



Optimal Climate Change Policies When Governments Cannot Commit

[Link to publication record in Manchester Research Explorer](#)

Citation for published version (APA):

Ulph, A., & Ulph, D. (2010). *Optimal Climate Change Policies When Governments Cannot Commit*. (SCI discussion paper series; No. 4).

Citing this paper

Please note that where the full-text provided on Manchester Research Explorer is the Author Accepted Manuscript or Proof version this may differ from the final Published version. If citing, it is advised that you check and use the publisher's definitive version.

General rights

Copyright and moral rights for the publications made accessible in the Research Explorer are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

Takedown policy

If you believe that this document breaches copyright please refer to the University of Manchester's Takedown Procedures [<http://man.ac.uk/04Y6Bo>] or contact uml.scholarlycommunications@manchester.ac.uk providing relevant details, so we can investigate your claim.



SCI DISCUSSION PAPER

Optimal Climate Change Policies When Governments Cannot Commit

By:

Alistair Ulph

University of Vice-President and Dean
of the Faculty of Humanities,
University of Manchester.

David Ulph

Professor of Economics, University of
St Andrews

Senior External Research Fellow,
Centre for Business Taxation,
University of Oxford

July 2010

We are grateful to Partha
Dasgupta and particularly
Nick Stern for earlier
discussions on this topic.

Prepared for *Journal of
Environmental Economics
and Management*

The **Sustainable Consumption Institute (SCI)** was established in 2007 as a world leading, interdisciplinary research institute at The University of Manchester. Research, teaching and outreach at the SCI aims to resolve how to move society towards a low carbon economy. A key to our success will be to work cooperatively with government, business and the consumer. For more information, visit: <http://www.sci.manchester.ac.uk>

SCI DISCUSSION PAPERS are intended to stimulate discussion among researchers, practitioners and policy makers on a range of current and emerging sustainability issues.

Published by:

Sustainable Consumption Institute
The University of Manchester
188 Waterloo Place
Oxford Road
Manchester, UK
M13 9PL
www.sci.manchester.ac.uk

More information is available by contacting Lynda McIntosh in Communications and Marketing via email at Lynda.mcintosh@manchester.ac.uk or telephone +44 (0) 161 275 0187.

The views expressed in this publication are those of the author(s) and not necessarily those of the SCI, the funders or The University of Manchester.

Optimal Climate Change Policies When Governments Cannot Commit¹

Alistair Ulph²

and

David Ulph³

Abstract

This paper examines the optimal design of climate change policies in the context where governments want to encourage the private sector to undertake significant immediate investment in developing cleaner technologies, but the carbon taxes and other environmental policies that could in principle stimulate such investment will be imposed over a very long future. The conventional claim by environmental economists is that environmental policies *alone* are sufficient to induce firms to undertake optimal investment. However this argument requires governments to be able to commit to these future taxes, and it is far from clear that governments have this degree of commitment. We assume instead that governments cannot commit, and so both they and the private sector have to contemplate the possibility of there being governments in power in the future that give different (relative) weights to the environment. We show that this lack of commitment has a significant asymmetric effect. Compared to the situation where governments can commit it *increases* the incentive of the current government to have the investment undertaken, but *reduces* the incentive of the private sector to invest. Consequently governments may need to use additional policy instruments – such as R&D subsidies – to stimulate the required investment.

August 2009

¹ We are grateful to Partha Dasgupta and particularly Nick Stern for earlier discussions on this topic.

² Vice-President and Dean of the Faculty of Humanities, University of Manchester.

Contact details: Oxford Road, University of Manchester, Manchester M13 9PL, UK.

E-Mail: alistair.ulph@manchester.ac.uk

³ Professor of Economics, University of St Andrews and Senior External Research Fellow, Centre for Business Taxation, University of Oxford.

Contact details: School of Economics & Finance, University of St Andrews, St Andrews, Fife KY16 9AL, Scotland. E-Mail: du1@st-andrews.ac.uk

Optimal Climate Change Policies When Governments Cannot Commit

1. Introduction

The *Stern Review on the Economics of Climate Change* - Stern (2007) - has argued powerfully for strong early action to combat climate change. In particular the report argues for early investment equivalent to 1% of GDP forever – later updated to 2% - in discovering and introducing new cleaner technologies and other forms of mitigation activity that will limit the ultimate rise in temperature. This is contrasted with potential losses equivalent to at least 5% of GDP forever from doing nothing. The *Review* notes that climate change takes place in a global economy with many distortions and consequently advocates a range of policies required to bring about the required investment, including carbon pricing through a mix of emission taxes and carbon trading and policies designed to accelerate both innovation and adoption of new low-emission technologies. The *Review* also emphasizes that since such policies will have impacts over a long time horizon, the inter-temporal evaluation of such policies needs to consider how society should evaluate two key factors: the substantial risks that climate change may pose for future generations, and how these risks might affect some of the poorest people in the world.

The *Stern Review* has had a critical reception from some leading economists – e.g. Dasgupta (2007), Nordhaus (2007), Weitzman (2007) – mostly focusing on the approach to inter-temporal policy evaluation, though this has not always led to a serious questioning of the policy conclusions – Weitzman (2007). Stern (2008) has given a robust defence of his approach to inter-temporal policy evaluation, noting that climate change, and policies to deal with it, do not have marginal impacts on the economy and take place in a global economy with many distortions. In such a context, Stern (2008) re-emphasises that the framework for conducting inter-temporal policy evaluation needs to consider how society will manage the very substantial risks from climate change and so such evaluation needs to be considered from first principles.

In this paper we elaborate on another feature of the *Stern Review* which has received less attention, although it is linked to a particular aspect of evaluating future risk. As we have noted, the *Stern Review* argues strongly for the use of a range of policy instruments, including both carbon pricing and policies to stimulate environmental R&D. This is in contrast with recommendations that come from more standard environmental economic modeling which is grounded in a framework in which a single government chooses a time-path of policy instruments maximizing a welfare function over an infinite horizon. Within such a framework environmental policy instruments such as emissions/carbon taxes play a dual function: they induce the optimal level of emissions in every period, but they also give firms incentives to undertaken investment in R&D etc that will produce cleaner technologies and so lower the taxes they will have to face in the future. In this stylized framework, the

major conclusion is that environmental policy instruments alone⁴ are sufficient to support the optimal R&D investment, output and emission paths.

Of course it is widely recognized that one reason why this simple argument breaks down is that there are multiple market failures, and in particular as well as the environmental externality failure there are failures in the market for R&D⁵. There is a large literature on how environmental policies and R&D policies interact in such a second-best world (e.g. Jaffee, Newell and Stavins (2003) for an excellent survey). This second-best argument is part of the justification given in the *Stern Review* for deploying a range of both environmental and R&D policy instruments.

