional Tests of Asset Pricing Models on Fama French 25 size-B/M or 55 portfolios with 30 industry portfolios(1960:3-2001:8)

The table presents the estimated results of the second step cross-sectional regression using the excess returns on 25 portfolios sorted by book-to-market(Panel A) or 55 portfolios with 30 industry portfolios.(Panel B) The full-sample factor loadings, which are the independent variables in the regressions, are computed in one multiple time-series regression following the suggestions of Lettau and Ludvigson (2001b). The Adjusted R^2 follows the specification of Jagannathan and Wang (1996). The standard errors are obtained from posterior distribution of estimated parameters. The last column reports the root mean squared pricing errors of the model. Note: Rmrf, SMB, and HML are the Fama and French (1993)'s market and size and B/M factors; Latent 1,2 and 3 are extracted structural shocks from FAVAR model. Monetary stands for identified monetary shock from FAVAR.

Panel A. 25 portfolios							
CAPM	constant	Rmrf				Adj.R2	RMSE
	0.014	-0.624				0.214	0.0019
S.E.	0.004	0.432					
Fama-French	constant	Rmrf	SMB	HML		Adj.R2	RMSE
	0.013	-0.756	0.129	0.439		0.752	0.001
S.E.	0.003	0.394	0.134	0.146			
FAVAR	constant	latent1	latent2	latent3	Monetary	Adj.R2	RMSE
	-0.001	-0.035	0.057	0.21	-0.539	0.508	0.0014
S.E.	0.003	0.022	0.029	0.093	0.192		
Panel B. 55portfolios							
CAPM	constant	Rmrf				Adj.R2	RMSE
	0.009	-0.228				0.03	0.0018
S.E.	0.003	0.349					
Fama-French	constant	Rmrf	SMB	HML		Adj.R2	RMSE
	0.009	-0.347	0.113	0.249		0.244	0.0016
S.E.	0.003	0.335	0.147	0.137			
FAVAR	constant	latent1	latent2	latent3	Monetary	Adj.R2	RMSE
	0.003	-0.007	0.004	0.051	-0.194	0.146	0.0017
S.E.	0.003	0.01	0.016	0.043	0.099		

Chapter 2

Asset Pricing Models with Stochastic Risk Aversion

2.1 Introduction

Investors demand compensation for holding assets with uncertain payoffs. The degree of risk aversion determines the amount of this compensation or risk premium. Since time-varying risk premia in financial markets are a stylized fact, time-varying risk aversion is equally emphasized in asset pricing literature. Notably, the external habit formation model of Campbell and Cochrane (1999) uses the surplus consumption ratio to proxy for time-varying risk aversion and their model successfully matches the historical equity premium. Furthermore, Wachter (2006) and Verdelhan (2006) extend the Campbell and Cochrane (1999) model to the bond market and the foreign exchange market, respectively, to explain the expectation hypotheses puzzle. The relationship between the stock and the bond markets has also been modeled with a latent time-varying risk aversion process in Bekaert, Engstrom, and Grenadier (2006).

While time-varying risk aversion is proxied by the surplus consumption ratio in con-

sumption asset pricing literature, most of the empirical asset pricing studies in finance use financial market variables as instruments for time-varying risk aversion. Although these studies typically motivate their specifications of time-varying risk aversion using the external habit specification of Campbell and Cochrane (1999), it seems rather arbitrary that they choose any return forecasting variables as proxies in an attempt to improve pricing performances of their models. Moreover, each candidate model employs one or two variables as proxies even without any specification test;¹ the surplus consumption ratio (Duffee (2005)), the consumption wealth ratio (Lettau and Ludvigson (2001b)), the dividend price ratio (Duffee (2005) and (Ferson and Harvey (1999))), the yield spread (Brennan, Wang, and Xia (2004) and (Ferson and Harvey (1999))), the default spread (Jagannathan and Wang (1996) and (Ferson and Harvey (1999))), the inflation rate (Brandt and Wang (2003)), the real GDP growth (Hodrick and Zhang (2001)), the stochastically detrended short term interest rate (Ferson and Harvey (1999))²

The first goal of this paper is to develop direct time series specification tests of time-varying risk aversion under the generalized versions of conditional CAPM framework, which allow several nonlinear features and heteroscedasticity. I examine whether empirically proposed variables in previous studies are indeed significant determinants of time-varying risk aversion and compare them with the theoretically motivated surplus consumption ratio. In order to check the hypothesis, I construct the surplus consumption ratio data following Duffee (2005) and Wachter (2006). Since many empirical asset pricing models motivate time-varying risk aversion with the surplus consumption ratio, it would be illuminating to investigate how previously proposed variables sustain their explanatory powers on risk aversion once I include the surplus consumption ratio along with those variables.

Recently, in a closely related paper to the present study, Guo, Wang, and Yang (2006)

¹Return forecastability is the major concerns to select these proxies in empirical asset pricing studies.

²See the data appendix for the detailed explanation about construction and definition.

use semi-parametric techniques to investigate time-varying risk aversion hypothesis. They find that risk aversion is constant once they include CAY of Lettau and Ludvigson (2001a) as proxy for hedging components in the ICAPM. In fact, the same variables used in the conditional CAPMs with time-varying risk aversion are often selected as proxy variables for hedging components in the ICAPM. In this sense, conditional CAPMs are usually interpreted as alternative models for the ICAPMs. Since Merton (1973)'s theoretical presentation of the ICAPM, it has been recognized that there exist state variables that capture the variation in future investment opportunities, and assets' covariations with such variables should be priced in the cross-section of average returns. Recently Campbell (1996), Brennan, Wang, and Xia (2004), and Petkova (2006) build the models based on Merton (1973), in which only the factors that forecast future investment opportunities or stock returns are admitted. Therefore, the same predictive variables proposed for time-varying risk aversion have been also included as the ICAPM factors. Without appropriate treatment of this ICAPM intuition, i.e., the hedging components, time-varying risk aversion might be a spurious fact indicated as in Guo, Wang, and Yang (2006).³

In this paper, I present a testing ground based on a generalized version of CAPM and ICAPM in order to extract meaningful relative risk aversion. Specifically, I develop econometric models which allow both the positivity of risk aversion and the conditional heteroscedasticity. I also estimate several volatility models and risk aversion specifications to check the robustness of my results. Furthermore, in order to elaborate on the results, I investigate the implications of my models for the cross-section of stock returns. I examine how selected CAPMs and ICAPMs are capable of explaining the cross-section of the Fama-French 25 size and B/M sorted portfolios alone or with 30 industry portfolios.⁴

³"Could it be because of the time-varying relative risk aversion or the changing investor opportunities?". Cochrane (2001) criticizes most of the ICAPM studies for the ad-hoc choice of multiple factors ("fishing license" argument). I would argue that without correct identification of time-varying risk aversion, the ICAPM is still subject to this "fishing license" problem.

⁴Lewellen, Nagel, and Shanken (2006) criticizes most of the cross-sectional asset pricing studies for the

My empirical findings from both time series and cross-sectional investigations unequivocally suggest that time varying relative risk aversion are important for explaining the risk-return relation in the stock market. I find, among other things, that only consumption related variables are significant determinants for relative risk aversion while other return forecasting variables frequently suggested in finance literature lose their statistical significance once I include those variables along with the surplus consumption ratio. Therefore, even though consumption CAPM model might not be useful to understand stock return behavior, I argue that we must include these consumption variables in conditioning information sets. And these results are quite robust across six different time-series model specifications and the two cross-sectional tests.

I summarize the main findings as follows. First, I uncover that the surplus consumption ratio has the most explanatory power, along with correct negative sign, for time varying relative risk aversion. Typically low surplus consumption ratio is interpreted as the indicator of recession. Negative estimates imply that during the bad times, investors' sensitivity to risk increases since those are the time when investors' marginal utility is the highest and they are eager to increase their consumption and avoid the risky investment.

Secondly, I construct the consumption wealth ratio without a look-ahead bias (CAYA) to re-evaluate its significance on time-varying relative risk aversion and hedging components. While Lettau and Ludvigson (2001b) suggest that consumption wealth ratio (CAY) is a crucial variable for time-varying risk aversion, several papers have questioned the usefulness of CAY because it uses the whole sample data to construct the data and that information is not available when investors try to use it. In spite of the criticisms on CAY, I find that the consumption wealth ratio without a look-ahead bias still captures part of time-varying

choice of the Fama-French 25 portfolios. (Possible Data Snooping problem) Especially, they show that many empirical asset pricing models could price only the Fama-French 25 portfolios (R^2 is above 75 %) but not the 55 portfolios including 30 industry portfolios. (R^2 is typically below 10%) Therefore I check the robustness of the proposed CAPMs and ICAPMs for the value premium and for the capability to explain the industry portfolios.

relative risk aversion. My results support empirical specifications of Lettau and Ludvigson (2001b) while they do not support the interpretation of Guo and Whitelaw (2006) that CAY mostly explains the hedging component.

Thirdly, only the stochastically detrended short term interest rate(RREL) has some explanatory power on hedging components. This result confirms the suggestion of Merton (1973) that the interest rate should be the main determinant of hedging components. However, other possible candidates such as the dividend price ratio, the default spread, the inflation and the real GDP growth do not have any incremental impact on either components while some of the variables are critical to explaining the volatility of stock returns. Even though some of the variables are capable of explaining one of the components alone, they lose statistical significance in the presence of the surplus consumption ratio, CAYA, or RREL.

Finally, I compare the proposed CAPMs and ICAPMs from the time series tests with Fama-French three factor model on the ability to explain the cross-section of the average returns. I find that the selected conditional ICAPM with both time-varying relative risk aversion and hedging components is not only comparable to Fama-French three factor model in explaining the value premium but also satisfy the robustness criteria of Lewellen, Nagel, and Shanken (2006) since they have a higher explanatory power for the 55 portfolios than all the other models do.

The rest of the paper is organized as follows. Section 2 presents the ICAPM framework of this study and outlines the empirical methods used to identify time-varying risk aversion and hedging components. Section 3 presents the data and examines the time series specification test results of the conditional CAPM and ICAPM. Section 4 discusses the cross-sectional implications of my empirical models for 25 size and B/M portfolios alone or with 30 industry portfolios. Section 5 summarizes the main findings and concludes.

2.2 Models

This section discusses how I develop empirical models; the first subsection discusses discrete time ICAPM and CAPM frameworks and the second subsection presents the implied time series econometric models.

2.2.1 The general ICAPM framework

The analysis in this paper assumes that asset returns are governed by a variant of pricing kernels motivated by Merton (1973) to deduce the following empirical asset pricing implications.

$$r_{t+1} = \alpha_0 + \gamma_{t+1}\sigma_{m,t+1} + \alpha_1'z_t + \varepsilon_{t+1}$$

$$\gamma_{t+1} = \phi_0 + \phi_1\gamma_t + \phi_2'x_t + \nu_{t+1}$$

where $\varepsilon_{t+1}|\psi_t \sim N(0, h_{t+1})$, $h_{t+1} = \omega + \beta_1 h_t + \beta_2 \varepsilon_t^2$, $\nu_{t+1} \sim N(0, \sigma_v^2)$, r_{t+1} is the market excess return ($R_{m,t+1} - R_{f,t+1}$) and $\sigma_{m,t+1}$ is the conditional variance of the market excess return given information up to time t. ($VAR_t[R_{m,t+1}]$). γ_{t+1} stands for relative risk aversion and z_t and x_t are state variables for hedging components and risk aversion respectively.

In the next section, I briefly explain how I get my empirical specifications by extending the ICAPM proposed by Guo and Whitelaw (2006). Their ICAPM is convenient since they specify hedging components as linear function of exogenous variables using Campbell (1996)'s ICAPM. This simplification can avoid the complex joint estimation of multivariate GARCH models and nonlinear state space models.⁵

⁵I am currently working on this specification for my next paper

2.2.1.1 A simple illustration of discrete time version of Merton ICAPM

Most ICAPM papers specify their versions of discrete ICAPM without much explanation. Probably most illustrations given in the following sections would be common sense in finance community. However, I could not find any reference showing them step by step. So, I decide to derive and explain each step from the basic argument.

I show how we can get a discrete version of Merton ICAPM from his original continuous model as a basic framework to deduce my empirical models.

First, I assume that the value function associated with the investor's problem in a continuous-time setting is a function of wealth W_t and the state variables z_t that forecast future expected market returns or changes in the investment opportunity set. State variables in the ICAPM arise because investors desire to hedge against adverse changes in available investment opportunities.

The stochastic discount factor of ICAPM is given by:⁶

$$\Lambda_t = e^{-\rho t} V_W(W_t, z_t) \quad (2.2.1)$$

where ρ is the subjective discount rate and V_W is the value function with subscripts denoting partial derivatives.

By applying Ito's Lemma,

$$\frac{d\Lambda_t}{\Lambda_t} = -\rho dt + \frac{W_t V_{WW}(W_t, z_t)}{V_W(W_t, z_t)} \frac{dW_t}{W_t} + \frac{W_t V_{Wz}(W_t, z_t)}{V_W(W_t, z_t)} dz_t \quad (2.2.2)$$

where the second derivative terms have been dropped since they will not affect the asset pricing equation.

The elasticity of marginal utility with respect to wealth is the coefficient of relative risk

⁶I closely follow the terminology and the derivation of continuous time asset pricing formula given in chapter 9 of Cochrane (2001). And I present a simple derivation of multi-period pricing kernel in order to help the intuition behind it in Appendix B.

aversion from the first order condition or the envelop theorem ($U_C = V_W$). Because utility function of risk averse investor implies $V_W > 0$ and $V_{WW} < 0$, the model suggests a positive relative risk aversion.

$$\gamma_t \equiv -\frac{W_t V_{WW}(W_t, z_t)}{V_W(W_t, z_t)}$$

The following continuous time asset pricing model formula is given in Cochrane (2001).

$$E_t \left[\frac{dp_{i,t}}{p_{i,t}} \right] + \frac{D_{i,t}}{p_{i,t}} - R_t^f dt = -E_t \left[\frac{d\Lambda_t}{\Lambda_t} \frac{dp_{i,t}}{p_{i,t}} \right], \quad (2.2.3)$$

where $\frac{dp_{i,t}}{p_{i,t}}$ denotes the return of asset i and $\frac{D_{i,t}}{p_{i,t}}$ is the dividend yield of asset i . R_t^f stands for risk free rate. Finally, E_t indicate the rational expectation conditional on the information up to time t .

By substituting (2.2.2) into (2.2.3), the price of asset i follows in continuous time,

$$E_t \left[\frac{dp_{i,t}}{p_{i,t}} \right] + \frac{D_{i,t}}{p_{i,t}} - R_t^f dt = \gamma_t E_t \left[\frac{dW_t}{W_t} \frac{dp_{i,t}}{p_{i,t}} \right] - \frac{V_{Wz,t}}{V_{W,t}} E_t \left[dz_t \frac{dp_{i,t}}{p_{i,t}} \right], \quad (2.2.4)$$

Since there is no difference between second moments and covariances in continuous time ($E_t = Cov_t$), we can use the following form as an approximation of Merton's ICAPM in discrete time.

$$E_t [R_{i,t+1}] - R_{t+1}^f = \gamma_t cov_t (R_{i,t+1}, \Delta W_{t+1}/W_t) + \lambda_{z,t} cov_t (R_{i,t+1}, \Delta z_{t+1}) \quad (2.2.5)$$

Where $\lambda_{z,t} \equiv -\frac{V_{Wz,t}}{V_{W,t}}$ being the risk prices for the innovations in the state variables z_t .

Finally, the growth in wealth ($\Delta W_{t+1}/W_t$) is approximated by the stock market portfolio return ($R_{m,t}$) as usual, and by the same reasoning, the changes in the hedging factors can be approximated by the returns on the corresponding factor-mimicking portfolios.

Since equation (2.2.5) must hold for any asset, the conditional excess market return ($E_t [R_{m,t+1}] - R_{t+1}^f$) can be written as a function of its conditional variance ($\text{var}_t (R_{m,t+1})$) and its covariance with changes in state variables.

$$E_t [R_{m,t+1}] - R_{t+1}^f = \gamma_t \text{var}_t (R_{m,t+1}) + \lambda_{z,t} \text{cov}_t (R_{m,t+1}, \Delta z_{t+1}) \quad (2.2.6)$$

Under certain conditions, Merton (1980) argues that hedging component is negligible and the conditional excess market return is proportional to its conditional variance. This form is interpreted as the conditional CAPM because every result of the CAPM is preserved.

$$E_t [R_{m,t+1}] - R_{t+1}^f = \gamma_t \text{var}_t (R_{m,t+1}) \quad (2.2.7)$$

In its most general form, all of the terms in (2.2.6) of the ICAPM could be time-varying. In this paper, however, I only assume that the coefficient of relative risk aversion is time-varying but hedging coefficients ($\lambda_{z,t}$) are constant since the present study mainly focuses on the source of time varying relative risk aversion and the current literature mainly use this fact to explain various empirical puzzles in finance.

To my knowledge, none of the papers have estimated the full version of the conditional ICAPM and CAPM with time-varying risk aversion except for Guo, Wang, and Yang (2006). Recent empirical asset pricing studies such as Scruggs (1998) and Scruggs and Glabadanidis (2003) use a version of (2.2.7) and elaborate only hedging components with multivariate GARCH-M model with constant risk aversion. However, they find that extension to that direction could be problematic for explaining the risk-return trade off in asset markets.

In order to model hedging components appropriately, I follow the approach taken by Guo and Whitelaw (2006). They propose to model hedging component with alternative linear specifications motivated by Campbell's ICAPM instead of multivariate GARCH model.

In the next section, I briefly summarize how Campbell's ICAPM motivate this alterna-

tive specification with hedging components as a linear function of a vector of state variables z_t .

2.2.1.2 Specification of hedging components implied by Campbell ICAPM

Campbell (1996) proposes a discrete time multi-period asset pricing model in the spirit of Merton (1973). This model uses a loglinear approximation to the budget constraint to substitute out consumption from a standard intertemporal asset pricing model. Risk premia are related to the covariances of asset returns with the market return and with news about the discounted value of all future market returns. This is a discrete-time analogue of Merton (1973) continuous-time model in which assets are priced using their covariances with certain "hedge portfolios" that index changes in the investment opportunity set.

Campbell uses the following recursive utility function.

$$U_t = ((1 - \beta)C_t^{1-\rho} + \beta[E_t(U_{t+1}^{1-\gamma})]^{\frac{1-\rho}{1-\gamma}})^{\frac{1}{1-\rho}} \quad (2.2.8)$$

where β is the subjective discount rate and γ is the coefficient of relative risk aversion and finally ρ is the elasticity of intertemporal substitution.

The stochastic discount factor is then defined from (2.2.8) as

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{U_{t+1}}{[E_t(U_{t+1}^{1-\gamma})]^{\frac{1}{1-\gamma}}} \right)^{\rho-\gamma}$$

By replacing the certainty equivalent utility with the value of the market portfolio,

$$M_{t+1} = [\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho}]^{\theta} \left(\frac{1}{R_{m,t+1}} \right)^{1-\theta}$$

where $\theta = \frac{1-\gamma}{1-\rho}$

This pricing kernel implies the following Euler equations for the market portfolio ($R_{m,t+1}$) and individual asset i 's return ($R_{i,t+1}$)

$$1 = E_t[\{\beta(\frac{C_{t+1}}{C_t})^{-1/\sigma} R_{m,t+1}\}^\theta] \quad (2.2.9)$$

$$1 = E_t[\{\beta(\frac{C_{t+1}}{C_t})^{-1/\sigma}\}^\theta \{\frac{1}{R_{m,t+1}}\}^{1-\theta} R_{i,t+1}] \quad (2.2.10)$$

Campbell first log-linearizes (2.2.9) and (2.2.10) under the assumption of conditionally lognormal and homoskedastic asset returns, and combines them with log linearized budget constraint. ($W_{t+1} = R_{m,t+1}(W_t - C_t)$) to get an asset pricing formula without any reference to consumption but relating assets' returns to their covariances with the market return and news about future market returns.⁷

$$E_t r_{i,t+1} - r_{f,t+1} + \frac{V_{ii}}{2} = \gamma V_{im} + (\gamma - 1) V_{ih} \quad (2.2.11)$$

where lower case returns ($r_{i,t+1}$ and $r_{f,t+1}$ mean log returns and $V_{ii} \equiv Cov(r_{i,t+1} - E_t r_{i,t+1}, r_{i,t+1} - E_t r_{i,t+1})$ and $V_{im} \equiv Cov(r_{i,t+1} - E_t r_{i,t+1}, r_{m,t+1} - E_t r_{m,t+1})$ and $V_{ih} \equiv Cov(r_{i,t+1} - E_t r_{i,t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{m,t+1+j})$ and $\rho \equiv 1 - \exp(c - w)$.

In order to obtain innovations of the state variables, he suggests a vector autoregressive factor (VAR)-pricing model. Specifically, he assumes that the return on the market can be written as the first element of a K -element state vector z_{t+1} and the other elements are variables that are known to the market by the end of period $t + 1$ and are relevant for forecasting future returns on the market.

$$z_{t+1} = A z_t + \varepsilon_{t+1}$$

This VAR pricing model for stock market returns can be used as a motivation for

⁷His model can be generalized to the case in which asset returns are conditionally heteroscedastic under certain conditions. (Campbell (1996) for the details)

⁸In Campbell's ICAPM, relative risk aversion is constant.

alternative specification of hedging components since it implies hedging components might be a linear function of return forecasting variables(z_t). Following Guo and Whitelaw (2006), I model hedge components as a linear function of a vector of state variables motivated by Campbell's ICAPM.

$$\text{cov}_t(R_{m,t+1}, \Delta z_{t+1}) = \alpha_0 + \alpha_1' z_t \quad (2.2.12)$$

This model has several differences from discrete time Merton's ICAPM. First, in implementation of Campbell model, researchers typically omit volatility terms in VAR specification. However, we can include a measure of volatility directly in (2.2.12) and interpret this model as GARCH-in-mean combined with VAR. In addition to that, time-varying relative risk aversion or hedging coefficients are not possible in Campbell's framework.

Finally, Campbell derives the multi-factor asset pricing model using information from VAR.

$$E_t r_{i,t+1} - r_{f,t+1} + \frac{V_{ii}}{2} = \gamma V_{i1} + (\gamma - 1) \sum_{k=1}^K \lambda_k V_{ik} \quad (2.2.13)$$

where λ measure the importance of each state variable in forecasting future returns on the market and $V_{ik} \equiv \text{Cov}(r_{i,t+1}, \varepsilon_{k,t+1})$ and $\varepsilon_{k,t+1}$ are k th residual from VAR estimation.

In this model, if some variables do not forecast returns, they will have zero risk prices and can be omitted from the model. Therefore, Campbell ICAPM shares the intuition of Merton ICAPM and suggests that variables correlated with changes in future investment opportunities should be risk factors that explain the cross-section of portfolio returns.

2.2.1.3 The source of Time-Varying Relative Risk Aversion(RRA)

Merton (1973) model allows time-varying relative risk aversion without economic justifica-

tion.⁹ To be worse, Campbell (1996) just assumes constant relative risk aversion.¹⁰

Many empirical asset pricing models use an external habit specification in utility function to motivate time-varying risk aversion.¹¹ As a reference, I summarize briefly the asset pricing model proposed by Campbell and Cochrane (1999). They develop a version of consumption asset pricing model in which investors' preferences exhibit an external habit formation. The habit feature of this model generates the time-varying RRA.

They use the following utility function.

$$U(C_t, H_t) = \frac{1}{1-\gamma} (C_t - H_t)^{1-\gamma} \quad (2.2.14)$$

where H_t represents the habit level of the representative investor.

And the implied pricing kernel from (2.2.14) is denoted as

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \frac{S_{t+1}}{S_t} \right)^{-\gamma}$$

where β is the subjective discount factor and surplus consumption ratio (S_t) is defined as $S_t = \frac{C_t - H_t}{C_t}$.

In this model, the relative risk aversion can be expressed as the following.

$$\gamma_t \equiv -\frac{W_t V_{WW}(W_t, z_t)}{V_W(W_t, z_t)} = -\frac{\partial \ln(V_W(W_t, z_t))}{\partial \ln(W_t)} = -\frac{\partial \ln(U_C(C_t, H_t))}{\partial \ln(C_t)} \frac{\partial \ln(C_t)}{\partial \ln(W_t)} = -\frac{C_t U_{CC}(C_t, H_t)}{U_C(C_t, H_t)} \frac{\partial \ln(C_t)}{\partial \ln(W_t)}$$

With given utility function,

⁹Return forecasting variables are main candidates to determine time-varying RRA in the ICAPM

¹⁰In actual implementation of this model, implied conditional version is freely assumed and estimated but in that case, simple VAR model can not be used to get state variables which are developed under constant risk aversion.

¹¹Since I use three different pricing kernels to motivate my empirical models, model inconsistency might exist. However, this type of empirical models are common in the empirical asset pricing literature. It seems that there is a tension between internally consistent but empirically problematic models and seemingly reasonable and practical pricing models.

$$\gamma_t = \frac{\gamma}{S_t} \frac{\partial \ln(C_t)}{\partial \ln(W_t)}$$

In various asset pricing papers, this external habit model is often credited to the rational interpretation of the source of time varying relative risk aversion. For example, Lettau and Ludvigson (2001b) motivate their conditional CAPM with this specification and suggest to use the consumption wealth ratio as a proxy to capture the changing information set.

In this paper, I follow their idea and presume that main source of time-varying RRA is motivated from the Campbell and Cochrane (1999) model. Rather than using CAY, however, I construct the direct measure of the surplus consumption ratio following Duffee (2005) and Wachter (2006).

Recently, several empirical studies(for example, Ferson and Harvey (1999)) just incorporate return forecasting variables for modeling time-varying risk aversion as the following.

$$\gamma_t = \phi_0 + \phi_1' x_t$$

where x_t are return forecasting variables

Since this specification is purely empirical, it would be interesting to check which variables would be the primary source of time-varying RRA under one common empirical framework. In the following section, I develop an econometric tool to directly compare different empirical models.

2.2.2 Empirical Specification

From the theoretical considerations given in the previous sections, I propose to use a specific version of empirical ICAPMs.

2.2.2.1 Baseline Empirical Models

$$r_{t+1} = \alpha_0 + \gamma_{t+1}\sigma_{m,t+1} + \alpha_1'Z_t + \varepsilon_{t+1} \quad (2.2.15)$$

$$\gamma_{t+1} = \phi_0 + \phi_1\gamma_t + \phi_2'X_t + \nu_{t+1} \quad (2.2.16)$$

where $\varepsilon_{t+1}|\psi_t \sim N(0, h_{t+1})$, $h_{t+1} = \omega + \beta_1 h_t + \beta_2 \varepsilon_t^2$, $\nu_{t+1} \sim N(0, \sigma_v^2)$, r_{t+1} is the market excess return ($R_{m,t+1} - R_{f,t+1}$) and $\sigma_{m,t+1}$ is the conditional variance of the market excess return given information up to time t . ($VAR_t[R_{m,t+1}]$). γ_{t+1} stands for relative risk aversion and z_t and x_t are state variables for hedging components and risk aversion respectively.

