Local feedback dissipativity and dissipativity-based stabilization of nonlinear discrete-time systems

LOCAL FEEDBACK DISSIPATIVITY AND DISSIPATIVITY-BASED STABILIZATION OF NONLINEAR DISCRETE-TIME SYSTEMS

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Abstract: This paper is endeavoured to the problem of local feedback dissipativity and the use of feedback dissipativity for stabilization purposes in the nonlinear discrete-time setting. Sufficient conditions under which a class of non-affine discrete-time control systems can be rendered locally dissipative are derived. A methodology in order to render nonlinear single-input single-output discrete-time systems locally dissipative is given, it is based on the feedback losslessness situation. The feedback dissipativity methodology proposed will be applied to an academic nonlinear example. Stability properties of a class of feedback dissipative systems are used for stabilization purposes in the example.

Keywords: Discrete-time systems, Nonlinear systems, Energy control, Passive elements, Feedback stabilization.

1. INTRODUCTION

The study of dissipativity-related concepts in the nonlinear discrete-time setting is an interesting field for which a lot of problems remain unsolved. A main problem not having attracted broadly attention is the establishment of conditions for a nonlinear discrete-time system to be rendered dissipative or passive via state feedback.

Dissipative and passive systems present highly desirable properties which may simplify the system analysis and control design. This fact impels to transform a system which is not dissipative (passive) into a dissipative (passive) one. The action of rendering a system dissipative (passive) by means of a static state feedback is known as *feedback dissipativity* (feedback passivity or *passivation*). Systems which can be rendered dissipative (passive) are regarded as feedback dissipative (feedback passive) systems.

The problem of the establishment of conditions for a nonlinear discrete-time system to be rendered passive or dissipative via a state feedback has not been solved yet in a general manner. This problem has only been solved for lossless systems (Byrnes and Lin, 1994). A non-general solution to the problem of feedback dissipativity for single-input single-output systems is given in (Navarro-López *et al.*, 2001), (Navarro-López *et al.*, 2002b). An alternative methodology dealing with feedback passivity in multiple-input multiple-output affine-in-the-input systems has been proposed (Navarro-López and Fossas-Colet, 2002a) through the properties of the relative degree and the zero dynamics, in the line of (Byrnes and Lin, 1994). This paper

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proposes a new approach treating the feedback dissipativity problem in a local way for nonlinear discrete-time systems of general form.

For the linear case, the problems of feedback passivity and feedback dissipativity for (Q,S,R)-dissipative systems, i.e., dissipative systems with supply functions of the form $s(y,u) = y^T Qy + 2y^T Su + u^T Ru$, have been solved in the framework of the positive real control problem (de Souza and Xie, 1992), (de Souza *et al.*, 1993) and the (Q,S,R)-dissipative control problem (Tan *et al.*, 1999), (Tan *et al.*, 2000) in connection with H_∞ design.

In this paper, a procedure for dealing with the feedback dissipativity problem is posed. The proposed feedback dissipativity approach is based on the establishment of the input u which satisfies the fundamental dissipativity inequality; it is the idea underlying in (Sira-Ramírez, 1998) for the continuous-time case. The approach is of approximate type and is based on the proposal of a control u which renders the system (V,s)-dissipative. In this case, dissipativity is seen as a "perturbation" of the system losslessness situation. The errors of the approximation are bounded, and sufficient conditions are given under which the approximation made is valid.

The paper is organized as follows. Section (2) presents basic definitons used in the sequel. Section (3) solves the local feedback dissipativity problem in two steps: first, the system is rendered lossless, second, the control which makes the system dissipative is proposed in order to satisfy an approximation of the fundamental dissipativity inequality. The feedback dissipativity methodology is illustrated by means of an academic nonlinear discrete-time system in Section (4). For the example, the feedback passivity or passivation problem is treated. Stability properties of a class of feedback dissipative systems are dealt with in Section (5). These properties are used for the stabilization of the fixed point of the system, which is unstable in open loop. Conclusions and suggestions for further research are presented in the last section.