In this paper we wish to focus on another weakness of the conventional economic analysis, which is that it pre-supposes that current governments can commit to tax policies into the far distant future, and this violates a basic principle of political decision-making which is that current governments cannot tie the hands of future governments. Of course this need not matter if future governments were just like the current government, and, in particular, placed exactly the same weight on environmental considerations relative to other issues such as growth and employment as did the current government. But the lack of commitment matters if we take seriously the idea that future governments can have different objectives from the current government.

So even if the current government announces the taxes that it would like to impose in the future, when firms are contemplating the investment decisions they are making they need to take into account the possibility that future governments may place a greater or smaller (relative) weight on environmental issues and hence on the taxes they might impose. The *Stern Review* notes that “Lack of certainty over the future pricing of the carbon externality will reduce the incentive to innovate”, and this lack of credibility in future government policies provides a rationale (additional to the second-best market failure argument) for why carbon pricing alone will be insufficient to induce optimal R&D and why R&D policy is also required⁶.

But of course this uncertainty about possible future environmental policies affects the current government as well. It is conceivable that the uncertainty about the policies of future governments which deters firms from making R&D investments would also deter current governments from undertaking policies to induce such investments. So the question we want to address is what policies should the current government implement with respect to R&D investment when it recognizes that future governments may implement different policies from those that it would choose if it could commit and when it recognizes that firms also face this policy uncertainty which will affect their investment intentions.

So while the *Stern Review* identifies the importance of commitment issues with respect to future environmental pricing policies, and that this would make it desirable

⁴ This is consistent with the conventional neoclassical notion of innovation being induced by changes in relative prices (Jaffee, Newell and Stavins (2003)), in this case by changes in relative prices resulting from environmental policies.

⁵ Standard failures include capital market failures, appropriability problems, ‘rent-stealing’ – see Jaffee et al (2003) for a survey.

⁶ See Stern (2007) pp 393 and 399.

for current governments to also act on environmental R&D policies, it does not provide an analysis that ensures that current governments will have an incentive to take such action.⁷ That is the purpose of this paper.

Now there have been a number of studies of the nature of government policies in a variety of policy contexts that contrast the outcomes when governments can and cannot commit.⁸ In these studies commitment and lack of commitment are modeled as different move structures in a game between governments and firms undertaking an investment. If the government can commit it announces a single value of its policy instrument, which for purposes of discussion we will take to be an emission tax rate⁹, before firms make their investment decision; whereas if governments cannot commit the tax rate is set after firms have made their investment decision, and so will depend on the investment decision that is made. Compared to the situation where governments can commit, lack of commitment brings the advantage of a more responsive policy – effectively a wider range of policy instruments – but has the disadvantage that investment decisions will now be distorted by the desire to influence tax policy. So there may be circumstances where the inability to commit is welfare superior to the situation where governments can commit.

What these studies do not explain is why, if the government does have the ability to commit, it can only commit to a single tax *rate* and cannot commit to a tax *policy* whereby it announces in advance what taxes will be set conditional on the investment decisions made by firms. This combines the benefits of having more instruments with those of non-manipulability and is certainly welfare superior¹⁰. In this paper we avoid these issues by assuming that if the government can commit it can commit to a *policy* in which the taxes it will set will depend on the R&D investment decision that is made.

An approach that comes close to that pursued here is that by Boyer and Laffont (1999). They analyse the trade-off that can arise if future governments have better information about environmental damage costs, but may also attach different weights to environmental damages; how far would a current government want to restrict what future governments might do ('tying their hands') by mandating that the future government sets environmental taxes equal to expected damage costs. This would prevent the effects of 'capture' but at the expense of limiting a future government's ability to respond to better information. Similar issues arise in the context of environmental policies being taken by different levels of government (e.g. EU / member state; national / local) - see for example Johal and Ulph (2002,2003).

For simplicity, in this paper we have ignored the issue of future governments getting better information about damage costs¹¹. So in the context of the Boyer and Laffont

⁷ Of course there are many other policy challenges in tackling climate change – the need for international agreement, the issue of stock externalities (and hence the appropriate time-path of carbon taxes), and, as mentioned in footnote 11 below, the issue of learning etc. These undoubtedly interact with the commitment issue, but for simplicity we have ignored these.

⁸ See for example Leahy and Neary (1996, 2000), Petrakis & Xepapadeas (1999, 2000).

⁹ Or other policy instrument

¹⁰ See Ulph and Ulph (2001) for a demonstration

¹¹ We have analysed some of the implications of this in Ulph & Ulph (1997). If a government can commit then it can address this issue by announcing today a tax *policy* which sets out what tax *rates* it intends to set in the future conditional on what it learns about the seriousness of the climate change

analysis, a current government would always want to use its powers to commit future governments to set the emission taxes the current government would want to set. But what Boyer and Laffont fail to explain is *how* governments can tie the hand of future governments – i.e. can commit. The issue we address is what policies a current government should pursue when there are no constitutional powers that allow it to prevent future governments setting whatever emission taxes they would want to set. The point we make is that governments may still have a weak indirect ability to influence future tax-setting policy through their ability to influence the investment decisions of the private sector particularly if that investment takes an irreversible form of something like R&D. The issue we are concerned with is that in addition to setting environmental taxes to control future emissions and outputs, there is an additional policy issue of inducing the appropriate level of R&D investment.

As noted above there are many reasons why governments will want to employ environmental R&D policies in addition to environmental taxes. To highlight the role that lack of commitment plays in driving governments to use such policies, we develop a framework in which all the standard features that would justify environmental R&D policies are absent¹². We show the following:

- If the government can commit to future emission taxes, then it can induce what it regards as the optimal R&D investment. If the scale of investment is sufficiently low it may be sufficient to commit to a single tax rate – that which it will impose conditional on the R&D investment taking place. Otherwise if the scale of investment is very large then it needs to commit to a policy and use the threat of setting a much higher tax rate if firms fail to invest to induce the optimal investment.
- However if the government cannot commit then this has a very asymmetric effect on the incentives of the private sector and the government to have the investment undertaken. As the *Stern Review* recognised, lack of commitment *reduces* the incentive of the private sector to invest – essentially because it introduces an intrinsic degree of risk aversion to the incentive. However, lack of commitment *increases* the incentive for the government to have investment undertaken – essentially because having a cleaner technology in place reduces the sensitivity of its view of welfare to the fact that other governments might operate different policies¹³.
- This leads to the fundamental conclusion of this paper which is that *if governments, who would want R&D investment undertaken if they could commit, cannot commit to future environmental then environmental policies alone may no longer be sufficient to induce the optimal R&D investment in which case it is always in the interests of such governments to resort to technology policy instruments like R&D subsidies.*

problem. There is still the commitment problem if there are different governments that will operate different *policies* conditional on learning more about the nature of climate change – its speed and potential damage.

¹² For example we assume there is a single firm – so there are no spillover or strategic investment issues – and that this firm can fully capture all consumer benefits – so there are no undervaluation effects.