I suggest six different models to check the robustness of empirical results since, at quarterly frequency, it is not clear which model will capture heteroscedasticity or time-varying RRA accurately.

Usually, simple versions of the models in (2.2.15) are estimated by univariate or multivariate GARCH-in-mean models at weekly or monthly horizon. (see Scruggs and Glabadanidis (2003) and references therein) GARCH models at quarterly horizon are not rare either. (Duffee (2005)) However, at quarterly horizon, it is known that GARCH models are not so successful to characterize conditional heteroscedasticity since GARCH effect typically seems to vanish at that horizon.¹² This phenomenon could be problematic to get correct estimates of the relative risk aversion and hedging components. In fact, several GARCH-in-mean studies using monthly return series even find negative relative risk aversion. Harvey (2001) concludes that incorrect specification of volatility could be one of the main causes behind this phenomenon.

Recent progress in time series literature introduces new models of stock return volatility.

¹²You can check it in several tables presented in empirical results section. In most of the case, persistence parameter is below 0.6 and estimated parameters are marginally significant.

Andersen and Bollerslev (1998) shows that the use of high frequency data should give us a better and less noisy measure of volatility. This "realized variance" approach has since become very popular for modeling volatility. However, it is not clear how microstructure noises such as jump characteristics of high frequency asset market data affect the estimates of realized variance. Probably, quarterly frequency provides a reasonable balance between efficiency and robustness of constructed realized variance to microstructure noise.¹³

Given the insights from the aforementioned studies, I propose to estimate a general version of the conditional ICAPM by utilizing both GARCH and "realized variance" approach.

Model 1 uses only realized variance and I call this "realized variance-in-mean with time-varying relative risk aversion" model. When I estimate the realized variance, I follow the approach taken by Guo and Whitelaw (2006) and include all return-forecasting variables in realized variance equation as (u_t) . Specifically, log volatility model is specified since I find that the logarithms of realized variance series confirms to normality assumption better than the level of realized variance does.

$$\sigma_{m,t+1} = h_{t+1} = \exp(E_t(\ln \hat{\sigma}_{m,t+1}) + \frac{1}{2}\sigma_\eta^2), \ln \hat{\sigma}_{m,t+1} = \delta_0 + \delta_1 \ln \hat{\sigma}_{m,t} + \delta_2' u_t + \eta_{t+1} \quad (2.2.17)$$

where $\eta_{t+1} \sim N(0, \sigma_\eta^2)$ and $\hat{\sigma}_{m,t}$ is the measure of realized variance defined in data appendix.

Guo and Whitelaw (2006) argue that, from the stock return predictability literature, certain variables might just explain time-varying heteroscedasticity (realized variance) but not hedging components. By including possible variables for hedging components in realized variance equation, they argue that they can obtain the correct hedging components.¹⁴ Same criticism can be applied to the models of time varying risk aversion. Following their argu-

¹³See Andersen, Bollerslev, Diebold, and Wu (2005) and Guo and Whitelaw (2006) and references therein for other recent applications of quarterly realized variance in the asset pricing literature.

¹⁴In actual implementation, they just include RREL and CAY without any specification test in either realized variance or hedging components.

ments, I project realized variance on these variables and extract all the (linear) information about future volatility (realized variance) contained by them. The residual predictive power that these variables have for expected returns should be due either to risk aversion or to hedge components.

Since GARCH-in-mean models are standard in this literature, I also report results using GARCH-in-mean model (Model2).

$$\sigma_{m,t+1} = h_{t+1}, h_{t+1} = \omega + \beta_1 h_t + \beta_2 \varepsilon_t^2 \quad (2.2.18)$$

Finally model 3 is a hybrid of realized variance-in-mean and GARCH in error terms. Although both terms should be same in theory, this model will clarify in which dimension GARCH fails to explain the data.

$$\sigma_{m,t+1} = \exp(E_t(\ln \hat{\sigma}_{m,t+1}) + \frac{1}{2} \sigma_\eta^2), h_{t+1} = \omega + \beta_1 h_t + \beta_2 \varepsilon_t^2 \quad (2.2.19)$$

In order to guarantee that RRA has the positive sign, I suggest a nonlinear specification for relative risk aversion in each of the models 1,2 and 3. I denote them as models 4,5 and 6 respectively.¹⁵

Usually, an ad-hoc approach is used to identify time varying relative risk aversion by projecting it into various instruments. (see Ferson and Harvey (1999)) But this approach is valid only if the econometrician knows the full set of state variables available to investors. Conditional models are attractive to capture time-varying risk premiums. But, they can be misspecified with the wrong conditioning variables.¹⁶ Therefore, I allow possible misspecification of risk aversion with an autoregressive specification.

¹⁵Model 7 with endogeneity consideration is also derived and presented in appendix C for completeness

¹⁶In a related research, Ghysels (1998) finds that conditional models are fragile and may have bigger pricing errors than unconditional models.

$$r_{t+1} = \alpha_0 + \exp(\gamma_t)\sigma_{m,t+1} + \alpha_1' z_t + \varepsilon_{t+1} \quad (2.2.20)$$

In this paper, I examine whether empirically proposed variables in previous studies are indeed significant determinants of time-varying RRA, in comparison with the theoretically motivated surplus consumption ratio. I use several candidates (z_t) for time-varying risk aversion. Each candidate model employs one or two variables as proxies without any specification test; the surplus consumption ratio (Campbell and Cochrane (1999) and Duffee (2005)), the consumption wealth ratio (Lettau and Ludvigson (2001b)), the dividend price ratio (Duffee (2005) and (Ferson and Harvey (1999))), the yield spread (Brennan, Wang, and Xia (2004) and (Ferson and Harvey (1999))), the default spread (Jagannathan and Wang (1996) and (Ferson and Harvey (1999))), the inflation rate (Brandt and Wang (2003)), the real GDP growth (Hodrick and Zhang (2001)), the stochastically detrended short term interest rate (Ferson and Harvey (1999))¹⁷

These instruments (z_t) for time-varying RRA are also well known to be forecasting variables for stock or bond returns from return predictability literature. Some representative researches are as follows: a measure of the surplus consumption ratio (Li (2001)), the consumption-wealth ratio (Lettau and Ludvigson (2001a)), the dividend-price ratio (Fama and French (1988)), the term spread and the default spread (Fama and French (1989)), the stochastically detrended interest rate (Campbell (1991)), the inflation (Brandt and Wang (2003) and the real GDP growth (Ang, Piazzesi, and Wei (2006))).

Guo and Whitelaw (2006) and Guo, Wang, and Yang (2006) find that the correct specification of hedging component is crucial for estimating relative risk aversion correctly. Especially, Guo, Wang, and Yang (2006) argue that after including CAY into hedging components,¹⁸ they can not reject constant relative risk aversion hypothesis. In order to check

¹⁷See the data appendix for the detailed explanation about construction and definition.

¹⁸They just include CAY but not other variables

this possibility, I include all the instruments for relative risk aversion as candidates of the proxies hedging components.¹⁹

In particular, Merton (1973) suggests that "one should interpret the effects of a changing interest rate.. in the way economists have generally done in the past: namely, as a single variable representation of shifts in the investment opportunity set." Therefore, interest rate variables such as short-term interest rate or yield spread would be natural instruments for hedging components.

2.2.2.2 Nonlinear State-Space Model with GARCH

My empirical specification is characterized as the following state space formulation. For simplicity, I just present an estimation method for the most complex model(model6). Therefore, I put $(\sigma_{t+1}=\hat{\sigma}_{t+1})$ in the mean but $(\sigma_{t+1})=h_{t+1})$ in the variance specification respectively.

Measurement equation:

$$r_{t+1} = \alpha_0 + f(\gamma_{t+1})\hat{\sigma}_{m,t+1} + \alpha_1'z_t + \varepsilon_{t+1}$$

where r_{t+1} is the market excess return($R_{m,t+1} - R_{f,t+1}$) and $\sigma_{m,t+1}$ is expectation of realized variance of the market excess return given information up to time t. ($VAR_t[R_{m,t+1}]$), $\varepsilon_{t+1} \sim N(0, h_{t+1})$, h_{t+1} stands for the GARCH, z_t are lagged exogenous variables. And $f(\gamma_{t+1})$ is γ_{t+1} for models 1,2 and 3 but $exp(\gamma_{t+1})$ for models 4,5 and 6. Finally, γ_{t+1} stands for time-varying RRA.

Transition equation

$$\gamma_{t+1} = \phi_0 + \phi_1\gamma_t + \phi_2'x_t + \nu_{t+1}$$

¹⁹I follow the ICAPM specification of Hodrick and Zhang (2001), Guo and Whitelaw (2006) and Guo, Wang, and Yang (2006) and use contemporaneous values of the predictive variables rather than their innovations. Recently, Petkova (2006) advocates to use innovations.

where x_t are lagged exogenous variables.

The estimation of this model is cumbersome since I have both nonlinear measurement equation and GARCH type heteroscedasticity. Furthermore, I need to address the generated regressors problem once if I jointly maximize this state space model and realized variance equation for $\sigma_{m,t+1}$.

Following Kim and Nelson (2005), I combine the approximation method of Harvey, Ruiz, and Sentana (1992) with extended Kalman filtering technique to develop a filtering method and to construct the likelihood function.²⁰ Finally, I explain the joint maximum likelihood estimation techniques developed to solve generated regressor problem based on the original ideas of Pagan (1984).

For the easy exposition, I first explain the estimation methods in case where generated regressor problem does not exist. Then, in the following step, I describe joint maximum likelihood function based on Pagan (1984) and the chapter five of Kim and Nelson (1999) to account for generated regressor problem.

2.2.3 Extended Kalman Filtering with GARCH disturbances

In this section, I explain how to approximate nonlinear measurement equation with Extended Kalman filtering technique, which is a Taylor expansion of latent variables(γ_{t+1}) around previous state variable estimates($\gamma_{t+1|t}$). And then I simply extend the state-space model with ARCH disturbances proposed by Harvey, Ruiz, and Sentana (1992) to my nonlinear model.

2.2.3.1 Extended Kalman Filtering

First, I use extended Kalman filtering technique to linearize measurement equation. After denoting $\exp(\gamma_{t+1})\hat{\sigma}_{m,t+1}$ as $f(\hat{\sigma}_{m,t+1}; \gamma_{t+1})$, I take a Taylor series expansion of the non-

²⁰Following Kim and Nelson (2005) I also develop the most complex model specification(model 7) with endogeneity issues, which is presented in the appendix C

linear function($f(\hat{\sigma}_{m,t+1} : \gamma_{t+1})$) around $\gamma_{t+1} = \gamma_{t+1|t}$. In this expression, $\gamma_{t+1|t}$ indicates $E[\gamma_{t+1}|\Psi_t]$ where Ψ_t denotes the information set available up to time t .

After linearization, I get the following measurement equation:

$$r_{t+1} = f(\hat{\sigma}_{m,t+1} : \gamma_{t+1}) + \frac{\partial f(\hat{\sigma}_{m,t+1} : \gamma_{t+1})}{\partial \gamma_{t+1}}(\gamma_{t+1} - \gamma_{t+1|t}) + \alpha'_1 z_t + \varepsilon_{t+1} \quad (2.2.21)$$

where $f(\hat{\sigma}_{m,t+1} : \gamma_{t+1}) = \exp(\gamma_{t+1|t})\hat{\sigma}_{m,t+1}$, $\frac{\partial f(\hat{\sigma}_{m,t+1};\gamma_{t+1})}{\partial \gamma_{t+1}} \equiv \exp(\gamma_{t+1|t})\hat{\sigma}_{m,t+1}$

By redefining some of variables, I get the following linearized measurement equation

$$Y_{t+1} = \hat{X}_{t+1}\gamma_{t+1} + \alpha'_1 z_t + \varepsilon_{t+1} \quad (2.2.22)$$

where $Y_{t+1} = r_{t+1} - \exp(\gamma_{t+1|t})\hat{\sigma}_{m,t+1} + \exp(\gamma_{t+1|t})\hat{\sigma}_{m,t+1}\gamma_{t+1|t}$ and $\hat{X}_{t+1} = \exp(\gamma_{t+1|t})\hat{\sigma}_{m,t+1}$

Without heteroscedasticity, I could use the usual Kalman filtering to construct a maximum likelihood estimator. However, I must address how to estimate GARCH specification.

2.2.3.2 GARCH approximation

In order to estimate GARCH in ε_{t+1} , I include ε_{t+1} in transition equation following Harvey, Ruiz, and Sentana (1992).²¹ In matrix forms,

$$Y_{t+1} = \begin{bmatrix} \hat{X}_{t+1}, 1 \end{bmatrix} \begin{bmatrix} \gamma_{t+1} \\ \varepsilon_{t+1} \end{bmatrix} + \alpha'_1 z_t$$

$$\begin{bmatrix} \gamma_{t+1} \\ \varepsilon_{t+1} \end{bmatrix} = \begin{bmatrix} \phi_0 + \phi'_2 x_t \\ 0 \end{bmatrix} + \begin{bmatrix} \phi_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \gamma_t \\ \varepsilon_t \end{bmatrix} + \begin{bmatrix} \nu_{t+1} \\ \varepsilon_{t+1|t} \end{bmatrix},$$

$$\begin{bmatrix} \nu_{t+1} \\ \varepsilon_{t+1} \end{bmatrix} \sim N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\nu^2 & 0 \\ 0 & h_{t+1} \end{pmatrix} \right]$$

²¹See Kim and Nelson (1999), chapter 3 and 6 for the more detail explanation.

I use the following compact forms to explain modified Kalman filtering algorithm.

$$Y_{t+1} = \tilde{X}_{t+1}\tilde{\beta}_{t+1} + \alpha'_1 z_t$$

$$\tilde{\beta}_{t+1} = \mu_{t+1} + F\tilde{\beta}_t + \tilde{v}_{t+1}, \tilde{v}_{t+1|t} \sim (0, \tilde{Q}_{t+1})$$

where $\tilde{X}_{t+1} = [\hat{X}_{t+1}, 1]$, $\tilde{\beta}_{t+1} = \begin{bmatrix} \gamma_{t+1} \\ \varepsilon_{t+1} \end{bmatrix}$, $\mu_{t+1} = \begin{bmatrix} \phi_0 + \phi'_2 x_t \\ 0 \end{bmatrix}$, $F = \begin{bmatrix} \phi_1 & 0 \\ 0 & 0 \end{bmatrix}$, $\tilde{v}_{t+1} = \begin{bmatrix} \nu_{t+1} \\ \varepsilon_{t+1} \end{bmatrix}$ and $\tilde{Q}_{t+1} = \begin{pmatrix} \sigma_\nu^2 & 0 \\ 0 & h_{t+1} \end{pmatrix}$

At each iteration of the Kalman filter, I obtain a linear approximation of the model around $\gamma_{t+1} = \gamma_{t+1|t}$, and calculate Y_{t+1} and \tilde{X}_{t+1} for the following Kalman filter.

$$\tilde{\beta}_{t+1|t} = F\tilde{\beta}_{t|t} + \mu_{t+1}$$

$$p_{t+1|t} = Fp_{t|t}F' + \tilde{Q}_{t+1}$$

$$\eta_{t+1|t} = Y_{t+1} - \tilde{X}'_{t+1}\tilde{\beta}_{t+1|t} - \alpha'_1 z_t$$

$$H_{t+1|t} = \tilde{X}'_{t+1}p_{t+1|t}\tilde{X}_{t+1}$$

$$\tilde{\beta}_{t+1|t+1} = \tilde{\beta}_{t+1|t} + p_{t+1|t}\tilde{X}_{t+1}H_{t+1|t}^{-1}\eta_{t+1|t}$$

$$p_{t+1|t+1} = p_{t+1|t} - p_{t+1|t}\tilde{X}_{t+1}H_{t+1|t}^{-1}\tilde{X}'_{t+1}p_{t+1|t}$$

where Ψ_t is the information set up to time t, $\tilde{\beta}_{t+1|t}$ is conditional estimate of $\tilde{\beta}_{t+1}$ on information up to t ($E[\tilde{\beta}_{t+1}|\Psi_t]$), $\tilde{\beta}_{t+1|t+1}$ is conditional estimate of $\tilde{\beta}_{t+1}$ on information up to t+1 ($E[\tilde{\beta}_{t+1}|\Psi_{t+1}]$), $p_{t+1|t}$ is covariance matrix of $\tilde{\beta}_{t+1}$ conditional on information up to t ($E[(\tilde{\beta}_{t+1} - \tilde{\beta}_{t+1|t})(\tilde{\beta}_{t+1} - \tilde{\beta}_{t+1|t})']$), $p_{t+1|t+1}$ is covariance matrix of $\tilde{\beta}_{t+1}$ conditional on information up to t+1 ($E[(\tilde{\beta}_{t+1} - \tilde{\beta}_{t+1|t+1})(\tilde{\beta}_{t+1} - \tilde{\beta}_{t+1|t+1})']$) and $H_{t+1|t}$ is conditional variance of prediction error ($E[\eta_{t+1|t}^2]$)

In order to process the above Kalman filter, I need ε_t^2 term in order to calculate GARCH(h_{t+1}) in \tilde{Q}_{t+1} matrix. As in Harvey, Ruiz, and Sentana (1992), the term ε_t^2 is approximated by $E[\varepsilon_t^2|\Psi_t]$, where Ψ_t is information up to time t.

In order to get the form of $E[\varepsilon_t^2|\Psi_t]$, Harvey, Ruiz, and Sentana (1992) use the following definition.

$$\varepsilon_t = E[\varepsilon_t|\Psi_t] + (\varepsilon_t - E[\varepsilon_t|\Psi_t])$$

After straightforward calculation, it can be shown that:

$$E[\varepsilon_t^2|\Psi_t] = E[\varepsilon_t|\Psi_t]^2 + E\left[(\varepsilon_t - E[\varepsilon_t|\Psi_t])^2\right]$$

where $E[\varepsilon_t|\Psi_t]$ is obtained from the last element of $\tilde{\beta}_{t|t}$ and its mean squared error $E\left[(\varepsilon_t - E[\varepsilon_t|\Psi_t])^2\right]$ is given by the last diagonal element of $p_{t|t}$.

As by-products of the above Kalman filter, I obtain the prediction error $\eta_{t+1|t}$ and its variance $H_{t+1|t}$. Based on this prediction error decomposition, the approximate log likelihood can easily be calculated as

$$\ln L(Y_{t+1}|\hat{\sigma}_{m,t+1}) = -\frac{1}{2} \sum_{t=1}^T \ln((2\pi)|H_{t+1|t}|) - \frac{1}{2} \sum_{t=1}^T \eta'_{t+1|t} H_{t+1|t}^{-1} \eta_{t+1|t} \quad (2.2.23)$$

2.2.3.3 Generated regressor problem

In this subsection, I explain how to estimate $\hat{\sigma}_{m,t+1}$ and log likelihood function given in (2.2.23) jointly.²² While previously I assume $\hat{\sigma}_{m,t+1}$ is given as data, only a proxy for $\hat{\sigma}_{m,t+1}$ is available. In fact, I use constructed conditional expectation of realized variance. This implies that I have classical generated regressor problem and without joint estimation, I would get biased standard errors. Furthermore, all Kalman filter algorithm should be revised in this case since the usual Kalman filter assumes that there is no endogeneity issue.²³ Therefore simple two step estimation with standard error correction is impossible.

²²See Kim and Nelson (1999), chapter 5 for the more details.

²³Refer to Kim (2006) for this issue

Based on Pagan (1984) and Kim and Nelson (1999), I suggest the following joint maximum likelihood estimation.

$$F(Y_{t+1}, \sigma_{m,t+1}) = F(Y_{t+1}|\sigma_{m,t+1})F(\sigma_{m,t+1})$$

where F stands for a density function.

After taking logs, I get

$$\ln F(Y_{t+1}, \sigma_{m,t+1}) = \ln F(Y_{t+1}|\sigma_{m,t+1}) + \ln F(\sigma_{m,t+1})$$

where $\ln F(Y_{t+1}|\hat{\sigma}_{m,t+1})$ is given in (2.2.23) and $\ln F(\sigma_{m,t+1})$ has the following.

$$\ln L = -\frac{1}{2} \sum_{t=2}^T \ln((2\pi)|\sigma_{\eta}^2|) - \frac{1}{2} \sum_{t=2}^T (\eta_t)' \sigma_{\eta}^{-2} (\eta_t), \quad (2.2.24)$$

where $\eta_{t+1} = \ln \hat{\sigma}_{m,t+1} - \delta_0 - \delta_1 \ln \hat{\sigma}_{m,t} - \delta_2' u_t$

Then I replace $\sigma_{m,t+1}$ in (2.2.23) with $Exp(E_t(\ln \hat{\sigma}) + 0.5 * Var_t(\ln \hat{\sigma}))$ to correct for Jensen's inequality terms. Finally, I jointly maximize the sum of log likelihood values of (2.2.23) and (2.2.24) with respect to all parameters.²⁴

2.3 Data and Empirical Results

2.3.1 Data

In this study, I use quarterly data for the period 1957:1 to 2005:4.²⁵ The beginning of the period is set to 1957:1 to get the surplus consumption ratio data(SURP). In addition to

²⁴I experiment with several starting values for each empirical model to ensure global convergence of the parameters. I use the GAUSS 7.0 and Optmum or CML procedure to numerically maximize the joint likelihood function with BFGS as base optimization algorithm and utilize various numerical techniques such as transformation function techniques and penalty algorithms to limit the boundaries of parameters. I also apply parameter rescaling techniques for numerical stability to attain fast convergence.

²⁵See the data appendix for the details on the construction and the definition of the data

the surplus consumption ratio, I consider the following variables; the dividend price ratio on NYSE-AMEX-Nasdaq value-weighted stock return from CRSP(DP); the default premium(DEF), defined as the difference in yields between BAA and AAA corporate bonds; Lettau and Ludvigson (2001a)'s CAY without a look ahead bias(CAYA); the difference between the risk-free rate and its average in the previous 4 quarters(RREL); the term premium, denoted as the yield spread between 10-year Treasury bonds and 3-month Treasury bills(TERM); the inflation, measured by GDP deflator(INFLA); the real GDP growth with seasonal adjustment(REGD).

Figure 2.1 plots these return predictors, with the shaded areas denoting business recessions dated by the National Bureau of Economic Research (NBER). Several facts deserve to be noticed. First, all the variables except for REGD are quite persistent and exhibit strong cyclical patterns. While RREL,REGD, and SURP tend to decrease during business recessions, the other variables move countercyclically. The summary statistics presented in Table 2.1 also confirm these facts. Most of the variables are highly serially correlated, with the autocorrelation coefficients above 85 %. Secondly, Figure 2.1 presents the consumption wealth ratio(CAY) of Lettau and Ludvigson (2001a) along with the similar variable without a look ahead bias(CAYA). Two variables have a similar time series patterns and their correlation is around 76 %. Thirdly, SURP and CAYA have a negative correlation. According to Campbell and Cochrane (1999), logarithm of the surplus consumption ratio(SURP) is defined as the following autoregressive specification.²⁶

$$\text{SURP}_{t+1} = \mu + \phi \text{SURP}_t + \lambda(\text{SURP}_t)\varepsilon_{t+1}$$

where ϕ is the habit persistent parameter, and $\lambda(\text{SURP}_t)$ is the sensitivity function, and ε_{t+1} is the innovation in consumption growth.

Campbell and Cochrane (1999) show that $\lambda(\text{SURP}_t)$ is inversely related to surplus

²⁶Here I follow the summary of Li and Zhong (2006).

consumption ratio and that when surplus consumption ratio falls, the sensitivity function $\lambda(\text{SURP}_t)$ and expected excess returns rise. Lettau and Ludvigson (2001b) suggest that consumption-wealth ratio may be a good proxy of $\lambda(\text{SURP}_t)$. So, SURP and CAYA seem to have a negative correlation as it is confirmed in the data.

In time series analysis, my stock return measure is the standard value-weighted return of NYSE-AMEX-NASDAQ index from CRSP. To compute excess equity returns, I subtract the lagged 3 month continuously compounded T-Bill yield earned over the same period. Consistent with quarterly data, I calculate the realized variance using the daily CRSP value-weighted stock returns and the pseudo daily risk-free rate by assuming that risk-free rate is constant within a quarter. The daily excess market return is the difference between the daily risk-free rate and the daily market return. Realized stock market variance (REVOL) is defined as the variance of daily excess stock market returns in a quarter. And in order to check the robustness of my results, I construct four different measures of realized variance series. For example, I replace the largest value in realized variance series with the second largest value following Guo and Whitelaw (2006) and I use auto-correlation corrected measure of the realized variance following French, Schwert, and Stambaugh (1987).

Table 2.2 summarizes descriptive statistics for this four different versions of realized variance series. All series have similar sample statistics and their correlations are around 85%. Figure 2.2 plots 4 versions of realized variance along with the estimated GARCH(1,1) series²⁷ The shaded areas also denote recessions dated by the National Bureau of Economic Research. The figure shows that volatility moves countercyclically and also tends to increase dramatically during several crises such as the 1962 Cuban missile crisis, the 1987 stock market crash, the 1997 East Asia crisis and the 1998 Russian bond default. For comparison with realized variance, I also estimate simple GARCH(1,1) model using only quarterly data. Even though all four versions have similar sample characteristics, Figure 2.2 suggests that

²⁷Estimated models are given in Table 2.2.

only REVOL2 and REVOL2auto have comparable magnitude with the estimated GARCH series. It seems reasonable since GARCH models estimate current quarterly volatility with the past information set. However, REVOL1 estimates the volatility of the current quarter with daily data in that quarter only which might get too extreme values in crash periods. REVOL2 removes the effects of this outlier.

In cross-sectional analysis, I use ,as test assets, the returns on Fama-French 25 portfolios sorted by size and book-to-market and 30 industry portfolios. Even though the 25 portfolios have become the benchmark in testing competing asset pricing models, Lewellen, Nagel, and Shanken (2006) show that the 55 portfolios are the more appropriate to rigorously compare the models. All the portfolio returns and the Fama-French three-factors -the returns of the market portfolio(Rmrf), HML, and SMB are downloaded from French's website.

2.3.2 Asset pricing models with time-varying RRA

In this section, I report estimation results for six different model specifications. Previous research just assumed one or two variables could serve as the proxy for time-varying RRA without time-series specification tests. However, this practice might be warned against possible data snooping biases. I argue that the empirical results presented here are relevant since I compare different specifications of time-varying RRA under a unified empirical framework.

For models using realized variance, I report the estimation results only with one measure of realized variance(Revoll) since all other measures provide qualitatively similar results. All the exogenous variables are normalized to have mean of zero and standard deviation of one to facilitate the interpretations. Finally, I don't want to entertain a "kitchen sink" regression, including all of the variables in the specification and searching for the correct form. Moreover, I find that the estimation with all the variables has a difficulty to get convergence since many variables capture the similar recession state. Instead, using an

educated guess from theoretical arguments from Campbell and Cochrane (1999), I take "SURP" as the single crucial variable against which other variables are compared since the external habit formation specification addressed with this variable is the main theoretical motivation for conditional CAPMs.

