



Interactive starting points for problem solving in undergraduate mathematics (PSUM)

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Title: Interactive starting points for problem solving in undergraduate mathematics (PSUM)

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Abstract:

The Liverpool Hope University-NRICH collaboration aimed to

- develop an innovative and sustainable online bank of starting points for problem-solving, presented in an interactive, visual and engaging way that will nurture mathematical thinking, logical processes and modelling. The starting points will permit a range of teaching approaches – individual, small group and whole class. They will be fully functional on a range of digital technologies including handhelds.
- contribute to the guide for incorporating problems into courses, developing problem-solving skills and assessment and providing case studies of effective integration of problem-solving into undergraduate mathematics courses.

Four starting points for problem solving have been designed, developed and tested and will be available on the NRICH website from September 2012.

Background and Rationale:

The HE Mathematics Curriculum Summit was held at the University of Birmingham in January 2011 (Rowlett, 2011). Operated by the Maths, Stats and OR (MSOR) Network as part of the National HE STEM Programme's Mathematical Sciences HE Curriculum Innovation Project, the Summit brought together Heads of Departments or their representatives from 26 university mathematics departments in England and Wales. Education representatives from the Institute of Mathematics and its Applications, the Royal Statistical Society, the Operational Research Society and the Council for the Mathematical Sciences, members of the National HE STEM Programme, sigma and the MSOR Network, and several individuals also attended.

The Summit's ultimate aim was to produce a set of recommendations for targeting financial support in mathematics degree programmes for curriculum development. Of the summit's 14 recommendations, the first three pertained directly to problem-solving, namely –

1. Sharing good practice – Collecting case studies on embedding problem-solving in undergraduate mathematics degree programmes, and developing a good practice guide for teaching and assessing problem-solving.
2. Developing a bank of problems with solutions and extensions – These problems would allow teachers looking to implement problem-solving in their degree programmes access to robust and unfamiliar problems with which to teach students.
3. Development of a collection of teaching resources on the development of mathematics – These would help to develop students' awareness of the culture of mathematics, and dispel the idea that mathematics is a static and complete body of knowledge.

The QAA (2007) benchmark statement for mathematics, statistics and operational research (MSOR) is clear about the importance of problem-solving in any undergraduate

programme. It specifies problem-solving as a generic subject specific skill to be developed in all undergraduate courses:

graduates will have the ability to demonstrate knowledge of key mathematical concepts and topics, both explicitly and by applying them to the solution of problems. They will be able to comprehend problems, abstract the essentials of problems and formulate them mathematically and in symbolic form, so as to facilitate their analysis and solution, and grasp how mathematical processes may be applied to them, including where appropriate an understanding that this might give only a partial solution.(p.15)

and includes the following standards (p.20-21):

Threshold: demonstrate a reasonable level of skill in comprehending problems, formulating them mathematically and obtaining solutions by appropriate methods

Typical: demonstrate skill in abstracting the essentials of problems, formulating them mathematically and obtaining solutions by appropriate methods

Having devised and specified four starting points for problem solving a programmer was contracted to develop these so they could be used on a range of technological devices in different ways as suits the lecturer and students. NRICH's considerable expertise in making starting points available for mathematical problem-solving has informed the way in which the problems have been developed.

These were being trialled with students working at undergraduate level and refined to ensure they are robust on different platforms and can be used effectively by individuals, small groups or with a class, as suits different lecturers. Students were invited to complete anonymous online questionnaires and participate in informal discussion groups about their experiences. The outcomes of the surveys are reported in the next section of this report.

In 1980 Papert set out a vision for how computers might transform the way in which mathematics is learnt. Almost 20 years ago BecTa (1993) declared that technology allows learners to explore mathematics in a way that enables them to –

- *Learn from feedback*: The computer often provides fast and reliable feedback that is non-judgemental and impartial. This can encourage students to make their own conjectures and to test out and modify their ideas.
- *Observe patterns*: The speed of computers and calculators enables students to produce many examples when exploring mathematical problems. This supports their observation of patterns and the making and justifying of generalisations.
- *See connections*: The computer enables formulae, tables of numbers and graphs to be linked readily. Changing one representation and seeing changes in the others helps students to understand the connections between them.
- *Work with dynamic images*: Students can use computers to manipulate diagrams dynamically. This encourages them to visualise the geometry as they generate their own mental images.
- *Explore data*: Computers enable students to work with real data that can be represented in a variety of ways. This supports interpretation and analysis.
- *'Teach' the computer*: When students design an algorithm (a set of instructions) to make a computer achieve a particular result, they are compelled to express their commands unambiguously and in the correct order; they make their thinking explicit as they refine their ideas.

