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The Time-varying Risk Return Tradeoff in the Long-Run: UK evidence

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Abstract

The risk-return tradeoff implied by time-invariant conditional CAPM and ICAPM is rather weak with the two century history of UK data from 1836 to 2010, contrary to the findings of Lundblad (2007). I develop a nonlinear ICAPM with multivariate GARCH-M based on Harvey et al. (1992) to allow for the time-varying risk-return tradeoff and hedging coefficients. I find that the risk return relation is largely positive over the time. More importantly, I show that the seemingly negative risk-return relation could be entirely spurious because it is not statistically different from zeros with the 95% confidence bounds. I conclude that the time-varying risk-return tradeoff is the main reason for the weak relation.

JEL Classification: G12, C15, C22

Keywords: Asset pricing, Time-varying risk return tradeoff, GARCH

1. Introduction

Mainstream asset pricing theories such as the CAPM or ICAPM implies a positive risk-return tradeoff. While the risk-return tradeoff is fundamental to finance, the empirical evidence has been rather inconclusive. For example, French, Schwert, and Stambaugh (1987) and Scruggs (1998) find a positive relation between the expected excess market return and conditional variance, whereas Glosten, Jagannathan, and Runkle (1993), Scruggs and Glabadanidis (2003) find either a negative or insignificant relation.

Using the nearly two century history of the US equity market data, Lundblad (2007) finds that the risk-return relationship is positive and significant regardless of GARCH specifications. Based on Monte Carlo analysis he first shows that researchers need more than 100 years of data to estimate the relation between the market risk premium and conditional volatility with any precision. He argues that the weak empirical relationship found from previous papers may be viewed as a statistical artefact of small samples. Lundblad asserts that in the GARCH-M context, one simply requires sufficiently a long span of data in order to detect this relationship.

In this paper, I re-examine the issue with continuously compounded UK stock and bond returns with a two century of data from 1836:01 to 2010:12. Contrary to the findings of Lundblad (2007), I present evidence that a long span of time-series does not seem to guarantee a significantly positive risk return relation. I also extend his specification and estimate a version of ICAPM with two factors employed in Scruggs and Glabadanidis (2003). I find that these models with a time-invariant univariate GARCH-M or bivariate GARCH-M models produce rather weak evidence.

Lundblad (2007) presents exploratory evidence to show that the time-varying risk return tradeoff might be important. Motivated with this observation, I develop a

nonlinear ICAPM with GARCH-M models to allow for the time-varying risk-return tradeoff and hedging coefficients. I am agnostic about the source of this time-varying relation. Due to historical data limitation, it is not easy to determine the source of time-varying relation empirically. Theoretically, the risk-return tradeoff can be time-varying with any sign. In most popular asset pricing models such as the CAPM or ICAPM, the source of time-varying relation is time-varying positive risk aversion. However, the negative relation can be easily justifiable. For example, Whitelaw (2000) shows that the negative relation exists when the market excess returns acts as a proxy for hedging components in a regime switching consumption based model. Therefore, the risk return tradeoff should be investigated empirically.

In summary, I show that the risk-return relation is largely positive across time with this general nonlinear ICAPM specification. Further, even when the point estimate indicates the negative relation, it is not statistically different from zeros with the 95% confidence bounds. I find that the time-varying risk-return tradeoff is the main reason for the weak relation.

The remainder of this paper is organized as follows: Section 2 provides the theoretical framework and the empirical models for the risk return relationship. Section 3 presents the econometric methodology for estimating a nonlinear ICAPM. In Section 4, the historical data and the sources are discussed and time-series evidence on the risk return relationship with the usual conditional CAPM and ICAPM are provided. In Section 5, I present empirical results with a nonlinear ICAPM with a time-varying risk return tradeoff and hedging coefficient. Finally, Section 6 concludes.

2. Risk Return in Equilibrium

Merton (1973) derives the dynamic risk-return trade-off between the conditional mean of the return on the wealth portfolio, $E_t[r_{M,t+1}]$, in relation to its conditional variance,

$\sigma_{M,t}^2$ and the conditional covariance with variation in the investment opportunity set, $\sigma_{MF,t}$:

$$E_t[r_{M,t+1} - r_{f,t}] = \left[\frac{-J_{WW}W}{J_W} \right] \sigma_{M,t}^2 + \left[\frac{-J_{WF}}{J_W} \right] \sigma_{MF,t} \quad (1)$$

where $J(W(t), F(t), t)$ is the indirect utility function in wealth, $W(t)$, and $F(t)$, describing the evolution of the investment opportunity set over time; subscripts denote partial derivatives, and $\left[\frac{-J_{WW}W}{J_W} \right]$ is the coefficient of relative risk aversion, denoted as λ_M . which is typically assumed to be positive. The $\left[\frac{-J_{WF}W}{J_W} \right]$ in the second component describes the hedging coefficient. The sign of the hedging coefficient is indeterminate because it depends on the relationship between the marginal utility of wealth and the state of the world, and the conditional covariance.

If the investment opportunity set is time-invariant, Merton (1980) shows that the hedging component is negligible and the conditional excess market return is proportional to its conditional variance.

$$E_t[r_{M,t+1} - r_{f,t}] = \left[\frac{-J_{WW}W}{J_W} \right] \sigma_{M,t}^2 \quad (2)$$

Since Merton(1980), this conditional CAPM specification has been subject to dozens of empirical investigations. In this paper, I first employ a version of this model to be consistent with Lundblad (2007).

Empirically, I estimate the following GARCH-M (Model 1).

$$r_{M,t+1} - r_{f,t} = \lambda_0 + \lambda_M \sigma_{M,t}^2 + \varepsilon_{t+1}$$

Where ε_{t+1} is mean zero with conditional variance ($\sigma_{M,t}^2$), $\sigma_{M,t+1}^2 = \delta_0 + \delta_1 \varepsilon_t^2 + \delta_2 \sigma_{M,t}^2$

$r_{M,t+1} - r_{f,t}$ is the stock market return in excess of the conditionally risk free rate.

If Lundblad (2007)'s argument is correct, λ_M should be positive and statistically significant. I also add a constant, λ_0 , which could exist due to transaction costs or taxes. In actual empirical implementation, I experimented with various asymmetric GARCH specifications but the asymmetric terms are statistically insignificant for stock and bond returns data. Therefore, I present only empirical results with the usual GARCH specification.