2. PRELIMINARY DEFINITIONS

Let a nonlinear single-input single-output discrete-time system of the form

$$x(k+1) = f(x(k),u(k)), \quad x \in \mathscr{X}, \;\; u \in \mathscr{U} \qquad (1)$$

$$y(k) = h(x(k),u(k)), \quad y \in \mathscr{Y} \qquad (2)$$

where $f : \mathscr{X} \times \mathscr{U} \to \mathscr{X}$ and $h : \mathscr{X} \times \mathscr{U} \to \mathscr{Y}$ are smooth maps with $\mathscr{X} \subset \mathscr{R}^n$, $\mathscr{U}, \mathscr{Y} \subset \mathscr{R}$ open sets. $k \in Z_+ := \{0,1,2,\ldots\}$. All considerations will be restricted to an open set of $\mathscr{X} \times \mathscr{U}$ containing $(\overline{x},\overline{u})$, having $\overline{x}$ as an isolated fixed point of $f(x,\overline{u})$, with $\overline{u}$ a constant, i.e., $f(\overline{x},\overline{u}) = \overline{x}$. A positive definite smooth function $V : \mathscr{X} \to \mathscr{R}$, with $V(0) = 0$, associated with the system (1)-(2) and addressed as the *storage function* is considered. Function V is considered to have a strict local minimum in $\overline{x}$. A second smooth function is also taken into account, called the *supply function*, denoted by $s(y,u)$, with $s : \mathscr{Y} \times \mathscr{U} \to \mathscr{R}$.

Definition 1. The supply function $s(y,u)$ is said to satisfy the zero-input-output (ZIO) property if

$$\begin{aligned} s(0,u) &= 0, \quad \forall\, u \in \mathscr{U} \\ s(y,0) &= 0, \quad \forall\, y \in \mathscr{Y} \end{aligned} \qquad (3)$$

Definition 2. The system (1)-(2) with storage function $V(x)$ and supply function $s(y,u)$ is said to be (V,s)-dissipative (resp., strictly (V,s)-dissipative) if the following inequality (resp., strict inequality) is satisfied

$$\begin{aligned} & V(f(x,u)) - V(x) \leq s(h(x,u),u), \\ & \forall (x,u) \in \mathscr{X} \times \mathscr{U} \end{aligned} \qquad (4)$$

Definition 3. The system (1)-(2) is said to be V-passive if it is (V,s)-dissipative with respect to the supply rate $s(y,u) = yu$. The system is said to be (V,s)-lossless if (4) is an equality.

Let $\alpha : \mathscr{X} \times \mathscr{U} \to \mathscr{U}$ be a $\mathscr{C}^1$ function with $\alpha(\overline{x},0) = \overline{u}$. A nonlinear static state feedback control law is denoted by the expression $u = \alpha(x,v)$. The system $x(k+1) = f(x(k),\alpha(x(k),v(k)))$ is referred by the *feedback transformed* system, which may be also denoted by $x(k+1) = \overline{f}(x(k),v(k))$. In addition, $\overline{h}(x,v)$ denotes the function $h(x,\alpha(x,v))$.

Definition 4. A feedback control law $u = \alpha(x,v)$ is locally *regular* if for all $(x,v) \in \mathscr{X} \times \mathscr{U}$, it follows that $\partial\alpha/\partial v \neq 0$.

Definition 5. Consider the system (1)-(2) and two scalar functions $V(x)$ and $s(y,v)$ as a storage function and a supply function, respectively. The system is said to be *feedback dissipative* (resp., feedback stricly dissipative) with the functions V and s if there exists a *regular* static state feedback control law of the form $u = \alpha(x,v)$, with v as the new input, such that the feedback transformed system is (V,s)-dissipative (resp., strictly (V,s)-dissipative).

Definition 6. A system of the form (1)-(2) is said to be feedback passive if it is feedback dissipative with $s(y,v) = yv$.

3. FEEDBACK DISSIPATIVITY SEEN AS A PERTURBATION OF THE LOOSSLESSNESS SITUATION

This section deals with the problem of feedback dissipativity for a class of nonlinear discrete-time systems.