The starting points have been designed in such a way that students can explore the situation and collect information that requires them to make choices about which technologies (including pencil and paper) and approaches to use in order to solve the problem. The use of technology to support learning and provide a context within which students can explore mathematics and develop understanding has been much researched (eg. Hoyles et al (2003, 2004), Povey et al (2000)) and is not straightforward. The way in which tasks are designed and used is at least as important as using technology per se.

Experiences from the piloting and trialling of the starting points were used to form case studies that will be incorporated into the general guidance and tutor notes that will be available alongside the starting points on the website.

Evaluation:

The project produced interactive starting points for problem-solving in undergraduate mathematics. The starting points were carefully specified and designed by the team and then implemented by an experienced mathematician-programmer with a view to being available on a range of technological platforms including handhelds. All starting points were trialled with undergraduates or other students working on undergraduate level mathematics (eg graduates on a subject knowledge enhancement course prior to embarking on a PGCE). All students and tutors completed anonymous online surveys which provided evidence of efficacy, and some students participated in focus group discussions with members of the team.

Preliminary data was collected for all of the interactive starting points, whilst in the early stages of development. The survey respondents were predominantly male (68%) with an average age of 28. The majority of responses were mature students on a Mathematics Subject Enhancement course at Liverpool Hope University. Although most of the respondents were not undergraduates, they did have equivalent mathematical qualifications to the average undergraduate student i.e. A level mathematics, with about one in five having completed A level further mathematics.

A somewhat surprising finding was that students predominantly chose to solve the problems using traditional pen and paper methods, with software applications (such as autograph) being far more popular than calculators. More people chose to work with others than individually (see figure 1).