Table 2.3 shows estimation outcomes for the realized-volatility-in model(model1). I restate this version of conditional CAPM here as a reference.

$$r_{M,t+1} = \alpha_0 + \gamma_{t+1} E_t(\ln \hat{\sigma}_{m,t+1} + \frac{1}{2} \sigma_\eta^2) + \varepsilon_{t+1}$$

$$\gamma_{t+1} = \phi_0 + \phi_1 X_{1t} + \phi_2 X_{2t} + v_{t+1}$$

$$\ln \hat{\sigma}_{m,t+1} = \delta_0 + \delta_1 \ln \hat{\sigma}_{m,t} + \delta_2' u_t + \eta_{t+1}$$

where $\varepsilon_{t+1} \sim N(0, E_t(\ln \hat{\sigma}_{m,t+1} + \frac{1}{2} \sigma_\eta^2))$, $v_{t+1} \sim N(0, \sigma_v^2)$, $\eta_{t+1} \sim N(0, \sigma_\eta^2)$ and u_t are entered in the following order(CAYA_t, DP_t, TERM_t, DEF_t, INFL_t)

Since SURP, REGD and RREL in volatility equation($\ln \hat{\sigma}_{m,t+1}$) and α_0 and ϕ_0 are not statistically significant in the preliminary estimation, I re-estimate the model and report results without these variables.²⁸First, in volatility equation, CAYA(δ_{21}), DEF(δ_{24}) and INFL(δ_{25}) have statistically significant negative signs while past market volatility(δ_1), DP(δ_{22}) and TERM(δ_{23}) have statistically significant positive signs. These results are quite robust across the models that use realized volatility and also consistent with the results of previous research. Lettau and Ludvigson (2002) show that past volatility is positively related to future volatility and CAY is negatively related to it. And even though it is not directly related to the stock return case, Bansal, Khatchatrian, and Yaron (2005), in a related research, report that dividend price ratio positively affect consumption growth volatility.

²⁸I re-estimate and report results in this way for all models presented in this section. Furthermore, the value of σ_v is too small and causes convergence problem in the models. So except for model1, I fix σ_v as 0.01 for identification.

Second, after the estimating univariate specification for relative risk aversion, I re-estimate the same model with SURP and one of other possible candidates given from univariate specification tests in order to evaluate the importance of each conditioning variables. While SURP, CAYA, TERM and RREL are statistically significant in univariate specification tests, SURP drives out TERM and RREL in bivariate tests. Only CAYA retains statistical significance with positive sign. The implied risk aversion is countercyclical with positive coefficient of SURP and negative coefficient of CAYA since low surplus consumption ratio and high CAYA is typically interpreted as an indicator of recession or other bad states.

Table 2.4 reports estimation results for the hybrid model(model2) with realized-volatility-in-mean and GARCH in error terms. In this model, I modify model1 by assuming the conditional variance of ε_{t+1} as h_{t+1} .

For realized volatility parameters, the results of Table 2.3 are preserved such that the signs and magnitudes of coefficients are quite similar. For GARCH parameters, I summarize the results as follows. First, the estimated parameters from GARCH model are typically less persistent than we usually obtain from estimations of monthly or weekly data. The persistence parameter(β_1) is just around 0.5. Furthermore, β_2 is marginally significant with typical t-statistics around 1.8. and these results hold in other model specifications using GARCH. These results confirm prior assertion that GARCH-in-mean models are probably not a good description of stock returns at quarterly horizon. For relative risk aversion specification, I find that consistent with Table 2.3, SURP, CAYA, TERM and RREL are statistically significant and have the same signs as before. However, with bivariate analysis, both CAYA and TERM survive the competition with SURP for relative risk aversion.

Finally, Table 2.5 shows the estimation results for model 3. In this model, I replace $E_t(\ln \hat{\sigma}_{m,t+1} + \frac{1}{2}\sigma_\eta^2)$ with $h_{m,t+1}$. GARCH parameters are similar to those in Table 2.4 and in relative risk specification, SURP and RREL are the two significant variables in

this GARCH-in-mean specification. However, these results seem unreliable since one of the GARCH parameter(β_2) is not statistically significant across all different models. Even though the same insignificant GARCH phenomenon exists in model2, the model 3 seems to have more severe problems. In model 2, we can think that only heteroscedasticity is badly modeled while in model3, both the mean and the volatility terms are problematic with this inaccurate GARCH specification.

I summarize my findings as follows. First, realized volatility specification seems better than GARCH specification because in most estimations, all the estimated parameters in realized volatility equations are quite significant and robust but GARCH parameters are either marginally significant or insignificant at all. Second, the surplus consumption ratio is always statistically significant with correct negative sign no matter which models I use. Third, CAYA seems to capture additional explanatory power on relative risk aversion except for unreliable GARCH-in-mean with time-varying relative risk aversion case(model3). Several financial variables seem to forecast just volatility terms even though there is some evidence for bond market variables such as TERM and RREL as proxies for relative risk aversion. TERM and RREL can also be interpreted as recession state variables. In recessions, premia on long-term bonds tend to be high and yields on short bonds tend to be low. Therefore, increase in the term spread and decrease in the short term interest rate indicates more serious recessions, implying higher sensitivity to their risk.

For models 1,2 and 3, I do not impose the positivity of relative risk aversion even though theory clearly requires it. In order to check the robustness of the results, I augment each version of the model with positivity restriction on relative risk aversion. I restate this version of conditional CAPM here as a reference.

$$r_{M,t+1} = \alpha_0 + \exp(\gamma_{t+1})E_t(\ln \hat{\sigma}_{m,t+1} + \frac{1}{2}\sigma_\eta^2) + \varepsilon_{t+1}$$

$$\gamma_{t+1} = \phi_0 + \phi_1 X_{1t} + \phi_2 X_{2t} + v_{t+1}$$

$$\ln \hat{\sigma}_{m,t+1} = \delta_0 + \delta_1 \ln \hat{\sigma}_{m,t} + \delta_2' u_t + \eta_{t+1}$$

where $\varepsilon_{t+1} \sim N(0, E_t(\hat{\sigma}_{m,t+1}))$, $v_{t+1} \sim N(0, \sigma_v^2)$, $\eta_{t+1} \sim N(0, \sigma_\eta^2)$ and u_t are entered in the following order(CAYA_t,DP_t,TERM_t,DEF_t,INFL_t)

Table 2.6 and 2.7 show the estimation outcomes for the realized-volatility-in model with positivity restriction(model4). First, in volatility equation, consistent with model 1 and model 2, CAYA(δ_{21}),DEF(δ_{24}) and INFL(δ_{25}) have statistically significant negative signs while past market volatility(δ_1), DP(δ_{22}) and TERM(δ_{23}) have statistically significant positive signs. These results are robust across the models even with positivity restriction. Second, for relative risk aversion, Table 2.6 indicates all the variables are significant except for REGD. However, once these variables are included in risk aversion equation along with SURP, only CAYA and TERM are statistically significant in Table 2.7. All three variables have signs consistent with the results without positivity restriction such that the implied risk aversion is countercyclical with positivity restriction.

Table 2.8 and 2.9 report the estimation results for the hybrid model(model5) with positivity restriction. In other words, I modify model4 by specifying the conditional variance of ε_{t+1} as h_{t+1} . The estimation outputs for volatility equations precisely resemble those of model2. The signs and magnitudes are quite similar with and without positivity restriction. However, for relative risk aversion specification, almost all variables show statistical significance except for REGD. Table 2.9 indicates that CAYA and INFL are significant against SURP while all the other variables lose statistical significance. In this estimation, inflation has a statistically significant negative coefficient. When aggregate risk aversion is high, consumers demand greater real returns on holding risky securities. Therefore, when real returns are unexpectedly high with unexpectedly low inflation, aggregate risk aversion should fall due to the implied good news about inflation. While the negative coefficient seems to betray this basic understanding, I don't investigate this issue further since coefficient become insignificant once I put INFL with SURP and CAYA.

Finally, Table 2.10 shows the estimation results for model 6. In this model, I replace $E_t(\ln \hat{\sigma}_{m,t+1} + \frac{1}{2}\sigma_\eta^2)$ in model 5 with $h_{m,t+1}$. GARCH parameters are similar to those in Table 2.8 and in relative risk specification, all variables except for TERM are statistically significant while SURP is the only significant variable when I include other variables with it. As with model 3, the estimates obtained from model 6 seem unreliable since one of GARCH parameter(β_2) is not statistically significant across all estimations.

I summarize findings with positivity restriction. First, realized volatility specification is still better than GARCH since all the estimated parameters in realized volatility equations are quite significant and robust while GARCH parameters are insignificant in several cases. Second, once I impose the positivity constraint on relative risk aversion, almost all forecasting variables are statistically significant. Since those variables exhibit strong cyclical patterns and it is well known that the sensitivity of risk does vary with recession, positivity restriction seems to provide more sensible estimates of the relative risk aversion. Consistent with the results in models without positivity restriction, the surplus consumption ratio is always statistically significant with correct negative sign. Third, CAYA captures additional explanatory power on relative risk aversion except for the unreliable GARCH-in-mean with time-varying relative risk aversion case(model6). Several financial variables just forecast volatility terms.

Several interesting points are raised for empirical asset pricing models at this point. With direct comparison, first, only consumption related variables are important determinants of relative risk aversion and financial market variables typically used in conditional asset pricing studies do not look as primary instruments for it. Especially, the surplus consumption ratio motivated from the external habit formation model is the most important determinant for explaining the time varying nature of relative risk aversion. These results seem to suggest a warning for asset pricing studies without consumption related variables. For example, typical bond market research imposes that market prices of risks or relative

risk aversion for bond market are a function of yield variables only. My estimates suggest that the estimation of market price of risk or relative risk aversion might be difficult to be identified if we use only yield variables since TERM and RREL are marginally significant.

2.3.3 Robustness checks with hedging components

Merton (1973) points out that in addition to the stock market variance, hedging demand for time-varying investment opportunities is also an important determinant of the expected stock market risk premium. In empirical investigation of that idea, Scruggs (1998) and Guo and Whitelaw (2006) find that ignoring hedge components in the ICAPM might introduce a downward bias in the estimated risk-return relation (or the relative risk aversion) because the volatility and the hedge demand could be negatively correlated. This is classical omitted variable bias problem in which the set of predictor variables is misspecified. Especially, Guo, Wang, and Yang (2006) argue that relative risk aversion is constant once we correctly model the ICAPM with CAY. In order to test this hypothesis, I include each return forecasting variable as a proxy for hedging component and test that relative risk aversion is constant or not. Here I use the ICAPM specification of Hodrick and Zhang (2001), Guo and Whitelaw (2006) and Guo, Wang, and Yang (2006) and use contemporaneous values of the predictive variables rather than their innovations. I extend previous models with the following specification.

For the parameters of realized volatility equation and GARCH, I don't discuss results further since their signs and magnitudes are quite close to the corresponding models provided in conditional CAPMs.

Table 2.11 report the estimated results for model 1 with additional hedging component. Since SURP seems robust as a proxy for relative risk aversion, I include all the other variables except for SURP and check the robustness of the results. In summary, the surplus consumption ratio is always statistically significant and has a correct negative sign even after

I include additional variables as hedging components. However, CAYA does not provide consistent results. First, three out of seven cases are significant at 5 % but are marginally significant for other three cases. Especially, with CAYA in hedging components, both terms lose statistical significance.²⁹ Finally, none of the variables has any statistical significance as a hedging component.

For other models without positivity restrictions, two results are noteworthy. First, Table 2.12 indicates that RREL is statistically significant as the hedging component. Second, CAYA seems to lose its significance as a proxy for relative risk aversion with any variable as the hedging component but CAYA in model 3 again recovers its statistical significance as a proxy for relative risk aversion. Furthermore, it is significant even when I include CAYA as the hedging component.

Since positivity restriction might help to identify hedging components more precisely, I estimate models 4,5 and 6 with additional hedging components. Table 2.14 shows that both SURP and CAYA retain statistical significance and the same signs but CAYA also loses its significance when I put CAYA as the hedging component. Now RREL becomes marginally significant with SURP and CAYA as proxies for relative risk aversion. In Table 2.15, only two different results strike out compared with Table 2.14. Now CAYA in relative risk aversion specification is still significant even when CAYA is also included as a hedging component. Finally, RREL is statistically significant at 5% with both SURP and CAYA are included as proxies for relative risk aversion. Finally, none of the variables in a hedging component are significant in model 6.

In summary, several points deserve to be mentioned. First, these tables strongly indicate that relative risk aversion is indeed time-varying with or without a correct modeling of hedging components. Among other things, the surplus consumption ratio(SURP) is almost always statistically significant with the negative sign. Second, the consumption-wealth ratio

²⁹In unreported tables, if I put just SURP in relative risk aversion specification, then CAYA in a hedging component becomes marginally significant.

without a look ahead bias,(CAYA) captures only relative risk aversion and not hedging components. Often CAY is used as proxy for relative risk aversion(Lettau and Ludvigson (2001b)) but hedging components interpretation is also utilized(Guo and Whitelaw (2006)). The estimated results in this paper support strongly relative risk aversion interpretation. Third, RREL has some explanatory power as a proxy for hedging components. Guo and Whitelaw (2006) also include RREL as a proxy for investment opportunities. Fourth, TERM is insignificant in most cases even though it is widely used either a conditioning variable or a proxy for a hedging component. In fact, several recent studies find that the yield spread(TERM) is not a very good predictor of economic activity after 1985. Notably, Ang, Piazzesi, and Wei (2006) after imposing no arbitrage restrictions, find that the short term interest rate, not the term spread is the main forecasting instrument to future economic activity. Also, Ang and Bekaert (2006) confirms that after extensive statistical analysis, only the short term interest rate or RREL strongly negatively predicts excess returns among chosen predictability variables. Since Merton (1980), it has been well known that the interest rate accurately describes the changing investor opportunity set. Finally, other return forecasting variables do not have any statistical power to explain both relative risk aversion or hedging components while some of the variables forecast market volatility.³⁰

2.3.4 The filtered estimates of Relative Risk Aversion

In this section, I report the filtered estimates of the relative risk aversion from the two models with positivity restriction since qualitative implications are similar across the models with slight difference in magnitudes.³¹

Since relative risk aversion relates to SURP and CAYA, both of which are strong cyclical

³⁰Without time-varying relative risk aversion, some of variables are statistically significant in unreported results. This seems natural since these are return forecasting variables by definition.

³¹Since I am using complex nonlinear state space model with GARCH, I can not use simple Kalman smoothing algorithm. So, I just report filtered estimates of relative risk aversion

indicators of economic conditions, it is reasonable to suspect a business cycle pattern in risk aversion. Figure 2.3 plots estimates of relative risk aversion from the conditional CAPM with SURP and from the conditional ICAPM with SURP and CAYA as instruments for risk aversion and RREL as a proxy for hedging components along with shade indicating NBER business cycle contraction. I find that the relative risk aversions implied by both models are mostly countercyclical even though they miss the short recessions around 1970 and 1974. Intuitively, we expect periods of strong economic conditions to be associated with low or falling risk aversion, while recessions are associated with high or rising risk aversion. Table 2.17 presents that the filtered relative risk aversion is around 2 on average with a standard deviation of 3, which is consistent with sensible estimates that many economists are willing to accept. It appears to capture the turbulent financial markets during 1990s in which the relative risk aversion could be high not because of the recession but because of extremely volatile movements in international financial markets.³²

2.3.5 Cross-sectional comparison of the conditional Asset pricing models

I have shown that the relative risk aversion identified with the surplus consumption ratio and the consumption wealth ratio moves countercyclically and such a relation is still statistically significant even after I control for the hedging component and time-varying market volatility. These results appear to be robust because I reach the same conclusion using several different specifications. However, in time series asset pricing analysis, I extract all the implications from just one variable, excess returns estimation. Without further verification, it looks premature to conclude that time-varying relative risk aversion is really important for asset pricing applications.

To further elaborate on the results, in this section, I follow Lettau and Wachter (2006)'s

³²Recently Chue (2005) shows that the time-varying relative risk aversion with surplus consumption ratio could be important to understand the financial crises or contagion.

suggestion and investigate the models' implications for the cross-section of stock returns.³³ Fama (1991) conjectures that we should relate the cross-section properties of expected returns to the variation of expected returns through time. Usually, two different approaches have been suggested. We can use conditional versions of unconditional single factor models, such as conditional CAPM or conditional consumption CAPM while unconditional multi-factor models are also frequently used. From my time-series specification tests, I find that probably we need both terms to fully understand the risk premium in stock market. Consumption-related variables are significant determinants for relative risk aversion but the short term interest rate has some explanatory power as the hedging component.

Conditional models are appealing because unconditional models may not capture time-varying risk premiums appropriately. Theoretically, Hansen and Richard (1987) show that, even if the unconditional versions of some models fail, the corresponding conditional models with correctly specified information sets could be perfectly valid for capturing the dynamics of risk premiums and they will outperform the unconditional versions of the models. However, as Ghysels (1998) argues, if the model's implied time-varying risk premiums are misspecified due to the wrong conditioning variable without any specification test, then these conditional models may have bigger pricing errors than their counterparts in unconditional specification. In this sense, current risk-return trade-off research seems to have problems since they just focus on time-series information. Therefore, cross-sectional verifications of my time-series models should be implemented.

In this paper, as test assets, I use Fama-French 25 portfolios. It has become standard practice in the cross-sectional asset-pricing literature to evaluate models based on how well they explain average returns on size- and book-to-market-sorted portfolios. In addition to that, I use 55 portfolios with Fama-French 25 portfolios and 30 industry portfolios. As Lewellen, Nagel, and Shanken (2006) suggest, many models are proposed to capture the

³³I follow precisely the testing methods suggested in Petkova (2006) to compare empirical models.

value premium but they typically fails to match the risk premium implied by 55 portfolios. Here I show that my empirically chosen models are comparable to the Fama-French three factor model in explaining the value premium and furthermore seem to capture part of this industry premium better than the Fama-French three factor model does.

I compare the conditional asset pricing models proposed in the present study directly with the purely empirical Fama-French three-factor model. Fama and French (1993) argue that HML and SMB represent compensations for risk consistent with Merton (1973)'s ICAPM. However, it is still not clear whether the HML and SMB factors have specific economic interpretations. Cochrane (2001) argues that asset pricing models that use portfolio returns as factors may be successful in describing asset returns, but those models will never be able to explain portfolio returns in economic sense since these models leave unanswered the question of what explains the return-based factors themselves. Therefore, I expect that the conditional ICAPM suggested in this paper might shed some lights on this issue since part of premium could be attributed to time-varying relative risk aversion or hedging components.

Here I interpret the results in this section with a caution. Lewellen, Nagel, and Shanken (2006) show that all the comparison measures utilized in this paper can be misleading in some cases. However, I follow several remedies proposed by them and report my empirical results with several diagnostics to minimize those problems.³⁴

In order to determine whether the suggested empirical models can account for the cross section of returns on the 25 Fama-French size and B/M sorted portfolios for the period from 1957:1 to 2005:4, I utilize the Fama and MacBeth (1973) procedure.³⁵ I use this Fama-Macbeth type beta pricing approach for the following reasons. The excess returns

³⁴See their Table 1 for the details. Almost all recently suggested models are not better than Fama-French three factor model after extensive simulations and various robustness checks.

³⁵See chapter 12 of Cochrane (2001) for the detailed explanation. Alternative GMM approaches are nicely explained in chapter 11.

on the test assets commonly chosen in empirical work often exhibit high contemporaneous correlations. This can make some of the numerical calculations of the standard GMM approach unstable for a large cross-section of assets typically with small span of data set. This instability has been a main concern of many recent papers on conditional pricing models that use versions of the beta pricing approach. (Lettau and Ludvigson (2001b))

In the first stage time series regression, I regress the portfolio returns on the market excess return, and several additional variables specified from the model to obtain the betas. As in Lettau and Ludvigson (2001b), the full-sample loadings, which are the independent variables in the second stage regressions, are computed in one multiple time-series regression. The most general form of the conditional ICAPM in this paper is as follows.³⁶

$$\begin{aligned}
 R_{i,t} - R_{f,t} = & \alpha_i + \beta_{iM}(R_{M,t} - R_{f,t}) + \beta_{i2}\text{SURP}_{t-1} * (R_{M,t} - R_{f,t}) \\
 & + \beta_{i3}\text{CAYA}_{t-1} * (R_{M,t} - R_{f,t}) + \beta_{i4}\text{CAYA}_{t-1} + \beta_{i5}\text{RREL}_{t-1} + \varepsilon_{it}
 \end{aligned} \tag{2.3.1}$$

where $R_{i,t}$ is portfolio returns; $R_{f,t}$ is treasury bill returns; $R_{M,t}$ is market returns;

(2.3.1) needs to be estimated from GLS since portfolio returns typically have at least heteroscedasticity. However, we can interpret (2.3.1) as an seemingly unrelated regression system with same independent variables. In this case, it is well known that ordinary least squares results are the same as more efficient general least squares estimates with general covariance structure of errors.

Estimated betas from the first-stage regressions are subsequently used as independent variables in the second-stage cross-sectional regression for all time periods. Hence, the risk premium estimates in the second-stage regression are subject to an errors-in-variables bias. To correct for this problem, I adjust the standard errors from the second stage regressions

³⁶I use the usual market excess return as the market factor rather than the lagged realized variance employed by Guo, Wang, and Yang (2006) in order to do direct comparison with the results of other asset pricing studies. I need to use estimated expected value of realized variance instead of the lagged realized variance. This might cause another source of errors in variable problem.

as proposed in Shanken (1992). However, I also report the Fama-MacBeth standard errors since Jagannathan and Wang (1998) show that with conditional heteroscedasticity, the standard errors produced by the Fama-MacBeth procedure do not necessarily overstate the precision of the risk premium estimates.³⁷ The second-stage regression for (2.3.1) is presented as follows.

$$R_{i,t} - R_{f,t} = \lambda_0 + \lambda_M \widehat{\beta}_{iM} + \lambda_2 \widehat{\beta}_{i2} + \lambda_3 \widehat{\beta}_{i3} + \lambda_4 \widehat{\beta}_{i4} + \lambda_5 \widehat{\beta}_{i5} + u_{i,t} \quad i = 1, \dots, 25, \quad (2.3.2)$$

where λ stands for the risk price.

Following Fama and MacBeth, I run this cross-sectional regressions each quarter, generating time-series of estimates for risk prices(λ). Means, standard errors, and t-statistics are then computed from these time series and inferences proceed in the usual manner, as if the time series are independently and identically distributed. It is well known that security returns are cross-sectionally correlated, due to the common market and industry factors, and also heteroscedastic. As a result, the usual formulas for standard errors are not appropriate for the OLS cross-sectional regressions(CSR). Fama-Macbeth approach can be interpreted as a remedy for this phenomenon. Since the true variance of each quarterly estimator depends on the covariance matrix of returns, cross-sectional correlation and heteroskedasticity are reflected in the time series of quarterly estimates.

My empirical asset pricing models are based on the time series analysis in previous section and the first-stage regression for each model is presented.³⁸

1. CCAPM1: $R_{i,t} - R_{f,t} = \alpha_i + \beta_{iM}(R_{M,t} - R_{f,t}) + \beta_{i2}\text{SURP}_{t-1} * (R_{M,t} - R_{f,t})$
2. CCAPM2: $\text{CCAPM1} + \beta_{i3}\text{CAYA}_{t-1} * (R_{M,t} - R_{f,t})$
3. CCAPM3: $\text{CCAPM2} + \beta_{i4}\text{CAYA}_{t-1}$
4. CICAPM1: $\text{CCAPM1} + \beta_{i4}\text{CAYA}_{t-1}$

³⁷I find only weak evidence of remaining GARCH effect for the quarterly excess returns in the first-stage regression(unreported)

³⁸I omit SURP since this variable does not seem to add any expansibility on cross-sectional regressions.

5. CICAPM2: $CCAPM1 + \beta_{i5}RREL_{t-1}$
6. CICAPM3: $CCAPM2 + \beta_{i4}RREL_{t-1}$
7. CICAPM4: $CCAPM2 + \beta_{i4}CAYA_{t-1} + \beta_{i5}RREL_{t-1}$

where CCAPM denotes the conditional CAPM; CICAPM denotes the conditional ICAPM.

To better evaluate the performances of the conditional CAPMs and ICAPMs, I also estimate and report results for the simple unconditional CAPM, the unconditional ICAPM, and the Fama-French three-factor model.

1. CAPM: $R_{i,t} - R_{f,t} = \alpha_i + \beta_{iM}(R_{M,t} - R_{f,t})$
2. ICAPM: $CAPM + \beta_{i2}CAYA_{t-1} + \beta_{i3}RREL_{t-1} + \beta_{i4}TERM_{t-1}$
3. Fama-French three factor model: $CAPM + \beta_{i2}SMB_t + \beta_{i3}HML_t$

To judge the goodness of fit of the suggested empirical models, I use the cross-sectional R^2 measure employed first by Jagannathan and Wang (1996). This R^2 shows the fraction of cross-sectional variation in average returns that is explained by the model. This measure is calculated as

$$R^2 = \frac{\sigma_C^2(\bar{R}) - \sigma_C^2(\bar{e})}{\sigma_C^2(\bar{R})}$$

where σ_C^2 represents the in-sample cross-sectional variance, \bar{R} is a vector of average excess returns, and \bar{e} stands for the vector of average residuals in cross-sectional regression.

I also report the root mean square of pricing errors (α) in cross-sectional regression (RMSE) as another intuitive diagnostic to compare the models. I use $\sqrt{\frac{1}{25} \sum_{i=1}^{25} \alpha_i^2} = \sqrt{\frac{1}{N} \alpha' \alpha}$ for all the models. Cochrane advocates this measure by arguing that even though Hansen-Jagannathan (HJ) distance measure is invariant to portfolio formation, this simple RMSE could be more informative if the original portfolios were primary concerns and the second

moment matrix of the test assets is quite close to singular³⁹ since the HJ distance places too much weight on pricing near-riskless portfolios rather than pricing the original assets.

Finally, Lewellen, Nagel, and Shanken (2006) suggest to report Shanken (1985)'s Hotelling T^2 statistics since the cross-sectional R^2 is not invariant to portfolio formation. Following Petkova (2006), I compute and report the transformed Hotelling T^2 statistic which is adjusted for the errors-in variables problem and has an approximate F-distribution in small samples. The transformed test statistic is computed as

$$Q = \frac{T\bar{e}'\hat{\Sigma}^{-1}\bar{e}}{(1+c)}$$

where T is the number of time-series observations, \bar{e} stands for the average residual vector in the cross section, and $\hat{\Sigma}$ is the estimated covariance matrix of the residuals in the first-stage time series regression and c is the Shanken correction term.