Scruggs (1998) argues that the partial relationship between market risk premia and conditional volatility can be masked in the univariate context by failing to account for the additional hedging demands associated with a time varying investment opportunity set. Based on this observation, I also estimate a time-invariant ICAPM:

$$E_t [r_{M,t+1} - r_{f,t}] = \lambda_0 + \lambda_M \sigma_{M,t}^2 + \lambda_F \sigma_{MF,t}$$

I use the long bond return in excess of the risk free as a proxy for hedging portfolios following Scruggs (1998) and Scruggs and Glabadanidis (2003). Empirically, I employ the following bivariate GARCH-M with a diagonal BEKK specification (Model 2).

$$r_{M,t+1} - r_{f,t} = \lambda_{0,M} + \lambda_M \sigma_{M,t}^2 + \lambda_F \sigma_{MF,t} + \varepsilon_{M,t+1}$$

$$r_{F,t+1} - r_{f,t} = \lambda_{0,F} + \lambda_M \sigma_{MF,t}^2 + \lambda_F \sigma_{F,t}^2 + \varepsilon_{F,t+1}$$

$$\text{cov}[\varepsilon_{M,t}, \varepsilon_{F,t} | F_{t-1}] = \Sigma_t, \Sigma_t = \begin{pmatrix} \sigma_{M,t}^2 & \sigma_{MF,t} \\ \sigma_{MF,t} & \sigma_{F,t}^2 \end{pmatrix}$$

where $r_{F,t+1} - r_{f,t}$ is the long bond return in excess of the conditionally risk free rate.

In estimation, following Scruggs and Glabadanidis (2003), I constrain the prices of risk to be identical across markets consistent with the ICAPM. To describe the time-

series evolution of the stock and bond market return conditional covariance matrix, I employ the following diagonal BEKK specification.

$$\Sigma_t = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}' \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} + \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}' \begin{pmatrix} \varepsilon_{M,t-1}^2 & \varepsilon_{M,t-1}\varepsilon_{F,t-1} \\ \varepsilon_{M,t-1}\varepsilon_{F,t-1} & \varepsilon_{F,t-1}^2 \end{pmatrix} \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}' \Sigma_{t-1} \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}$$

This specification guarantees the positive definiteness of the conditional covariance matrix, and yet allow time variation in conditional variances, covariances, and correlations across these markets. Consistent with the univariate GARCH-M analysis of the market portfolio returns and bond returns (unreported), I do not include asymmetric terms. While this model is a restricted form of the general multivariate GARCH-M employed in Kroner and Ng (1998), the more general model is very difficult to estimate; this restricted model facilitates many of the features that seem to be empirically relevant.

Finally, I generalize the ICAPM with a time-varying risk return tradeoff and hedging coefficient. Lundblad (2007) provides some preliminary evidence to show that the fundamental risk return relationship has changed over time. While he presents evidence only with univariate context, I employ the following bivariate model allowing both time-varying risk return relation and hedging coefficient (Model 3).

$$\begin{aligned} r_{M,t+1} - r_{f,t} &= \lambda_{0,M} + \lambda_{M,t}\sigma_{M,t}^2 + \lambda_{F,t}\sigma_{MF,t} + \varepsilon_{M,t+1} \\ r_{F,t+1} - r_{f,t} &= \lambda_{0,F} + \lambda_{M,t}\sigma_{MF,t} + \lambda_{F,t}\sigma_{F,t}^2 + \varepsilon_{F,t+1} \\ \text{cov}[\varepsilon_{M,t}, \varepsilon_{F,t} | F_{t-1}] &= \Sigma_t, \Sigma_t = \begin{pmatrix} \sigma_{M,t}^2 & \sigma_{MF,t} \\ \sigma_{MF,t} & \sigma_{F,t}^2 \end{pmatrix} \end{aligned}$$

Where $\lambda_{M,t} = \lambda_{M,t-1} + \varepsilon_t, \varepsilon_t \sim N(0, \sigma_v^2), \lambda_{F,t} = \lambda_{F,t-1} + \eta_t, \eta_t \sim N(0, \sigma_F^2)$

$$\Sigma_t = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}' \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} + \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}' \begin{pmatrix} \varepsilon_{M,t-1}^2 & \varepsilon_{M,t-1}\varepsilon_{F,t-1} \\ \varepsilon_{M,t-1}\varepsilon_{F,t-1} & \varepsilon_{F,t-1}^2 \end{pmatrix} \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}' \Sigma_{t-1} \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}$$

Typically, the random walk specifications are used to capture a persistent yet slow movement in the time-varying coefficient model with stochastic volatility (e.g. Cogley

and Sargent (2005)). I follow this tradition to facilitate the estimation by using parsimonious empirical models.

3. Estimation Methods for a Nonlinear ICAPM

I will first present a general estimation framework and then demonstrate that a nonlinear ICAPM is an example of this method. A general state space framework with GARCH terms can be expressed following Harvey, Ruiz, and Sentana (1992).

Measurement equation: $y_t = H_t \beta_t + Az_t + \Lambda \varepsilon_t$

Transition equation: $\beta_t = \mu + F \beta_{t-1} + \omega_t$,

Where $\varepsilon_t | \psi_{t-1} \sim N(0, h_{1t})$, $\omega_t | \psi_{t-1} \sim N(0, h_{2t})$, $h_{1t} = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 h_{1t-1}$, $h_{2t} = \theta_0 + \theta_1 \omega_{t-1}^2 + \theta_2 h_{2t-1}$

where y_t is an $n \times 1$ vectors of variables observed at time t ; β_t is a $k \times 1$ vector of unobserved state variables; H_t is an $n \times k$ matrix that links the observed y_t vector and the unobserved β_t ; z_t is an $r \times 1$ vector of exogenous or predetermined observed variables; μ is $k \times 1$; ω_t and ε_t are 1×1 ; Λ is $n \times 1$.

Given the model's parameters, the linear Kalman filter for the state-space model consists of the following six equations:

Prediction: $\beta_{t|t-1} = \mu + F \beta_{t-1|t-1}$, $p_{t|t-1} = F p_{t-1|t-1} F' + h_{2t}$,

$$\eta_{t|t-1} = y_t - H_t \beta_{t|t-1} - Az_t, \quad f_{t|t-1} = H_t p_{t|t-1} H_t' + \Lambda h_{1t} \Lambda'$$

Updating : $\beta_{t|t} = \beta_{t|t-1} + p_{t|t-1} H_t' f_{t|t-1}^{-1} \eta_{t|t-1}$, $p_{t|t} = p_{t|t-1} - p_{t|t-1} H_t' f_{t|t-1}^{-1} H_t p_{t|t-1}$,

To process above prediction, I approximate h_{1t} and h_{2t} by Harvey, Ruiz, and Sentana (1992).

$$h_{1t} = \alpha_0 + \alpha_1 E[\varepsilon_{t-1}^2 | \psi_{t-1}] + \alpha_2 h_{1t-1}, \quad h_{2t} = \theta_0 + \theta_1 E[\omega_{t-1}^2 | \psi_{t-1}] + \theta_2 h_{2t-1},$$

To get the conditional expectations of the squares of the unobserved shocks of interest, Harvey, Ruiz, and Sentana augment the heteroskedastic shocks into the original state vector in the transition equation.