The approach is local since the (V,s)-dissipativity of the feedback transformed system is assured in a compact subset of $\mathscr{X}\times\mathscr{U}$ containing the fixed point of the system. The orbits of the feedback transformed system are assured not to leave this compact by means of the stability properties of the class of dissipativity treated. The feedback dissipativity methodology proposed is based upon the dissipativity inequality (4) using the first-order Taylor expansion formula at u of $V(f(x,u))$ and $s(h(x,u),v)$.

The underlying idea is that dissipativity is seen as a "perturbation" of the system losslessness situation, in the sense that the control which makes the system dissipative (u) is based on the control that renders the system lossless (u^*), and u is locally valid in a neighbourhood of u^*. This method can be also seen as an alternative way of treating the feedback losslessness problem to the one proposed in (Byrnes and Lin, 1994). The local feedback losslessness methodology proposed is derived from the feedback dissipativity one proposed in (Navarro-López *et al.*, 2002b).

3.1 Description of the methodology

Before proposing the control which renders the system (1)-(2) (V,s)-dissipative, the feedback losslessness problem is defined in terms of the dissipativity inequality.

Definition 7. Consider a system of the form (1)-(2) and two scalar functions $V(x)$ and $s(y,v)$ considered as a storage function and a supply function, respectively. The system is said to be *feedback lossless* with the functions V and s, if there exists a *regular* static state feedback control law of the form, $u=\alpha(x,v)$, with v the new input, such that the feedback transformed system is (V,s)-lossless.

The existence of a feedback control law of the form $u^*=\alpha(x,v)$ for which the system is rendered (V,s)-lossless must be assessed from the existence of solutions, for the control input u^*, of the following equation,

$$V(f(x,u^*))-V(x)=s(h(x,u^*),v) \tag{5}$$

The following proposition states sufficient conditions under which local feedback losslessness is possible.

Proposition 8. Consider a system of the form (1)-(2) and two scalar functions $V(x)$ and $s(y,v)$ considered as a storage function and a supply function, respectively. Let $(x_0,u_0^*,v_0)\in\mathscr{X}\times\mathscr{U}\times\mathscr{U}$. Suppose that the following two conditions are satisfied:

(i) $\exists(x_0,u_0^*,v_0)$ such that equality (5) holds, i.e., $V(f(x_0,u_0^*))-V(x_0)-s(h(x_0,u_0^*),v_0)=0$

(ii) $\frac{\partial}{\partial u^*}[V(f(x,u^*))-s(h(x,u^*),v)]\Big|_{(x_0,u_0^*,v_0)}\neq 0$

Then, there exists a unique static state feedback control law of the form $u^*=\alpha^*(x,v)$ defined in a neighbourhood of (x_0,v_0) and valued in a neighbourhood of u_0^* such that the feedback transformed system $x(k+1)=\overline{f}(x(k),v(k))$, $y(k)=\overline{h}(x(k),v(k))$ is (V,s)-lossless.

Proof The proof follows from the implicit function theorem.

Consider the system (1)-(2). Suppose there exists a regular static state feedback $u^*:\mathscr{X}\times\mathscr{U}\to\mathscr{U}$ which renders the system (V,s)-lossless. Let a function $\delta u^*(x,u^*,v)$ such that $\delta u^*:\mathscr{X}\times\mathscr{U}\times\mathscr{U}\to\mathscr{U}$. Define the following state dependent input coordinate transformation,

$$u=u^*(x,v)+\delta u^*(x,u^*,v) \tag{6}$$

with,

$$\begin{aligned}&\delta u^*(x,u^*,v)=\\&=-\mu\,\frac{\partial}{\partial u}[V(f(x,u))-s(h(x,u),v)]\Big|_{u=u^*}\end{aligned} \tag{7}$$

where μ is a positive constant.