Table 2.18 reports the estimated coefficients, Fama-MacBeth(FM) and Shanken-corrected standard errors and the degrees of freedom-adjusted R^2 and the RMSE and F-statistics for the cross-sectional regressions using the excess returns on 25 portfolios sorted by book-to-market and size.

First, in most cases, the market factor receives a negative and statistically insignificant risk premium consistent with the findings of Fama and French (1992).⁴⁰ While it appears to be a severe problem for the CAPM, the negative market risk premium has not been understood yet. Since that issue is beyond the scope of this paper, I defer it for future studies, for that issue is beyond the scope of this paper.

Second, the proposed conditional CAPMs(CCAPM1, CCAPM2 and CCAPM3) seem to indicate clear improvements over CAPM. CCAPM1 and CCAPM2 deliver much higher

³⁹Lewellen, Nagel, and Shanken (2006) suggest that Fama-French 25 size and B/M sorted portfolios have essentially three degree of freedom.

⁴⁰Jagannathan and Wang (1996) and Lettau and Ludvigson (2001b) also report negative estimates for the market risk premium, using monthly or quarterly data.

cross-sectional R^2 of 40 % and 60%, respectively and smaller RMSE than CAPM does. Especially, CCAPM3 provides remarkable improvement(R^2 is 70%). These results show that it is likely for CAPM to hold conditionally since as Lettau and Ludvigson (2001b) argue, if the CAPM holds conditionally, but not unconditionally, better performance may obtain if the CAPM is scaled by variables that capture relevant conditioning information. However, the take-away point is that a large number of macroeconomic variables can be added to ad-hoc linear factor models($M_{t+1} = a - bf_{t+1}$) in this way to price the Fama-French 25 portfolios. Here I want to emphasize the difference of my conditional CAPM from the models proposed by Lettau and Ludvigson (2001b) or Ferson and Harvey (1999). In this paper, I get conditioning variables with non-ad-hoc time-series specification tests with several robustness checks while Lettau and Ludvigson (2001b) use just simple intuition to use CAY and Ferson and Harvey (1999) do not try differentiate hedging components with risk aversion components or volatility components. Therefore, while my unconditional model still might be subject to this data snooping issue, I argue that my models are subject to it clearly in a lesser degree than other models do.

Third, the results from the conditional ICAPM(CICAPM4 and CICAPM5) show that the cross-sectional R^2 is around 70% and RMSE is around 0.0036. These performances are a little bit lower than or comparable to those from the Fama-French model, while these ICAPM models offer comparable performance over the conditional CAPM.

Fourth, unconditional ICAPM with CAYA,TERM, and RREL is not bad for this application. It has comparable R^2 of 68% and RMSE(0.0036). Finally, with Shanken (1985), all models are statistically rejected at 1% level even though Figure 2.4 shows some success to capture the variations across the portfolios. This rejection is largely from the smallest growth portfolio(11) as usual. It is unclear that this portfolio should be interpreted as an outlier.

Figure 2.4 plots the realized versus predicted returns of the models examined of the

selected models. The numbers on the x-axis are the portfolios' names. The first digit number in a portfolio name is the size group it belongs to, and the second digit is the B/m group it belongs to. Both the size groups and the B/M groups are in ascending order. The closer a portfolio lies to the 45-degree line, the better the model explains the returns of that portfolio. It can be seen from the graph that the conditional ICAPM(CICAPM5) explains the value effect comparable to Fama-French three-factor model: In general, the fitted expected returns on value portfolios (larger second digit) are higher than the fitted expected returns on growth portfolios (smaller second digit).

The value premium has been an important but controversial subject in the asset pricing literature. Consistent with Fama and French's ICAPM conjecture, Liew and Vassalou (2000) find that the value premium forecasts output growth. Brennan, Wang, and Xia (2004) and Petkova (2006) also show that the value premium is correlated with their measures of investment opportunities. However, there are alternative explanations in the conditional CAPM literature for the value premium. In particular, Lettau and Ludvigson (2001b) find that the conditional CAPM with CAY helps explain the value premium and argue that the Fama-French factors are mimicking portfolios for risk factors associated with time-variation in market price of risk(or risk aversion).⁴¹

The estimation results in Table 2.18 shed light on the on-going debate about the value premium since I develop the cross-sectional asset pricing models based on more thorough time-series specification tests. First, the selected conditional CAPM with SURP and CAYA indeed explains the strong value premium for 1957 to 2005. Likewise, once augmented with proxies for hedging components, the proposed models also seem to capture the value premium comparable to the Fama-French three-factor model. Therefore, just for explaining the value premium, either conditional CAPM or conditional ICAPM looks good. In fact, many models have been successfully developed to explain this value premium.

⁴¹Zhang (2005) also develops equilibrium model with adjustment costs for investment to explain the value premium when relative risk aversion is high

Recently, Lewellen, Nagel, and Shanken (2006) argue that the proposed models for the value premium do not seem to explain premium of industry portfolios. Typically, they find that most of the models are even worse than the Fama-French three factor model in explaining industry premium. Therefore, Lewellen, Nagel, and Shanken (2006) recommend that when three factors explain nearly all of the time-series variation in returns of size-B/M portfolios, we should augment them with 30 industry portfolios which don't correlate with SMB and HML as much for correct comparison of the models. Furthermore, since there are essentially three degrees of freedom in Fama-French 25 portfolios, Cochrane (2006) suggests that asset pricing models with more than three factors, should be carefully investigated even though those models tend to explain Fama-French 25 portfolios.⁴² Following the suggestions of Lewellen, Nagel, and Shanken (2006), I test the robustness of the proposed empirical models by examining the ability of the competing models to price industrial portfolios. I expect that if we have meaningful asset pricing models, my proposed models should describe these asset returns better than the Fama-French model does.

Table 2.19 reports the cross-sectional regression results on the 55 portfolios returns. Now the conditional CAPM(CCAPM3) and ICAPM(CICAPM4) appear to perform strictly better than all the models, including the conditional CAPM, the Fama-French three factor model and unconditional ICAPM, in explaining the test assets in terms of the intuitive measures(both R^2 and RMSE). The plots of Figure 2.5 also confirm these facts.

In summery, the carefully selected conditional CAPMs and ICAPMs with time-varying relative risk aversion and hedging components are not only comparable to Fama-French three factor model in explaining the value premium but also clearly satisfy the robustness criteria of Lewellen, Nagel, and Shanken (2006) since they have a higher explanatory power in terms of the 55 portfolios than all the other models. However, none of the models seems to price industry portfolios even though my models are clearly better than the Fama-French

⁴²Brennan, Wang, and Xia (2004) also tests their model with this 55 portfolios and finds that their model is statistically rejected. However, they don't report any intuitive statistics.

three factor models. Several researches (for example, Lettau and Wachter (2006) argues that the external habit formation model need an extra hedging component (cash flow state variables) to explain asset returns. In this paper, while I include RREL as a candidate for that, it seems not clear whether we have any incremental explanatory power with this specification. My paper can be interpreted as a first step to find the complete empirical models since I obtain the determinants of relative risk aversion with extensive empirical analysis. In my future study, I will try to incorporate other variables for explaining hedging components more accurately.

2.4 Conclusion

This paper contributes to the asset pricing literature in several respects. Recent applications of the ICAPM try to avoid "fishing license problem" by employing return forecasting variables. However, without understanding the nature of the forecastability, it seems possible to use either the conditional CAPM or the ICAPM. I develop novel time series methods of identifying the determinants of stochastic risk aversion and hedging components separately under the unified framework of Merton (1973) ICAPM.

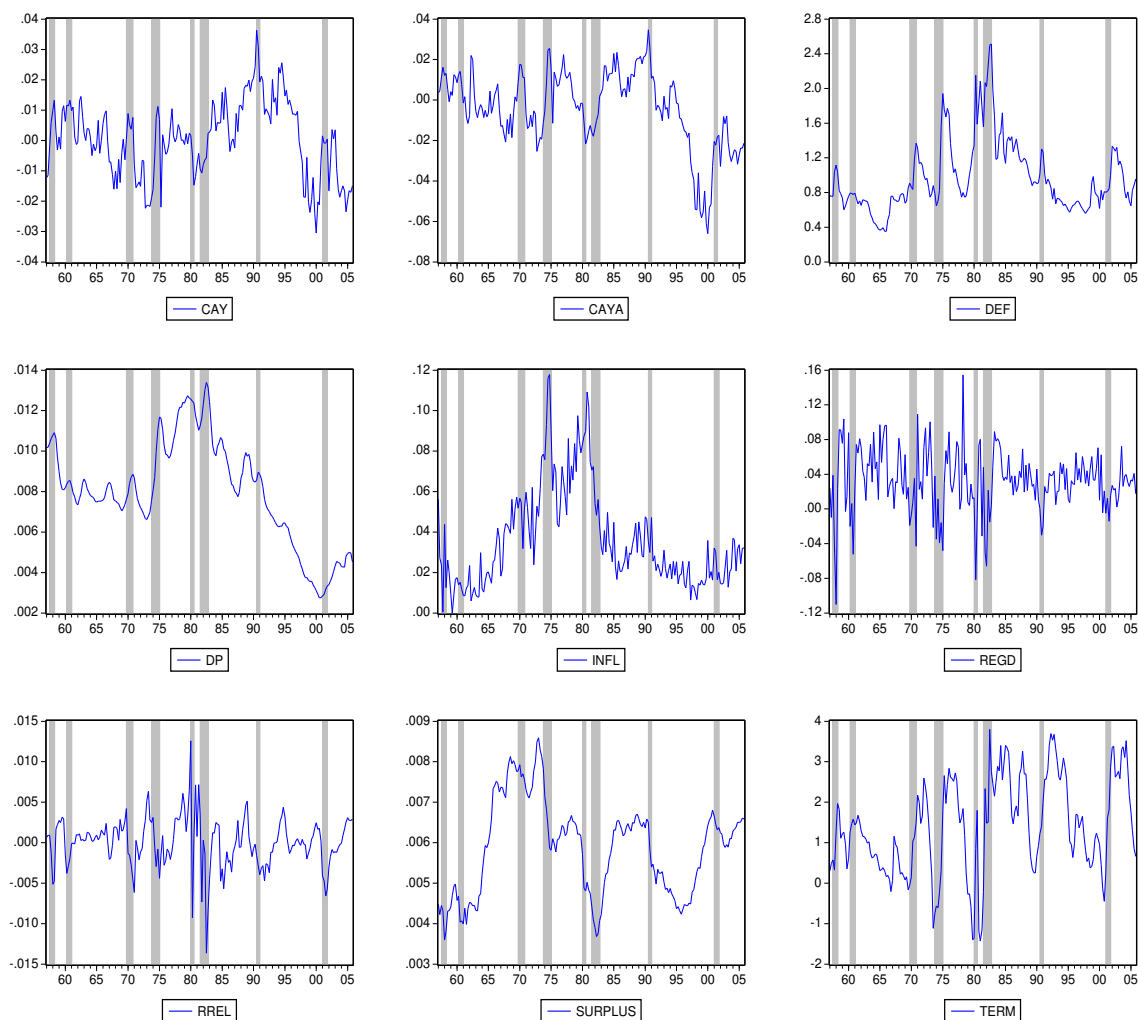
First, I find that only consumption related macroeconomic variables are needed to explain the time-varying relative risk aversion. Both the surplus consumption ratio and the consumption wealth ratio without a look ahead bias motivated from the external habit formation model have the most successful explanatory power, with correct signs, on time varying relative risk aversion. Other return forecasting variables including dividend price ratio, default spread, term spread, short term interest rate, inflation and real GDP growth lose their statistical significance especially in the presence of the surplus consumption ratio. Consistent with Lettau and Ludvigson (2001b) CAY seems to capture only part of relative risk aversion but not the hedging component suggested by Guo and Whitelaw (2006). Without careful decomposition of volatility, relative risk aversion and hedging components,

we can't determine whether certain variables are a proxy for relative risk aversion, volatility or hedging components. Second, only RREL captures part of the changing investment opportunities argued by Merton (1973). Other return forecasting variables only explain the time-varying volatility and become statistically insignificant in the presence of consumption related variables(SURP and CAYA) if I put those variables into relative risk aversion or hedging components specification.

In addition, I also compare the cross-sectional implication of the selected conditional CAPMs and ICAPMs with several bench mark asset pricing models including Fama-French three-factor model. The models are compared on a common set of returns: either the Fama-French 25 size and B/M sorted portfolios alone or with 30 industry portfolios. I find that the carefully selected conditional CAPMs and ICAPMs with time-varying relative risk aversion and hedging components are not only comparable to Fama-French three factor model in explaining the value premium but also meet the robustness criteria of Lewellen, Nagel, and Shanken (2006) since they have a higher explanatory power in terms of the 55 portfolios than all other models do. However, I find that stochastic risk aversion and a hedging component with RREL is not enough to explain industry risk premium. Results reported in this paper suggest several implications to develop models to price industry premium. Among other things, correct identification of relative risk aversion by consumption variables is crucial for the success of that model.

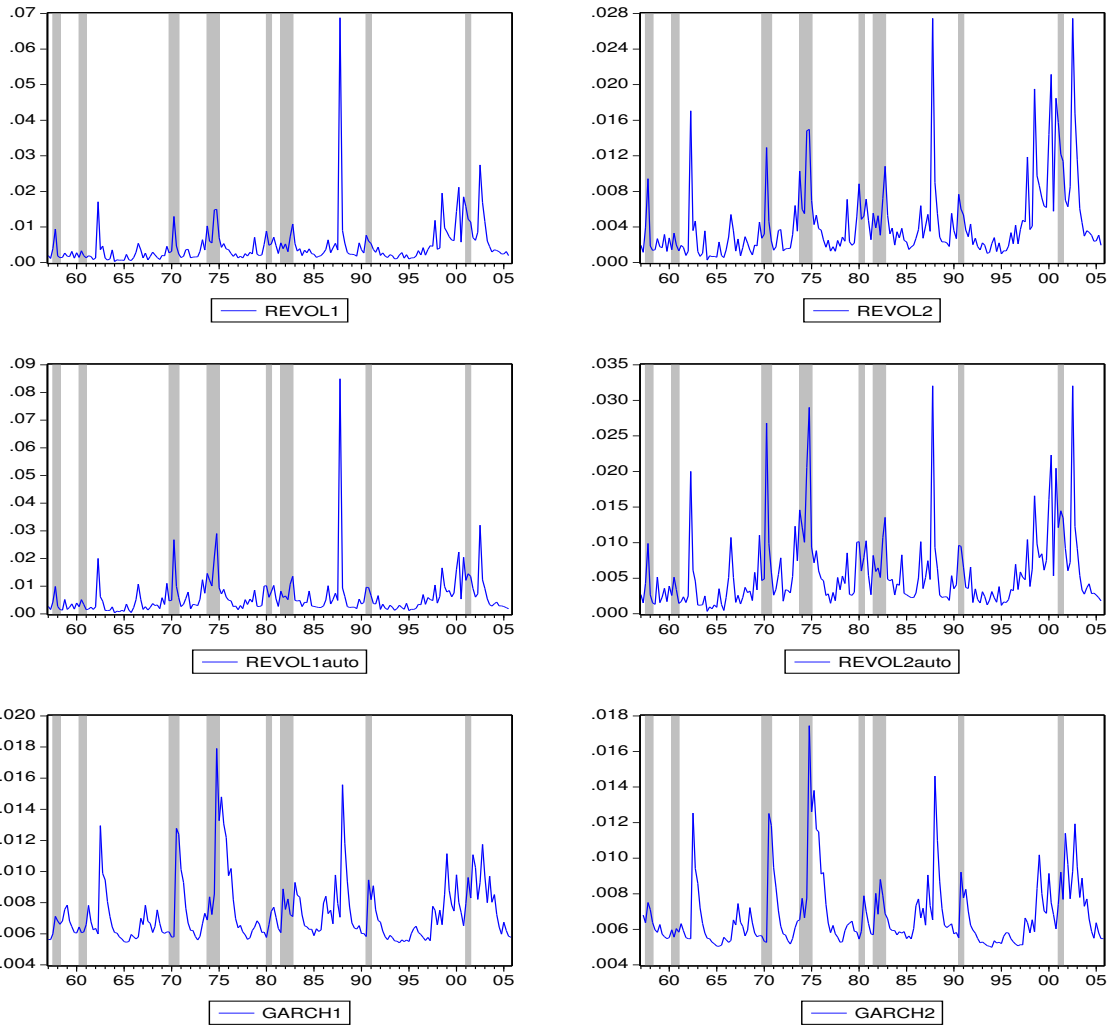
Recently, Guo (2006) employs a simpler version of Guo, Wang, and Yang (2006) to check the risk-return trade off in the international stock markets. The time-varying relative risk aversion identified in the present study is expected to clarify some of the issues in that area. In fact, the cross-sectional implication of the changing risk aversion or hedging components would be more interesting. Since Fama and French (1998) present the value premium in the international stock markets and cast the doubt on the validity of the traditional international asset pricing models, several conditional and unconditional models have been suggested and

estimated to explain it without thorough specification tests. Notably, Zhang (2006) finds that the world CAPM augmented with both exchange rate risk and the conditioning variable for describing world business cycle(industry production) is the best performing model. I am currently working on the extension of the empirical models employed in this paper to this context and expects that those models could illuminate the precise nature of the risk premium in the international stock market.



Note: CAY is the consumption-wealth ratio; CAYA is the consumption-wealth ratio without a look ahead bias; DEF is the yield spread between Baa- and Aaa-rated corporate bonds; DP is the deseasonalized dividend over price ratio; INFL is the quarterly inflation rate measured by GDP deflator; REGD is the quarterly real GDP growth; RREL is the difference between the 3 month treasury bill rate and its average in the previous 4 quarters; SURP is the surplus consumption ratio; and TERM is the yield spread between 10-year treasury bonds and 3-month treasury bills. Shared areas indicate NBER business recessions.

Figure 2.1: Exogenous Variables and Business Cycles(1957:2-2005:4)



Note: Realized volatility(REVOL1) is defined in the data appendix; REVOL2 replaces a data point(1987:4) in REVOL1 with the second largest one; REVOL1auto is the autocorrelation-corrected measure of REVOL1; REVOL2auto replaces a data point(1987:4) in REVOL1auto with the second largest one; $r_{M,t+1}$ is the CRSP value-weighted stock return minus lagged 3-month treasury bill rate. GARCH1 is estimated with simple GARCH(1,1) model of the market excess return(Model1); GARCH2 is estimated with the conditional ICAPM model with surplus consumption ratio(SURP) and RREL(Model2). All exogenous variables(SURP, RREL, and u_t) are normalized to have means of zero and standard deviations of one. Shared areas indicate NBER business recessions.

$$\text{Model1} : r_{M,t+1} = \gamma_1 h_{t+1} + \varepsilon_{t+1},$$

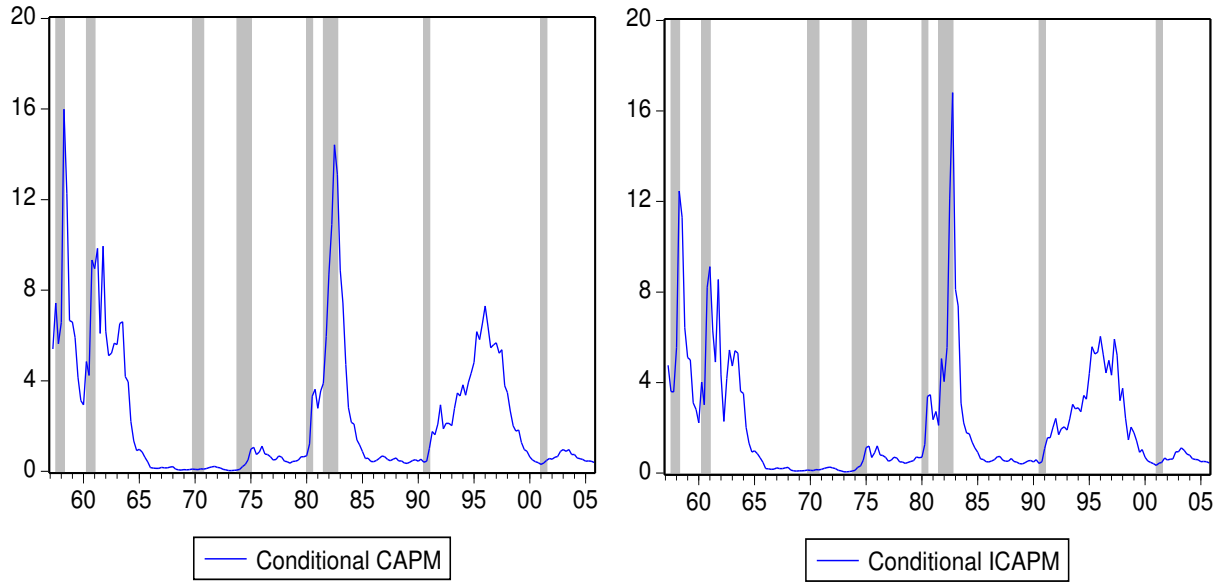
$$\text{Model2} : r_{M,t+1} = \exp(\gamma_{t+1}) \exp(E_t(\ln \hat{\sigma}_{m,t+1}) + \frac{1}{2} \sigma_\eta^2) + \alpha_1 (\text{RREL})_t + \varepsilon_{t+1},$$

$$\gamma_{t+1} = \phi_1 (\text{SURP})_t + v_{t+1},$$

$$\ln \hat{\sigma}_{m,t+1} = \delta_0 + \delta_1 \ln \hat{\sigma}_{m,t} + \delta_2' u_t + \eta_{t+1}$$

where $\varepsilon_{t+1} | \psi_t \sim N(0, h_{t+1})$, $v_{t+1} \sim N(0, 0.01)$, $\eta_{t+1} \sim N(0, \sigma_\eta^2)$, $h_{t+1} = \omega + \beta_1 h_t + \beta_2 \varepsilon_t^2$

Figure 2.2: Realized Volatility and GARCH(1957:2-2005:4)



This figure plots the quarterly time series of relative risk aversion series($\exp(\gamma_{t+1})$) implied by conditional CAPM with the surplus consumption ratio(Model1) and conditional ICAPM with the surplus consumption ratio and the consumption-wealth ratio as instruments for relative risk aversion and RREL as a proxy for the hedging component(Model2). All exogenous variables(SURP,RREL, and u_t) are normalized to have means of zero and standard deviations of one. Shaded areas indicate NBER business recessions.

$$\text{Model1} : r_{M,t+1} = \exp(\gamma_{t+1}) \exp(E_t(\ln \hat{\sigma}_{m,t+1}) + \frac{1}{2}\sigma_\eta^2) + \varepsilon_{t+1},$$

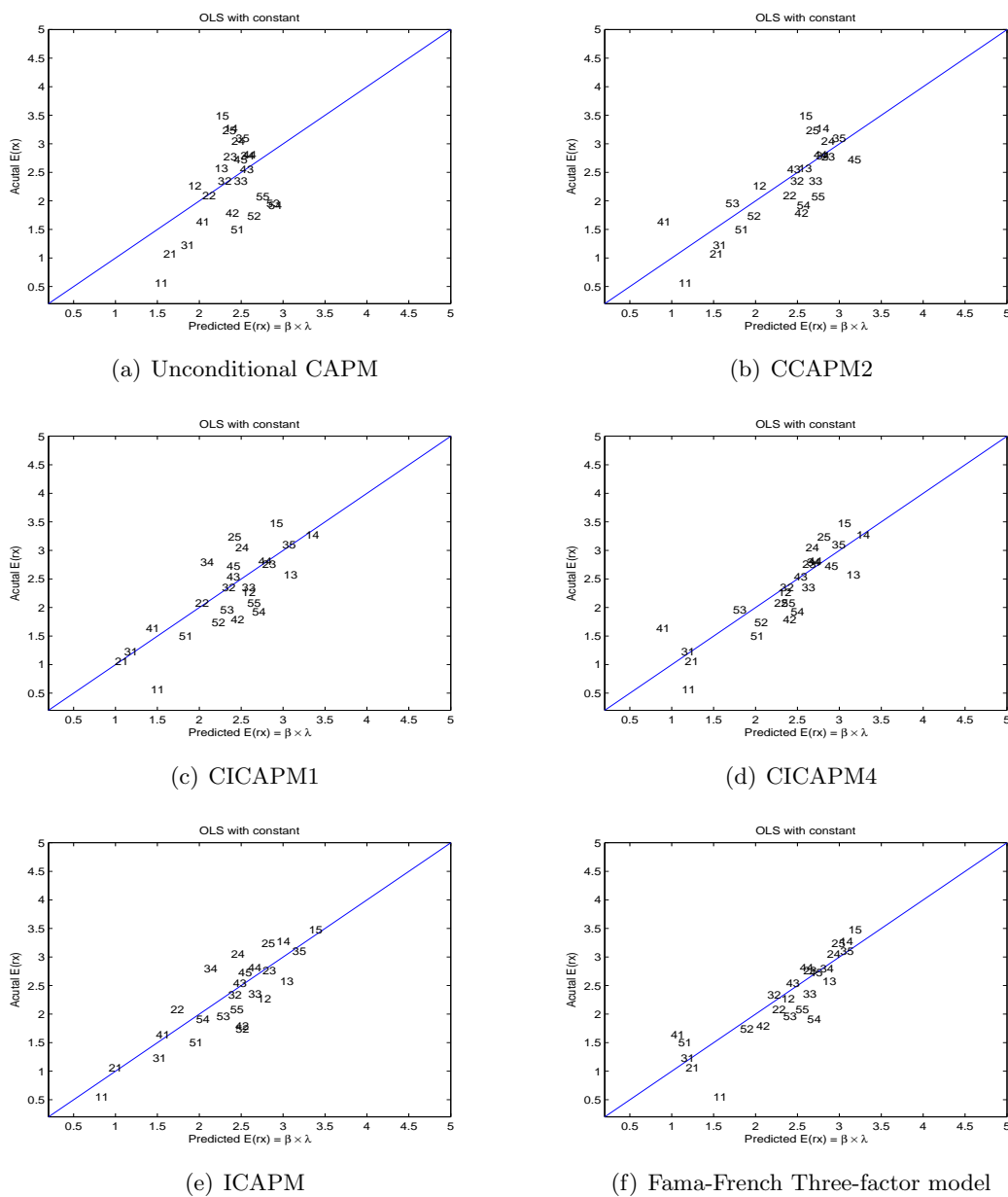
$$\text{Model2} : r_{M,t+1} = \exp(\gamma_{t+1}) \exp(E_t(\ln \hat{\sigma}_{m,t+1}) + \frac{1}{2}\sigma_\eta^2) + \alpha_1(\text{RREL})_t + \varepsilon_{t+1},$$

$$\gamma_{t+1} = \phi_1(\text{SURP})_t + v_{t+1},$$

$$\ln \hat{\sigma}_{m,t+1} = \delta_0 + \delta_1 \ln \hat{\sigma}_{m,t} + \delta_2' u_t + \eta_{t+1}$$