$$\begin{bmatrix} \beta_t \\ \varepsilon_t \\ \omega_t \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} F & 0_k & 0_k \\ 0_k & 0 & 0 \\ 0_k & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta_{t-1} \\ \varepsilon_{t-1} \\ \omega_{t-1} \end{bmatrix} + \begin{bmatrix} I_k & 0_k & \lambda \\ 0_k & 1 & 0 \\ 0_k & 0 & 1 \end{bmatrix} \begin{bmatrix} \omega_t \\ \varepsilon_t \\ \omega_t \end{bmatrix},$$

In matrix terms, $\beta_{t|t}^* = \mu^* + F^* \beta_{t|t-1}^* + G^* v_t^*$,

$$\text{Where } E[v_t^* v_t^{*'} | \psi_{\tau-1}] = \begin{bmatrix} Q & 0_k & 0_k \\ 0_k & h_t & 0 \\ 0_k & 0 & h_{2t} \end{bmatrix} = Q_t^*, \quad 0_k \text{ is } k \times 1 \text{ vector of zeros.}$$

$$\text{Then, the measurement equation becomes } y_t = \begin{bmatrix} H_t & \Lambda & 0_n \end{bmatrix} \begin{bmatrix} \beta_t \\ \varepsilon_t \\ \omega_t \end{bmatrix} + A z_t$$

where 0_n is an $n \times 1$ vector of zeros. In matrix terms, $y_t = H_t^* \beta_t^* + A z_t$.

By applying the Kalman filter recursions to the above transformed model, we have the following six equations.

$$\text{Prediction: } \beta_{t|t-1}^* = \mu^* + F^* \beta_{t-1|t-1}^*, \quad p_{t|t-1}^* = F^* p_{t-1|t-1}^* F^{*'} + G^* Q_t^* G^{*'},$$

$$\eta_{t|t-1}^* = y_t - H_t^* \beta_{t|t-1}^* - A^* z_t^*, \quad f_{t|t-1}^* = H_t^* p_{t|t-1}^* H_t^{*'} + R,$$

$$\text{Updating: } \beta_{t|t}^* = \beta_{t|t-1}^* + p_{t|t-1}^* H_t^{*'} f_{t|t-1}^{*-1} \eta_{t|t-1}^*, \quad p_{t|t}^* = p_{t|t-1}^* - p_{t-1|t-1}^* H_t^{*'} f_{t|t-1}^{*-1} H_t^* p_{t|t-1}^*,$$

To process the above Kalman filter, we need $E[\varepsilon_{t-1}^2 | \psi_{\tau-1}]$, $E[\omega_{t-1}^2 | \psi_{\tau-1}]$ in the Q_t^* .

Because we know

$$\varepsilon_{t-1} = E[\varepsilon_{t-1} | \psi_{\tau-1}] + \varepsilon_{t-1} - E[\varepsilon_{t-1} | \psi_{\tau-1}], \quad \omega_{t-1} = E[\omega_{t-1} | \psi_{\tau-1}] + \omega_{t-1} - E[\omega_{t-1} | \psi_{\tau-1}],$$

we get easily $E[\varepsilon_{t-1}^2 | \psi_{\tau-1}] = E[\varepsilon_{t-1} | \psi_{\tau-1}]^2 + E[(\varepsilon_{t-1} - E[\varepsilon_{t-1} | \psi_{\tau-1}])^2]$,

$$E[\omega_{t-1}^2 | \psi_{\tau-1}] = E[\omega_{t-1} | \psi_{\tau-1}]^2 + E[(\omega_{t-1} - E[\omega_{t-1} | \psi_{\tau-1}])^2],$$

where $E[\varepsilon_{t-1} | \psi_{\tau-1}]$ and $E[\omega_{t-1} | \psi_{\tau-1}]$ are obtained from the last two elements of $\beta_{t-1|t-1}^*$ and $E[(\varepsilon_{t-1} - E[\varepsilon_{t-1} | \psi_{\tau-1}])^2]$ and $E[(\omega_{t-1} - E[\omega_{t-1} | \psi_{\tau-1}])^2]$ are obtained from the last two diagonal elements of $p_{t-1|t-1}^*$.

As by-products of the above Kalman filter, we obtain the prediction error $\eta_{t|t-1}^*$ and its variance $f_{t|t-1}^*$. Based on this prediction error decomposition, the approximate log likelihood can easily be calculated as

$$\ln L = -\frac{1}{2} \sum_{t=1}^T \ln((2\pi)^n |f_{t|t-1}^*|) - \frac{1}{2} \sum_{t=1}^T \eta_{t|t-1}^{*'} f_{t|t-1}^{*-1} \eta_{t|t-1}^*,$$

which can be maximized with respect to the unknown parameters of the model for an approximate Quasi-MLE.

In this paper, I can use the following state space expression for a nonlinear ICAPM, and estimate the model with the econometric methods presented in this section.

Measurement equation: $y_t = H_t \beta_t + A z_t + \Lambda \varepsilon_t$

$$\begin{bmatrix} r_{M,t+1} - r_{f,t} \\ r_{F,t+1} - r_{f,t} \end{bmatrix} = \begin{bmatrix} \sigma_{M,t}^2 & \sigma_{MF,t} \\ \sigma_{MF,t} & \sigma_{F,t}^2 \end{bmatrix} * \begin{bmatrix} \lambda_{M,t} \\ \lambda_{F,t} \end{bmatrix} + \begin{bmatrix} \lambda_{0,M} \\ \lambda_{0,F} \end{bmatrix} + \begin{bmatrix} \varepsilon_{M,t+1} \\ \varepsilon_{F,t+1} \end{bmatrix}$$

$$\text{cov}[\varepsilon_{M,t}, \varepsilon_{F,t} | F_{t-1}] = \Sigma_t, \Sigma_t = \begin{pmatrix} \sigma_{M,t}^2 & \sigma_{MF,t} \\ \sigma_{MF,t} & \sigma_{F,t}^2 \end{pmatrix}$$

Transition equation: $\beta_t = \mu + F \beta_{t-1} + \omega_t$,

$$\begin{bmatrix} \lambda_{M,t} \\ \lambda_{F,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} \lambda_{M,t-1} \\ \lambda_{F,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix}, \text{cov}[\varepsilon_t, \eta_t | F_{t-1}] = \begin{pmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_F^2 \end{pmatrix}$$

Finally, in a diagonal BEKK specification, I need to approximate the following terms. I should denote this as an approximate BEKK model. However, the GARCH estimates presented in the Section 5 from this model are close to those from a bivariate

GARCH-M with BEKK specification. Therefore, the approximation errors seem minimal.

$$\varepsilon_{M,t-1}\varepsilon_{F,t-1} = (E[\varepsilon_{M,t-1}^2|\psi_{\tau-1}] * E[\varepsilon_{F,t-1}^2|\psi_{\tau-1}])^{0.5}$$

4. Empirical Analysis

4.1 Data Description

As Lundblad (2007) carefully demonstrates, we need a big span of data to investigate the risk return tradeoff to enhance the power of the time-series analysis. To maximize the power of the time-series analysis, I employ the longest monthly UK equity and bond market data from 1836:01 to 2010:12. Earlier U.K. data extending back to 1800 are also available, but I exclude this sample because the stock market data only represent a simple equal weighted average of three shares: the Bank of England, the East India Company, and the South Sea Company. For a more detailed explanation on the data sources, see the documentation from www.globalfindata.com.