Proposition 9. Let $V(x)$ and $s(y,v)$ a storage function and a supply function with s satisfying the ZIO property. Suppose conditions (i)-(ii) of Proposition 8 are satisfied. Let $\widetilde{\mathscr{X}}\subset\mathscr{X}$ and $\widetilde{\mathscr{U}},\widetilde{\mathscr{V}},\widetilde{\mathscr{P}}\subset\mathscr{U}$ be compact sets containing $\overline{x}$ and $\overline{u}$, respectively. Then, the system (1)-(2) is locally feedback dissipative with the functions V and s by means of a feedback of the form (6), with $u^*:\widetilde{\mathscr{X}}\times\widetilde{\mathscr{V}}\to\widetilde{\mathscr{U}}$ obtained from (5), $\delta u^*:\widetilde{\mathscr{X}}\times\widetilde{\mathscr{U}}\times\widetilde{\mathscr{V}}\to\widetilde{\mathscr{P}}$ given by (7) , and $u^*+\delta u^*$ defined in a neighbourhood of u^* if there exists a positive constant μ, for which the following conditions are satisfied

$$\begin{aligned}&|\max R_V-\min R_s|\leq\\&\leq\mu\left\{\frac{\partial}{\partial u}[V(f(x,u))-s(h(x,u),v)]\Big|_{u=u^*}\right\}^2\end{aligned} \tag{8}$$

$$\mu\left|\frac{\partial}{\partial u}[V(f(x,u))-s(h(x,u),v)]\Big|_{u=u^*}\right|\leq\rho \tag{9}$$

with ρ a positive constant small enough, and R_V, R_s the remainder of the Taylor expansion of $V(f(x,u^*+\delta u^*))$ and $s(h(x,u^*+\delta u^*),v)$ at u^*, respectively.

Proof Let consider control $u=u^*+\delta u^*$ is applied to the system (1)-(2), with u^* such a control that makes the system (V,s)-lossless with V and s as storage energy and supply functions, respectively. Control δu^* is proposed in such a way that makes the feedback transformed system be (V,s)-dissipative, with v the new input. By considering the first-order Taylor approximation at u^* of $V(f(x,u^*+\delta u^*))$ and $s(h(x,u^*+\delta u^*),v)$, one yields to

$$V\left(f(x,u^*)\right)+\frac{\partial}{\partial u}V(f(x,u))\Big|_{u=u^*}\delta u^*+R_V-V(x)\leq$$
$$\leq s(h(x,u^*),v)+\frac{\partial}{\partial u}s(h(x,u),v)\Big|_{u=u^*}\delta u^*+R_s \quad (10)$$

From (5) and (7), relation (10) takes the form,

$$\frac{\partial}{\partial u}\left[V(f(x,u))-s(h(x,u),v)\right]\Big|_{u=u^*}\delta u^*-R_s+R_V$$
$$\leq-\mu\left\{\frac{\partial}{\partial u}\left[V(f(x,u))-s(h(x,u),v)\right]\Big|_{u=u^*}\right\}^2-$$
$$-\min R_s+\max R_V\leq 0$$

which is assured by means of condition (8).

Control δu^* is also needed to be bounded and small enough in order to have $u^*+\delta u^*$ defined in a neighbourhood of u^*, i.e., let ρ be a positive constant, $u^*+\delta u^*\in[u^*-\rho,u^*+\rho]$. This holds if

$$\mu\left|\frac{\partial}{\partial u}\left[V(f(x,u))-s(h(x,u),v)\right]\Big|_{u=u^*}\right|\leq\rho$$

which is what condition (9) proposes with ρ small enough. Indeed, due to the fact that the smooth functions V, s are defined in compact sets, it is the same for their derivatives and consequently, they are bounded. $|\delta u^*|$ can be as small as wanted, by means of constant μ.

Summing up, if conditions (8)-(9) are satisfied then u given by (6), with u^* obtained from (5) and δu^* given by (7), renders the system (1)-(2) locally (V,s)-dissipative. In addition, the orbits of the feedback transformed system are assured not to leave the compact $\widetilde{\mathscr{X}}$, where (V,s)-dissipativity is achieved, if they start in $\widetilde{\mathscr{X}}$. Two cases have to be considered: (i) if $v=0$, as the system is (V,s)-dissipative (strictly (V,s)-dissipative) with s satisfying the ZIO property, there exists a region where the fixed point is stable (asymptotically stable), see Theorem 12 (ii) if $v\neq 0$, v must be bounded, and v will be established from the relation $s(\overline{h}(x,v),v)=0$ which assures that $V(x(k+1))-V(x(k)\leq 0$. The bounds of v will depend on the compact where (V,s)-dissipativity is achieved, as the example will illustrate.