¹³ Put differently lack of commitment increases the incentive of governments to have the investment undertaken because it gives the current government some weak control over the tax policies of future governments.

2. The Model

As indicated in the introduction we take it that there are a number of key features of the problems posed by climate change:

- there is a sector of the economy - energy/transport - that is largely responsible for the production of greenhouse gas emissions;
- the output of this sector is essential and consumers are reluctant to cut back their consumption of the output of this sector;
- so the major hope of reducing emissions lies in discovering cleaner technologies for producing the output of this sector;
- collectively the sector needs to make a very significant upfront investment in R&D to discover, develop and bring to market these new technologies that will emit fewer greenhouse gasses;
- however there is some degree of urgency in doing this, so there is a one-off opportunity to make the investment.

We capture these ideas in the following model. There is a single firm which produces a commodity the production/consumption of which generates pollution. So if $x \geq 0$ denotes output and $e > 0$ emissions per unit of output, total emissions will be $E = e.x$.

There are two periods. In period 1 the firm decides whether or not to make a fixed investment $F > 0$ in an environmental R&D project the sole benefit of which is to produce a cleaner technology. So if the firm makes the investment emissions per unit of output will be $e_L > 0$, but if it does not make the investment emissions will remain at the higher level $e_H > e_L$.

For simplicity we assume that all the consumer benefits and environmental damage created by this firm accrue in period 2. We also assume that any discounting is incorporated into the benefit and damage functions discussed below.

The only thing that the firm chooses in period 2 is the amount of output, x . This yields consumer benefits $B(x) > 0$ (net of production costs). We assume that on some interval $[0, \bar{x})$ his benefit function satisfies the standard conditions that $B'(x) > 0$, $B''(x) < 0$, where \bar{x} is defined by $B'(\bar{x}) = 0$. To capture the idea that consumers are reluctant to cut back consumption we make the stronger assumption that throughout $[0, \bar{x})$

$$\beta(x) \equiv -\frac{x B''(x)}{B'(x)} > 1. \quad (1)$$

Later in the paper it will be helpful to focus on the case where β is a constant, not a function of x .

For simplicity we will assume that the costs of production are sufficiently low (relative to benefits) that they can be ignored.

Although we have focused on a single firm, we want to abstract from any competition issues arising from the potential exercise of monopoly power by this firm, so we assume that the firm can perfectly price discriminate and thus capture the entire consumer benefit $B(x)$.

The environmental damage created by emissions are captured by the damage cost function $D(E) > 0$ which also satisfies the standard conditions that $D(0) = 0$, $D'(E) > 0$, $D''(E) > 0$. Let

$$\delta(E) \equiv \frac{E \cdot D''(E)}{D'(E)} > 0. \quad (2)$$

Once again, later in the paper it will be helpful to focus on the case where δ is a constant, not a function of E .

Governments are in power for just a single period and can set policy variables only for the period in which they are in power and only in relation to decisions that are being made by the private sector in that period.

So there is a government in period 1 which, without loss of generality, we can assume attaches a weight $\omega = 1$ to environmental damage costs, relative to the profits / consumer surplus generated by output in its social welfare function. The only decision that this period 1 government can influence is whether or not the firm makes the investment in R&D, and we assume that the only instrument at its disposal is an R&D subsidy which takes the form of a lump-sum payment S .¹⁴

The government that is in power in period 2 can influence only the level of output chosen by the firm in period 2 which it does through the choice of an emissions tax which we assume takes the form of a simple tax t per unit of emissions. Notice that the value of t chosen by the period 2 government will in principle depend on the emissions technology that is being used and hence on the R&D decision made by the firm in period 1.

To capture the issue of commitment we allow the possibility that the weight, ω , attached to environmental damage by the period 2 government may be different from that of the period 1 government – i.e. that $\omega \neq 1$. Then we say that **governments have the ability to commit** if the government in period 1 knows for sure that the weight given to damage by the period 2 government is also $\omega = 1$. In this case effectively the same government is choosing both the period 1 technology policy and the period 2 environmental policy. We will say that **governments do not have the ability to commit** if the government in period 1 knows that there is a possibility that the government in period 2 will put a different weight on environmental damage and so may choose tax rates that are different from those that it would have chosen had it remained in power.

Notice that the investment decision made by the firm in period 1 will depend on both the technology policy chosen by the period 1 government and by the anticipated

¹⁴ This will be an R&D tax if S is negative.

environmental policies chosen by the period 2 government, and so will also be affected by whether or not the government in period 1 can commit. Notice also that even if the government in period 1 cannot commit and so cannot *directly* choose the environmental policy in period 2, it can still exert some indirect influence over that policy through its ability to influence the investment decision in period 1. Consequently the questions we are interested in are: how does the ability of the government in period 1 to commit affect

- (i) the investment decision it regards as optimal;
- (ii) the technology policy that it chooses?

3. Analysis of the Model

We undertake the analysis by considering first the decisions of the government and firm in period 2 and then working backwards to the decisions of the government and firm in period 1.

3.1 Second Period Analysis.

We assume that the second-period government has a weight, ω , on the environment and the firm has a technology which emits e units of pollution per unit of output. The government wishes the firm to choose the output level that maximizes its (the government's) objective function. So let

$$\hat{x}(e, \omega) \equiv \underset{x}{\text{ARGMAX}} [B(x) - \omega D(e, x)] \quad (3)$$

be the optimal output level and $\hat{E}(e, \omega) = e \cdot \hat{x}(e, \omega)$ be the associated aggregate level of emissions. The first-order condition for the optimum output is:

$$B'(\hat{x}(e, \omega)) = e \cdot \omega D'(e, \hat{x}(e, \omega)), \quad (4)$$

i.e. Marginal Benefit equals Marginal Damage.

From (1), (2) and (4) we have the following comparative static elasticities for the optimal levels of output and emissions:

$$0 < -\frac{e \partial \hat{x}}{\hat{x} \partial e} = \frac{1 + \hat{\delta}}{\hat{\beta} + \hat{\delta}} < 1; \quad \frac{e}{\hat{E}} \frac{\partial \hat{E}}{\partial e} = 1 + \frac{e}{\hat{x}} \frac{\partial \hat{x}}{\partial e} = \frac{\hat{\beta} - 1}{\hat{\beta} + \hat{\delta}} > 0 \quad (5)$$

and

$$\frac{\omega}{\hat{E}} \frac{\partial \hat{E}}{\partial \omega} = \frac{\omega}{\hat{x}} \frac{\partial \hat{x}}{\partial \omega} = -\frac{1}{\hat{\beta} + \hat{\delta}} < 0 \quad (6)$$

where $\hat{\beta}$ and $\hat{\delta}$ are just the values of the elasticities in (1) and (2) evaluated at the optimal levels of output and emissions respectively.