The UK historical stock data (ticker symbol: TFTASD) are taken from the Global Financial Data provider, and represent the FTSE All Shares historical index. I also collect total return data for the UK short-term bill (ticker symbol: TRGBRBIM) and long-term consol bond (ticker symbol: TRGBRGCM) from the same provider. Short-term bill data will serve as the conditionally risk-free rate in my analysis. As originally suggested by Merton (1973) (and implemented in Scruggs (1998)), I collect long-term U.K. bond returns to capture variation in the investment opportunity set over time.

Table 1 reports summary statistics on the total returns for the U.K. equity market, $r_{M,t}$, the bond market, $r_{F,t}$, and the short bill return (the conditionally risk-free rate), $r_{f,t}$, for the full sample. All variables are expressed as continuously compounded returns. The return data for each series are also displayed in Figure 1. In the whole sample, the

mean return on the U.K. stock market portfolio is about 57 basis points per month. As expected, the stock market return is highly volatile (3.6% per month). Long term bond and short term bill returns have similar lower mean return around 36 basis points per month and also lower volatility as expected.

4.2 Expected Return - Volatility Tradeoff

4.2.1 the Conditional CAPM

Many asset pricing models imply a positive relationship between the risk premium on the market portfolio and the conditional variance of its return. A number of previous studies employ the following GARCH-M framework to explore this relationship (Model 1).

$$E_t[r_{M,t+1} - r_{f,t}] = \lambda_0 + \lambda_M \sigma_{M,t}^2, \sigma_{M,t+1}^2 = \delta_0 + \delta_1 \varepsilon_t^2 + \delta_2 \sigma_{M,t}^2$$

In summary, these studies typically find a statistically insignificant or a negative relationship between the market risk premium and its expected volatility. A notable exception is Lundblad (2007). Using simulations, he demonstrates that even 100 years of data constitute a small sample that may easily lead to this puzzling insignificant or negative risk return relation even though the true risk return tradeoff is positive. Using the nearly two century history of U.S. equity market returns, Lundblad estimates a positive and statistically significant risk return tradeoff across every specification considered. While he also presents similar evidence with U.K. data, I find that the evidence doesn't seem to be robust with continuously compounded returns.

Table 2 presents evidence on the risk-return tradeoff in the univariate context with a long span of U.K. data from 1836:01 to 2010:12. The point estimate, presented in panel A, for λ_M is 1.66 with a t statistics of 1.81, which is quite different from Lundblad's estimate (2.469 with a standard error of 0.906 in his table). To reconcile the difference, I also estimate the same model with simple returns, and the estimates

are provided in panel B. In this case, the mean variance tradeoff is positive (2.2522) and significant. Here I report the estimates with the usual GARCH-M because I find that asymmetric terms in the GARCH specifications are not statistically significant. Table 4 in Lundblad (2007) also present evidence that asymmetric GARCH models are unnecessary for U.K. data. Figure 2 displays the estimates of conditional market variance implied by the GARCH process. Elevated volatility is most pronounced during the Great Depression, 1973-74 stock market crash, the recent global financial crises of the late 1990.

Particularly, during the 1973-74 stock market crash, London Stock Exchange's FT 30 lost 73% of its value. The UK went into recession in 1974, with GDP falling by 1.1%. At the time, the UK's property market was going through a major crisis, and a secondary banking crisis forced the Bank of England to bail out a number of lenders. After the definitive market low for the FT30 Index on January 6th 1975 when the index closed at 146, the market almost doubled over next 3 months.

4.2.2 the Intertemporal CAPM

Scruggs (1998) argues that the partial relationship between risk premia and conditional volatility can be masked in the univariate context by failing to account for the additional hedging demands associated with a time varying investment opportunity set (essentially generating an omitted variable bias). I will explore this issue in this section.

Based on Scruggs (1998), I estimate two equations associated with the conditional mean of the market portfolio return and the bond market return implied by the ICAPM presented in equation (1). To describe the time-series evolution of the stock and bond market return conditional covariance matrix, I employ the following diagonal BEKK specification. I constrain the prices of risk to be identical across markets consistent with the ICAPM. For the easier explanation, I reproduce the empirical model below (Model 2).

$$r_{M,t+1} - r_{f,t} = \lambda_{0,M} + \lambda_M \sigma_{M,t}^2 + \lambda_F \sigma_{MF,t} + \varepsilon_{M,t+1}$$

$$r_{F,t+1} - r_{f,t} = \lambda_{0,F} + \lambda_M \sigma_{MF,t}^2 + \lambda_F \sigma_{F,t}^2 + \varepsilon_{F,t+1}$$

$$\text{cov}[\varepsilon_{M,t}, \varepsilon_{F,t} | F_{t-1}] = \Sigma_t, \Sigma_t = \begin{pmatrix} \sigma_{M,t}^2 & \sigma_{MF,t} \\ \sigma_{MF,t} & \sigma_{F,t}^2 \end{pmatrix}$$

$$\Sigma_t = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}' \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} + \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}' \begin{pmatrix} \varepsilon_{M,t-1}^2 & \varepsilon_{M,t-1} \varepsilon_{F,t-1} \\ \varepsilon_{M,t-1} \varepsilon_{F,t-1} & \varepsilon_{F,t-1}^2 \end{pmatrix} \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}' \Sigma_{t-1} \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}$$

This specification guarantees the positive definiteness of the conditional covariance matrix, and yet allow time variation in conditional variances, covariances, and correlations across these markets. Consistent with the univariate GARCH-M analysis of the market portfolio returns and bond returns (unreported), I do not include asymmetric terms. While this model is a restricted form of the general multivariate GARCH-M employed in Kroner and Ng (1998), the general model is very difficult to estimate; this restricted model facilitates many of the features that seem to be empirically relevant.

Table 3 presents evidence for the bivariate diagonal BEKK specification. First, the conditional variances for the bond and stock markets are persistent (above 0.8) in the full historical record. Figure 3 presents time-series plots of the conditional stock and bond return heteroskedasticity and the conditional covariance. The equity variance displays similar patterns presented in the univariate model (see Figure 2). Bond variance is quite low compared with the stock variance but it increases dramatically after 1980's and especially during the recent crisis.