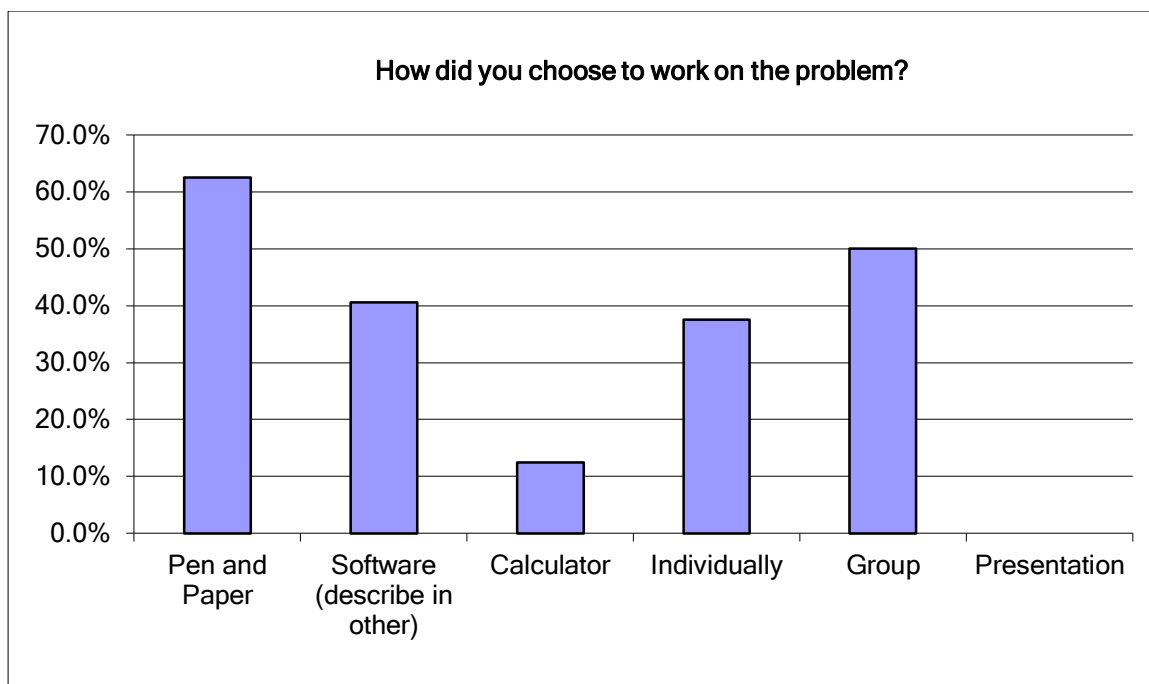
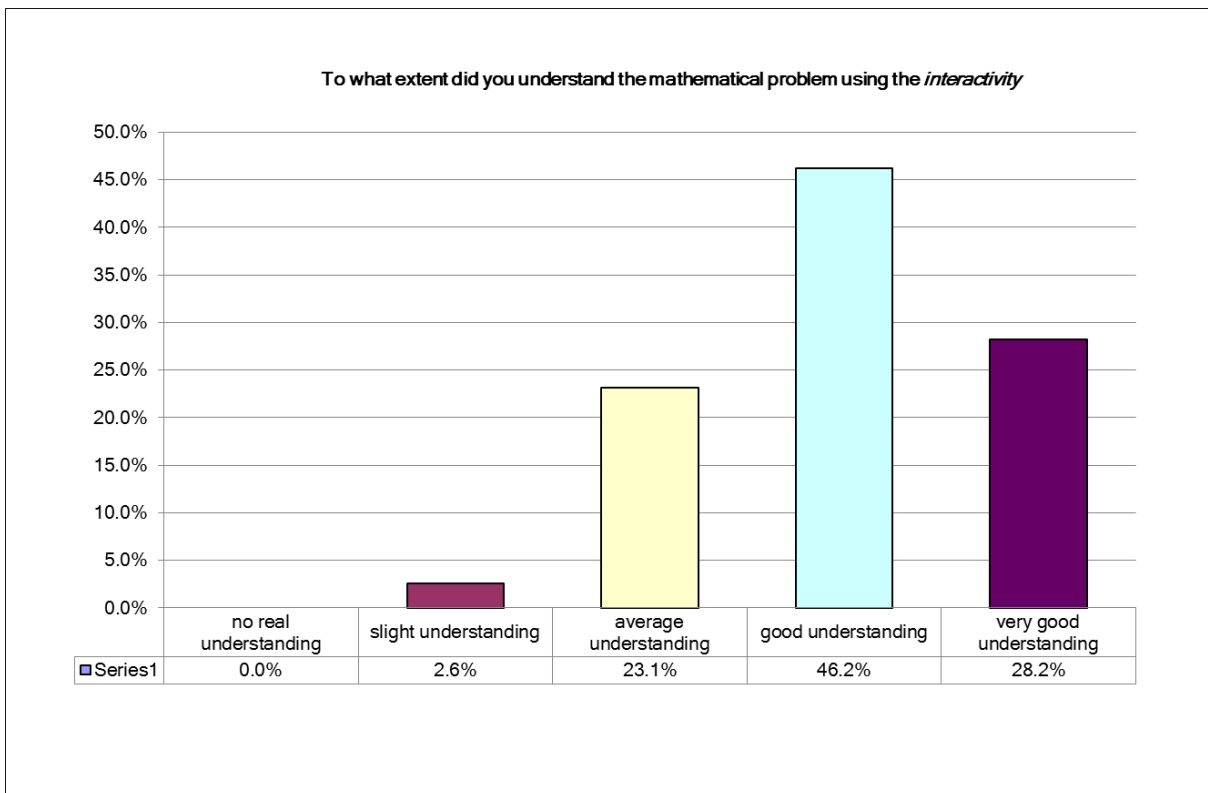