Remark 10. For the validity of this method, it is necessary to check how good the first-order Taylor approximations at u^* used for $V(f(x,u^*+\delta u^*))$ and for $s(h(x,u^*+\delta u^*),v)$ are. The validity of the method can be also checked by means of the boundedness of

$$|V_1-V_2|=|R_V| \quad (11)$$
$$|s_1-s_2|=|R_s| \quad (12)$$

with,

$$V_1=V(f(x,u^*+\delta u^*)),$$
$$V_2=V(f(x,u^*))+\frac{\partial}{\partial u}V(f(x,u))\Big|_{u=u^*}\delta u^*,$$
$$s_1=s(h(x,u^*+\delta u^*),v),$$
$$s_2=s(h(x,u^*),v)+\frac{\partial}{\partial u}s(h(x,u),v)\Big|_{u=u^*}\delta u^*.$$

4. ILLUSTRATING EXAMPLE

The feedback dissipativity methodology presented will be applied to the passivation of the following system extracted from (Sira-Ramírez, 1991),

$$\begin{aligned}x_1(k+1)&=\left[x_1^2(k)+x_2^2(k)+u(k)\right]\cos\left[x_2(k)\right]\\x_2(k+1)&=\left[x_1^2(k)+x_2^2(k)+u(k)\right]\sin\left[x_2(k)\right]\\y(k)&=x_1^2(k)+x_2^2(k)+u(k)\end{aligned} \quad (13)$$

System (13) is aimed to be rendered V-passive with storage function $V=x_1^2+x_2^2$ and supply function $s=yv$. Let $x_1\in[-\varepsilon_{x_1},\varepsilon_{x_1}]$, $x_2\in[-\varepsilon_{x_2},\varepsilon_{x_2}]$, $u,u^*\in[-\varepsilon_u,\varepsilon_u]$, $\delta u^*\in[-\rho,\rho]$, $v\in[-\varepsilon_v,\varepsilon_v]$ with ε_{x_1}, ε_{x_2}, ε_u, ρ, ε_v positive constants.

Now, the existence and validity of controls u^* and δu^* will be analyzed.

In order to obtain control u^*, equation (5), for the example, is calculated and takes the following form,

$$a_{u^*}(u^*)^2+b_{u^*}(x_1,x_2,v)u^*+c_{u^*}(x_1,x_2,v)=0 \quad (14)$$

with,

$$\begin{aligned}a_{u^*}&=1\\b_{u^*}(x_1,x_2,v)&=2(x_1^2+x_2^2)-v\\c_{u^*}(x_1,x_2,v)&=(x_1^2+x_2^2)^2-(x_1^2+x_2^2)-(x_1^2+x_2^2)v\end{aligned}$$

If sufficient feedback losslessness conditions (i) and (ii) of Proposition 8 are met for (13) for some (x_1,x_2,u^*,v) then a control u^* satisfying (14) exists. This u^* can be obtained from the explicit solution of (14), that is,

$$u_1^*(x_1,x_2,v)=\frac{-b_{u^*}+\sqrt{b_{u^*}^2-4a_{u^*}c_{u^*}}}{2a_{u^*}},$$
$$u_2^*(x_1,x_2,v)=\frac{-b_{u^*}-\sqrt{b_{u^*}^2-4a_{u^*}c_{u^*}}}{2a_{u^*}} \quad (15)$$

Then, it is necessary to assure that $\varphi(x_1,x_2,v)=b_{u^*}^2-4a_{u^*}c_{u^*}\geq 0$, which will be always achieved.

Concerning the computation of control δu^*, conditions (8)-(9) must be verified. They will be achieved by means of choosing an appropriate value of μ.