From (5) we see that, given our assumption in (1) that $\hat{\beta} > 1$, it follows that having a dirtier technology causes governments to cut back output, but less than

proportionately, so total emissions increase. Conversely it is only by moving to a cleaner technology that governments can hope to reduce emissions¹⁵. From (6) we see that governments that care more about the environment will want to have lower levels of output and emissions.

The maximum level of social welfare arising from the optimal choice of output by a period 2 government is:

$$\hat{W}^2(e, \omega) \equiv \underset{x}{\text{MAX}} [B(x) - \omega D(e, x)] \equiv B(\hat{x}(e, \omega)) - \omega D(e, \hat{x}(e, \omega)) \quad (7)$$

It is straightforward to show that:

$$-\frac{\partial \hat{W}^2}{\partial e} = \omega \hat{x}(e, \omega) D'[\hat{E}(e, \omega)] > 0 \quad (8)$$

$$\frac{\partial \hat{W}^2}{\partial \omega} = -D[\hat{E}(e, \omega)] < 0 \quad (9)$$

$$-\frac{\partial^2 \hat{W}^2}{\partial \omega \partial e} = D'[\hat{E}(e, \omega)] \cdot \frac{\partial \hat{E}}{\partial e} > 0 \quad (10)$$

The expression in (8) shows that as long as a government puts a positive weight on the environment it will have a positive (marginal) incentive to reduce emissions, while (10) shows that the greater is a government's concern about the environment the greater is its (marginal) incentive to reduce emissions.

To induce the profit-motivated firm to choose the optimal output level the second-period government needs to set the appropriate emission tax. So let

$$\Pi(\tau) = \underset{x}{\text{MAX}} [B(x) - \tau x], \quad (11)$$

and

$$\tilde{x}(\tau) = \underset{x}{\text{ARGMAX}} [B(x) - \tau x] \quad (12)$$

be, respectively, the maximum operating (period 2) profits and the associated profit-maximising level of output of the firm when it faces any given output tax τ .

The first-order condition associated with (12) is

$$B'(\tilde{x}) = \tau. \quad (13)$$

So from (4) and (13) it follows that to induce the firm to produce output level the government sets an emission tax $\hat{t}(e, \omega)$ equal to marginal damage – i.e. such that:

¹⁵ Despite the fact that (5) shows that optimal output will expand – but not by much – through having a cleaner technology.

$$\hat{t}(e, \omega) = \omega D' \left[\hat{E}(\omega, e) \right] \quad (14)$$

with associated optimal output tax

$$\hat{\tau}(e, \omega) = e \hat{t}(e, \omega) = \omega e D' \left[\hat{E}(e, \omega) \right] = B' \left[\hat{x}(e, \omega) \right]. \quad (15)$$

From (13) and (15) it is easy to see that

$$\tilde{x} \left[\hat{\tau}(e, \omega) \right] = \hat{x}(e, \omega). \quad (16)$$

From (14), (15), (5) and (6) we can derive the following comparative static elasticities of the emissions and output tax rates:

$$\frac{e \partial \hat{t}}{\hat{t} \partial e} = \hat{\delta} \cdot \frac{e \partial \hat{E}}{\hat{E} \partial e} > 0; \quad \frac{e \partial \hat{\tau}}{\hat{\tau} \partial e} = 1 + \frac{e \partial \hat{t}}{\hat{t} \partial e} > 1 \quad (17)$$

$$\frac{\omega \partial \hat{t}}{\hat{t} \partial \omega} = \frac{\omega \partial \hat{\tau}}{\hat{\tau} \partial \omega} = 1 + \hat{\delta} \frac{\omega \partial \hat{E}}{\hat{E} \partial \omega} = \frac{\hat{\beta}}{\hat{\beta} + \hat{\delta}} > 0. \quad (18)$$

From (17) we see that if the firm introduces a cleaner technology with lower emissions per unit of output, then this cause the government to lower the tax rate on *emissions*. This result follows from the result we established in (5) that total emissions will fall with a cleaner technology. Having a cleaner technology will lower the *output* tax the firm will face in two ways:

- (i) for a given level of emissions tax it will lower the amount of tax it pays on every unit of output;
- (ii) it will induce whatever government it faces to lower the emissions tax.

As we will see later on, the fact that there are these two ways of lowering the effective tax on output that a firm faces has important implications for the design of policy.

Combining (11) and (14) we can define

$$\hat{\Pi}(e, \omega) = \Pi \left[\hat{\tau}(e, \omega) \right] \quad (19)$$

as the operating (period 2) profits the firm will make if it has a technology which emits e units of pollution per unit of output and faces a government that puts a weight ω on environmental damage. It is straightforward to show that:

$$-\frac{\partial \hat{\Pi}}{\partial e} = \hat{x}(e, \omega) \frac{\partial \hat{\tau}}{\partial e} > 0; \quad \frac{\partial \hat{\Pi}}{\partial \omega} = -\hat{x}(e, \omega) \frac{\partial \hat{\tau}}{\partial \omega} < 0. \quad (20)$$

Thus profits, net of emission taxes, are decreasing in the level of emissions per unit of output and in the weight the government places on the environment.

Finally we also need to consider the level of welfare that the government in period 1¹⁶ derives when it evaluates the implications of the policies it anticipates will be implemented in period 2 if the government that is then in power has a weight ω on environmental issues, and the firm has a technology with emissions per unit of output e . This is given by

$$W^1(e, \omega) = B[\hat{x}(e, \omega)] - D[e\hat{x}(e, \omega)]. \quad (21)$$

Obviously we have the following link between $W^1(e, \omega)$ and $\hat{W}^2(e, \omega)$:

$$W^1(e, 1) = \hat{W}^2(e, 1). \quad (22)$$

For later purposes it is useful to notice that, from (6) and (21)

$$\frac{\omega \cdot \partial W^1(e, \omega)}{\partial \omega} = - \frac{[B'(\hat{x}) - eD'(e\hat{x})] \cdot \hat{x}}{\hat{\beta} + \hat{\delta}} \quad (23)$$

and that

$$\frac{\partial W^1(e, \omega)}{\partial \omega} \begin{matrix} > \\ < \end{matrix} 0 \text{ as } \omega \begin{matrix} < \\ > \end{matrix} 1. \quad (24)$$

So, not surprisingly, the period 1 government's welfare is maximized when it anticipates that future governments will act like it.

3.2 First Period Analysis.

In the first period the only decision to be made by the firm is whether or not to undertake the R&D investment; the only decision to be made by the government is whether or not to influence the R&D decision of the firm by taxing or subsidizing R&D. We consider these decisions first on the assumption that the government in period 1 can commit to emission taxes in period 2, (or, equivalently, that the same government is in power in both periods 1 and 2) and then on the assumption that the government in period 1 can make no commitment about emission taxes in period 2 (or, equivalently, that the government in power in period 2 may not be the same as the one in period 1).