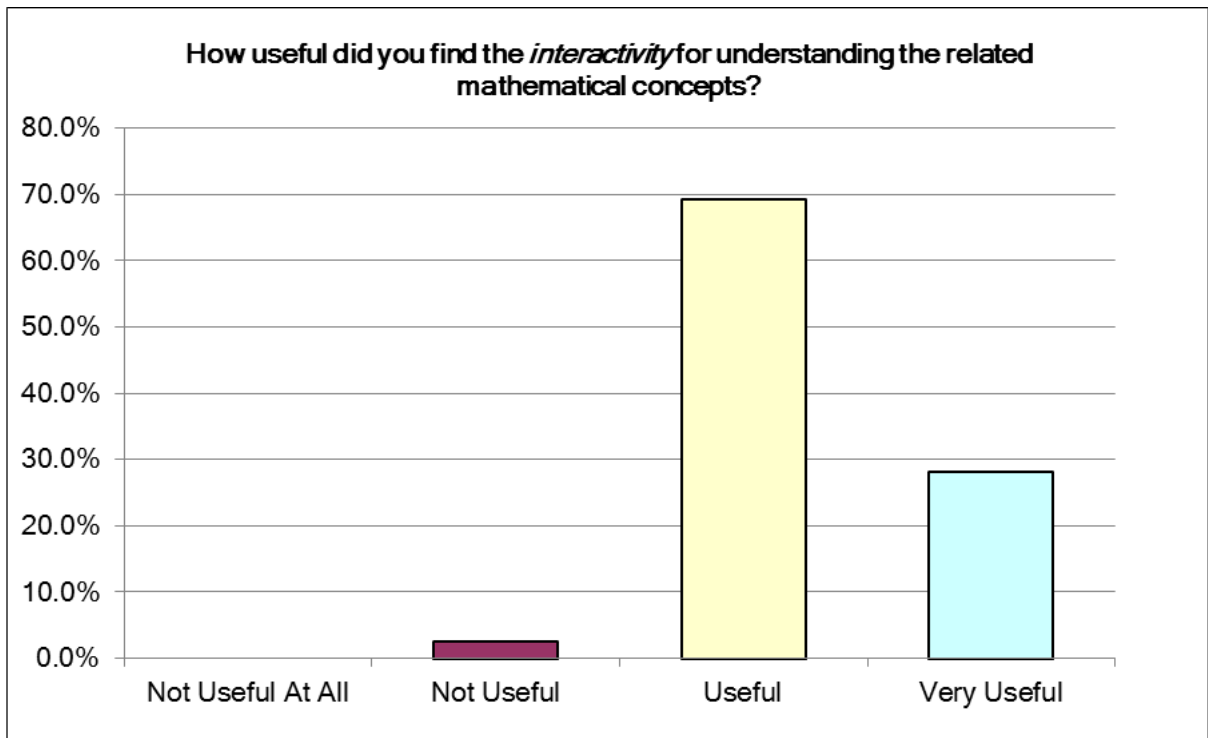
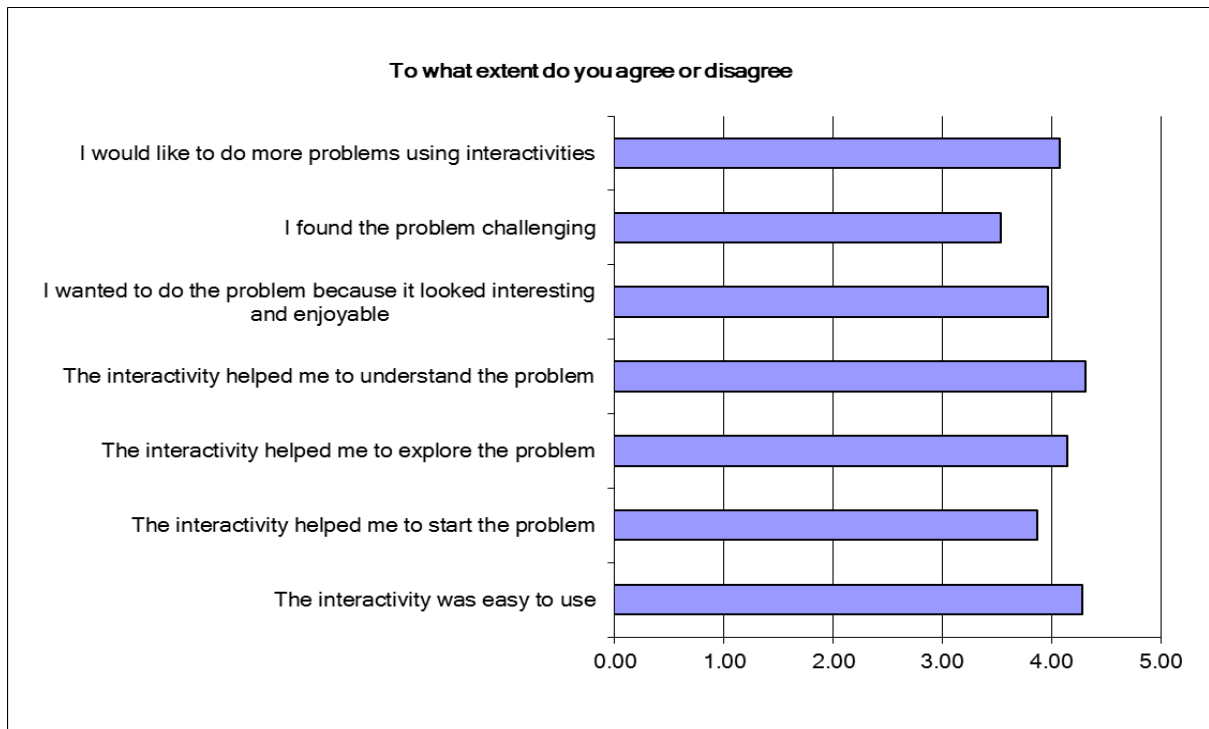