Considering $R_V=(\delta u^*)^2$, $R_s=0$ and taking into account (7), conditions (8) and (9) take the following form, respectively,

$$\mu \leq \frac{\min\left\{\left[2(u^* + x_1^2 + x_2^2) - v\right]^2\right\}}{\max\left\{\left[2(u^* + x_1^2 + x_2^2) - v\right]^2\right\}} \quad (16)$$

$$\mu \leq \frac{\rho}{\max\left|2(u^* + x_1^2 + x_2^2) - v\right|} \quad (17)$$

for some positive constant ρ. Then, an upper bound of constant μ can be established from the minimum value given by (16) and (17), and will depend on the bounds of the states and the controls.

It is also necessary to give a bound for v in order to ensure that the orbits of the feedback transformed system will remain in the compacts where the feedback dissipativity is considered. An option is proposing v in such a way to have $s(\overline{h}(x,v),v) = (x_1^2 + x_2^2 + u^* + \delta u^*)v = 0$. Control u^* is approximated by its linearization at $x_1 = x_2 = v = 0$, obtaining $u^* = \frac{1}{2}v$, and using this in $s(\overline{y},v) = 0$, it is obtained that $v = 0$ or,

$$v = -2(1-2\mu)(x_1^2 + x_2^2) \quad (18)$$

Control v will be bounded by means of μ, x_1 and x_2. Considering (17) and the fact that ρ must be small enough, it can be concluded that the denominator appearing in (17) will be greater than ρ and, consequently, μ will be less than one. Then, from (18), one yields to,

$$|v| < 2(\varepsilon_{x_1}^2 + \varepsilon_{x_2}^2) \quad (19)$$

A value for ε_v is proposed as $\varepsilon_v = 2(\varepsilon_{x_1}^2 + \varepsilon_{x_2}^2)$, and relations (16) and (17) yield to,

$$\mu \leq \left[\frac{-\varepsilon_u}{2(\varepsilon_{x_1}^2 + \varepsilon_{x_2}^2) + \varepsilon_u}\right]^2 = \overline{\mu}_1 \quad (20)$$

$$\mu \leq \frac{\rho}{4(\varepsilon_{x_1}^2 + \varepsilon_{x_2}^2) + 2\varepsilon_u} = \overline{\mu}_2 \quad (21)$$

Finally, an upper bound for μ is proposed as follows,

$$0 < \mu \leq \min(\overline{\mu}_1, \overline{\mu}_2) \quad (22)$$

This bound of μ ensures conditions (8)-(9) to hold and control v to be bounded.

Remark 11. As it was expected, v depends on the sets where the states are defined, and there is a clear relation between the sets where the controls are defined (ε_v, ε_u), paremeter μ and the sets where the states are defined (ε_{x_1}, ε_{x_2}).

Under the conditions studied, control δu^* is given by expression (7), i.e.,

$$\delta u^*(x_1, x_2, u^*, v) = -\mu\left[2(u^* + x_1^2 + x_2^2) - v\right] \quad (23)$$

with μ satisfying (22), and control u^* as defined by equation (14), for which two possibilities are the controls (15). The control which renders the system V-passive is given by $u = \alpha(x,v) = u^* + \delta u^*$. Two solutions for the passifying control can be considered:

$$u_1 = \alpha_1(x,v) = u_1^*(x,v) + \delta u_1^*(x, u_1^*, v) \quad (24)$$

$$u_2 = \alpha_2(x,v) = u_2^*(x,v) + \delta u_2^*(x, u_2^*, v) \quad (25)$$

5. DISSIPATIVITY-BASED STABILIZATION

A nonlinear regular static state feedback control law of the form $u = \alpha(x,v)$, which achieves either local (V,s)-*dissipativity* or local *strict* (V,s)-*dissipativity*, induces an *implicit damping injection* which makes the system fixed point locally stable (resp., locally asymptotically stable if strict (V,s)-dissipativity is considered) for certain particular values of the transformed control input. The following theorem clarifies this assertion.