Next, I present evidence on the intertemporal relationship between risk and expected return. Parameter estimates for the conditional mean are also provided in Table 3. In Panel A, for the whole sample, the partial relationships between the expected market excess return and market conditional variance is positive and statistically significant with t statistics of 2.09. However, the partial relationship between expected market excess returns and covariance with variation in the investment opportunity set is not statistically significant. Strictly speaking, a two factor ICAPM does not seem to be supported by the data.

In sum, with variation in the investment opportunity set, the risk return tradeoff becomes positive and statistically significant. However, this ICAPM specification doesn't seem to be supported by the data because the hedging coefficient is not statistically significant.

4.2.3 A Time-varying Risk Return Tradeoff?

There are several concerns associated with the potentially strong assumption of a time-invariant risk return tradeoff. First, in equilibrium, the mean variance tradeoff can be interpreted as risk aversion and may exhibit cyclical variation through time as implied by habit models such as Campbell and Cochrane (1999). Further, risk aversion may also vary because of the evolution of financial markets and improved risk sharing. Hence, I conduct an empirical analysis of a time-varying risk return tradeoff, and investigate whether the proposed nonlinear ICAPM shed light on the puzzling risk return relation.

Lundblad (2007) also conducts an exploratory analysis to allow the risk return tradeoff coefficient to vary with several observable financial and macroeconomic indicators of the development of the U.S. financial market and economy. However, Lundblad employs only univariate models in this context. Without fully applying time-varying coefficients both in the risk return tradeoff and the hedging coefficient, it is unclear if the time-varying relation exist or it just indicates misspecifications. Due to the limit of historical data, I choose econometric models with latent factors to estimate the unstable relation rather than to use the models with exogenous variables.

Before conducting a formal econometric analysis in the next section, I present preliminary evidence of instability by estimating the two-factor ICAPM in the previous section recursively at least with 10 years of data starting from 1846:01. Figure 4 presents rolling sample estimates of the risk return tradeoff and the hedging coefficient from the ICAPM. While it is difficult to argue for the time-varying risk

return relation with any formal statistics at this stage, it looks as if both the risk return tradeoff and the hedging coefficients vary a lot.

5. A Time-varying Risk Return Tradeoff with a Nonlinear ICAPM

Motivated by theoretical arguments and preliminary rolling estimates of the ICAPM, I develop and estimate a nonlinear ICAPM with the time-varying risk return tradeoff and hedging coefficients (Model 3). I reproduce the empirical model for the easier explanation.

$$\begin{aligned}
r_{M,t+1} - r_{f,t} &= \lambda_{0,M} + \lambda_{M,t} \sigma_{M,t}^2 + \lambda_{F,t} \sigma_{MF,t} + \varepsilon_{M,t+1} \\
r_{F,t+1} - r_{f,t} &= \lambda_{0,F} + \lambda_{M,t} \sigma_{MF,t}^2 + \lambda_{F,t} \sigma_{F,t}^2 + \varepsilon_{F,t+1} \\
\text{cov}[\varepsilon_{M,t}, \varepsilon_{F,t} | F_{t-1}] &= \Sigma_t, \Sigma_t = \begin{pmatrix} \sigma_{M,t}^2 & \sigma_{MF,t} \\ \sigma_{MF,t} & \sigma_{F,t}^2 \end{pmatrix}
\end{aligned}$$

Where $\lambda_{M,t} = \lambda_{M,t-1} + \varepsilon_t, \varepsilon_t \sim N(0, \sigma_v^2), \lambda_{F,t} = \lambda_{F,t-1} + \eta_t, \eta_t \sim N(0, \sigma_F^2)$

$$\Sigma_t = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}' \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} + \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}' \begin{pmatrix} \varepsilon_{M,t-1}^2 & \varepsilon_{M,t-1} \varepsilon_{F,t-1} \\ \varepsilon_{M,t-1} \varepsilon_{F,t-1} & \varepsilon_{F,t-1}^2 \end{pmatrix} \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}' \Sigma_{t-1} \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}$$

Typically, the random walk specifications are used to capture a persistent yet slow movement in the time-varying coefficient model with stochastic volatility (e.g. Cogley and Sargent (2005)). I follow this tradition with the above parsimonious empirical model.

Table 4 presents parameter estimates of the model 3. First, I find that variance estimates in the time-varying risk return (σ_v) and hedging coefficient (σ_F) are statistically significant at 5% level. If these terms are not statistically significant, it would be difficult to argue for this model to explain the puzzling risk return relationship. Figure 5 presents time-series plots of the conditional stock and bond return variances and the conditional stock-bond return covariance. The stock return

variance follows the same patterns presented in the conditional CAPM and ICAPM (see Figure 2 and 3). Bond variance displays clustering consistent with the estimates from ICAPM (see Figure 3) as well, increasing dramatically during the early 1980's. Over the full historical record, the conditional covariance between the stock and bond market is a small, but positive number.

Figure 6 shows the time-varying risk return relation and 95% confidence bands estimated from the nonlinear ICAPM. The estimated partial expected market return volatility tradeoff is largely positive and statistically significant for the full historical record. Further, this figure shows that the seemingly negative relation could be entirely spurious because the negative relation is not statistically different from zeros with the 95% confidence bounds. The estimated hedging coefficient is also time-varying and negative over the time (Figure 7). This evidence indicates that incorporating time-varying risk return coefficient along with the changing coefficients in the hedging demands associated with variation in the investment opportunity set is crucial to understand the puzzling risk return tradeoff. In sum, I find contrary to the evidence presented in the previous section with the time-invariant conditional CAPM and ICAPM, the expected market return conditional volatility tradeoff is positive and statistically significant for the full historical record. I conclude that the time-varying risk-return trade-off is the main reason for the weak relation.

6. Conclusion

While the risk–return tradeoff is fundamental to finance, the empirical evidence on the relationship between the risk premium on aggregate stock market and the variance of its return is ambiguous at best. Lundblad (2007) argues the main culprit of this puzzling relationships is the small sample problem. He finds a statistically significant positive risk return tradeoff using information from two century history of stock market returns in all of the econometric specifications.

In this paper, I first present new evidence that the risk-return trade-off is rather weak even with the two century history of UK continuously compounded return data, when I employ a time-invariant conditional CAPM or ICAPM with GARCH-M models. In the conditional CAPM, the risk return tradeoff parameter is positive yet statistically insignificant at 5% level. When the time-varying investment opportunity set is explicitly accounted as the hedging component, the risk return tradeoff becomes positive and statistically significant at 5% level. But one of the crucial implication of the ICAPM is rejected; the hedging coefficient is insignificant even at 10% level.