Figure 1: Ways of working on the problem.

The large majority of student participants found that they obtained a good to very good understanding of the mathematical concepts contained within each problem solving scenario and 97.4% of student participants found that the simulations were helpful for learning mathematics.

The student's response towards the interactive starting points was overwhelmingly positive. They found the problems attractive, interesting, enjoyable and easy to use. When solving the problems students found that the opportunity to interact and the visual presentation of the problems encouraged them to explore the context thoroughly thus gaining insights which helped enable them to be successful at solving the problem. Students were keen to explore more problems presented in this way.





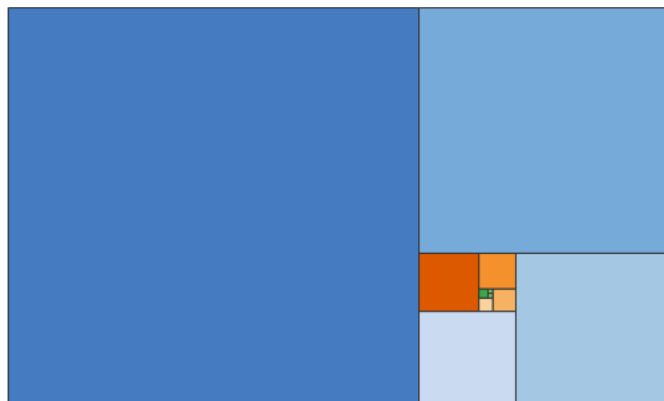
Survey responses to the individual starting points:

Picture This!

Picture this!

Here is a diagram corresponding to two positive integers: 144 and 89.