3.2.1 Commitment.

We determine both the social and the private incentive for the investment to be undertaken, and, by comparing the two, determine the optimal policy. Since this paper addresses the question of how the inability of the government to commit might affect both its desire to have the investment undertaken, and the policies it might need to use to achieve this, we certainly want to assume that, if it can commit, it would want the investment to take place.

¹⁶ For whom, recall, $\omega = 1$

The gain in social welfare that the period 1 government anticipates obtaining if it is in power in period (2) and the investment is carried out is given by

$$\Delta W^1(1) \equiv W^1(e_L, 1) - W^1(e_H, 1). \quad (25)$$

From (22) we know that

$$\Delta W^1(1) \equiv \Delta \hat{W}^2(1) = \hat{W}^2(e_L, 1) - \hat{W}^2(e_H, 1) > 0 \quad (26)$$

which, from (8), we know to be positive. Consequently the government will want to have the investment undertaken if and only if

$$\Delta \hat{W}^2(1) > F. \quad (27)$$

As indicated above we certainly want to assume that, if it can commit, the government wants the investment to be undertaken. So in everything that follows we assume that (27) holds.

Assuming that the firm knows for sure that the period 1 government will be in power in period 2 and fully anticipates how the emissions tax it sets will depend on the emissions technology that is being used, the private incentive to undertake the investment is given by

$$\Delta \hat{\Pi}(1) \equiv \hat{\Pi}(e_L, 1) - \hat{\Pi}(e_H, 1) = \Pi[\hat{\tau}(e_L, 1)] - \Pi[\hat{\tau}(e_H, 1)] > 0, \quad (28)$$

which, from (20) we know to be positive.

From (7), (11), (15), (19), (25) and (26) it is easy to see that

$$\Delta \hat{\Pi}(1) - \Delta \hat{W}^2(1) = H[\hat{E}(e_H, 1)] - H[\hat{E}(e_L, 1)] \quad (29)$$

where

$$H(E) \equiv E \cdot D'(E) - D(E).$$

It is easy to see that, given our assumptions about the damage function, $H(E)$ is a positive and strictly increasing function, and since, from (5), we know that total emissions are increasing in e it follows that

$$\Delta \hat{\Pi}(1) > \Delta \hat{W}^2(1) \quad (30)$$

and so the firm, anticipating the environmental taxes that will be imposed by the government, has a stronger incentive to invest than the government. The intuition is clear. Because marginal damage is increasing, if firms face taxes that are equal to marginal damage then the total tax paid on any level of emissions is greater than the costs of damage to society, and, moreover, this difference is greater the higher the

¹⁷ Indeed it is easy to see that the above argument establishes the more general result that

$$\Delta \hat{\Pi}(\omega) > \Delta \hat{W}(\omega) \quad \forall \omega$$

level of emissions. So firms have more powerful incentives to reduce emissions than does society.

It therefore follows from (27) that $\Delta\hat{\Pi}(1) > F$, and so we have established

Proposition 1 If the government in period 1 can commit to environmental taxes in period 2, then environmental policies alone are sufficient to induce the optimal investment decision.

This is just a restatement of the conventional wisdom of environmental economics.

Now notice that from (15) and (27) we can re-write

$$\Delta\hat{\Pi}(1) = \left\{ \Pi[e_L\hat{t}(e_L)] - \Pi[e_H\hat{t}(e_L)] \right\} + \left\{ \Pi[e_H\hat{t}(e_L)] - \Pi[e_H\hat{t}(e_H)] \right\}. \quad (31)$$

Given that profits are a decreasing function of the tax rate, both terms on the RHS of (31) are positive. The first is the increase in profits the firm would get from undertaking the investment if the government announced just a single environmental tax rate – that which it would impose if the investment were made. The second term is the increase in profits that the firm faces from the threat of the government to set a higher tax if the firm fails to undertake the investment. So we have:

Corollary 1 (i) If $\Pi[e_L\hat{t}(e_L)] - \Pi[e_H\hat{t}(e_L)] \geq F$ then the government can induce the optimal investment by committing to a *tax rate* – that which it would impose if the investment were undertaken.

(ii) If $\Pi[e_L\hat{t}(e_L)] - \Pi[e_H\hat{t}(e_L)] < F$ then to induce the optimal investment the government in period 1 has to commit to a *tax policy* which shows how the taxes it will set will vary with the investment decisions of firms. In particular it has to commit to setting higher taxes if firms fail to invest.

Since much of the literature on environmental policy focuses on marginal investment decisions, it is often assumed that it is sufficient to commit to future tax rates. This corollary suggests that governments may need a stronger degree of commitment to induce very significant investments.

3.2.2 No Commitment.

We begin by establishing the incentives of the government and the firm to invest if they knew for sure that the government in power in period 2 had a weight $\omega \geq 0$ on environmental damage, and the consequent implications for policy. We will then discuss how the analysis goes through when there is uncertainty about the possible future government.

3.2.2.1 The Case of Certainty

The incentive of the government to invest is given by

$$\Delta W^1(\omega) = W^1(e_L, \omega) - W^1(e_H, \omega) \quad (32)$$

while that of the firm is

$$\Delta \hat{\Pi}(\omega) \equiv \hat{\Pi}(e_L, \omega) - \hat{\Pi}(e_H, \omega) = \Pi[\hat{\tau}(e_L, \omega)] - \Pi[\hat{\tau}(e_H, \omega)] \geq 0. \quad (33)$$

Now we know from (21), (25), (26), (27), (32) and (33) that when $\omega = 1$ then

$$\hat{\Pi}(1) > \Delta W^1(1) = \Delta \hat{W}^2(1) > F, \quad (34)$$

while from (24) it follows that

$$\frac{\partial \Delta W^1(1)}{\partial \omega} = 0. \quad (35)$$

Notice next that if $\omega = 0$ then, from (14) and (15) $\hat{\tau}(e_L, 0) = \hat{\tau}(e_H, 0) = 0$, which, from (16), implies $\hat{x}(e_L, 0) = \hat{x}(e_H, 0) = \tilde{x}(0) = \bar{x}$, so we have,

$$\Delta W^1(0) = D[e_H \bar{x}] - D[e_L \bar{x}] > 0 = \Delta \hat{\Pi}(0) \quad (36)$$

so when $\omega = 0$ the social incentive to invest is greater than the private incentive, which is zero and so certainly less than F .