Motivated by theoretical arguments and preliminary rolling estimates of the ICAPM, I develop and estimate a nonlinear ICAPM with the time-varying risk return tradeoff and hedging coefficients. Consistent with the implication of the model, I find that variance estimates in the time-varying risk return and hedging coefficients are statistically significant at 5% level. In this model, the risk return relation is largely positive over the time. Further, I show that the seemingly negative relation could be entirely spurious because the estimated relation is not statistically different from zeros with the 95% confidence bounds. I conclude that the time-varying risk-return trade-off is the main reason for the weak relation.

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Table 1
Descriptive Statistics (1836:01 - 2010:12)

	$r_{M,t}$	$r_{F,t}$	$r_{f,t}$
Mean	0.005727	0.003733	0.003548
Std. Dev.	0.036466	0.022212	0.002527
Skewness	0.009363	0.357253	1.37425
Kurtosis	18.49204	7.466997	4.92976
Auto(1)	0.087	0.111	0.985

	$r_{M,t}$	$r_{F,t}$	$r_{f,t}$
$r_{M,t}$	1		
$r_{F,t}$	0.233426	1	
$r_{f,t}$	0.032399	0.068808	1

Table 1 reports summary statistics and autocorrelation and correlations for returns on U.K. stock market portfolios ($r_{M,t}$), long term bond returns ($r_{F,t}$), and the return on the short term bill ($r_{f,t}$) available from the Global Financial Data provider. All variables are expressed as continuously compounded returns.

Table 2 Risk-Return Tradeoff: the conditional CAPM

	λ_0	λ_M	δ_0	δ_1	δ_2
Panel A	0.0004	1.6668	0.0000	0.1088	0.8951
t-stat	0.7005	1.8132	2.5851	6.1362	64.8426
Panel B	0.0005	2.2522	0.0000	0.1109	0.8918
t-stat	0.7151	2.2554	2.7842	6.2697	63.8226

Table 2 presents evidence on the conditional mean and volatility of the U.K. stock market portfolio implied by the GARCH-M from 1836:01 to 2010:12. Panel A (B) presents estimation results with continuously compounded returns (simple returns). The mean equation is:

$$r_{M,t+1} - r_{f,t} = \lambda_0 + \lambda_M \sigma_{M,t}^2 + \varepsilon_{t+1}, \text{ where } \varepsilon_{t+1} \text{ is mean zero with the conditional variance } (\sigma_{M,t}^2)$$

$$\sigma_{M,t+1}^2 = \delta_0 + \delta_1 \varepsilon_t^2 + \delta_2 \sigma_{M,t}^2$$

Table 3 Risk Return Tradeoff in the ICAPM

Panel A. Conditional Mean			
		S.E.	t-stat
$\lambda_{0,M}$	0.000425	0.000554	0.766158
λ_M	1.324719	0.63499	2.086204
$\lambda_{0,F}$	-0.00024	0.000427	-0.56964
λ_F	0.987953	1.34776	0.733034

Panel B. Conditional Variance			
		S.E.	t-stat
c11	0.002974	0.000304	9.792615
c12	0.000573	0.000183	3.136242
c22	0.001546	0.000119	12.94215
a11	0.356504	0.011707	30.45326
a22	0.238791	0.006908	34.56903
b11	0.937731	0.00369	254.1151
b22	0.970183	0.001405	690.5052

Table 3 provides parameter estimates for the following two-factor ICAPM with BEKK (1,1) specification.

$$r_{M,t+1} - r_{f,t} = \lambda_{0,M} + \lambda_M \sigma_{M,t}^2 + \lambda_F \sigma_{MF,t} + \varepsilon_{M,t+1}$$

$$r_{F,t+1} - r_{f,t} = \lambda_{0,F} + \lambda_M \sigma_{MF,t}^2 + \lambda_F \sigma_{F,t}^2 + \varepsilon_{F,t+1}$$

$$\text{cov}[\varepsilon_{M,t}, \varepsilon_{F,t} | F_{t-1}] = \Sigma_t, \Sigma_t = \begin{pmatrix} \sigma_{M,t}^2 & \sigma_{MF,t} \\ \sigma_{MF,t} & \sigma_{F,t}^2 \end{pmatrix}$$

$$\Sigma_t = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}' \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} + \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}' \begin{pmatrix} \varepsilon_{M,t-1}^2 & \varepsilon_{M,t-1} \varepsilon_{F,t-1} \\ \varepsilon_{M,t-1} \varepsilon_{F,t-1} & \varepsilon_{F,t-1}^2 \end{pmatrix} \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}' \Sigma_{t-1} \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}$$

Table 4. Risk Return Tradeoff in a Nonlinear ICAPM

Panel A. Conditional Mean			
		S.E.	t-stat
$\lambda_{0,M}$	-0.002	0.0013	-1.82274
$\lambda_{0,F}$	0.0058	0.0006	10.16276

Panel B. Conditional Variance			
		S.E.	t-stat
c11	0.0011	0.0003	4.0086
c12	0.0049	0.0004	11.23176
c22	0.0009	0.0015	0.609475
a11	0.2558	0.0036	72.04135
a22	0.3483	0.021	16.60006
b11	0.9727	0.001	1015.334
b22	0.9138	0.0127	72.00469

Panel C. TVP variance			
		S.E.	t-stat
σ_v	0.421	0.1159	3.631739
σ_F	0.2861	0.073	3.917361

Table 4 provides parameter estimates for the following two-factor nonlinear ICAPM with BEKK (1,1) specification.

$$r_{M,t+1} - r_{f,t} = \lambda_{0,M} + \lambda_{M,t}\sigma_{M,t}^2 + \lambda_{F,t}\sigma_{MF,t} + \varepsilon_{M,t+1}$$

$$r_{F,t+1} - r_{f,t} = \lambda_{0,F} + \lambda_{M,t}\sigma_{MF,t}^2 + \lambda_{F,t}\sigma_{F,t}^2 + \varepsilon_{F,t+1}$$

$$\text{cov}[\varepsilon_{M,t}, \varepsilon_{F,t} | F_{t-1}] = \Sigma_t, \Sigma_t = \begin{pmatrix} \sigma_{M,t}^2 & \sigma_{MF,t} \\ \sigma_{MF,t} & \sigma_{F,t}^2 \end{pmatrix}$$

Where $\lambda_{M,t} = \lambda_{M,t-1} + \varepsilon_t, \varepsilon_t \sim N(0, \sigma_v^2), \lambda_{F,t} = \lambda_{F,t-1} + \eta_t, \eta_t \sim N(0, \sigma_F^2)$

$$\Sigma_t = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}' \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} + \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}' \begin{pmatrix} \varepsilon_{M,t-1}^2 & \varepsilon_{M,t-1}\varepsilon_{F,t-1} \\ \varepsilon_{M,t-1}\varepsilon_{F,t-1} & \varepsilon_{F,t-1}^2 \end{pmatrix} \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}' \Sigma_{t-1} \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}$$