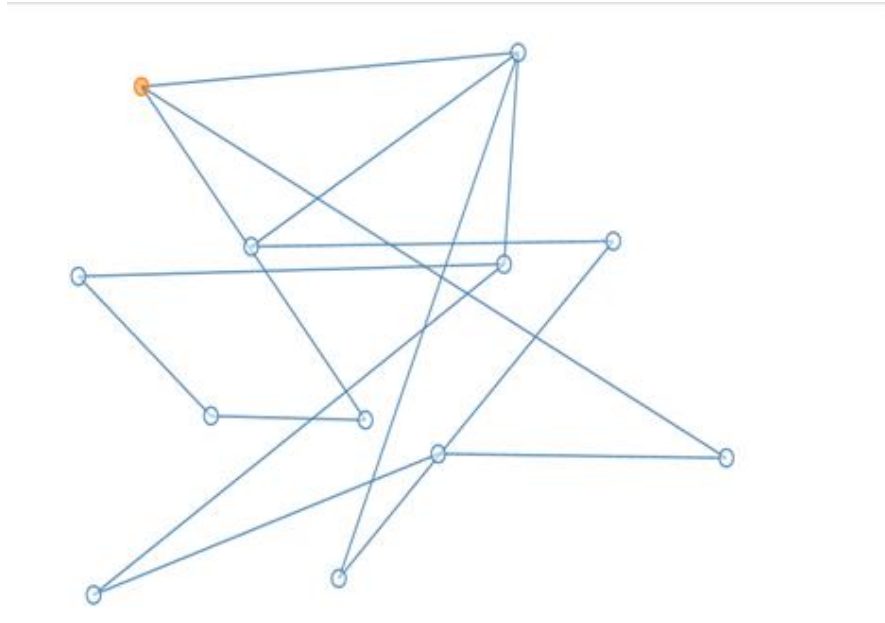
144 89



All of the respondents found this starting point useful for exploring the problem. The qualitative feedback captures how the computer delivery supported problem solving. A clear theme was the benefit of instantaneous feedback: “(The resource was) Interactive, quick to generate answers”, “you could easily change the starting numbers, and gaining an accurate visual representation is useful”, and “you could try different integers and see how they affected the diagram”, “Quick visual results to confirm findings.” and “(The interactivity), enabled me to check my answers quickly.”

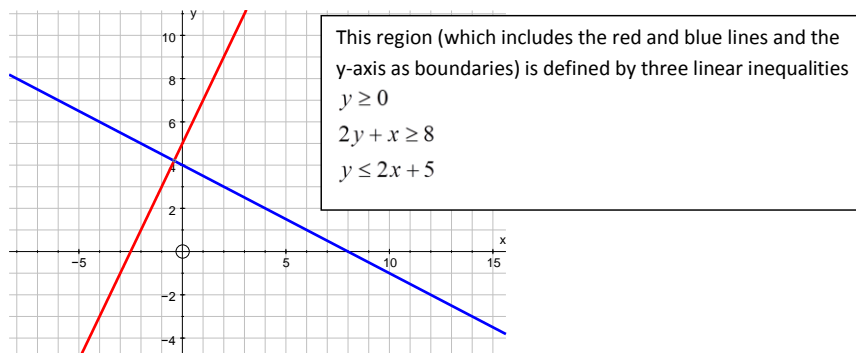
Respondents generally reported deepening and reinforcing their knowledge of mathematics. “[I] understood the concept of golden ratio prior to using it, however it made it easy to illustrate how it works using remainders”, “(The) Visual (aspect, helped me) to understand the concept, (it also) made it easier to see where you'd made any mistakes.”, “It helped to reinforce Euclid's algorithm.”, “(It) provided an alternative way of explanation.”. More generally students commented that, the interactivity was “very pleasing on the eye and user-friendly.” However, one student reported “Wasn't quite sure what I was meant to be doing, even after we had finished...”. This is something tutor may need to take account of.

Networks and Graphs



All respondents found this starting point helpful for learning the mathematics and exploring the problem. Qualitative responses from the students focused on the visual, interactive environment as “easy to manipulate,” and “nice visual representation,” which allowed students to “experiment with different arrangements quickly.” The students also confirmed that using technology has distinct advantages over paper based experimentation because, “(I can) move nodes around” and “(it is) easier than drawing,” and it allows “seeing where you are going and have been.”

Linear Programming



All respondents found this starting point helpful. The visual representation and interactivity was warmly received: "It allowed quick changes to be made, and allowed a visual freedom I've never experienced before in linear modelling", "Simple to use. Being able to alter in real time is very helpful."

Nearly all of the respondents felt that this starting point helped them learn the relevant mathematics, because "(it) makes you feel like you're doing more" and "(I) like to see visual demonstrations of theory." Other students reported how the use of technology facilitated the problem solving process, e.g. "It allowed quick updates to the solution space, which in turned allowed me to determine if I've made silly mistakes, and hence improve my intuition as a maths student."