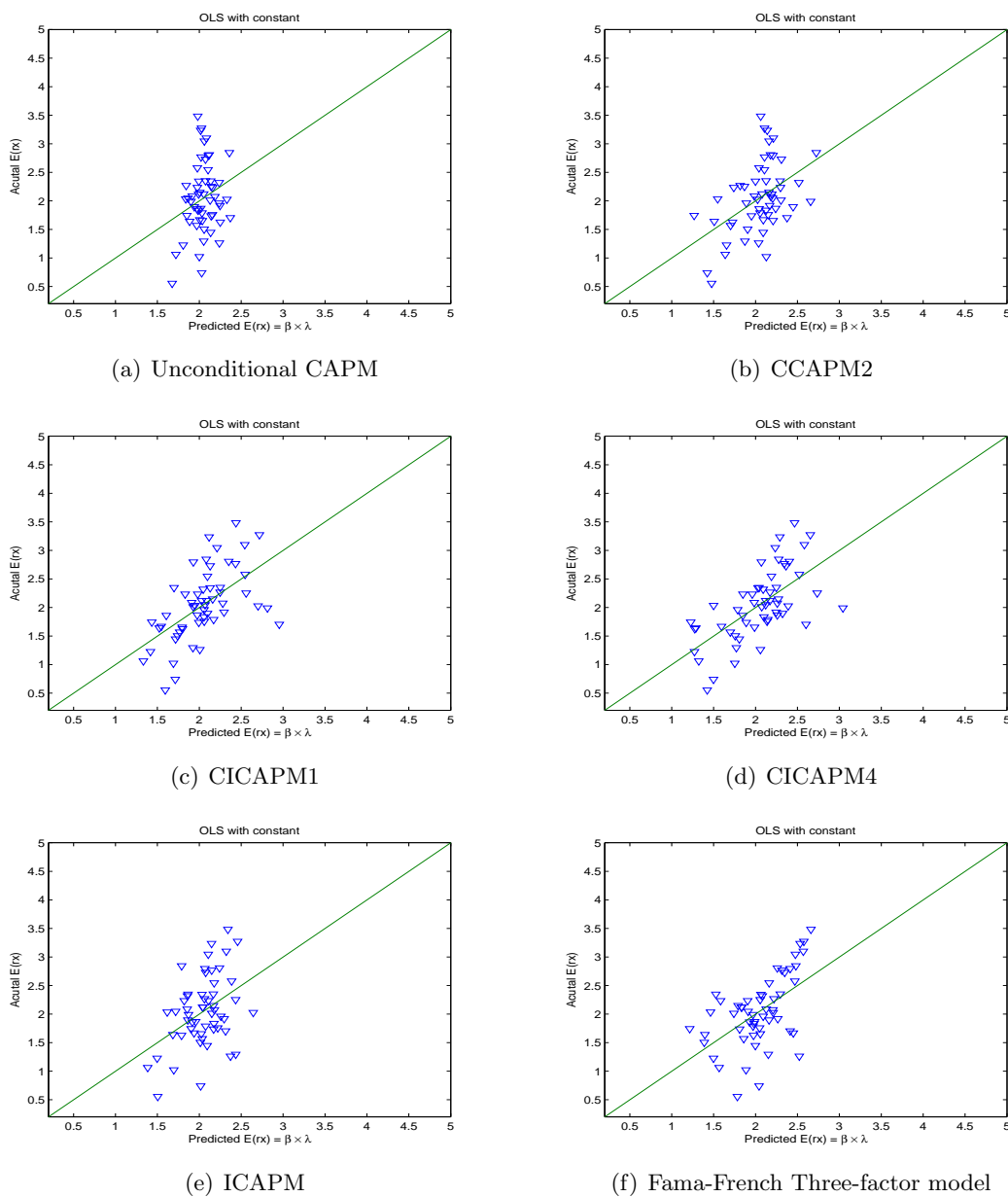
where $\varepsilon_{t+1}|\psi_t \sim N(0, h_{t+1})$, $v_{t+1} \sim N(0, 0.01)$, $\eta_{t+1} \sim N(0, \sigma_\eta^2)$, $h_{t+1} = \omega + \beta_1 h_t + \beta_2 \varepsilon_t^2$

Figure 2.3: Time-series of relative risk aversion.(1957:2-2005:4)



The plot shows realized average returns (in percent) on the vertical axis and fitted expected returns (in percent) on the horizontal axis for 25 size and book-to-market sorted portfolios. The first digit refers to the size quintile (1 being the smallest and 5 the largest), while the second digit refers to the book-to-market quintile (1 being the lowest and 5 the highest). For each portfolio, the realized average return is the time-series average of the portfolio return and the fitted expected return is the fitted value for the expected return from the corresponding model. The straight line is the 45-degree line from the origin. All models are defined in section 2.3.5.

Figure 2.4: Fitted Expected Returns Versus Average Realized Returns for the Fama French 25 portfolios(1957:2-2005:4)



The plot shows realized average returns (in percent) on the vertical axis and fitted expected returns (in percent) on the horizontal axis for 25 size and book-to-market sorted portfolios and 30 industry portfolios. For each portfolio, the realized average return is the time-series average of the portfolio return and the fitted expected return is the fitted value for the expected return from the corresponding model. The straight line is the 45-degree line from the origin. All models are defined in section 2.3.5.

Figure 2.5: Fitted Expected Returns Versus Average Realized Returns for the Fama French 25 portfolios and 30 Industry portfolios(1957:2-2005:4)

Table 2.1: Summary Statistics

Summary statistics for data used in the paper. Data is sampled quarterly from 1957:2 to 2005:4. The Auto(1) give the first autocorrelation. Note: EXCESS is the CRSP value-weighted stock return minus lagged 3-month treasury bill rate; CAYA is the consumption-wealth ratio without a look ahead bias; DEF is the yield spread between Baa- and Aaa-rated corporate bonds; DP is the deseasonalized dividend price ratio; INFL is the quarterly inflation rate measured by GDP deflator; REGD is the quarterly real GDP growth; RREL is the difference between the 3 month treasury bill rate and its average in the previous 4 quarters; SURP is the surplus consumption ratio; and TERM is the yield spread between 10-year treasury bonds and 3-month treasury bills.

	EXCESS	CAYA	DEF	DP	INFL	REGD	RREL	SURP	TERM
Panel A: Correlation Matrix									
EXCESS	1.000								
CAYA	-0.140	1.000							
DEF	0.120	0.171	1.000						
DP	0.061	0.641	0.527	1.000					
INFL	-0.126	0.194	0.435	0.563	1.000				
REGD	0.026	-0.088	-0.238	-0.097	-0.227	1.000			
RREL	-0.179	-0.091	-0.358	-0.002	0.162	0.289	1.000		
SURP	-0.154	-0.038	-0.138	-0.152	0.303	0.084	0.193	1.000	
TERM	0.167	0.141	0.260	-0.081	-0.300	0.117	-0.441	-0.196	1.000
Panel B: Univariate Summary Statistics									
Mean	0.011	-0.004	0.975	0.008	0.036	0.032	0.000	0.006	1.421
Std. Dev.	0.085	0.019	0.411	0.003	0.024	0.036	0.003	0.001	1.185
Skewness	-0.850	-0.863	1.367	-0.115	1.161	-0.390	-0.342	0.109	-0.074
Kurtosis	4.412	3.823	4.948	2.404	3.910	4.555	6.013	2.161	2.382
Auto(1)	0.030	0.909	0.921	0.985	0.865	0.293	0.530	0.975	0.874

Table 2.2: Summary Statistics for Realized Variance and GARCH

Summary statistics for the Realized variance and the estimated GARCH. Data is sampled quarterly from 1957:2 to 2005:4. Note: $r_{M,t+1}$ is the CRSP value-weighted stock return minus lagged 3-month treasury bill rate; Realized volatility (REVOL1) is defined in the data appendix; REVOL2 replaces a data point (1987:4) in REVOL1 with the second largest one; REVOL1auto is the autocorrelation-corrected measure of REVOL1; REVOL2auto replaces a data point (1987:4) in REVOL1auto with the second largest one; GARCH1 is estimated with simple GARCH(1,1) model of the market excess return (Model1); GARCH2 is estimated with the conditional ICAPM model with surplus consumption ratio (SURP) and RREL (Model2). All exogenous variables (SURP, RREL, and u_t) are normalized to have means of zero and standard deviations of one.

$$\text{Model1} : r_{M,t+1} = \gamma_1 h_{t+1} + \varepsilon_{t+1},$$

$$\text{Model2} : r_{M,t+1} = \exp(\gamma_{t+1}) \exp(E_t(\ln \hat{\sigma}_{m,t+1}) + \frac{1}{2} \sigma_\eta^2) + \alpha_1 (\text{RREL})_t + \varepsilon_{t+1},$$

$$\gamma_{t+1} = \phi_1 (\text{SURP})_t + v_{t+1},$$

$$\ln \hat{\sigma}_{m,t+1} = \delta_0 + \delta_1 \ln \hat{\sigma}_{m,t} + \delta_2' u_t + \eta_{t+1}$$

where $\varepsilon_{t+1} | \psi_t \sim N(0, h_{t+1})$, $v_{t+1} \sim N(0, 0.01)$, $\eta_{t+1} \sim N(0, \sigma_\eta^2)$, $h_{t+1} = \omega + \beta_1 h_t + \beta_2 \varepsilon_t^2$

	REVOL1	REVOL2	REVOL1auto	REVOL2auto	GARCH1	GARCH2
Panel A : Correlation Matrix						
REVOL1	1.000					
REVOL2	0.896	1.000				
REVOL1auto	0.963	0.841	1.000			
REVOL2auto	0.836	0.927	0.884	1.000		
GARCH1	0.224	0.319	0.236	0.342	1.000	
GARCH2	0.239	0.340	0.253	0.366	0.977	1.000
Panel B : Univariate Summary Statistics						
Mean	0.005	0.004	0.006	0.006	0.007	0.007
Std. Dev.	0.006	0.004	0.008	0.005	0.002	0.002
Skewness	6.408	2.657	6.505	2.564	2.185	2.293
Kurtosis	61.443	11.346	63.255	11.073	8.940	9.424

Table 2.3: Time-series specification test: time-varying RRA (Model1)

The Table shows the estimation results of several different specifications of conditional CAPM without positivity restriction. Note: $r_{M,t+1}$ is the CRSP value-weighted stock return minus lagged 3-month treasury bill rate; Realized volatility ($\hat{\sigma}_{m,t+1}$) is defined in the data appendix; CAYA is the consumption-wealth ratio without a look ahead bias; DEF is the yield spread between Baa- and Aaa-rated corporate bonds; DP is the deseasonalized dividend price ratio; INFL is the quarterly inflation rate measured by GDP deflator; REGD is the quarterly real GDP growth;RREL is the difference between the 3 month treasury bill rate and its average in the previous 4 quarters; SURP is the surplus consumption ratio; and TERM is the yield spread between 10-year treasury bonds and 3-month treasury bills. All exogenous variables (X_{1t} , X_{2t} , and u_t) are normalized to have means of zero and standard deviations of one to facilitate the interpretation.

$$\text{Model : } r_{M,t+1} = \gamma_{t+1} E_t(\ln \hat{\sigma}_{m,t+1} + \frac{1}{2} \sigma_\eta^2) + \varepsilon_{t+1},$$

$$\gamma_{t+1} = \phi_0 + \phi_1 X_{1t} + \phi_2 X_{2t} + v_{t+1}, \ln \hat{\sigma}_{m,t+1} = \delta_0 + \delta_1 \ln \hat{\sigma}_{m,t} + \delta'_2 u_t + \eta_{t+1},$$

where $\varepsilon_{t+1} | \psi_t \sim N(0, E_t(\hat{\sigma}_{m,t+1}))$, $v_{t+1} \sim N(0, \sigma_v^2)$, $\eta_{t+1} \sim N(0, \sigma_\eta^2)$, u_t are (CAYA, DP, TERM, DEF, INFL, t)

X_{1t}	ϕ_0	ϕ_1	σ_v	δ_0	δ_1	δ_{21}	δ_{22}	δ_{23}	δ_{24}	δ_{25}	σ_η	
SURP	2.662	-3.603	5.227	-3.664	0.330	0.183	-0.453	-0.167	0.322	0.176	-0.600	
tstats	1.834	-1.976	2.328	-10.532	5.272	3.261	-5.780	-3.358	4.925	3.243	-19.585	
CAYA	2.274	1.788	5.710	-3.682	0.327	0.175	-0.445	-0.169	0.324	0.171	-0.600	
tstats	2.135	1.823	2.672	-10.592	5.221	3.115	-5.691	-3.393	4.936	3.147	-19.563	
DP	2.234	1.244	5.945	-3.674	0.329	0.179	-0.449	-0.169	0.322	0.171	-0.600	
tstats	2.085	1.281	2.878	-10.570	5.246	3.196	-5.730	-3.394	4.919	3.139	-19.575	
TERM	2.331	2.122	5.571	-3.665	0.330	0.179	-0.445	-0.173	0.322	0.170	-0.600	
tstats	2.199	2.147	2.598	-10.550	5.272	3.180	-5.669	-3.463	4.912	3.118	-19.560	
DEF	2.024	0.688	6.081	-3.673	0.329	0.179	-0.446	-0.169	0.321	0.171	-0.600	
tstats	1.846	0.692	2.986	-10.576	5.253	3.184	-5.692	-3.390	4.890	3.134	-19.580	
INFL	2.498	-1.660	6.171	-3.684	0.327	0.179	-0.446	-0.168	0.325	0.175	-0.599	
tstats	2.281	-1.716	3.149	-10.641	5.241	3.198	-5.703	-3.376	4.967	3.219	-19.643	
REGD	2.219	0.267	6.188	-3.673	0.329	0.179	-0.445	-0.169	0.324	0.171	-0.600	
tstats	2.039	0.259	3.090	-10.586	5.256	3.186	-5.686	-3.396	4.942	3.141	-19.595	
RREL	1.989	-2.186	5.663	-3.677	0.328	0.180	-0.447	-0.169	0.317	0.172	-0.600	
tstats	1.861	-2.230	2.677	-10.579	5.237	3.189	-5.684	-3.388	4.819	3.169	-19.580	
X_{1t}, X_{2t}	ϕ_0	ϕ_1	ϕ_2	σ_v	δ_0	δ_1	δ_{21}	δ_{22}	δ_{23}	δ_{24}	δ_{25}	σ_η
SURP,CAYA	2.604	-3.337	1.788	4.589	-3.671	0.329	0.180	-0.455	-0.168	0.323	0.178	-0.600
tstats	2.513	-3.148	1.895	1.876	-10.550	5.246	3.200	-5.796	-3.372	4.939	3.268	-19.551
SURP,TERM	2.595	-3.002	1.589	4.771	-3.663	0.330	0.183	-0.453	-0.171	0.323	0.175	-0.600
tstats	2.494	-2.771	1.625	2.028	-10.544	5.275	3.246	-5.761	-3.433	4.934	3.214	-19.563
SURP,RREL	2.321	-3.006	-1.688	4.822	-3.669	0.330	0.186	-0.456	-0.169	0.318	0.177	-0.600
tstats	2.215	-2.775	-1.750	2.069	-10.552	5.261	3.296	-5.811	-3.386	4.850	3.256	-19.570

Table 2.4: Time-series specification test: time-varying RRA (Model2)

The Table shows the estimation results of several different specifications of conditional CAPM without positivity restriction. Note: $r_{M,t+1}$ is the CRSP value-weighted stock return minus lagged 3-month treasury bill rate; Realized volatility($\hat{\sigma}_{m,t+1}$) is defined in the data appendix; CAYA is the consumption-wealth ratio without a look ahead bias; DEF is the yield spread between Baa- and Aaa-rated corporate bonds; DP is the deseasonalized dividend price ratio; INFL is the quarterly inflation rate measured by GDP deflator; REGD is the quarterly real GDP growth;RREL is the difference between the 3 month treasury bill rate and its average in the previous 4 quarters; SURP is the surplus consumption ratio; and TERM is the yield spread between 10-year treasury bonds and 3-month treasury bills. All exogenous variables($X_{1,t}$, $X_{2,t}$, and u_t) are normalized to have means of zero and standard deviations of one to facilitate the interpretation.

$$\text{Model : } r_{M,t+1} = \gamma_{t+1}(E_t(\ln \hat{\sigma}_{m,t+1}) + \frac{1}{2}\sigma_\eta^2) + \varepsilon_{t+1},$$

$$\gamma_{t+1} = \phi_0 + \phi_1 X_{1,t} + \phi_2 X_{2,t} + v_{t+1}, \ln \hat{\sigma}_{m,t+1} = \delta_0 + \delta_1 \ln \hat{\sigma}_{m,t} + \delta'_2 u_t + \eta_{t+1},$$

where $\varepsilon_{t+1}|\psi_t \sim N(0, h_{t+1})$, $v_{t+1} \sim N(0, \sigma_\eta^2)$, $\eta_{t+1} \sim N(0, \sigma_\eta^2)$, $h_{t+1} = \omega + \beta_1 h_t + \beta_2 \varepsilon_t^2$

$X_{1,t}$	ϕ_0	ϕ_1	ϕ_2	δ_0	δ_1	δ_2	δ_{21}	δ_{22}	δ_{23}	δ_{24}	δ_{25}	σ_η	ω	β_1	β_2
SURP	2.295	-3.835		-3.525	0.357	0.183	-0.461	-0.150	0.316	0.187	-0.598	0.003	0.461	0.461	0.178
tstats	2.302	-3.106		-9.399	5.253	2.891	-5.243	-2.819	4.430	3.115	-19.674	1.927	1.999	1.999	1.497
CAYA	1.708	2.087		-3.573	0.348	0.169	-0.449	-0.157	0.348	0.174	-0.597	0.003	0.592	0.592	0.045
tstats	1.771	2.198		-9.470	5.092	2.665	-5.054	-2.900	4.740	2.880	-19.679	1.155	1.827	1.827	0.431
DP	1.838	1.245		-3.527	0.357	0.170	-0.446	-0.151	0.332	0.172	-0.597	0.002	0.576	0.576	0.123
tstats	1.754	1.529		-9.310	5.193	2.712	-5.074	-2.806	4.602	2.832	-19.679	1.910	3.188	3.188	1.238
TERM	1.980	2.063		-3.619	0.340	0.175	-0.452	-0.176	0.342	0.179	-0.597	0.002	0.575	0.575	0.104
tstats	1.990	2.510		-9.668	5.012	2.765	-5.163	-3.243	4.713	2.956	-19.685	1.797	2.848	2.848	1.310
DEF	1.178	1.068		-3.544	0.354	0.174	-0.448	-0.151	0.328	0.181	-0.597	0.002	0.580	0.580	0.121
tstats	0.973	1.140		-9.359	5.153	2.735	-5.054	-2.805	4.513	2.953	-19.684	1.893	3.216	3.216	1.218
INFL	2.280	-0.995		-3.576	0.348	0.166	-0.437	-0.158	0.330	0.173	-0.597	0.002	0.576	0.576	0.154
tstats	2.001	-1.131		-9.454	5.073	2.644	-4.942	-2.910	4.519	2.853	-19.686	2.133	3.743	3.743	1.656
REGD	1.860	-0.016		-3.562	0.350	0.168	-0.442	-0.154	0.333	0.175	-0.597	0.002	0.584	0.584	0.145
tstats	1.685	-0.068		-9.409	5.104	2.671	-5.004	-2.847	4.584	2.883	-19.687	2.054	3.681	3.681	1.460
RREL	1.307	-1.813		-3.610	0.342	0.172	-0.447	-0.160	0.346	0.171	-0.597	0.002	0.631	0.631	0.077
tstats	1.253	-2.202		-9.540	4.981	2.724	-5.085	-2.951	4.758	2.822	-19.681	1.689	3.205	3.205	0.917
$X_{1,t}, X_{2,t}$	ϕ_0	ϕ_1	ϕ_2	δ_0	δ_1	δ_2	δ_{21}	δ_{22}	δ_{23}	δ_{24}	δ_{25}	σ_η	ω	β_1	β_2
SURP,CAYA	2.234	-3.239	2.290	-3.513	0.359	0.185	-0.462	-0.149	0.325	0.188	-0.597	0.004	0.423	0.423	0.000
tstats	2.450	-3.190	2.968	-9.378	5.293	2.979	-5.296	-2.804	4.501	3.183	-19.677	0.087	0.064	0.064	0.478
SURP,TERM	2.298	-3.087	1.868	-3.594	0.345	0.185	-0.468	-0.174	0.332	0.190	-0.597	0.003	0.490	0.490	0.122
tstats	2.361	-2.729	2.140	-9.552	5.052	2.944	-5.375	-3.171	4.573	3.158	-19.683	1.789	1.979	1.979	1.252
SURP,RREL	1.723	-3.017	-1.496	-3.550	0.352	0.188	-0.466	-0.158	0.324	0.183	-0.597	0.002	0.583	0.583	0.075
tstats	1.701	-2.627	-1.647	-9.405	5.150	2.984	-5.327	-2.936	4.494	3.042	-19.677	1.472	2.212	2.212	0.776

Table 2.5: Time-series specification test: time-varying RRA(Model3)

The Table shows the estimation results of several different specifications of Conditional CAPM without positivity restriction. Note: $r_{M,t+1}$ is the CRSP value-weighted stock return minus lagged 3-month treasury bill rate; CAYA is the consumption-wealth ratio without a look ahead bias; DEF is the yield spread between Baa- and Aaa-rated corporate bonds; DP is the deseasonalized dividend price ratio; INFL is the quarterly inflation rate measured by GDP deflator; REGD is the quarterly real GDP growth; RREL is the difference between the 3 month treasury bill rate and its average in the previous 4 quarters; SURP is the surplus consumption ratio; and TERM is the yield spread between 10-year treasury bonds and 3-month treasury bills. All exogenous variables (X_{1t}, X_{2t} , and u_t) are normalized to have means of zero and standard deviations of one to facilitate the interpretation.

$$\text{Model : } r_{M,t+1} = \gamma_{t+1}h_{t+1} + \varepsilon_{t+1}, \gamma_{t+1} = \phi_0 + \phi_1 X_{1t} + \phi_2 X_{2t} + v_{t+1}$$

$$\text{where } \varepsilon_{t+1} | \psi_t \sim N(0, h_{t+1}), v_{t+1} \sim N(0, 0.01), h_{t+1} = \omega + \beta_1 h_t + \beta_2 \varepsilon_t^2$$

	X_{1t}	ϕ_0	ϕ_1	ω	β_1	β_2	
SURP		2.271	-2.026	0.002	0.571	0.143	
tstats		2.589	-2.230	2.093	3.551	1.652	
CAYA		2.183	1.109	0.002	0.600	0.155	
tstats		2.571	1.344	2.160	4.397	1.697	
DP		2.275	0.828	0.002	0.583	0.172	
tstats		2.664	1.005	2.251	4.349	1.801	
TERM		2.133	1.178	0.002	0.562	0.141	
tstats		2.422	1.256	2.119	3.577	1.552	
DEF		2.211	0.675	0.002	0.579	0.166	
tstats		2.566	0.818	2.248	4.213	1.739	
INFL		2.355	-0.854	0.002	0.558	0.172	
tstats		2.725	-1.077	2.228	3.872	1.843	
REGD		2.281	-0.307	0.002	0.576	0.180	
tstats		2.692	-0.410	2.283	4.340	1.901	
RREL		2.070	-1.976	0.002	0.593	0.132	
tstats		2.372	-2.177	2.162	3.989	1.572	
	X_{1t}, X_{2t}	ϕ_0	ϕ_1	ϕ_2	ω	β_1	β_2
SURP,RREL		2.089	-1.898	-1.825	0.002	0.586	0.116
tstats		2.349	-2.075	-2.019	2.003	3.439	1.495

Table 2.6: Time-series specification test: time-varying RRA(Model4)-I

The Table shows the estimation results of several different specifications of conditional CAPM with positivity restriction. Note: $r_{M,t+1}$ is the CRSP value-weighted stock return minus lagged 3-month treasury bill rate; Realized volatility($\hat{\sigma}_{m,t+1}$) is defined in the data appendix; CAYA is the consumption-wealth ratio without a look ahead bias; DEF is the yield spread between Baa- and Aaa-rated corporate bonds; DP is the deseasonalized dividend price ratio; INFL is the quarterly inflation rate measured by GDP deflator; REGD is the quarterly real GDP growth;RREL is the difference between the 3 month treasury bill rate and its average in the previous 4 quarters; SURP is the surplus consumption ratio; and TERM is the yield spread between 10-year treasury bonds and 3-month treasury bills. All exogenous variables(x_t and u_t) are normalized to have means of zero and standard deviations of one to facilitate the interpretation.

$$\text{Model : } r_{M,t+1} = \exp(\gamma_{t+1})(E_t(\ln \hat{\sigma}_{m,t+1}) + \frac{1}{2}\sigma_\eta^2) + \varepsilon_{t+1},$$

$$\gamma_{t+1} = \phi_1 x_t + v_{t+1}, \ln \hat{\sigma}_{m,t+1} = \delta_0 + \delta_1 \ln \hat{\sigma}_{m,t} + \delta_2' u_t + \eta_{t+1},$$

where $\varepsilon_{t+1}|\psi_t \sim N(0, E_t(\hat{\sigma}_{m,t+1}))$, $v_{t+1} \sim N(0, 0.01)$, $\eta_{t+1} \sim N(0, \sigma_\eta^2)$

	x_t	ϕ_1	δ_0	δ_1	δ_{21}	δ_{22}	δ_{23}	δ_{24}	δ_{25}	σ_η
SURP	-1.461	-3.594	0.339	0.195	-0.475	-0.171	0.324	0.188	-0.608	
tstats	-7.877	-10.235	5.324	3.385	-5.924	-3.378	4.862	3.359	-19.461	
CAYA	1.218	-3.592	0.338	0.174	-0.451	-0.172	0.331	0.171	-0.610	
tstats	4.654	-10.171	5.283	2.994	-5.600	-3.361	4.913	3.079	-19.386	
DP	1.052	-3.578	0.341	0.191	-0.466	-0.172	0.327	0.175	-0.610	
tstats	4.088	-10.128	5.319	3.309	-5.807	-3.378	4.870	3.132	-19.370	
TERM	0.864	-3.576	0.341	0.187	-0.458	-0.178	0.333	0.175	-0.611	
tstats	2.231	-10.099	5.304	3.234	-5.704	-3.469	4.942	3.120	-19.328	
DEF	0.600	-3.569	0.342	0.189	-0.460	-0.172	0.323	0.176	-0.611	
tstats	3.226	-10.072	5.325	3.270	-5.731	-3.370	4.763	3.150	-19.344	
INFL	-1.686	-3.603	0.336	0.188	-0.461	-0.172	0.334	0.184	-0.609	
tstats	-4.969	-10.207	5.256	3.272	-5.753	-3.369	4.968	3.288	-19.398	
REGD	-0.446	-3.580	0.340	0.186	-0.459	-0.174	0.333	0.175	-0.611	
tstats	-0.949	-10.082	5.277	3.225	-5.706	-3.395	4.937	3.120	-19.309	
RREL	-0.749	-3.558	0.345	0.195	-0.466	-0.178	0.317	0.177	-0.610	
tstats	-7.237	-10.049	5.367	3.367	-5.794	-3.489	4.680	3.165	-19.384	