We want to compare the incentive to invest by the government in period 1 if it anticipates there being a government in period 2 which attaches no weight to the environment, with the incentive to invest if it itself were in power. It is straightforward to show that

$$\begin{aligned} \Delta W^1(0) - \Delta W^1(1) = & \left\{ \left[D(e_H \bar{x}) - D(e_L \bar{x}) \right] - \left[D(e_H \hat{x}(1, e_H)) - D(e_L \hat{x}(1, e_H)) \right] \right\} \\ & - \left\{ B[\hat{x}(1, e_L)] - B[\hat{x}(1, e_H)] \right\} + \left\{ D[e_L \hat{x}(1, e_L)] - D[e_L \hat{x}(1, e_H)] \right\} \end{aligned} \quad (37)$$

The first term is positive and shows that there will be a greater reduction in damage brought about by having the new technology when we evaluate this at the level of output that would arise if the government took no action compared to the much lower level of output that would prevail if the firm failed to invest and the period-1 government imposed its desired emissions/output tax. The second term is negative and shows that, compared to the situation where the period-2 government put no weight on the environment, one of the benefits that would arise were the period-1 government in power in period 2 would be that output would be higher if firms undertook the investment. However the final term is positive reflecting the fact that, conditional on the firm using the cleaner technology, this latter factor carries the downside of higher emissions.

In what follows we assume that \bar{x} is sufficiently large that the first term in (37) dominates and consequently we have:

$$\Delta W^1(0) > \Delta W^1(1) = \Delta \hat{W}^2(1) > F \quad (38)$$

where the last inequality follows from (26).

So, from (36) and (38) we have established

Proposition 2 If the level of output that is chosen by a government in period 2 that attached no weight to the environment is sufficiently high, then the incentive of the period 1 government to have the investment undertaken would be **higher** than if it itself were in power in period 2, while the private incentives of the firm to invest will be zero.

We now make the stronger assumption that the elasticities β and δ are constant.

It is straightforward to show from (23) that

$$\left. \frac{\omega \partial W^1(\omega)}{\partial \omega} \right|_{\omega=0} = \frac{[e_L \bar{x} \cdot D'(e_L \bar{x})] - [e_H \bar{x} \cdot D'(e_H \bar{x})]}{\beta + \delta} < 0 \quad (39)$$

so $\Delta W^1(\omega)$ is strictly decreasing at $\omega = 0$.

In addition, from (18) it follows that for all $\omega \geq 0$

$$\frac{\omega \partial \hat{\Pi}(\omega)}{\partial \omega} = \frac{\beta}{\beta + \delta} \left\{ [\tilde{x}(\hat{\tau}(\omega, e_H)) \cdot \hat{\tau}(\omega, e_H)] - [\tilde{x}(\hat{\tau}(\omega, e_L)) \cdot \hat{\tau}(\omega, e_L)] \right\} > 0 \quad (40)$$

where the inequality follows from the fact that although the higher output tax when the firm fails to invest lowers output, because $\beta > 1$ output falls less than proportionately causing tax revenue to increase.

This is as far as we have been able to push the analysis at a high level of generality. To understand what happens to the functions $\Delta W^1(\omega)$ and $\Delta \hat{\Pi}(\omega)$ for values of ω other than 0 and 1, we have derived these two functions explicitly for the case where the two elasticities β and δ are constant.¹⁸ Details are given in Appendix 1, and the resulting functions are illustrated in Figure 1 which is presented in Appendix 2.

The key features are

¹⁸ One implication of assuming constant elasticities for all values of x and E is that $\bar{x} = \infty$ and so the inequality in (37) holds for sure. But as we have shown this inequality will hold under a far wider range of cases.

- the social incentive to invest takes a minimum at $\omega = 1$ and is *convex* over an interval $[0, \bar{\omega}]$ where $\bar{\omega} > 1$
- the private incentive to invest is zero when $\omega = 0$, strictly increasing and *concave*.

The intuition behind the first result is that the lower are emissions per unit of output the less sensitive is the welfare of the period-1 government to the type of government that is in power in period 2, so it sees a greater incentive to have the cleaner technology in place precisely when it contemplates the possibility of a different type of government's being in power.

The second conclusion follows because although emission/output taxes increase as the weight placed on environmental damage increases, they do so at a decreasing marginal rate – precisely because the firm responds by cutting back output. This introduces an intrinsic degree of effective risk aversion to the investment decision.

In the light of the second of the above two conclusions, we can now define $\underline{\omega}$ as the critical minimal weight placed on the environment by the period 2 government above which the private incentive of the firm to invest exceeds F .

So we have the following

Proposition 3(i) Whatever type of government is in power in period 2, the government in period 1 will always want the investment to be undertaken
3 (ii) If $\omega \geq \underline{\omega}$ then the firm will choose to invest and, once again, environmental policies alone are sufficient to induce the optimal investment. However if $\omega < \underline{\omega}$ then the use of environmental policies alone will not be sufficient to bring about the optimal investment and it will be necessary for the period-1 government to introduce an R&D subsidy to induce what it regards as the socially optimal investment.

Now let us turn to the case of uncertainty.

3.2.2.1 The Case of Uncertainty

Now suppose that the firm and the government are unsure what type of government will be in power in period 2 but share a common belief that ω is distributed with positive support on the real line¹⁹ according to the probability density function $f(\omega) > 0$ where

$$E(\omega) = \int_0^{\infty} \omega f(\omega) d\omega = 1. \quad (40)$$

Given the properties of $\Delta W^1(\omega)$ and $\hat{\Pi}(\omega)$ we have

¹⁹ If the distribution had positive support only for values of $\omega > \underline{\omega}$ then there once again environmental policies alone will be sufficient to achieve the optimum investment.

$$\int_0^{\infty} \Delta W^1(\omega) f(\omega) d\omega > \Delta W^1(1) = \Delta \hat{W}^2(1) > F \quad (41)$$

and

$$\int_0^{\infty} \Delta \hat{\Pi}(\omega) f(\omega) d\omega < \Delta \hat{\Pi}(1). \quad (42)$$

Given (26) and (29) it will only be if there is a sufficiently great degree of uncertainty with high probabilities that $\omega < \underline{\omega}$ that it is likely that $\int_0^{\infty} \Delta \hat{\Pi}(\omega) f(\omega) d\omega < F$.

So we have:

Proposition 4 (i) Whatever the distribution of possible types of period-2 governments, the period-1 government will always want to have the investment undertaken.

4 (ii) If the degree of uncertainty is particularly large – in particular if there is a significant risk of having governments with $\omega < \underline{\omega}$ - then environmental policies alone may not be sufficient to induce the optimal investment and the period-1 government will need to resort to an R&D subsidy.

4. Conclusions

We have shown that the inability of governments to commit to policies into the future has significant implications for the optimal set of policies, and calls into question the traditional prescription that environmental policies alone are sufficient to induce both the optimal output and the optimal investment. This is because the inability to commit has very different effects on the incentives of both governments and the private sector to have cleaner technologies introduced. It makes the government *more* determined to have the investment carried out but reduces the private incentive to invest because it introduces an intrinsic element of risk aversion to the investment incentive.

In demonstrating this result we have employed a very simple model which ignores many other features of climate change policy-making:

- the need to reach international agreement,
- the stock-externality problem and the optimal time-path of emission taxes;
- the fact that there is still considerable uncertainty about the climate change process and its likely effects, but there is learning taking place.