More general responses included: "I can see great use of this software in a teaching environment to get the idea of what is going on across faster....I found it refreshing to see something presented so simply. It was extremely good." and "[It] is a good idea, better than using autograph."

Filling Objects

Responses were more mixed to this starting point. Most found it useful, but only one respondent reported achieving a good understanding of the mathematics by the end of a two hour session. This may be linked to the demands of the mathematical content associated with this starting point. Less than half the respondents reported that the starting point was helpful for learning mathematics. However most found the simulation useful for exploring the problem and appreciating the visualisation: "The 3D illustration gives you a clue how to tackle the problem," and "watching the rate change helps to visualise the problem at various stages" and "the visual effect was great"

Summary

Respondents found all of the starting points useful for engaging with mathematical concepts through problem solving. They encouraged active exploration of the problem exemplifying many of the BeCTa (1993) benefits: learning from feedback, working with dynamic images (visualisation) and exploring data.

Discussion, Learning and Impact (Success):

The original intention was to develop an environment which would work on various technological platforms including handhelds. In practice, given the relatively modest funding, tight timescale and change of circumstances of key personnel four interactive starting points were generated. These starting points do work on various technological platforms, including handhelds and permit a range of teaching approaches. They took longer than anticipated to develop, which meant that the amount of trialling and quantity of feedback was substantially less than originally anticipated.

Despite this, all starting points were trialled with students working at undergraduate level. Students and tutors completed anonymous online surveys which provided evidence of efficacy and informed the online guidance which accompanies the online resources. The analysis of student responses is included in the section above. There were too few tutor responses to make any meaningful analysis possible.

Good links were made with the more substantial HE-STEM funded project based at Coventry University and contributions were made to the guidance produced. A joint

presentation was made at BSRLM in March 2012 and is planned for HE-STEM12 in September.

For any future projects, more time needs to be available to find and contract the necessary expertise. The design of the starting points needs to be done more quickly so that there is more time to test prototypes and get feedback from across a range of different institutions. Whilst this had been planned, it did not happen in practice.

Further Development and Sustainability:

The starting points will be available on the NRICH website, nrich.maths.org/psum. This will ensure they will be available in the future. The starting points have been developed using freeware which means that they can be developed further and new starting points could be developed to fit with these HE-STEM funded starting points. Awareness will be raised through case studies in the problem-solving guide and conference presentations.

Outputs:

The PSUM starting points and accompanying notes are available on the NRICH website (nrich.maths.org/psum).

The team has contributed to the guidance produced by the HE-STEM problem-solving project based at Coventry University (www.mathcentre.ac.uk/problemsolving).

References:

- BeCTa, 1993. Entitlement to ICT in Secondary Mathematics www.ictadvice.org.uk
- Hoyles, C. & Noss, R. 2003. What can digital technologies take from and bring to research in mathematics education? in Bishop, A.J., Clements, M.A., Keitel, C., Kilpatrick, J. & Leung, F.K.S., Eds, Second International Handbook of Mathematics Education (pp. 323–349). Dordrecht: Kluwer Academic Publishers.
- Hoyles, C. Noss, R. & Kent, P., 2004. On the integration of digital technologies into mathematics classrooms. *International Journal of Computers for Mathematical Learning* 9 (3) 309-326
- Margolis, E., Ed. 2001. *The hidden curriculum in higher education*. New York: Routledge
- Mason, J., Burton, L. & Stacey, K., 1985. *Thinking Mathematically*. Harlow: Addison-Wesley
- NRICH nrich.maths.org.uk
- Papert, S., 1980. *Mindstorms: Children, computers and powerful ideas*. Brighton: Harvester Press
- Polya, G., 1990. *How to solve it*. London: Penguin
- Povey, H. & Ransom, M., 2000. Some Undergraduate Students' Perceptions of Using Technology for Mathematics: Tales of Resistance. *International Journal of Computers for Mathematical Learning* 5(1) 47-63
- QAA, 2007. *Benchmark statement for mathematics, statistics and operational research*. Gloucester: QAA (www.qaa.ac.uk)
- Rowlett, P., ed. 2011, *HE Mathematics Curriculum Summit*. MSOR Network