Theorem 12. (Navarro-López *et al.*, 2002b) Consider the system (1)-(2), and two scalar functions $V(x)$ and $s(y,v)$ as a storage function and a supply function satisfying the ZIO property, respectively. Suppose $\overline{x}$ an isolated fixed point for $f(x,\overline{u})$, with $\overline{u}$ a constant. Suppose there exists a feedback control law, $u = \alpha(x,v)$, defined in an open neighbourhood $\mathscr{W} = \widetilde{\mathscr{X}} \times \widetilde{\mathscr{U}}$ with $\widetilde{\mathscr{X}} \subset \mathscr{X}$, $\widetilde{\mathscr{U}} \subset \mathscr{U}$ which renders the system (V,s)-dissipative (resp., strictly (V,s)-dissipative). Consider $x = \overline{x}$ the unique x for which $V(x) = 0$. Let $\mathscr{W}$ invariant with respect to $x(k+1) = f(x(k), \alpha(x(k), 0))$ and $(\overline{x}, \overline{u}) \in \mathscr{W}$. Then, for all $x \in \widetilde{\mathscr{X}}$, the control law $u = \alpha(x,0)$ locally stabilizes (resp., locally asymptotically stabilizes) the system to $\overline{x}$.

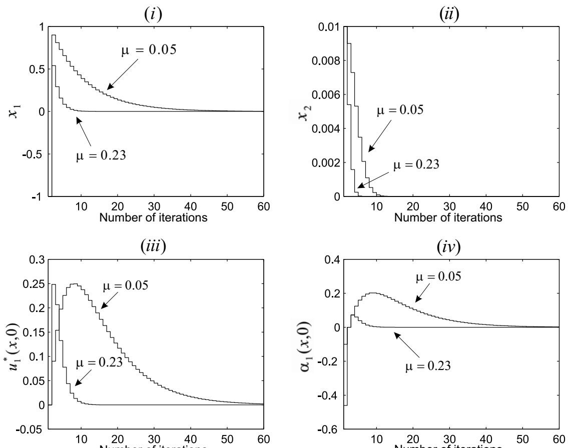

Fig. 1. Stabilized system response of (13) with $x_0 = (-1, 0.01)^T$ for $\mu = 0.05$ and $\mu = 0.23$ (i) x_1 (ii) x_2 (iii) control which renders the system V-lossless $u_1^*(x,0)$ (iv) passifying control for $v = 0$, $\alpha_1(x,0) = u_1^*(x,0) + \delta u_1^*(x, u_1^*, 0)$.

The passifying control $\alpha(x,v)$ with $v = 0$ is applied to system (13) using $\alpha_1(x,0)$, and as Theorem 12 establishes, the system converges to the fixed point $(0,0)$,

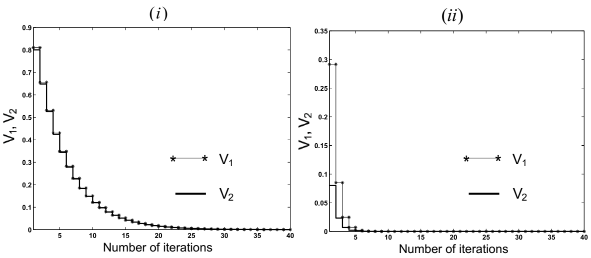

Fig. 2. Analysis of the approximations for $V(f(x, \alpha_1(x,0))$ with $x_0 = (-1, 0.01)^T$ (i) V_1, V_2 for $\mu = 0.05$ (ii) V_1, V_2 for $\mu = 0.23$.

which is unstale in open loop. It is considered $\varepsilon_{x_1} = 1$, $\varepsilon_{x_2} = 0.01$, $\varepsilon_u = 1.9$, $\rho = 1.8$ and the values for the upper bounds of μ are given by $\overline{\mu}_1 = 0.2373195$ and $\overline{\mu}_2 = 0.2307574$, consequently, admissible values of μ for which the passifying method is valid are $\mu \in (0, \overline{\mu}_2]$. The response of the system for $\alpha_1(x,0)$ and different values for the constant μ is depicted in Figure 1. Changes in the constant μ influence the response of the stabilized system.