Figure 1. Time Series Plots of Asset Return Data

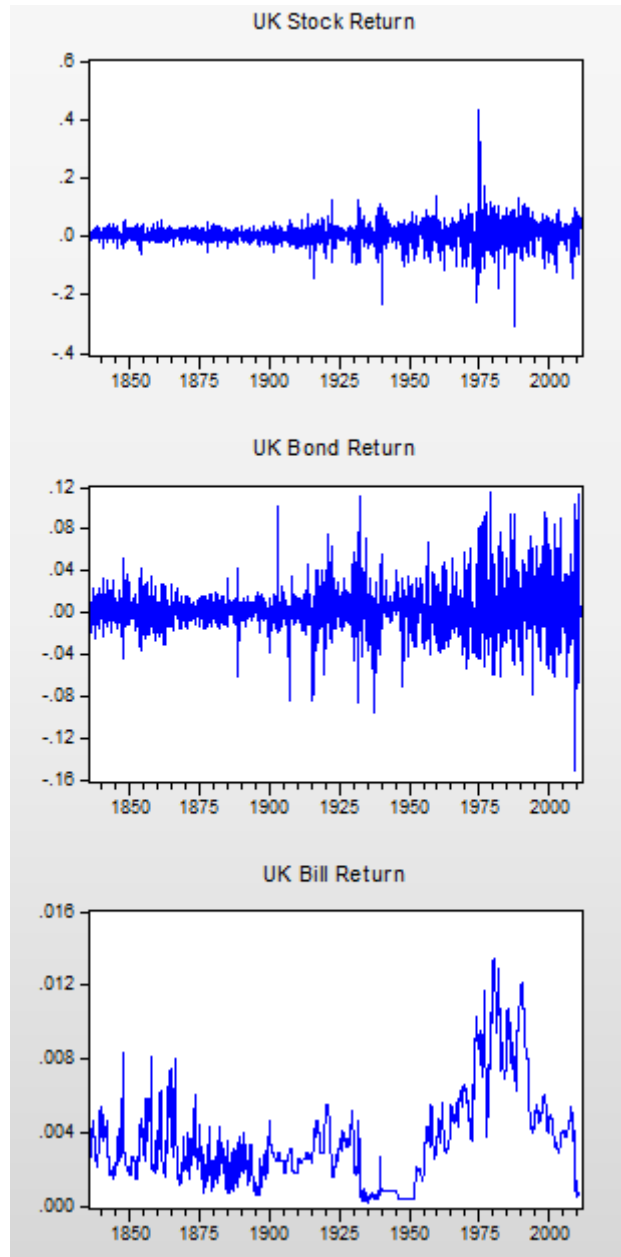


Figure 2. Conditional Equity Variance

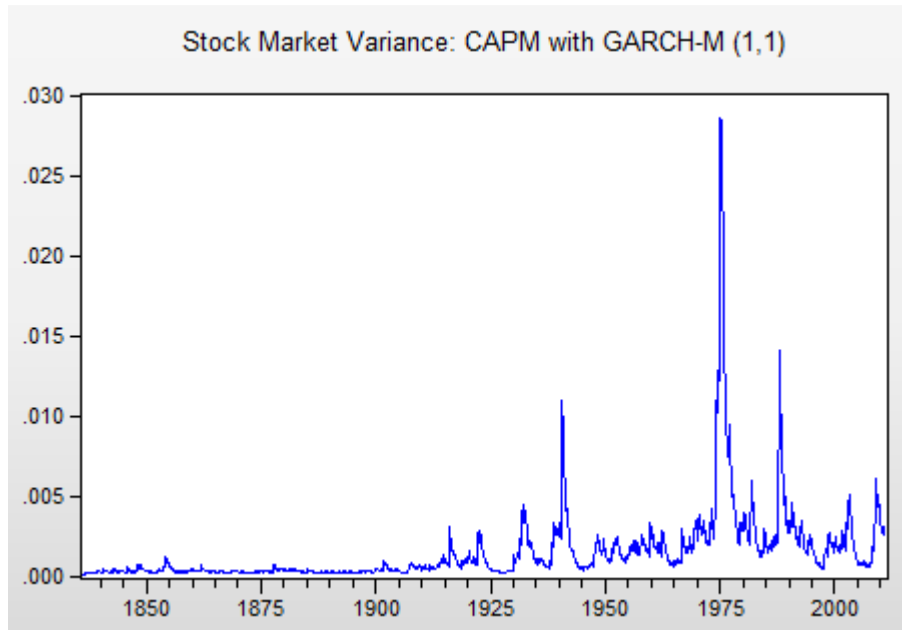


Figure 3. Conditional Variance from ICAPM

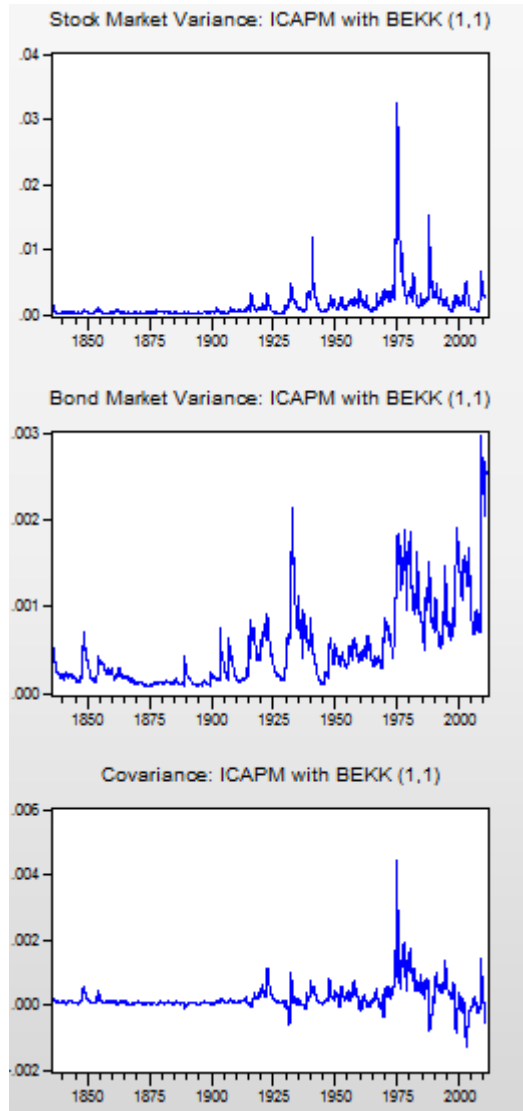


Figure 4. Rolling Estimates of the ICAPM