Table 2.7: Time-series specification test: time-varying RRA(Model4)-II

The Table shows the estimation results of several different specifications of conditional CAPM with positivity restriction. Note: $r_{M,t+1}$ is the CRSP value-weighted stock return minus lagged 3-month treasury bill rate; Realized volatility($\hat{\sigma}_{m,t+1}$) is defined in the data appendix; CAYA is the consumption-wealth ratio without a look ahead bias; DEF is the yield spread between Baa- and Aaa-rated corporate bonds; DP is the deseasonalized dividend price ratio; INFL is the quarterly inflation rate measured by GDP deflator; REGD is the quarterly real GDP growth;RREL is the difference between the 3 month treasury bill rate and its average in the previous 4 quarters; SURP is the surplus consumption ratio; and TERM is the yield spread between 10-year treasury bonds and 3-month treasury bills. All exogenous variables(x_t ,SURP $_t$, and u_t) are normalized to have means of zero and standard deviations of one to facilitate the interpretation.

$$\text{Model : } r_{M,t+1} = \exp(\gamma_{t+1})(E_t(\ln \hat{\sigma}_{m,t+1}) + \frac{1}{2}\sigma_\eta^2) + \varepsilon_{t+1},$$

$$\gamma_{t+1} = \phi_1(\text{SURP})_t + \phi_2 x_t + v_{t+1}, \ln \hat{\sigma}_{m,t+1} = \delta_0 + \delta_1 \ln \hat{\sigma}_{m,t} + \delta_2' u_t + \eta_{t+1},$$

where $\varepsilon_{t+1}|\psi_t \sim N(0, E_t(\hat{\sigma}_{m,t+1}))$, $v_{t+1} \sim N(0, 0.01)$, $\eta_{t+1} \sim N(0, \sigma_\eta^2)$

x_t		ϕ_1	ϕ_2	δ_0	δ_1	δ_{21}	δ_{22}	δ_{23}	δ_{24}	δ_{25}	σ_η
SURP,	CAYA	-1.205	1.009	-3.680	0.324	0.180	-0.476	-0.171	0.337	0.194	-0.607
	tstats	-3.883	2.171	-10.445	5.072	3.101	-5.926	-3.362	5.053	3.471	-19.514
SURP,	DP	-1.629	-0.309	-3.604	0.337	0.192	-0.470	-0.172	0.331	0.188	-0.607
	tstats	-8.411	-1.264	-10.287	5.315	3.342	-5.873	-3.393	4.959	3.367	-19.497
SURP,	TERM	-1.390	0.404	-3.579	0.342	0.199	-0.479	-0.178	0.317	0.188	-0.608
	tstats	-6.486	1.497	-10.174	5.361	3.455	-5.956	-3.491	4.730	3.359	-19.474
SURP,	DEF	-1.597	-0.269	-3.620	0.334	0.189	-0.474	-0.172	0.337	0.190	-0.607
	tstats	-8.917	-1.096	-10.329	5.266	3.302	-5.931	-3.395	5.027	3.400	-19.498
SURP,	INFL	-1.427	-0.614	-3.639	0.331	0.190	-0.479	-0.170	0.337	0.200	-0.606
	tstats	-6.400	-1.971	-10.403	5.230	3.325	-6.011	-3.364	5.055	3.564	-19.547
SURP,	RREL	-1.285	-0.221	-3.583	0.341	0.199	-0.478	-0.176	0.317	0.187	-0.608
	tstats	-4.624	-1.324	-10.183	5.345	3.452	-5.960	-3.456	4.723	3.344	-19.455

Table 2.8: Time-series specification test: time-varying RRA(Model5)-I

The Table shows the estimation results of several different specifications of Conditional CAPM with positivity restriction. Note: $r_{M,t+1}$ is the CRSP value-weighted stock return minus lagged 3-month treasury bill rate; Realized volatility($\hat{\sigma}_{m,t+1}$) is defined in the data appendix; CAYA is the consumption-wealth ratio without a look ahead bias; DEF is the yield spread between Baa- and Aaa-rated corporate bonds; DP is the deseasonalized dividend price ratio; INFL is the quarterly inflation rate measured by GDP deflator; REGD is the quarterly real GDP growth;RREL is the difference between the 3 month treasury bill rate and its average in the previous 4 quarters; SURP is the surplus consumption ratio; and TERM is the yield spread between 10-year treasury bonds and 3-month treasury bills. All exogenous variables(x_t and u_t) are normalized to have means of zero and standard deviations of one to facilitate the interpretation.

$$\text{Model : } r_{M,t+1} = \exp(\gamma_{t+1})(E_t(\ln \hat{\sigma}_{m,t+1}) + \frac{1}{2}\sigma_\eta^2) + \varepsilon_{t+1},$$

$$\gamma_{t+1} = \phi_1 x_t + v_{t+1}, \ln \hat{\sigma}_{m,t+1} = \delta_0 + \delta_1 \ln \hat{\sigma}_{m,t} + \delta_2' u_t + \eta_{t+1},$$

where $\varepsilon_{t+1}|\psi_t \sim N(0, h_{t+1})$, $v_{t+1} \sim N(0, 0.01)$, $\eta_{t+1} \sim N(0, \sigma_\eta^2)$, $h_{t+1} = \omega + \beta_1 h_t + \beta_2 \varepsilon_t^2$

	x_t	ϕ_1	δ_0	δ_1	δ_{21}	δ_{22}	δ_{23}	δ_{24}	δ_{25}	σ_η	ω	β_1	β_2
SURP	-1.464	-3.533	0.355	0.177	-0.455	-0.152	0.319	0.179	-0.598	0.002	0.525	0.183	
tstats	-6.682	-9.375	5.205	2.807	-5.168	-2.819	4.453	2.983	-19.660	2.194	3.061	1.562	
CAYA	1.278	-3.550	0.353	0.158	-0.438	-0.154	0.342	0.170	-0.597	0.002	0.630	0.019	
tstats	5.225	-9.436	5.169	2.501	-4.967	-2.844	4.708	2.852	-19.680	0.738	1.297	0.210	
DP	1.043	-3.529	0.357	0.175	-0.449	-0.147	0.327	0.177	-0.597	0.002	0.584	0.117	
tstats	4.067	-9.331	5.201	2.760	-5.126	-2.739	4.551	2.942	-19.691	1.909	3.258	1.246	
TERM	0.846	-3.599	0.343	0.168	-0.448	-0.164	0.339	0.180	-0.598	0.002	0.581	0.132	
tstats	1.928	-9.515	5.006	2.661	-5.083	-3.005	4.657	2.977	-19.661	2.000	3.458	1.515	
DEF	0.584	-3.545	0.354	0.172	-0.446	-0.148	0.325	0.179	-0.597	0.002	0.568	0.137	
tstats	3.361	-9.368	5.154	2.731	-5.097	-2.748	4.481	2.980	-19.686	2.005	3.311	1.411	
INFL	-1.699	-3.576	0.348	0.174	-0.442	-0.158	0.326	0.182	-0.598	0.002	0.569	0.156	
tstats	-3.798	-9.461	5.071	2.770	-5.057	-2.910	4.474	3.009	-19.671	2.081	3.503	1.677	
REGD	-0.526	-3.567	0.349	0.172	-0.448	-0.154	0.333	0.180	-0.597	0.002	0.594	0.127	
tstats	-1.701	-9.434	5.097	2.720	-5.104	-2.841	4.582	2.979	-19.687	1.976	3.587	1.453	
RREL	-0.718	-3.570	0.349	0.173	-0.448	-0.162	0.335	0.175	-0.597	0.002	0.605	0.112	
tstats	-6.528	-9.406	5.071	2.744	-5.103	-2.980	4.607	2.900	-19.680	1.928	3.540	1.320	

Table 2.9: Time-series specification test: the time-varying RRA(Model5)-II

The Table shows the estimation results of several different specifications of Conditional CAPM with positivity restriction. Note: $r_{M,t+1}$ is the CRSP value-weighted stock return minus lagged 3-month treasury bill rate; Realized volatility($\hat{\sigma}_{m,t+1}$) is defined in the data appendix; CAYA is the consumption-wealth ratio without a look ahead bias; DEF is the yield spread between Baa- and Aaa-rated corporate bonds; DP is the deseasonalized dividend price ratio; INFL is the quarterly inflation rate measured by GDP deflator; REGD is the quarterly real GDP growth;RREL is the difference between the 3 month treasury bill rate and its average in the previous 4 quarters; SURP is the surplus consumption ratio; and TERM is the yield spread between 10-year treasury bonds and 3-month treasury bills. All exogenous variables(X_{1t}, X_{2t} , $SURP_t$, and u_t) are normalized to have means of zero and standard deviations of one to facilitate the interpretation.

Model : $r_{M,t+1} = \exp(\gamma_{t+1})(E_t(\ln \hat{\sigma}_{m,t+1}) + \frac{1}{2}\sigma_\eta^2) + \varepsilon_{t+1}$,

$$\gamma_{t+1} = \phi_1(\text{SURP})_t + \phi_2 X_{1t} + \phi_3 X_{2t} + v_{t+1}, \ln \hat{\sigma}_{m,t+1} = \delta_0 + \delta_1 \ln \hat{\sigma}_{m,t} + \delta_2' u_t + \eta_{t+1},$$

where $\varepsilon_{t+1}|\psi_t \sim N(0, h_{t+1})$, $v_{t+1} \sim N(0, \sigma_\eta^2)$, $h_{t+1} = \omega + \beta_1 h_t + \beta_2 \varepsilon_t^2$

	X_{1t}	ϕ_1	ϕ_2	ϕ_3	δ_0	δ_1	δ_{21}	δ_{22}	δ_{23}	δ_{24}	δ_{25}	σ_η	ω	β_1	β_2
SURP,	CAYA	-1.032	1.280		-3.605	0.343	0.165	-0.456	-0.155	0.346	0.185	0.597	0.002	0.650	0.047
	tstats	-4.012	4.667		-9.666	5.071	2.607	-5.190	-2.878	4.811	3.132	19.691	1.123	2.247	0.559
SURP,	DP	-1.641	-0.317		-3.547	0.353	0.174	-0.450	-0.154	0.327	0.178	-0.597	0.002	0.529	0.183
	tstats	-7.816	-1.267		-9.457	5.198	2.768	-5.128	-2.849	4.533	2.959	-19.693	2.175	3.067	1.552
SURP,	TERM	-1.311	0.420		-3.540	0.354	0.180	-0.458	-0.162	0.321	0.179	-0.598	0.002	0.540	0.157
	tstats	-4.522	1.311		-9.355	5.165	2.854	-5.197	-2.975	4.424	2.979	-19.659	2.083	3.021	1.526
SURP,	DEF	-1.618	-0.301		-3.563	0.350	0.171	-0.454	-0.155	0.332	0.180	-0.597	0.002	0.546	0.168
	tstats	-8.104	-0.905		-9.511	5.160	2.714	-5.191	-2.863	4.597	2.986	-19.686	2.120	3.171	1.483
SURP,	INFL	-1.457	-0.692		-3.580	0.347	0.169	-0.454	-0.153	0.335	0.186	-0.597	0.002	0.542	0.172
	tstats	-6.551	-2.185		-9.604	5.142	2.709	-5.220	-2.839	4.656	3.117	-19.714	2.091	3.132	1.569
SURP,	RREL	-1.156	-0.254		-3.542	0.354	0.181	-0.458	-0.160	0.322	0.178	-0.598	0.002	0.546	0.152
	tstats	-2.717	-1.163		-9.364	5.159	2.861	-5.196	-2.950	4.459	2.954	-19.651	2.090	3.076	1.441
	X_{1t}, X_{2t}	ϕ_1	ϕ_2	ϕ_3	δ_0	δ_1	δ_{21}	δ_{22}	δ_{23}	δ_{24}	δ_{25}	σ_η	ω	β_1	β_2
SURP	INFL,CAYA	-1.430	0.187	-0.660	-3.591	0.345	0.166	-0.456	-0.153	0.337	0.188	0.597	0.002	0.553	0.160
	tstats	-6.065	0.500	-1.880	-9.629	5.105	2.636	-5.222	-2.835	4.676	3.133	19.702	2.054	3.158	1.508

Table 2.10: Time-series specification test: time-varying RRA(Model6)

The Table shows the estimation results of several different specifications of conditional CAPM with positivity restriction. Note: $r_{M,t+1}$ is the CRSP value-weighted stock return minus lagged 3-month treasury bill rate; CAYA is the consumption-wealth ratio without a look ahead bias; DEF is the yield spread between Baa- and Aaa-rated corporate bonds; DP is the deseasonalized dividend price ratio; INFL is the quarterly inflation rate measured by GDP deflator; REGD is the quarterly real GDP growth; RREL is the difference between the 3 month treasury bill rate and its average in the previous 4 quarters; SURP is the surplus consumption ratio; and TERM is the yield spread between 10-year treasury bonds and 3-month treasury bills. All exogenous variables(X_{1t} and X_{2t}) are normalized to have means of zero and standard deviations of one to facilitate the interpretation.

Model : $r_{M,t+1} = \exp(\gamma_{t+1})h_{t+1} + \varepsilon_{t+1}$, $\gamma_{t+1} = \phi_1 X_{1t} + \phi_2 X_{2t} + v_{t+1}$,

where $\varepsilon_{t+1}|\psi_t \sim N(0, h_{t+1})$, $v_{t+1} \sim N(0, 0.01)$, $h_{t+1} = \omega + \beta_1 h_t + \beta_2 \varepsilon_t^2$

Panel A	X_{1t}				Panel B	X_{1t}		X_{2t}			
	ϕ_1	ω	β_1	β_2		ϕ_1	ϕ_2	ω	β_1	β_2	
SURP	-1.216	0.002	0.585	0.139							
tstats	-5.822	2.120	3.771	1.705							
CAYA	1.074	0.002	0.619	0.127	SURP,	CAYA	-0.996	0.733	0.002	0.632	0.111
tstats	3.783	2.004	4.204	1.498	tstats	tstats	-1.806	0.865	1.911	3.898	1.409
DP	1.018	0.002	0.599	0.146	SURP,	DP	-1.400	-0.361	0.002	0.574	0.161
tstats	4.104	2.127	4.182	1.650	tstats	tstats	-6.027	-1.005	2.178	3.857	1.834
TERM	0.670	0.002	0.573	0.143	SURP,	TERM	-1.144	0.423	0.002	0.580	0.137
tstats	1.617	2.087	3.601	1.621	tstats	tstats	-5.035	1.450	2.087	3.621	1.713
DEF	0.653	0.002	0.592	0.140	SURP,	DEF	-1.269	-0.079	0.002	0.582	0.145
tstats	4.402	2.077	3.919	1.607	tstats	tstats	-5.398	-0.363	2.149	3.801	1.738
INFL	-1.347	0.002	0.555	0.156	SURP,	INFL	-1.186	-0.329	0.002	0.580	0.145
tstats	-3.796	2.146	3.448	1.777	tstats	tstats	-4.474	-0.944	2.147	3.798	1.831
REGD	-0.618	0.002	0.588	0.157	SURP,	REGD	-1.259	0.063	0.002	0.583	0.143
tstats	-2.798	2.138	4.058	1.771	tstats	tstats	-5.256	0.312	2.136	3.789	1.710
RREL	-0.705	0.002	0.591	0.137	SURP,	RREL	-0.973	-0.300	0.002	0.576	0.134
tstats	-7.033	2.036	3.768	1.611	tstats	tstats	-2.909	-1.626	2.017	3.411	1.648

Table 2.11: Time-series specification test: time-varying RRA(Model1) and a hedging component

The Table shows the estimation results of several different specifications of conditional ICAPM without positivity restriction. Note: $r_{M,t+1}$ is the CRSP value-weighted stock return minus lagged 3-month treasury bill rate; Realized volatility($\hat{\sigma}_{m,t+1}$) is defined in the data appendix; CAYA is the consumption-wealth ratio without a look ahead bias; DEF is the yield spread between Baa- and Aaa-rated corporate bonds; DP is the deseasonalized dividend price ratio; INFL is the quarterly inflation rate measured by GDP deflator; REGD is the quarterly real GDP growth;RREL is the difference between the 3 month treasury bill rate and its average in the previous 4 quarters; SURP is the surplus consumption ratio; and TERM is the yield spread between 10-year treasury bonds and 3-month treasury bills. All exogenous variables(z_t , $SURP_t$, $CAYA_t$, and u_t) are normalized to have means of zero and standard deviations of one to facilitate the interpretation.

Model : $r_{M,t+1} = \gamma_{t+1}(E_t(\ln \hat{\sigma}_{m,t+1}) + \frac{1}{2}\sigma_\eta^2) + \alpha_1 z_t + \varepsilon_{t+1}$,

$$\gamma_{t+1} = \phi_0 + \phi_1(\text{SURP})_t + \phi_2(\text{CAYA})_t + v_{t+1}, \ln \hat{\sigma}_{m,t+1} = \delta_0 + \delta_1 \ln \hat{\sigma}_{m,t} + \delta_2' u_t + \eta_{t+1},$$

where $\varepsilon_{t+1}|\psi_t \sim N(0, E_t(\hat{\sigma}_{m,t+1}))$, $v_{t+1} \sim N(0, \sigma_v^2)$, $\eta_{t+1} \sim N(0, \sigma_\eta^2)$

z_t	ϕ_0	ϕ_1	ϕ_2	σ_v	α_1	δ_0	δ_1	δ_{21}	δ_{22}	δ_{23}	δ_{24}	δ_{25}	σ_η
CAYA	2.632	-3.353	2.184	4.552	0.003	-3.672	0.329	0.180	-0.453	-0.167	0.323	0.176	-0.600
tstats	2.502	-3.153	0.895	1.841	0.176	-10.557	5.246	3.199	-5.732	-3.362	4.946	3.228	-19.544
DP	2.628	-3.412	2.081	4.509	0.003	-3.670	0.329	0.179	-0.451	-0.167	0.323	0.177	-0.600
tstats	2.534	-3.182	1.775	1.810	0.421	-10.550	5.253	3.176	-5.728	-3.350	4.952	3.263	-19.544
TERM	2.559	-3.117	1.668	4.395	-0.006	-3.656	0.332	0.179	-0.453	-0.168	0.321	0.177	-0.600
tstats	2.480	-2.908	1.766	1.730	-1.086	-10.477	5.276	3.168	-5.764	-3.360	4.877	3.248	-19.542
DEF	2.595	-3.390	1.850	4.534	0.002	-3.674	0.328	0.181	-0.455	-0.168	0.326	0.178	-0.600
tstats	2.509	-3.184	1.939	1.835	0.443	-10.550	5.235	3.213	-5.800	-3.369	4.956	3.278	-19.546
INFL	2.496	-2.705	2.080	4.000	0.010	-3.664	0.330	0.175	-0.452	-0.165	0.326	0.186	-0.600
tstats	2.451	-2.464	2.206	1.456	1.763	-10.448	5.222	3.109	-5.750	-3.306	4.962	3.387	-19.527
REGD	2.598	-3.369	1.812	4.535	-0.002	-3.672	0.329	0.181	-0.456	-0.168	0.324	0.177	-0.600
tstats	2.513	-3.174	1.920	1.836	-0.394	-10.555	5.245	3.216	-5.805	-3.372	4.945	3.257	-19.545
RREL	2.584	-2.993	1.671	4.277	0.010	-3.661	0.331	0.182	-0.458	-0.168	0.314	0.184	-0.601
tstats	2.509	-2.799	1.784	1.658	1.676	-10.461	5.248	3.216	-5.823	-3.360	4.747	3.350	-19.543

Table 2.12: Time-series specification test: time-varying RRA (Model2) and a hedging component

The Table shows the estimation results of several different specifications of conditional ICAPM without positivity restriction. Note: $r_{M,t+1}$ is the CRSP value-weighted stock return minus lagged 3-month treasury bill rate; Realized volatility($\hat{\sigma}_{m,t+1}$) is defined in the data appendix; CAYA is the consumption-wealth ratio without a look ahead bias; DEF is the yield spread between Baa- and Aaa-rated corporate bonds; DP is the deseasonalized dividend price ratio; INFL is the quarterly inflation rate measured by GDP deflator; REGD is the quarterly real GDP growth;RREL is the difference between the 3 month treasury bill rate and its average in the previous 4 quarters; SURP is the surplus consumption ratio; and TERM is the yield spread between 10-year treasury bonds and 3-month treasury bills. All exogenous variables(z_t , $SURP_t$, CAYA $_t$ and u_t) are normalized to have means of zero and standard deviations of one to facilitate the interpretation.

Model : $r_{M,t+1} = \gamma_{t+1}(E_t(\ln \hat{\sigma}_{m,t+1}) + \frac{1}{2}\sigma_\eta^2) + \alpha_1 z_t + \varepsilon_{t+1}$,

$$\gamma_{t+1} = \phi_0 + \phi_1(\text{SURP})_t + \phi_2(\text{CAYA})_t + v_{t+1}, \ln \hat{\sigma}_{m,t+1} = \delta_0 + \delta_1 \ln \hat{\sigma}_{m,t} + \delta_2' u_t + \eta_{t+1},$$

where $\varepsilon_{t+1}|\psi_t \sim N(0, h_{t+1})$, $v_{t+1} \sim N(0, \sigma_\eta^2)$, $\eta_{t+1} \sim N(0, \sigma_\eta^2)$, $h_{t+1} = \omega + \beta_1 h_t + \beta_2 \varepsilon_t^2$

z_t	ϕ_0	ϕ_1	ϕ_2	α_1	δ_0	δ_1	δ_{21}	δ_{22}	δ_{23}	δ_{24}	δ_{25}	σ_η	ω	β_1	β_2
CAYA	2.097	-2.190	6.912	0.041	-3.619	0.340	0.169	-0.459	-0.162	0.338	0.187	-0.597	0.002	0.694	0.081
tstats	2.472	-2.413	1.104	1.024	-9.554	4.951	2.675	-5.218	-2.980	4.638	3.097	-19.697	1.844	4.479	1.038
DP	2.159	-2.086	1.407	0.003	-3.619	0.340	0.169	-0.459	-0.162	0.338	0.187	-0.597	0.002	0.606	0.119
tstats	2.461	-2.250	1.323	0.392	-9.555	4.951	2.672	-5.212	-2.981	4.636	3.097	-19.697	1.945	3.744	1.428
TERM	2.080	-1.931	1.079	-0.008	-3.619	0.340	0.169	-0.459	-0.162	0.338	0.187	-0.597	0.002	0.592	0.108
tstats	2.346	-2.094	1.222	-1.351	-9.551	4.949	2.674	-5.214	-2.982	4.636	3.097	-19.698	1.889	3.310	1.355
DEF	2.143	-2.028	1.158	-0.002	-3.619	0.340	0.169	-0.459	-0.162	0.338	0.187	-0.597	0.002	0.600	0.111
tstats	2.421	-2.205	1.322	-0.307	-9.553	4.950	2.671	-5.213	-2.980	4.636	3.099	-19.697	1.921	3.532	1.331
INFL	2.142	-1.557	1.354	0.010	-3.619	0.340	0.169	-0.459	-0.162	0.338	0.187	-0.597	0.002	0.593	0.136
tstats	2.458	-1.593	1.532	1.467	-9.556	4.952	2.673	-5.218	-2.981	4.638	3.098	-19.697	2.060	3.891	1.575
REGD	2.149	-2.063	1.183	-0.001	-3.619	0.340	0.169	-0.459	-0.162	0.338	0.187	-0.597	0.002	0.601	0.116
tstats	2.438	-2.227	1.350	-0.160	-9.553	4.950	2.672	-5.214	-2.980	4.635	3.098	-19.697	1.955	3.627	1.405
RREL	2.129	-1.866	1.190	0.010	-3.619	0.340	0.169	-0.459	-0.162	0.338	0.187	-0.597	0.002	0.612	0.090
tstats	2.341	-2.002	1.326	1.813	-9.553	4.950	2.673	-5.212	-2.980	4.635	3.098	-19.697	1.814	3.296	1.151

Table 2.13: Time-series specification test: time-varying RRA(Model3) and a hedging component

The Table shows the estimation results of several different specifications of conditional ICAPM without positivity restriction. Note: $r_{M,t+1}$ is the CRSP value-weighted stock return minus lagged 3-month treasury bill rate; CAYA is the consumption-wealth ratio without a look ahead bias; DEF is the yield spread between Baa- and Aaa-rated corporate bonds; DP is the deseasonalized dividend price ratio; INFL is the quarterly inflation rate measured by GDP deflator; REGD is the quarterly real GDP growth; RREL is the difference between the 3 month treasury bill rate and its average in the previous 4 quarters; SURP is the surplus consumption ratio; and TERM is the yield spread between 10-year treasury bonds and 3-month treasury bills. All exogenous variables($SURP_t, CAYA_t$, and z_t) are normalized to have means of zero and standard deviations of one to facilitate the interpretation.