Now, the approximation made for function $V(f(x,u))$, with $u = \alpha(x,0)$ is analyzed. As $v = 0$, the function $s(h(x, \alpha(x,0)), 0) = 0$ and there is no need to study its first-order Taylor approximation. Consider functions V_1, V_2. As δu^* tends to zero in the steady state, the approximation errors are zero and the approximations made are valid. In Figure 2, functions V_1,V_2 are compared graphically, for different values of μ: $\mu = 0.05$, $\mu = 0.23$, with $\alpha_1(x,0)$. It can be noticed that unless the steady state is reached, V_2 is not equal to V_1. When the system has reached its steady state $\delta u^* = 0$, $V_2 = V_1$ is obtained. The smaller in modulus the value of δu^* is, the better the approximation is, that is why the stabilized system response with $\mu = 0.23$ gives the worst approximation V_2, however, with $\mu = 0.23$ $V_1 - V_2$ gets zero sooner due to the fact that the response of the system is faster than the response with $\mu = 0.05$.

6. CONCLUSIONS

An approach for dealing with the local feedback dissipativity problem in general discrete-time systems has been proposed in addition to its use for stabilization purposes. The feedback dissipativity problem has been solved in a non-general manner since the approach proposed is based on the establishment of the input u which satisfies the fundamental dissipativity inequality; it is therefore necessary to associate a priori function V to the system, i.e., a storage function with respect to which the feedback transformed system will be (V,s)-dissipative. The feedback dissipativity conditions guarantee the existence of the control u which satisfies the dissipativity inequality. At any rate, it can be considered as an application-oriented feedback dissipativity method, since, when dealing with physical systems, it is interesting to define the storage function as the energy of the system.

Some suggestions for future work can be proposed. The main problem presented in the feedback dissipativity methodology is that control $u^* + \delta u^*$ is locally valid in a neighbourhood of u^*. The approximations of $V(f(x,u))$, $s(h(x,u),v)$ could be improven, using a higher order Taylor approximation type. Alternative methods in order to calculate u^* can be proposed. A geometric interpretation of the underlying idea of this method would be interesting.

It would be appealing to extend the results here obtained to the case of general multiple-input multiple-output nonlinear systems.

REFERENCES

Byrnes, C.I. and W. Lin (1994). Losslessness, feedback equivalence, and the global stabilization of discrete-time nonlinear systems. *IEEE Trans. on Aut. Control* **39**(1), 83–98.

de Souza, C.E. and L. Xie (1992). On the discrete-time bounded real lemma with application in the characterization of static state feedback h_∞ controllers. *Systems and Control Letters* **18**, 61–71.

de Souza, C.E., M. Fu and L. Xie (1993). h_∞ analysis and synthesis of discrete-time system with time-varying parameter uncertainty. *IEEE Transactions on Automatic Control* **38**, 459–462.

Navarro-López, E.M. and E. Fossas-Colet (2002a). Dissipativity, passivity and feedback passivity in the nonlinear discrete-time setting. *accepted in the 15th IFAC World Congress*.

Navarro-López, E.M., E. Fossas-Colet and D. Cortés (2001). Passification of nonlinear discrete-time systems using the speed-gradient algorithm. *Proc. National Congress of the Mexican Automatic Control Association* pp. 141–146.

Navarro-López, E.M., H. Sira-Ramírez and E. Fossas-Colet (2002b). Dissipativity and feedback dissipativity properties of general nonlinear discrete-time systems. *Submitted to the European Journal of Control*.

Sira-Ramírez, H. (1991). Non-linear discrete variable structure systems in quasi-sliding mode. *International Journal of Control* **54**(5), 1171–1187.

Sira-Ramírez, H. (1998). A general canonical form for feedback passivity. *International Journal of Control, Special Issue: Recent Advances in the Control of Nonlinear Systems* **71**(5), 891–905.

Tan, Z., Y.C. Soh and L. Xie (1999). Dissipative control for linear discrete-time systems. *Automatica* **35**, 1557–1564.

Tan, Z., Y.C. Soh and L. Xie (2000). Dissipative control of linear discrete-time systems with dissipative uncertainty. *International Journal of Control* **73**(4), 317–328.