Each of these can introduce additional features that not only interact with the commitment issue but also give rise to their own commitment issues. For example it is interesting to ask how the framework can be applied in the context where there is uncertainty about whether and when an international agreement might be reached. So there is scope for considerable further research on this topic.

References

- Boyer, M. and J-J Laffont (1999): "Towards a Theory of the Emergence of Environmental Incentive Regulation", *Rand Journal of Economics*, 30, 137-157.
- Dasgupta, P. (2007): "Comments on the *Stern Review of the Economics of Climate Change*", *National Institute Economic Review*, **199**, 4-7
- Jaffee, A., R. Newell and R. Stavins (2003): "Technological Change and the Environment", in K. Maler and J. Vincent (eds.) *Handbook of Environmental Economics*, North-Holland, Amsterdam, 461-516.
- Johal, S. and A. Ulph (2002): "Globalisation, Political Lobbying and the Design of International Environmental Governance", *Review of International Economics*, 10 (3), 387-403.
- Johal, S. and A. Ulph (2003): "Limiting Political Discretion and International Environmental Policy Coordination with Active Lobbying" in L. Marsiliani, M. Rauscher, and C. Withagen (eds.) *Environmental Policy-Making in an International Environmental Perspective*, Kluwer, Dordrecht, 303-336
- Leahy, D. and P. Neary, (1996): "International R&D Rivalry and Industrial Strategy Without Government Commitment" *Review of International Economics*, **4**, 322-38
- Leahy, D and P. Neary (2000): "Strategic Trade and Industrial Policy in Dynamic Oligopolies", *Economic Journal*, **110**, 484-508.
- Nordhaus, W.D. (2007): "A Review of the *Stern Review of the Economics of Climate Change*", *Journal of Economic Literature*, **45**, 686-702
- Petrakis, E. and A. Xepapadeas (1999): "Does Government Precommitment Promote Environmental Innovation?", in E. Petrakis, E. Sartzetakis and A. Xepapadeas, (eds), *Environmental Regulation and Market Power*, Edward Elgar, pp. 145-161.

E. Petrakis and A. Xepapadeas (2000): "Location Decisions, Environmental Policy and Government Commitment", in N. Georgantzis and I. Barreda (eds.), *Spatial Economics and Ecosystems: The Interaction Between Economics and the Natural Environment*, Series: Advances in Ecological Sciences WIT (Wessex Institute of Technology) Press. pp. 115-127.

Stern, N. (2007): *Stern Review of the Economics of Climate Change*, HM Treasury

Stern, N. (2008): "The Economics of Climate Change", *American Economic Review Papers and Proceedings*, 98 (2), 1 – 37.

Ulph, A. and D. Ulph (1997), "Global Warming, Irreversibility and Learning", *Economic Journal*, **107**, 636-650.

Ulph, A. and D. Ulph (2001) "Strategic Trade and Industrial Policy in Dynamic Oligopolies – the True Value of Commitment", *mimeo*, University of Southampton.

Weitzman, M. (2007), "A Review of the *Stern Review of the Economics of Climate Change*", *Journal of Economic Literature*, **45**, 703-724

Appendix 1

In this appendix we set out the results for a special case of our model in which the elasticity of the marginal benefit function, $\beta(x)$, and the elasticity of the marginal damage cost function, $\delta(E)$, are both constant. Thus we assume that the benefit function takes the form:

$$B(x) \equiv B_0 + \frac{B}{(1-\beta)} x^{1-\beta}, \quad B_0, B > 0, \quad \beta > 1$$

where $\beta = -\frac{x B''(x)}{B'(x)}$ is the constant elasticity of marginal and B_0 is assumed to be sufficiently large that $B(x) \geq 0$ for all relevant values of x .

The damage cost function is assumed to take the form:

$$D(E) \equiv \frac{E^{(1+\delta)}}{(1+\delta)}, \quad \delta > 0$$

where $\delta = \frac{E D''(E)}{D'(E)}$ is the constant elasticity of marginal damage.

We first analyse the environmental taxes the first-period government would set in the second-period if it had the powers to commit to these taxes.

A1. Second Period Policy When First-Period Government Can Commit.

Suppose in the second period the level of emissions per unit of output is given by e . Then it is straightforward to show that the optimal output that would be chosen by the period-1 government is given by:

$$\hat{x}(e,1) = B^{\frac{1}{\beta+\delta}} \cdot e^{\frac{1+\delta}{\beta+\delta}}; \quad (\text{A1})$$

the corresponding optimal level of emissions is given by:

$$\hat{E}(e,1) = e \cdot \hat{x}(e,1) = B^{\frac{1}{\beta+\delta}} \cdot e^{\frac{\beta-1}{\beta+\delta}}; \quad (\text{A2})$$

the emission tax required to induce the firm to set the optimal output is:

$$\hat{t}(e,1) = B^{\frac{\delta}{\beta+\delta}} \cdot e^{\frac{\delta(\beta-1)}{\beta+\delta}}. \quad (\text{A3})$$

For later purposes it is useful to note that, by multiplying (A2) and (A3) the tax revenue raised in this situation is:

$$\hat{R}(e,1) = B^{\frac{1+\delta}{\beta+\delta}} \cdot e^{\frac{(1+\delta)(\beta-1)}{(\beta+\delta)}}, \quad (\text{A4})$$

which is a strictly increasing function of e and that total environmental damages are

$$\hat{D}(e,1) = \frac{\hat{R}(e,1)}{1+\delta}. \quad (\text{A5})$$

Government welfare is

$$\hat{W}^2(e,1) = B_0 - \frac{(\beta + \delta)\hat{R}(e,1)}{(\beta - 1)(1 + \delta)} \quad (\text{A6})$$

while the firm's profits are given by:

$$\hat{\Pi}(e,1) = B_0 - \frac{\beta\hat{R}(e,1)}{\beta - 1}. \quad (\text{A7})$$

Comparing (A4) and (A5c) it is straightforward to see that $\hat{R}(e,1) > \hat{D}(e,1)$, so, as would be expected, tax revenue exceeds total damage costs, while comparing (A6) and (A7) confirms that government welfare exceeds profits, precisely because tax revenues exceed total damage costs. This just exemplifies for this special case the very general analysis underlying the conclusion in equation (29) in the main text.

A2. Second Period Policy When First-Period Government Cannot Commit.