Model : $r_{M,t+1} = \gamma_{t+1}h_{t+1} + \alpha_1z_t + \varepsilon_{t+1}$, $\gamma_{t+1} = \phi_0 + \phi_1(SURP)_t + \phi_2(CAYA)_t + v_{t+1}$,

where $\varepsilon_{t+1}|\psi_t \sim N(0, h_{t+1})$, $v_{t+1} \sim N(0, 0.01)$, $h_{t+1} = \omega + \beta_1h_t + \beta_2\varepsilon_{t+1}^2$

	z_t	ϕ_0	ϕ_1	ϕ_2	α_1	ω	β_1	β_2
CAYA		2.029	-1.906	-1.852	-0.006	0.002	0.599	0.090
tstats		2.243	-2.059	-2.032	-0.890	1.811	3.136	1.151
DP		2.051	-1.818	-1.873	-0.004	0.002	0.585	0.106
tstats		2.292	-1.964	-2.056	-0.608	1.924	3.240	1.377
TERM		2.068	-1.847	-1.561	-0.005	0.002	0.580	0.114
tstats		2.321	-2.005	-1.612	-0.747	1.978	3.311	1.487
DEF		2.094	-1.925	-1.926	0.002	0.002	0.588	0.122
tstats		2.362	-2.093	-2.021	0.332	2.022	3.581	1.509
INFL		2.115	-1.602	-1.709	0.007	0.002	0.580	0.136
tstats		2.391	-1.667	-1.896	1.044	2.142	3.744	1.621
REGD		2.065	-1.991	-2.086	-0.005	0.002	0.584	0.115
tstats		2.320	-2.150	-2.171	-0.821	2.010	3.428	1.509
RREL		1.591	-2.216	-8.456	-0.043	0.002	0.600	0.082
tstats		1.563	-2.295	-0.828	-0.655	1.916	3.471	1.079

Table 2.14: Time-series specification test: time-varying RRA(Model4) and a hedging component

The Table shows the estimation results of several different specifications of conditional ICAPM with positivity restriction. Note: $r_{M,t+1}$ is the CRSP value-weighted stock return minus lagged 3-month treasury bill rate; Realized volatility($\hat{\sigma}_{m,t+1}$) is defined in the data appendix; CAYA is the consumption-wealth ratio without a look ahead bias; DEF is the yield spread between Baa- and Aaa-rated corporate bonds; DP is the deseasonalized dividend price ratio; INFL is the quarterly inflation rate measured by GDP deflator; REGD is the quarterly real GDP growth;RREL is the difference between the 3 month treasury bill rate and its average in the previous 4 quarters; SURP is the surplus consumption ratio; and TERM is the yield spread between 10-year treasury bonds and 3-month treasury bills. All exogenous variables(z_t ,SURP $_t$,CAYA $_t$, and u_t) are normalized to have means of zero and standard deviations of one to facilitate the interpretation.

$$\text{Model : } r_{M,t+1} = \exp(\gamma_{t+1})(E_t(\ln \hat{\sigma}_{m,t+1}) + \frac{1}{2}\sigma_\eta^2) + \alpha_1 z_t + \varepsilon_{t+1},$$

$$\gamma_{t+1} = \phi_1(\text{SURP})_t + \phi_2(\text{CAYA})_t + v_{t+1}, \ln \hat{\sigma}_{m,t+1} = \delta_0 + \delta_1 \ln \hat{\sigma}_{m,t} + \delta'_2 u_t + \eta_{t+1},$$

where $\varepsilon_{t+1}|\psi_t \sim N(0, E_t(\hat{\sigma}_{m,t+1}))$, $v_{t+1} \sim N(0, 0.01)$, $\eta_{t+1} \sim N(0, \sigma_\eta^2)$

	z_t	ϕ_1	ϕ_2	α_1	δ_0	δ_1	δ_{21}	δ_{22}	δ_{23}	δ_{24}	δ_{25}	σ_η
CAYA	-1.289	0.802	-0.004	-3.671	0.325	0.183	-0.479	-0.172	0.335	0.195	-0.607	
tstats	-3.826	1.192	-0.498	-10.375	5.075	3.146	-5.957	-3.377	4.993	3.481	-19.519	
DP	-1.203	1.001	-0.001	-3.680	0.324	0.180	-0.477	-0.171	0.336	0.194	-0.607	
tstats	-3.874	2.129	-0.136	-10.444	5.073	3.107	-5.926	-3.366	5.050	3.469	-19.514	
TERM	-1.202	0.994	-0.009	-3.667	0.327	0.178	-0.473	-0.171	0.332	0.193	-0.606	
tstats	-3.827	2.086	-1.677	-10.412	5.120	3.059	-5.883	-3.351	4.963	3.455	-19.560	
DEF	-1.205	1.007	0.000	-3.680	0.324	0.180	-0.476	-0.171	0.337	0.194	-0.607	
tstats	-3.862	2.156	0.035	-10.447	5.072	3.100	-5.926	-3.360	5.029	3.470	-19.514	
INFL	-1.023	1.165	0.010	-3.668	0.326	0.172	-0.467	-0.167	0.337	0.198	-0.606	
tstats	-2.741	2.678	1.720	-10.386	5.101	2.961	-5.787	-3.287	5.064	3.527	-19.561	
REGD	-1.237	1.005	-0.004	-3.690	0.322	0.182	-0.479	-0.171	0.339	0.194	-0.607	
tstats	-4.033	2.174	-0.769	-10.482	5.049	3.126	-5.950	-3.366	5.078	3.483	-19.525	
RREL	-1.100	1.068	0.010	-3.666	0.327	0.180	-0.475	-0.170	0.325	0.197	-0.606	
tstats	-3.501	2.515	1.829	-10.379	5.107	3.114	-5.922	-3.350	4.829	3.512	-19.549	

Table 2.15: Time-series specification test: time-varying RRA (Model5) and a hedging component

The Table shows the estimation results of several different specifications of Conditional CAPM with positivity restriction. Note: $r_{M,t+1}$ is the CRSP value-weighted stock return minus lagged 3-month treasury bill rate; Realized volatility($\hat{\sigma}_{m,t+1}$) is defined in the data appendix; CAYA is the consumption-wealth ratio without a look ahead bias; DEF is the yield spread between Baa- and Aaa-rated corporate bonds; DP is the deseasonalized dividend price ratio; INFL is the quarterly inflation rate measured by GDP deflator; REGD is the quarterly real GDP growth;RREL is the difference between the 3 month treasury bill rate and its average in the previous 4 quarters; SURP is the surplus consumption ratio; and TERM is the yield spread between 10-year treasury bonds and 3-month treasury bills. All exogenous variables(z_t , $SURP_t$, CAYA $_t$ and u_t) are normalized to have means of zero and standard deviations of one to facilitate the interpretation.

$$\text{Model : } r_{M,t+1} = \exp(\gamma_{t+1})E_t(\ln \hat{\sigma}_{m,t+1} + \frac{1}{2}\sigma_\eta^2) + \alpha_1 z_t + \varepsilon_{t+1},$$

$$\gamma_{t+1} = \phi_1(\text{SURP})_t + \phi_2(\text{CAYA})_t + v_{t+1}, \ln \hat{\sigma}_{m,t+1} = \delta_0 + \delta_1 \ln \hat{\sigma}_{m,t} + \delta_2' u_t + \eta_{t+1},$$

where $\varepsilon_{t+1}|\psi_t \sim N(0, h_{t+1})$, $v_{t+1} \sim N(0, \sigma_\eta^2)$, $\eta_{t+1} \sim N(0, \sigma_\eta^2)$, $h_{t+1} = \omega + \beta_1 h_t + \beta_2 \varepsilon_t^2$

z_t	ϕ_1	ϕ_2	α_1	δ_0	δ_1	δ_{21}	δ_{22}	δ_{23}	δ_{24}	δ_{25}	σ_η	ω	β_1	β_2
SURP	-0.690	1.373	0.012	-3.573	0.348	0.163	-0.449	-0.155	0.342	0.179	-0.597	0.002	0.607	0.014
tstats	-1.685	6.399	1.663	-9.539	5.133	2.583	-5.111	-2.873	4.750	3.034	-19.701	0.460	0.734	0.156
CAYA	-1.030	1.293	0.001	-3.605	0.342	0.165	-0.455	-0.155	0.346	0.185	-0.597	0.002	0.649	0.052
tstats	-4.047	4.488	0.131	-9.671	5.069	2.600	-5.178	-2.873	4.809	3.122	-19.690	1.176	2.383	0.560
DP	-1.034	1.281	0.000	-3.605	0.342	0.165	-0.455	-0.155	0.346	0.185	-0.597	0.002	0.650	0.048
tstats	-3.968	4.658	0.052	-9.662	5.066	2.602	-5.186	-2.876	4.809	3.132	-19.690	1.135	2.274	0.555
TERM	-0.964	1.343	-0.012	-3.600	0.344	0.166	-0.457	-0.162	0.346	0.186	-0.597	0.003	0.556	6.71E-06
tstats	-3.849	6.312	-2.131	-9.666	5.092	2.556	-5.071	-2.913	4.775	3.157	-19.704	0.008	0.011	6.34E+00
DEF	-1.025	1.282	-0.001	-3.603	0.343	0.165	-0.455	-0.155	0.345	0.185	-0.597	0.002	0.651	0.044
tstats	-3.938	4.735	-0.152	-9.662	5.074	2.620	-5.205	-2.880	4.779	3.129	-19.694	1.086	2.176	0.513
INFL	-0.894	1.397	0.013	-3.597	0.344	0.162	-0.455	-0.154	0.349	0.195	-0.598	0.002	0.646	0.035
tstats	-3.239	6.145	2.091	-9.651	5.091	2.560	-5.180	-2.844	4.822	3.317	-19.658	0.890	1.716	0.445
REGD	-1.038	1.358	-0.007	-3.603	0.343	0.164	-0.456	-0.155	0.348	0.187	-0.597	0.006	2.65E-12	0.019
tstats	-4.343	6.286	-1.092	-9.708	5.099	2.598	-5.203	-2.883	4.859	3.184	-19.680	7.576	3.93E-06	0.215
RREL	-0.915	1.313	0.012	-3.597	0.344	0.166	-0.455	-0.157	0.341	0.184	-0.597	0.002	0.762	6.23E-07
tstats	-3.529	6.034	1.964	-9.634	5.083	2.624	-5.182	-2.910	4.753	3.127	-19.708	0.040	0.129	4.10E-01

Table 2.16: Time-series specification test: time-varying RRA(Model6) and a hedging component

The Table shows the estimation results of several different specifications of conditional ICAPM with positivity restriction. Note: $r_{M,t+1}$ is the CRSP value-weighted stock return minus lagged 3-month treasury bill rate; CAYA is the consumption-wealth ratio without a look ahead bias; DEF is the yield spread between Baa- and Aaa-rated corporate bonds; DP is the deseasonalized dividend price ratio; INFL is the quarterly inflation rate measured by GDP deflator; REGD is the quarterly real GDP growth; RREL is the difference between the 3 month treasury bill rate and its average in the previous 4 quarters; SURP is the surplus consumption ratio; and TERM is the yield spread between 10-year treasury bonds and 3-month treasury bills. All exogenous variables(z_t and $SURP_t$) are normalized to have means of zero and standard deviations of one to facilitate the interpretation.

Model : $r_{M,t+1} = \exp(\gamma_{t+1})h_{t+1} + \alpha_1 z_t + \varepsilon_{t+1}$, $\gamma_{t+1} = \phi_1(SURP)_t x_t + v_{t+1}$,

where $\varepsilon_{t+1}|\psi_t \sim N(0, h_{t+1})$, $v_{t+1} \sim N(0, 0.01)$, $\eta_{t+1} \sim N(0, \sigma_\eta^2)$, $h_{t+1} = \omega + \beta_1 h_t + \beta_2 \varepsilon_t^2$

	z_t	ϕ_1	α_1	ω	β_1	β_2
CAYA	-1.215	-0.006	0.002	0.594	0.116	
tstats	-5.581	-0.906	1.983	3.532	1.393	
DP	-1.201	-0.002	0.002	0.584	0.135	
tstats	-5.403	-0.244	2.082	3.688	1.630	
TERM	-1.217	-0.010	0.002	0.574	0.125	
tstats	-5.636	-1.725	2.003	3.330	1.603	
DEF	-1.203	-0.001	0.002	0.584	0.134	
tstats	-5.445	-0.216	2.061	3.667	1.588	
INFL	-1.188	0.009	0.002	0.568	0.162	
tstats	-5.092	1.399	2.216	3.834	1.833	
REGD	-1.237	-0.002	0.002	0.583	0.141	
tstats	-5.834	-0.356	2.132	3.773	1.715	
RREL	-1.136	0.010	0.002	0.594	0.110	
tstats	-4.613	1.691	1.944	3.379	1.412	

Table 2.17: Summary Statistics for the estimated Time-varying Risk Aversion

Summary statistics for the estimated risk aversion from 1957:2 to 2005:4. The Auto(1) give the first autocorrelation. This table shows descriptive statistics of the estimated risk aversion series($\exp(\gamma_{t+1})$) from conditional CAPM with the surplus consumption ratio(Model1) and conditional ICAPM with the surplus consumption ratio and the consumption-wealth ratio as instruments for relative risk aversion and RREL as a proxy for hedging component(Model2). Note: $r_{M,t+1}$ is the CRSP value-weighted stock return minus lagged 3-month treasury bill rate; Realized volatility(REVOL1) is defined in the data appendix; All exogenous variables(SURP_t,RREL_t,and u_t) are normalized to have means of zero and standard deviations of one.

$$\text{Model1 : } r_{M,t+1} = \exp(\gamma_{t+1}) \exp(E_t(\ln \hat{\sigma}_{m,t+1}) + \frac{1}{2}\sigma_\eta^2) + \varepsilon_{t+1},$$

$$\text{Model2 : } r_{M,t+1} = \exp(\gamma_{t+1}) \exp(E_t(\ln \hat{\sigma}_{m,t+1}) + \frac{1}{2}\sigma_\eta^2) + \alpha_1(\text{RREL})_t + \varepsilon_{t+1},$$

$$\gamma_{t+1} = \phi_1 * (\text{SURP})_t + v_{t+1},$$

$$\ln \hat{\sigma}_{m,t+1} = \delta_0 + \delta_1 \ln \hat{\sigma}_{m,t} + \delta_2' u_t + \eta_{t+1}$$

where $\varepsilon_{t+1}|\psi_t \sim N(0, h_{t+1})$, $v_{t+1} \sim N(0, 0.01)$, $\eta_{t+1} \sim N(0, \sigma_\eta^2)$, $h_{t+1} = \omega + \beta_1 h_t + \beta_2 \varepsilon_t^2$

	Risk Aversion(C-CAPM)	Risk Aversion(C-ICAPM)
Mean	2.388	2.070
Median	0.771	0.850
Maximum	15.994	16.801
Minimum	0.035	0.055
Std. Dev.	3.020	2.580
Skewness	1.835	2.342
Kurtosis	6.544	10.271
Auto(1)	0.906	0.852
Corr	0.964	

Table 2.18: Cross-Sectional Tests of Asset Pricing Models on Fama French 25 portfolios(1957:2-2005:4)

The table presents the estimated results of Fama and MacBeth (1973) cross-sectional regression using the excess returns on 25 portfolios sorted by book-to-market and size. The full-sample factor loadings, which are the independent variables in the regressions, are computed in one multiple time-series regression following Lettau and Ludvigson (2001b). The Adjusted R^2 follows the specification of Jagannathan and Wang (1996). The first set of standard errors, indicated by FM, stands for the Fama-MacBeth estimates. The second set, indicated by Shanken, adjusts for errors-in-variables and follows Shanken (1992). Approximate F-statistics is the transformed Hotelling T^2 -statistics for the small sample test that the pricing errors in the model are jointly zero by Shanken (1985) and 1 percent critical value is given below the F-statistics. The last column reports the root mean squared pricing errors of the model. Note: Rmrf, SMB, and HML are the Fama and French (1993)'s market and size and B/M factors;CAYA is the consumption-wealth ratio without a look ahead bias; RREL is the difference between the 3 month treasury bill rate and its average in the previous 4 quarters; SURP is the surplus consumption ratio; and TERM is the yield spread between 10-year Treasury bonds and 3-month treasury bills. All exogenous variables(SURP,CAYA, RREL, and TERM) are normalized to have means of zero and standard deviations of one. All models are defined in section 2.3.5.

CAPM	const	Rmrf	Adj. R^2	F-test	RMSE		
	0.0401	-0.0163	0.1781	2.96	0.0064		
FM	0.0083	0.0104		Critical(1%)			
Shanken	0.0084	0.0105		1.92			
CCAPM1	const	Rmrf	SURP.*Rmrf	Adj. R^2	F-test	RMSE	
	0.0647	-0.048	0.092	0.3962	3.03	0.0054	
FM	0.0112	0.0127	0.0283		Critical(1%)		
Shanken	0.0185	0.0194	0.046		1.94		
CCAPM2	const	Rmrf	SURP.*Rmrf	CAYA.*Rmrf	Adj. R^2	F-test	RMSE
	0.0267	-0.0112	0.0483	0.0772	0.597	3.18	0.0043
FM	0.0072	0.0095	0.0212	0.0207		Critical(1%)	
Shanken	0.0102	0.012	0.0293	0.0285		1.96	
CCAPM3	const	Rmrf	SURP.*Rmrf	CAYA.*Rmrf	CAYA	F-test	RMSE
	0.0241	-0.008	0.0242	0.0615	-0.8622	2.78	0.0037
FM	0.0074	0.0097	0.0171	0.0189	0.2982		Critical(1%)
Shanken	0.0111	0.0128	0.0247	0.0273	0.4397		1.99

CICAPM1	const	Rmrf	SURP.*Rmrf	CAYA	Adj. R^2	F-test	RMSE		
	0.0522	-0.0348	0.0502	-1.1464	0.5664	2.65	0.0045		
FM	0.0102	0.0117	0.0207	0.3251		Critical(1%)			
Shanken	0.0171	0.0179	0.0338	0.5391		1.96			
CICAPM2	const	Rmrf	SURP.*Rmrf	RREL	Adj. R^2	F-test	RMSE		
	0.0592	-0.0366	0.0367	-0.9611	0.4718	2.93	0.0049		
FM	0.0111	0.0128	0.0239	0.4115		Critical(1%)			
Shanken	0.0184	0.0196	0.0386	0.6727		1.96			
CICAPM3	const	Rmrf	SURP.*Rmrf	CAYA.*Rmrf	RREL	Adj. R^2	F-test	RMSE	
	0.0199	0.0016	-0.0109	0.0762	-1.2041	0.6976	3.06	0.0036	
FM	0.0079	0.0107	0.0179	0.0206	0.4254		Critical(1%)		
Shanken	0.014	0.0168	0.0306	0.0354	0.7486		1.99		
CICAPM4	const	Rmrf	SURP.*Rmrf	CAYA.*Rmrf	CAYA	RREL	Adj. R^2	F-test	RMSE
	0.0211	-0.0015	-0.0018	0.0677	-0.4416	-0.7116	0.7018	2.97	0.0035
FM	0.0077	0.0104	0.019	0.0187	0.303	0.4186		Critical(1%)	
Shanken	0.0117	0.0141	0.0278	0.0273	0.4507	0.6274		2.01	
ICAPM	const	Rmrf	CAYA	TERM	RREL	Adj. R^2	F-test	RMSE	
	0.0156	0.0124	-1.2236	1.5598	-0.3291	0.6887	2.02	0.0037	
FM	0.0072	0.0105	0.3811	0.3286	0.3763		Critical(1%)		
Shanken	0.0172	0.0212	0.8967	0.7693	0.8852		1.99		
Fama-French	const	Rmrf	SMB	HML	Adj. R^2	F-test	RMSE		
	0.0459	-0.0296	0.0034	0.015	0.7369	2.52	0.0035		
FM	0.0099	0.0119	0.0043	0.0047		Critical(1%)			
Shanken	0.0108	0.0127	0.0043	0.0047		1.96			

Table 2.19: Cross-Sectional Tests of Asset Pricing Models on Fama French 25 size-B/M and 30 industry portfolios(1957:2-2005:4)

The table presents the estimated results of Fama and MacBeth (1973) cross-sectional regression using the excess returns on 25 portfolios sorted by book-to-market and size together with 30 industry portfolios. The full-sample factor loadings, which are the independent variables in the regressions, are computed in one multiple time-series regression following Lettau and Ludvigson (2001b). The Adjusted R^2 follows the specification of Jagannathan and Wang (1996). The first set of standard errors, indicated by FM, stands for the Fama-MacBeth estimates. The second set, indicated by Shanken, adjusts for errors-in-variables and follows Shanken (1992). Approximate F-statistics is the transformed Hotelling T^2 -statistics for the small sample test that the pricing errors in the model are jointly zero by Shanken (1985) and 1 percent critical value is given below the F-statistics. The last column reports the root mean squared pricing errors of the model. Note: Rmrf, SMB, and HML are the Fama and French (1993)'s market and size and B/M factors;CAYA is the consumption-wealth ratio without a look ahead bias; RREL is the difference between the 3 month treasury bill rate and its average in the previous 4 quarters; SURP is the surplus consumption ratio; and TERM is the yield spread between 10-year Treasury bonds and 3-month treasury bills. All exogenous variables(SURP,CAYA, RREL, and TERM) are normalized to have means of zero and standard deviations of one. All models are defined in section 2.3.5.

Models		Variables		Statistics		
	const	Rmrf		Adj. R^2	F-test	RMSE
CAPM				0.0372	2.72	0.006
FM	0.0275	-0.0068				
Shanken	0.007	0.0094			Critical(1%)	
	0.007	0.0095			1.66	
CCAPM1			SURP.*Rmrf	0.0969	3.38	0.0057
FM	0.0329	-0.0139	0.0203			
Shanken	0.0077	0.0099	0.0143		Critical(1%)	
	0.008	0.0102	0.0148		1.66	
CCAPM2			SURP.*Rmrf CAYA.*Rmrf	0.1722	3.18	0.0054
FM	0.0214	-0.0031	0.0073			
Shanken	0.0069	0.0097	0.0131		Critical(1%)	
	0.0072	0.01	0.0136		1.67	
CCAPM3			SURP.*Rmrf CAYA.*Rmrf CAYA	0.363	2.14	0.0047
FM	0.0287	-0.0121	0.0234			
Shanken	0.0067	0.0093	0.0131		Critical(1%)	
	0.0084	0.0108	0.0158		1.67	
			0.0277			
			0.0138			
			0.0167			
			-0.65			
			0.2482			
			0.3083			

CICAPM1	const	Rmrf	SURP.*Rmrf	CAYA	Adj. R^2	F-test	RMSE		
	0.0399	-0.0226	0.0363	-0.6703	0.2877	2.08	0.0051		
FM	0.0075	0.0096	0.0143	0.2483		Critical(1%)			
Shanken	0.0098	0.0115	0.018	0.3194		1.67			
CICAPM2	const	Rmrf	SURP.*Rmrf	RREL	Adj. R^2	F-test	RMSE		
	0.0334	-0.014	0.0175	-0.044	0.084	2.62	0.0057		
FM	0.0077	0.0099	0.0141	0.222		Critical(1%)			
Shanken	0.0081	0.0102	0.0147	0.231		1.67			
CICAPM3	const	Rmrf	SURP.*Rmrf	CAYA.*Rmrf	RREL	Adj. R^2	F-test	RMSE	
	0.0215	-0.0032	0.0071	0.0258	-0.0009	2.66	0.0054		
FM	0.0066	0.0095	0.0136	0.0136	0.2199	Critical(1%)			
Shanken	0.0069	0.0097	0.014	0.0141	0.2281	1.67			
CICAPM4	const	Rmrf	SURP.*Rmrf	CAYA.*Rmrf	CAYA	RREL	Adj. R^2	F-test	RMSE
	0.0283	-0.0119	0.0241	0.0281	-0.6535	0.1178	0.358	2.13	0.0047
FM	0.0065	0.0092	0.0132	0.0136	0.2467	0.2159		Critical(1%)	
Shanken	0.0083	0.0106	0.0161	0.0165	0.3077	0.2679		1.68	
ICAPM	const	Rmrf	CAYA	TERM	RREL	Adj. R^2	F-test	RMSE	
	0.031	-0.0097	-0.3996	0.1235	-0.1978	0.106	2.08	0.0056	
FM	0.0072	0.0094	0.2444	0.1976	0.2259		Critical(1%)		
Shanken	0.008	0.0101	0.2696	0.217	0.2488		1.67		
Fama-French	const	Rmrf	SMB	HML	Adj. R^2	F-test	RMSE		
	0.0325	-0.0153	0.0029	0.0087	0.2583	2.57	0.0052		
FM	0.0082	0.0104	0.0044	0.0048		Critical(1%)			
Shanken	0.0085	0.0106	0.0044	0.0048		1.67			

Appendices

A Alternative approaches in monetary economics

In this section, I briefly summarize stylized facts and alternative models proposed in monetary economics before new Keynesian dynamic general stochastic equilibrium models employed in this paper are developed.

A.1 Stylized facts from time-series analysis

Simple time-series analysis such as vector autoregression(VAR) or structural vector autoregressions(SVAR) have provided the following stylized facts which must be explained with well-suited monetary economic models.⁴³

- Monetary policy shocks have a delayed yet persistent effect on real output. Especially positive monetary shocks lead to a hump-shaped positive response of output in USA economy.(Walsh (2001))
- Both anticipated and unanticipated shocks affect real economic activity(Mishkin (1982)).
- An increase in the money supply can reduce the real interest rate(liquidity effect) since more liquidity tends to lower the price of money which is equivalent to lowering the interest rate.

⁴³Refer to chapter 1 of Walsh (2001) for a detailed explanation on time series evidence.

A.2 Various monetary equilibrium models with micro-foundation

From the general equilibrium models built on the joint foundations of individual optimization and flexible prices to the class of general equilibrium models built on optimizing behavior and nominal rigidities are employed in most discussions of monetary policy issues.

Lucas (1976) argue that traditional policy evaluation exercises using macroeconometric models were flawed by a failure to recognize that the relations typically estimated were reduced-form rather than truly structural relations. This problem can be addressed by making use of structural relations that explicitly represent the dependence of economic decisions upon expectations regarding future endogenous variables. The inclusion of significant forward-looking terms in key structural relations have substantial consequences for an analysis of the character of optimal policy.

1. Flexible price approaches

- Lucas (1973) islands model with rational expectation and imperfect information and competitive markets is the first general equilibrium model based on the microfoundation to explain monetary facts. But in this model, only unanticipated shocks matter and it does not generate any persistence change in output.
- Cash in advance(CIA) model or monetary real business cycle(RBC) models of Lucas (1982) and Svensson (1985)) generally share same problem with islands model because they essentially keep the same structure of the real economy as RBC models but superimpose a monetary sector. Whilst they are able to generate real-nominal interactions, the effect is not very persistent and output quickly returns to baseline. ⁴⁴ Furthermore, the CIA models suffer from the same problems that generate the equity premium puzzle and these basic CIA models

⁴⁴Money in the utility function model has similar problems. It is well known that under certain conditions there exists an equivalence between putting money in the utility function or specifying cash in advance constraint.

are also not very successful in generating plausible asset price and interest rate data.(Giovannini and Labadie (1991))

- CIA augmented with limited participation mechanism proposed by Fuerst (1992) explains the liquidity effect successfully but it is too complicated by altering the structure of CIA models with limited participation only for explaining the liquidity effect. It is criticized since the appropriate model will have to involve greater complexity than simply assuming certain prices are fixed.

2. RBC with sticky prices alone fails to solve the persistence problem.(V. V. Chari and McGrattan (2000)) They argue that real rigidity in labor market should be added since the wage rate and prices react too strongly in their model to money supply shocks to have persistent output effects.

3. Empirical results and other ideas

- Hodrick, Kocherlakota, and Lucas (1991) find that classical CIA models fail to explain the variability of velocity with reasonable assumptions on the levels of risk aversion.
- Cooley and Hansen (1989) introduce the concept of inflation tax :when inflation is high, the real value of money declines sharply so that agents will hold less money. Since cash is required to finance consumption purchases, this high inflation causes lower consumption and so a nominal variables will affect a real variable. Therefore, inflation acts as a tax on goods which require money to be purchased and as a subsidy on credit goods which do not require cash. However, this mechanism of non-neutrality is not enough to explain the interaction between real and nominal variables in the data.
- The nominal interest rates is often viewed through the Fisher hypothesis; The nominal interest rate equals the real interest rate plus the expected inflation

rate. Therefore, an increase in the money supply can have two effects: it can reduce the real interest rate (liquidity effect) and it forecasts higher future inflation. Therefore to generate a falling nominal interest rate in response to a positive money supply shock, the liquidity effect should outweigh the Fisher effect. However, in neoclassical models money does not influence real variables (the real interest rate), increases in the money supply just forecast higher inflation and so the nominal interest rate rises as there is no liquidity effect but only a Fisher effect.