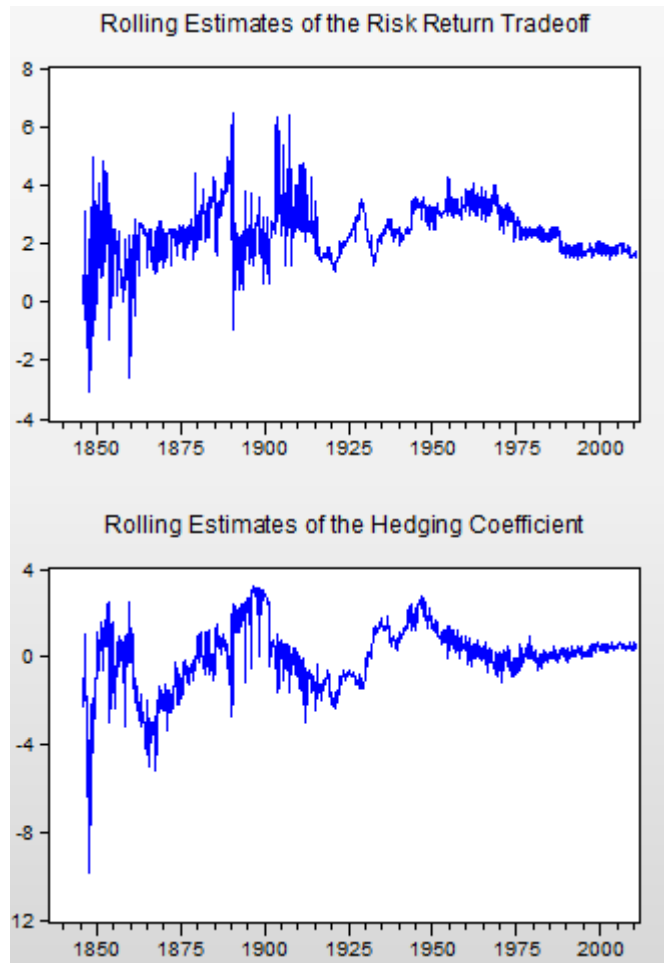


Figure 5. Conditional Variance from a Nonlinear ICAPM

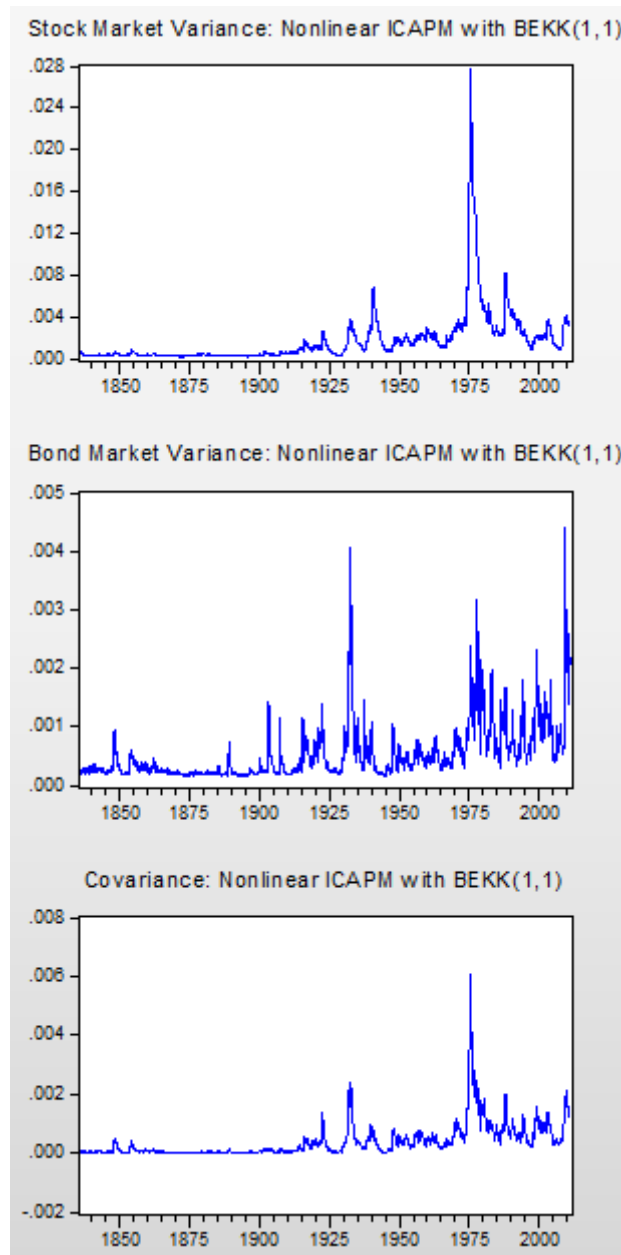


Figure 6. Risk Return Tradeoff from a Nonlinear ICAPM

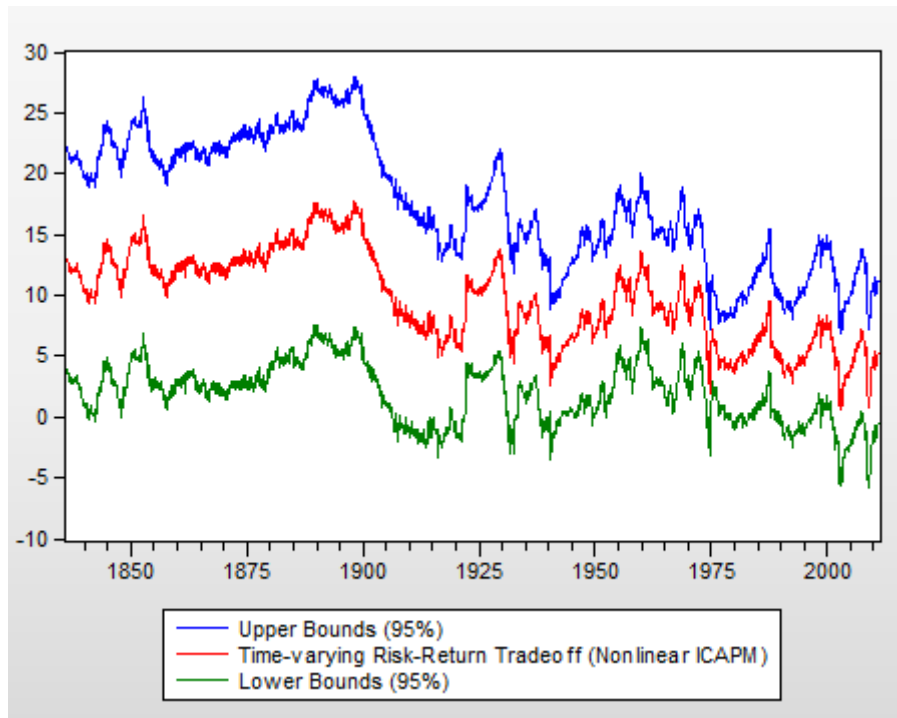


Figure 7. Hedging Coefficient from a Nonlinear ICAPM