We now suppose that in the second period the government in power has an objective function which attaches a weight ω to damage costs. Then it is straightforward to obtain the following results:

$$\hat{x}(e, \omega) = \omega^{\frac{1}{\beta+\delta}} \hat{x}(e, 1); \quad (\text{A8})$$

and

$$\hat{t}(e, \omega) = \omega^{\frac{\beta}{\beta+\delta}} \hat{t}(e, 1); \quad (\text{A9})$$

from which it follows that:

$$\hat{W}^2(e, \omega) = B_0 - \frac{(\beta + \delta)\omega^{\frac{\beta-1}{1+\delta}} \hat{R}(e, 1)}{(\beta - 1)(1 + \delta)}; \quad (\text{A10})$$

$$\hat{\Pi}(e, \omega) = B_0 - \frac{\beta\omega^{\frac{\beta-1}{\beta+\delta}} \hat{R}(e, 1)}{\beta - 1} \quad (\text{A11})$$

and

$$W^1(e, \omega) = B_0 - \hat{R}(e, 1) \left[\frac{\omega^{\frac{\beta-1}{\beta+\delta}}}{\beta - 1} + \frac{\omega^{\frac{1+\delta}{\beta+\delta}}}{1 + \delta} \right]. \quad (\text{A12})$$

From A(10) and A(12) it is straightforward to confirm the general result in equation (22) in the main text , namely $W^1(e,1) = \hat{W}^2(e,1)$

A3. First-Period Investment Decision.

We want to compare the incentives of the period-1 government to invest with those of the private firm. From (A11) and (A12) we get:

$$\Delta W^1(\omega) = W^1(e_L, \omega) - W^1(e_H, \omega) = \psi(\omega)\Delta\hat{R}, \quad (\text{A13})$$

and

$$\Delta\hat{\Pi}(\omega) = \hat{\Pi}(e_L, \omega) - \hat{\Pi}(e_H, \omega) = \eta(\omega)\Delta\hat{R}, \quad (\text{A14})$$

where

$$\Delta\hat{R} \equiv \hat{R}(e_H, 1) - \hat{R}(e_L, 1) > 0, \quad (\text{A15})$$

$$\psi(\omega) \equiv \frac{\omega^{\frac{\beta-1}{\beta+\delta}}}{\beta-1} + \frac{\omega^{\frac{1+\delta}{\beta+\delta}}}{1+\delta}, \quad (\text{A16})$$

and

$$\eta(\omega) \equiv \frac{\beta\omega^{\frac{\beta-1}{\beta+\delta}}}{\beta-1}. \quad (\text{A17})$$

These satisfy the following properties:

$$\omega \rightarrow 0 \Rightarrow \psi(\omega) \rightarrow \infty;$$

$$\psi'(\omega) = \frac{\omega^{\frac{1+2\delta+\beta}{\beta+\delta}}}{\beta+\delta}(\omega-1), \text{ so } \psi'(\omega) \begin{cases} > 0 \\ < 0 \end{cases} \text{ as } \omega \begin{cases} < \\ > \end{cases} 1;$$

$$\psi''(\omega) = \frac{\omega^{\frac{1+3\delta+2\beta}{\beta+\delta}}}{(\beta+\delta)^2} [(1+2\delta+\beta) - (1+\delta)\omega], \text{ so } \psi''(\omega) \begin{cases} < \\ > \end{cases} 0 \text{ as } \omega \begin{cases} < \\ > \end{cases} \bar{\omega} \equiv \frac{1+2\delta+\beta}{1+\delta} > 1$$

and

$$\eta(0) = 0; \quad \eta'(\omega) = \frac{\beta}{\beta+\delta} \omega^{\frac{1+\delta}{\beta+\delta}} > 0; \quad \eta''(\omega) = -\frac{\beta(1+\delta)}{(\beta+\delta)^2} \omega^{\frac{1+\beta+2\delta}{\beta+\delta}} < 0$$

So $\psi(\omega)$

- tends to infinity as ω tends to zero;
- is a decreasing strictly convex function of ω for ω less than 1;
- reaches a minimum when $\omega = 1$;

- is an increasing convex function of ω for ω between 1 and $\bar{\omega} > 1$;
- and an increasing but concave function of ω for $\omega \geq \bar{\omega}$.

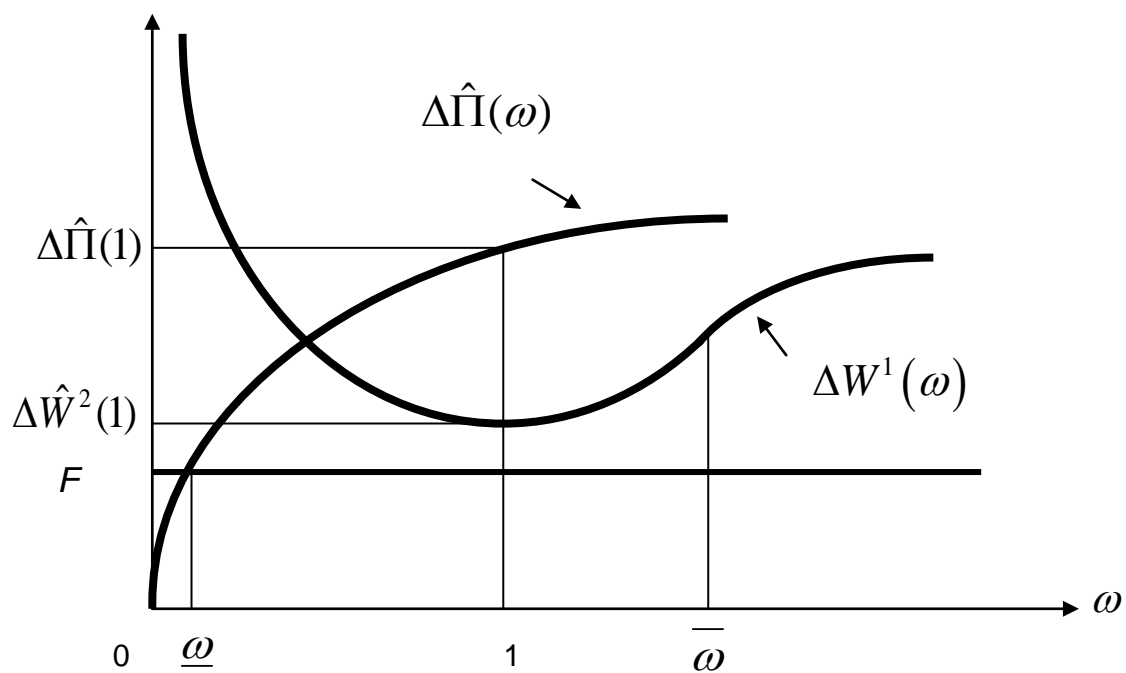
While $\eta(\omega)$

- is zero when ω is zero;
- for all non-negative values of ω is an increasing and strictly concave function.

These properties are precisely what are illustrated in Figure 1 in Appendix 2.

Appendix 2

Figure 1



The University of Manchester
Sustainable
Consumption Institute



For more information about the SCI Discussion Paper Series please contact:

Sustainable Consumption Institute
The University of Manchester
188 Waterloo Place
Oxford Road
Manchester, UK
M13 9PL

T: 0161 275 4030
F: 0161 275 0188
E: sci@manchester.ac.uk