4. **New-Keynesian approaches with microfoundation**⁴⁵

In order to explain a large degree of inflation inertia and persistent impact on output of monetary policy shocks, the New-Keynesian framework assumes that firms operate in monopolistic competitive markets and production is constrained by aggregate demand. Prices are assumed to be sticky and consequently do not move instantaneously to movements in marginal costs. Due to the price stickiness, the central bank affects aggregate demand through its influence on real interest rates. By lowering real interest rates, the central bank induces higher aggregate demand, marginal costs and prices than would otherwise materialize. For example, Rotemberg and Woodford (1999) use a dynamic IS curve based on intertemporal maximization and an aggregate supply curve based on the sticky prices in the New Keynesian Phillips curve. Rather than assuming a cash-in-advance constraint and facing the problems of generating substantial liquidity effects, they jump directly to a formulation in which the instrument of monetary policy is the interest rate itself. (Monetary policy rule) Following this approach, many New-Keynesian models are developed as they are summarized in this paper.

⁴⁵I refer to Woodford (2003) for details

B Pricing Kernel example

Without imposing any theoretical structure, the fundamental existence theorem of Harrison and Kreps (1979) states that, in the absence of arbitrage, there exists a positive stochastic discount factor, or pricing kernel, m_{t+1} , such that, for any traded asset with a net return at time t of $R_{i,t+1}$, the following equation holds:

$$1 = E_t[m_{t+1}(1 + R_{i,t+1})] \quad (\text{B-1})$$

where E_t denotes the mathematical expectation operator conditional on information available at time t .

In order to get some intuition behind this fundamental asset pricing formula, I present an example of multi-period consumption asset pricing kernel.⁴⁶

Consider a representative agent who maximizes:

$$\max E_0 \left[\sum_{t=0}^{\infty} \rho^t U(c_t) \right], 0 < \rho < 1 \quad (\text{B-2})$$

$$s.t. W_{t+1} = R_{t+1}(W_t - c_t) \quad (\text{B-3})$$

where c_t is consumption of an agent at time t , W_t is wealth valued in units of the consumption good, ρ is the subjective discount rate, and R_{t+1} is the real gross rate of return on the asset between dates t and $t+1$.

The implied dynamic program after plugging the budget constraint is given by:

$$V(W_t, R_t) = \max_{u_t} \{U(W_t - u_t) + \rho E_t V(u_t R_{t+1}, R_{t+1})\} \quad (\text{B-4})$$

where the control is $u_t = (W_t - c_t)$

⁴⁶I follow the explanation given in chapter 1 of Cochrane (2001)

The first order condition with respect to the optimal control is

$$-U'(c_t) + \rho E_t [\partial V(u_t R_{t+1}, R_{t+1}) / \partial u_t] R_{t+1} = 0 \quad (\text{B-5})$$

And the Benveniste-Scheinkman differentiability condition is $\partial V(u_t R_{t+1}, R_{t+1}) / \partial u_t = U'(c_{t+1})$

Therefore, the Euler condition can be summarized as

$$U'(c_t) = \rho E_t [R_{t+1} U'(c_{t+1})] \quad (\text{B-6})$$

$$\text{or } 1 = E_t [m_{t+1} R_{t+1}]$$

where $m_{t+1} \equiv \rho U'(c_{t+1}) / U'(c_t)$ is called the stochastic discount factor or the pricing kernel.

C Non-linear state-space model with endogeneity and GARCH

In this section, I present the state space model and computing algorithms for my empirical models (Model 7) by adapting empirical models of Kim and Nelson (2005) and Kim (2006) to my specification. Generally, endogeneity issue can exist once I allow misspecification in $E_t \ln \hat{\sigma}_{m,t+1}$. Guo and Whitelaw (2006) argue for this case. Slight different version of realized variance model is implemented for simplicity but Engle et al (1987, *Econometrica*) advocates this form.

C.1 Empirical model

$$r_t = \exp(\gamma_t) E_{t-1}(\ln \hat{\sigma}_{m,t}) + \alpha'_1 Z_{t-1} + \varepsilon_t \quad (\text{C-1})$$

where r_t is excess market return ($R_{m,t} - R_t^f$), $(\ln \hat{\sigma}_{m,t})$ is a measure of realized variance, Z_{t-1} are proxy variables for the hedging components, and $\exp(\gamma_t)$ stands for the time-varying RRA.

$$\gamma_t = \phi_0 + \phi_1 \gamma_{t-1} + \phi_2' X_{t-1} + \nu_t \quad (\text{C-2})$$

where X_{t-1} are proxy variables to capture the time-varying RRA.

By substituting observed realized variance $\ln \hat{\sigma}_{m,t}$ into (C-1) , I get

$$r_t = \exp(\gamma_t) \ln \hat{\sigma}_{m,t} + \alpha_1' Z_{t-1} + \varepsilon_t^* \quad (\text{C-3})$$

where $\varepsilon_t^* = \exp(\gamma_t) [E_{t-1} \ln \hat{\sigma}_{m,t} - \ln \hat{\sigma}_{m,t}] + \varepsilon_t$

Following Kim and Nelson (2005) and Kim (2006), I approximate distribution of ε_t^* as $N(0, p_t)$ and $p_t = \theta_0 + \theta_1 p_{t-1} + \theta_2 \varepsilon_{t-1}^{*2}$.

Incidentally, endogeneity becomes an important issue because $\text{cov}((\ln \hat{\sigma}_{m,t}), \varepsilon_t^*) \neq 0$. In fact, Brandt and Qiang (2004) suggest that there might be correlation between these two components using latent vector autoregressive regression model.

My specifications for $\ln \hat{\sigma}_{m,t}$ is :

$$\ln \hat{\sigma}_{m,t} = \delta_{0,t} + \delta_{1,t} \ln \hat{\sigma}_{m,t-1} + \delta_{2,t}' u_{t-1} + \eta_t \quad (\text{C-4})$$

$$\delta_{i,t} = \delta_{i,t-1} + \xi_t \quad (\text{C-5})$$

where u_{t-1} are exogenous variables which affect the time-varying volatility, $\xi_t \sim N(0, \sigma_\xi^2)$, and $\eta_t \sim N(0, \sigma_\eta^2)$ and $i=0$ or 1 .

Guo and Whitelaw (2006) show the evidence of instability of parameters in realized variance equation.

In general forms, I can express (C-4) as follows:

$$\ln \hat{\sigma}_{m,t} = E_{t-1}[\ln \hat{\sigma}_{m,t}] + \varsigma_{t|t-1} \quad (\text{C-6})$$

where $\varsigma_{t|t-1} = \Omega_{t|t-1}^{\frac{1}{2}} u_t^*$ and $u_t^* \sim i.i.d.N(0, 1)$

$\Omega_{t|t-1}$ can be obtained as the time-varying conditional variance covariance matrix for prediction error($\varsigma_{t|t-1}$) from the Kalman filter applied to models given (C-6).

Following Kim (2006), I assume the following specific structure for error terms in (C-3) and (C-6):

$$\begin{bmatrix} u_t^* \\ \varepsilon_t^* \end{bmatrix} \sim N \left[\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \cdot p_t^{\frac{1}{2}} \\ \rho \cdot p_t^{\frac{1}{2}} & p_t \end{bmatrix} \right]$$

After the Cholesky decomposition, I get:

$$\begin{bmatrix} u_t^* \\ \varepsilon_t^* \end{bmatrix} \sim N \begin{bmatrix} 1 & 0 \\ \rho p_t^{\frac{1}{2}} & \sqrt{(1-\rho^2)p_t} \end{bmatrix} \begin{bmatrix} u_t^* \\ w_t \end{bmatrix}, \begin{bmatrix} u_t^* \\ w_t \end{bmatrix} \sim i.i.d.N \left[\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right]$$

With this specific assumption, I can decompose ε_t^* into two distinct components to address the correlation explicitly:

$$\varepsilon_t^* = \rho p_t^{\frac{1}{2}} u_t^* + w_t$$

where $w_t \sim N(0, (1-\rho^2)p_t)$

Finally, I rewrite (C-3) as:

$$r_t = \exp(\gamma_t)(\ln \hat{\sigma}_{m,t}) + \alpha_1' Z_{t-1} + \rho p_t^{\frac{1}{2}} \cdot u_t^* + w_t, (B.1.6) \quad (C-7)$$

Kim (2006) show that once we get (u_t^*) , his modified Kalman filter can be utilized to estimate the empirical model given here. In next section, I construct modified Kalman filter and estimation method following his approach.

C.2 Estimation method

In this section, I develop two-step maximum likelihood estimation(MLE) based on Kalman filter using straightforward application of Kim (2006).

In the first step estimation, I have usual state space model with measurement equation(C-6) and transition equation(C-5). I estimate model using MLE with Kalman filter and obtain standardized residuals.(u_t^*) After incorporating u_t^* into (C-7), I need to estimate a nonlinear state space model with GARCH with a couple of approximations.⁴⁷

First, I use extended Kalman filtering technique to linearize measurement equation(C-7). After denoting $\exp(\gamma_t) \ln \hat{\sigma}_{m,t}$ as $f(\hat{\sigma}_{m,t}; \gamma_t)$, I take a Taylor series expansion of the nonlinear function($f(\hat{\sigma}_{m,t} : \gamma_t)$) around $\gamma_t = \gamma_{t|t-1}$. In this expression, $\gamma_{t|t-1}$ indicates $E[\gamma_t | \Psi_{t-1}]$ where Ψ_{t-1} denotes the information set available up to t-1.

After linearization, I get the following measurement equation:

$$r_t = f(\hat{\sigma}_{m,t} : \gamma_t) + \frac{\partial f(\hat{\sigma}_{m,t} : \gamma_t)}{\partial \gamma_t} (\gamma_t - \gamma_{t|t-1}) + \alpha'_1 Z_{t-1} + \rho p_t^{\frac{1}{2}} \cdot u_t^* + w_t$$

where $f(\hat{\sigma}_{m,t} : \gamma_t) = \exp(\gamma_{t|t-1}) \ln \hat{\sigma}_{m,t}$ $\frac{\partial f(\hat{\sigma}_{m,t}; \gamma_t)}{\partial \gamma_t} \equiv \exp(\gamma_{t|t-1}) \ln \hat{\sigma}_{m,t}$

By redefining some of variables, I get the following linearized measurement equation

$$Y_t = \hat{X}_t \gamma_t + \alpha'_1 Z_{t-1} + \rho p_t^{\frac{1}{2}} u_t^* + w_t \quad (\text{C-8})$$

where $Y_t = r_t - \exp(\gamma_{t|t-1}) \ln \hat{\sigma}_{m,t} + \exp(\gamma_{t|t-1}) \ln \hat{\sigma}_{m,t} \gamma_{t|t-1}$ and $\hat{X}_t = \exp(\gamma_{t|t-1}) \ln \hat{\sigma}_{m,t}$

In order to estimate GARCH in w_t , I include w_t into transition equation following Harvey, Ruiz, and Sentana (1992). In matrix forms,

$$Y_t = \begin{bmatrix} \hat{X}_t & 1 \end{bmatrix} \begin{bmatrix} \gamma_t \\ w_t \end{bmatrix} + \alpha'_1 q_{t-1} + e h_t^{\frac{1}{2}} u_t^*$$

⁴⁷I closely follow the approach taken by Kim and Nelson (2005)

$$\begin{bmatrix} \gamma_t \\ w_t \end{bmatrix} = \begin{bmatrix} \phi_0 + \phi'_2 X_{t-1} \\ 0 \end{bmatrix} + \begin{bmatrix} \phi_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \gamma_{t-1} \\ w_{t-1} \end{bmatrix} + \begin{bmatrix} v_t \\ w_t \end{bmatrix},$$

$$\begin{bmatrix} v_t \\ w_t \end{bmatrix} \sim N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_v^2 & 0 \\ 0 & (1 - \rho^2)p_t \end{pmatrix} \right]$$

I use the following compact forms to explain modified Kalman filtering algorithm.

$$Y_t = \tilde{X}_t \tilde{\beta}_t + \alpha'_1 q_{t-1} + \rho h_t^{\frac{1}{2}} u_t^*$$

$$\tilde{\beta}_t = \mu_t + F \tilde{\beta}_{t-1} + \tilde{v}_t, \tilde{v}_t \sim (0, \tilde{Q}_t)$$

where $\tilde{X}_t = [\tilde{X}_t, 1]$, $\tilde{\beta}_t = \begin{bmatrix} \gamma_t \\ w_t \end{bmatrix}$, $\mu_t = \begin{bmatrix} \phi_0 + \phi'_2 X_{t-1} \\ 0 \end{bmatrix}$, $F = \begin{bmatrix} \gamma_1 & 0 \\ 0 & 0 \end{bmatrix}$, $\tilde{v}_t = \begin{bmatrix} v_t \\ w_t \end{bmatrix}$

and $\tilde{Q}_t = \begin{pmatrix} \sigma_v^2 & 0 \\ 0 & (1 - \rho^2)p_t \end{pmatrix}$

At each iteration of the Kalman filter, I obtain a linear approximation of the model around $\gamma_t = \gamma_{t|t-1}$, and calculate Y_t and \tilde{X}_t for the following Kalman filter.

$$\begin{aligned} \tilde{\beta}_{t|t-1} &= F \tilde{\beta}_{t-1|t-1} + \mu_t \\ p_{t|t-1} &= F p_{t-1|t-1} F' + \tilde{Q}_t \\ \eta_{t|t-1} &= Y_t - \tilde{X}'_t \beta_{t|t-1} - \rho p_t^{\frac{1}{2}} (u_t^*) - \alpha'_1 Z_{t-1} \\ H_{t|t-1} &= \tilde{X}'_t p_{t|t-1} \tilde{X}_t \\ \tilde{\beta}_{t|t} &= \tilde{\beta}_{t|t-1} + p_{t|t-1} \tilde{X}_t H_{t|t-1}^{-1} \eta_{t|t-1} \\ p_{t|t} &= p_{t|t-1} - p_{t|t-1} \tilde{X}_t H_{t|t-1}^{-1} \tilde{X}'_t p_{t|t-1} \end{aligned}$$

where Ψ_{t-1} is the information set up to time t-1, $\tilde{\beta}_{t|t-1}$ is conditional estimate of $\tilde{\beta}_t$ on information up to t-1 ($E[\tilde{\beta}_t | \Psi_{t-1}]$), $\tilde{\beta}_{t|t}$ is conditional estimate of $\tilde{\beta}_t$ on information up to t ($E[\tilde{\beta}_t | \Psi_t]$), $p_{t|t-1}$ is covariance matrix of $\tilde{\beta}_t$ conditional on information up to t-1 ($E[(\tilde{\beta}_t - \tilde{\beta}_{t|t-1})(\tilde{\beta}_t - \tilde{\beta}_{t|t-1})']$), $p_{t|t}$ is covariance matrix of $\tilde{\beta}_t$ conditional on information up to t ($E[(\tilde{\beta}_t - \tilde{\beta}_{t|t})(\tilde{\beta}_t - \tilde{\beta}_{t|t})']$).

$\tilde{\beta}_{t|t})(\tilde{\beta}_t - \tilde{\beta}_{t|t})'$) and $H_{t|t-1}$ is conditional variance of prediction error ($E[\eta_{t|t-1}^2]$)

In order to process the above Kalman filter, I need ε_{t-1}^{*2} term in order to calculate GARCH(p_t) in \tilde{Q}_t matrix. As in Harvey, Ruiz, and Sentana (1992), the term ε_{t-1}^{*2} is approximated by $E[\varepsilon_{t-1}^{*2}|\Psi_{t-1}]$, where Ψ_{t-1} is information up to time t-1.

In order to get the form of $E[\varepsilon_{t-1}^{*2}|\Psi_{t-1}]$, Harvey, Ruiz, and Sentana (1992) use the following definition.

$$\varepsilon_{t-1}^* = E[\varepsilon_{t-1}^*|\Psi_{t-1}] + (\varepsilon_{t-1}^* - E[\varepsilon_{t-1}^*|\Psi_{t-1}])$$

After straightforward calculation, it can be shown that:

$$E[\varepsilon_{t-1}^{*2}|\Psi_{t-1}] = E[\varepsilon_{t-1}^*|\Psi_{t-1}]^2 + E\left[(\varepsilon_{t-1}^* - E[\varepsilon_{t-1}^*|\Psi_{t-1}])^2\right]$$

By using $\varepsilon_t^* = \rho p_t^{\frac{1}{2}} u_t^* + w_t$, I get the following form.

$$E[\varepsilon_{t-1}^{*2}|\Psi_{t-1}] = \left(u_t^* \rho p_t^{\frac{1}{2}} + E[w_{t-1}|\Psi_{t-1}]\right)^2 + E\left[(w_{t-1} - E[w_{t-1}|\Psi_{t-1}])^2\right]$$

where $E[w_{t-1}|\Psi_{t-1}]$ is obtained from the last element of $\tilde{\beta}_{t-1|t-1}$ and its mean squared error $E\left[(w_{t-1} - E[w_{t-1}|\Psi_{t-1}])^2\right]$ is given by the last diagonal element of $p_{t-1|t-1}$.

In order to correct for the endogeneity bias in Kalman filtering caused by including estimated residuals in second step measurement equation, Kim (2006) suggests the following modifications.

$$\begin{aligned} H_{t|t-1}^* &= \tilde{X}_t' p_{t|t-1} \tilde{X}_t + \rho^2 p_t^2 \\ p_{t|t}^* &= p_{t|t-1} - p_{t|t-1} \tilde{X}_t H_{t|t-1}^{*-1} \tilde{X}_t' p_{t|t-1} \\ p_{t-1|t}^* &= F p_{t|t}^* F' + \tilde{Q}_{t+1} \end{aligned}$$

D Data Appendix

In this section, I list all the variables used in the article and describe how they were computed from original data sources. I measure all variables at the quarterly frequency and my base sample period extends from 1957:1 to 2005:4.

D.1 CAY without a Look-ahead bias(CAYA)

CAY is suggested in Lettau and Ludvigson (2001a). Data for its construction is available from Lettau's website at quarterly frequency.⁴⁸ To estimate the parameters in cointegrating equation, they employ a dynamic least squares(DLS) technique proposed by Stocks and Watson (1993). The DLS specification adds leads and lags of the first difference of the right-hand side variables to a standard OLS regression and can eliminate the effects of regressor endogeneity on the distribution of the least squares estimator. Specifically they estimate the following forms:

$$c_t = \alpha + \beta_\alpha \cdot a_t + \beta_y \cdot y_t + \sum_{i=-k}^k b_{a,i} \Delta a_{t-i} + \sum_{i=-k}^k b_{y,i} \Delta y_{t-i} + \varepsilon_t, t = k + 1, \dots, T - k$$

where c is the aggregate consumption, a is the aggregate wealth, and y is the aggregate income.

Using estimated coefficients from the above equation provides

$$cay \equiv \widehat{cay}_t = c_t - \hat{\beta}_\alpha \cdot a_t - \hat{\beta}_y \cdot y, t = 1, \dots, T$$

Since Lettau-Ludvigson measure of cay is constructed using look-ahead(in sample)estimation regression coefficients. CAY is often criticized with possible look ahead bias. Recently, Goyal and Welch (2006) modified CAY with only prevailing data. They estimated a version of

⁴⁸I am grateful to Martin Lettau for making his series publicly available

CAY using only the data up to the current period(s) by estimating parameters with rolling regressions such that their measure of CAY(CAYA) does not use look-ahead estimation regression coefficients.

$$c_t = \alpha + \beta_w^s w_t + \beta_y^s y_t + \sum_{t=-8}^8 b_{w,i}^s \Delta w_{t-i} + \sum_{t=-8}^8 b_{y,i}^s \Delta y_{t-i} + \varepsilon_t, t = 9, \dots, s-8$$

where c is the aggregate consumption, w is the aggregate wealth, and y is the aggregate income. The superscript on the betas indicates that these are rolling estimates.

I downloaded basic data(1951:4-2005:4) from Ludvigson's website and constructed CAYA following Goyal and Welch's method to avoid the look-ahead bias and CAYA is defined as $\widehat{CAY} A_t = c_t - \beta_w^s w_t - \beta_y^s y_t, t = 1 \dots T$. Figure 2.1 shows that CAY and CAYA have similar patterns. However, their correlation is not perfect(0.76) in my sample.

D.2 Surplus consumption ratio

Campbell and Cochrane (1999) defines surplus consumption ratio as

$$\text{SURP}_t = \frac{C_t - X_t}{C_t}$$

where C_t is consumption and X_t is the habit level.

I followed Duffee (2005) and Wachter (2006)'s specification precisely and define a proxy for surplus consumption ratio at the quarterly frequency as

$$\text{SURP}_t = \frac{1 - \Psi}{1 - \Psi^{40}} \sum_{j=0}^{39} \Psi^j \Delta c(t-j)$$

where $\Psi=0.96$ and Δc refers to the log change in real per capita quarterly consumption on nondurables and services.

I downloaded all consumption and population data from Bureau of Economic Analysis from 1947:1 and constructed surplus consumption ratio series starting from 1957:1 since I

needed 40 quarters of data to construct the first observation. And I checked and verified my data with Duffee (2005)'s surplus consumption data for common periods.⁴⁹

D.3 Equity Market variables

I downloaded and used the CRSP daily and quarterly data files from 1956 to 2005 in order to create stock market variables.

Excess returns My stock return measure is the standard value-weighted return of NYSE-AMEX-NASDAQ index from CRSP. To compute excess equity returns, I subtracted the lagged 3month continuously compounded T-Bill yield earned over the same period from CRSP.

Dividend Price Ratio For the dividend price ratio, I follow Bekaert, Engstrom, and Xing (2006) by first calculating quarterly dividend yield series as,

$$DP_{t+1} = \left(\frac{P_{t+1}}{P_t} \right)^{-1} \left(\frac{P_{t+1} + D_{t+1}}{P_t} - \frac{P_{t+1}}{P_t} \right)$$

where $\frac{P_{t+1} + D_{t+1}}{P_t}$ and $\frac{P_{t+1}}{P_t}$ are available directly from the CRSP as the value weighted stock return series including and excluding dividends respectively.

And I use the four-period moving average as dividend price ratio,

$$dp_t^f = \frac{1}{4} [\ln(1 + DP_t) + \ln(1 + DP_{t-1}) + \ln(1 + DP_{t-2}) + \ln(1 + DP_{t-3})].$$

Realized variance series I use the daily CRSP value-weighted stock returns as a proxy for aggregate stock market returns consistent with my quarterly excess stock return measure. The quarterly risk-free rate is the lagged yield on 3 month T-bills and I constructed the daily risk-free rate by assuming that it is constant within a quarter. The excess stock market

⁴⁹I am grateful to G. Duffee for making his series publicly available

return is defined as the difference between the stock market return and the risk-free rate as usual.

Similar to Merton (1980), realized variance(REVOL1) is defined as the variance of the daily excess stock market returns in a quarter.

$$\sigma_{m,t}^2 = \frac{1}{N_t} \sum_{J=1}^{N_t} [r_{t,J} - \text{mean}(r_{t,J})]^2$$

where $r_{t,J}$: daily excess return , N_t : the number of trading days in a quarter

To check the robustness of my results, I constructed four different measures of realized variance series.

- REVOL1 : The usual realized variance series defined above
- REVOL2 : The 1987 stock market crash has a confounding effect on my variance measure; following Guo and Whitelaw (2006), I replace REVOL1 for 1987:4 with the second-largest observation in my sample. Guo and Whitelaw (2006) argue that REVOL2 is more appropriate since the variable(REVOL1) rose dramatically during this crash period but reverted to the normal level shortly after.

Non-synchronous trading of securities causes daily portfolio returns to be autocorrelated, particularly at lag one. Because of this autocorrelation, I estimate the variance of the quarterly excess market return as the sum of the squared daily excess returns plus twice the sum of the products of adjacent returns.(REVOL1auto) Following previous discussion, I also construct REVOL2auto by replacing REVOL1auto for 1987:4 with the second-largest observation in my sample.

$$\sigma_{m,t}^2 = \sum_{i=1}^{N_t} r_{i,t}^2 + \sum_{i=1}^{N_t-1} r_{i,t} r_{i+1,t}$$

where there are N_t daily returns, $r_{i,t}$, in quarter t.

Table 2.2 summarizes descriptive statistics for the four different versions of realized variance series. For the comparison, I also estimate simple GARCH(1,1) model. (estimated models are given below the Table 2.2) Even though all four versions have similar sample characteristics, Figure 2.2 suggests that only REVOL2 and REVOL2auto have comparable magnitude with the estimated GARCH series. It seems reasonable since GARCH models estimate current quarterly volatility with the past information set. However, REVOL1 estimates volatility of the current quarter with daily data in that quarter only which might get too extreme values in crash period. REVOL2 removes effects of this outlier.

Fama French Factors and Portfolios data

- Fama French Factors : Quarterly data on Rmrf, HML, and SMB, are obtained from Kenneth French's Web site.⁵⁰
- Fama French 25 size and B/M sorted portfolios and 30 industry portfolios : All portfolios are also obtained from Kenneth French's Web site. The original returns on the portfolios are monthly. So, I computed quarterly returns by compounding the three monthly returns of each quarter. I denote the 25 size and B/M sorted portfolios as 11, 12, 13, ..., 55, where the first digit indicates the portfolio's size group and the second digit the portfolio's B/M ratio group. The number 1 refers to the smallest size (lowest B/M ratio) and the number 5 to the biggest size (highest B/M ratio).

D.4 Other forecasting variables

- *Term spread(TERM)*: This is the difference between long-term and short-term government bond yields. The long-term government bond yield data are from the Federal Reserve Economic Database(FRED) and are based on a maturity of 10 years. The short-term yield is the 3-month Treasury bill rate (secondary market) and is also

⁵⁰I am thankful to Kenneth French for making the data available.

from the FRED. The original data are monthly. I obtained quarterly observations by averaging over the three months comprising a quarter.

- *Default spread(DEF)*: This is the difference between the Moody's seasoned Baa corporate bond yield and the Moody's seasoned Aaa corporate bond yield. The corporate bond yield data are from the FRED. The original corporate bond yield data are monthly. I obtained quarterly observations by averaging over the three months comprising a quarter.
- *Stochastically detrended short term interest rate(RREL)*: I computed RREL as the quarterly short-term interest rate in deviations from a four-quarter moving average consisting of the four previous quarters from the CRSP.
- *Real GDP growth(REGD)*: I computed real GDP growth using the seasonally adjusted data on real GDP in billions of chained 2000 dollars. Quarterly real GDP is from the FRED.
- *Inflation(INFL)*: I computed inflation series using GDP deflator with quarterly nominal and real GDP series from the FRED.

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