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# Sampled-data adaptive control for a class of nonlinear systems with parametric uncertainties<sup>★</sup>

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**Abstract:** Sampled-data control systems have been prevailing in various applications, in parallel with the development of digital computers and its applications in control activities. At the same time, adaptation has proved to improve performance of a control algorithm, particularly when uncertainties involve in the model of the systems. In this paper, a stabilization problem for a class of nonlinear systems with parametric uncertainties is addressed. A discrete-time adaptation algorithm based directly on the discrete-time model of the system is proposed. This adaptation algorithm is then used for constructing a discrete-time controller to stabilize (in a semiglobal practical sense) the original continuous-time system in closed-loop, in a sampled-data set-up. This proposed direct discrete-time technique is shown to improve the closed-loop performance of the system, compared to applying a discrete-time adaptive control which is obtained through emulation design (by means of sample and hold). An example is presented to illustrate the result, and to show the advantages of this direct discrete-time design for sampled-data implementation.

*Keywords:* Adaptive control; Nonlinear control; Sampled-data system; Parametric uncertainties.

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## 1. INTRODUCTION

Nowadays, sampled-data control systems have gained popularity due to the more common use of digital computers or microcontrollers to implement algorithms for controlling plants/processes which are generally continuous-time in nature. At the same time, the development of adaptive controllers that include some adaptation mechanism which has the capacity to adjust with respect to the system parametric, structural and environmental changes has also been benefited with the convenience of using digital devices with flexibility for modification and the use of memory (Tao, 2003).

Various results have been developed in nonlinear adaptive control methods for continuous-time systems, dealing with parametric uncertainties. The class of feedback linearizable systems that depend linearly on the unknown parameters are among the most widely studied (see Sastry and Bodson (1989), Marino and Tomei (1995), Krstić et al. (1995) and references therein).

Despite some indicated fundamental limitation in constructing results on discrete-time adaptive controller due to more complicated computation (Xie and Guo, 1999), a number of results have been developed, for instance in Yeh and Kokotović (1995); Kanellakopoulos (1994) and more recently in Monaco et al. (2000). System in parametric strict feedback form seems to also gain special attention as it has been the subject of a number of results (Zhao and Kanellakopoulos, 2002; González, 2003, 2009).

While there have been quite a number of results in continuous-time as well as discrete-time adaptive control, there are only very few results addressing sampled-data systems. Despite a quite good progress on research in sampled-data control design for various classes of nonlinear systems, there has been very little attention to the use of adaptation. The application papers (Warshaw et al., 1992; Warshaw and Schwartz, 1993), dealing with stabilization of a robotic manipulator extending the Slotine-Li controller (Slotine and Li, 1991) by including adaptation is among the very few results in sampled-data adaptive control. A quite recent paper (Wu and Ding, 2007) studied the adaptive backstepping in sampled-data framework and extended the result presented in Krstić et al. (1995).

In Karagiannis and Astolfi (2008), a new algorithm for the stabilization via state feedback of a class of linearly

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parameterized systems in feedback form is proposed. The result provides solution to two foregoing issues in classical adaptive control. That is, the convergence of the parameter estimation error and the stability of the corresponding non-adaptive controller. The construction relies on the nonlinear adaptive stabilization tools described in Astolfi and Ortega (2003), which results in an adaptive controller that is easier to tune.

The result presented in this paper is developed based on Karagiannis and Astolfi (2008). The adaptive controller obtained can be seen as a discrete-time counterpart of the one constructed in Karagiannis and Astolfi (2008). As we focus on the sampled-data implementation of the controller, intuitively the controller designed directly in the discrete-time framework will achieve better performance than discretizing a controller which is designed in the continuous-time framework (emulation). Starting from this point of view, while in Karagiannis and Astolfi (2008) the parameter estimator is continuous-time; in this paper, a discrete-time estimator is used aiming at obtaining an adaptive controller that has a better performance in sampled-data implementation than an emulation controller (Nešić and Laila, 2002; Nešić and Teel, 2004).

To test our proposed result, we compare the performance of our proposed design with the controller obtained by discretizing the controller designed in Karagiannis and Astolfi (2008). Some simulation results are presented, to show the advantages of our direct-discrete-time design. Some insight to possible further development of the current result is also given.

## 2. NOTATION AND DEFINITIONS

The set of real numbers is denoted by  $\mathbb{R}$ . A function  $\alpha : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$  is of class  $\mathcal{K}$  if it is continuous, strictly increasing and zero at zero. It is of class  $\mathcal{K}_\infty$  if it is of class  $\mathcal{K}$  and unbounded. Functions of class  $\mathcal{K}_\infty$  are invertible. We often drop the arguments of a function whenever they are clear from the context.

Consider a general input affine nonlinear system

$$\dot{x} = f(x) + g(x)u, \quad (1)$$

where  $x \in \mathbb{R}^n$  is the state and  $u \in \mathbb{R}^m$  is the control input. The functions  $f$  and  $g$  are smooth and  $f$  is zero at zero. If the input  $u$  is a state feedback controller, we write the closed loop system of (1) as

$$\dot{x} = \tilde{f}(x, u(x)) = \tilde{f}(x). \quad (2)$$

Suppose that the family of discrete-time models of the system (1) is

$$x_{k+1} = F_T(x_k, u_k), \quad (3)$$

where the parameter  $T > 0$  is a constant sampling period. We assume that the discrete-time control  $u = u_k$  is constant during sampling intervals  $[kT, (k+1)T)$ ,  $k \geq 0$ .

We use the following definitions throughout the paper.

Consider the nonlinear system (1) and the family of its discrete-time models (3). To obtain the exact discrete-time models  $x_{k+1} = F_T^e(x_k, u_k)$  of the system it requires solving the nonlinear initial value problem of the system. However, in general it is impossible to construct the model, as we need to solve the nonlinear initial value problem explicitly

over one sampling interval. Therefore, we need to use an approximate discrete-time model  $x_{k+1} = F_T^a(x_k, u_k)$  instead.

*Definition 2.1. (One step consistency):* The family of the approximate discrete-time models  $F_T^a$  is one step consistent with the family of the exact discrete-time models  $F_T^e$  if given any strictly positive real numbers  $(\Delta_x, \Delta_u)$ , there exist a function  $\rho \in \mathcal{K}_\infty$  and  $T^* > 0$  such that for all  $T \in (0, T^*)$ ,  $x \in \mathbb{R}^n$  and  $u \in \mathbb{R}^m$  with  $|x| \leq \Delta_x$ ,  $|u| \leq \Delta_u$  we have  $|F_T^e - F_T^a| \leq T\rho(T)$ . ■

*Definition 2.2. (Semiglobal practical asymptotic (SPA) stability):* A discrete-time system  $x_{k+1} = F_T(x_k)$  is SPA stable in a Lyapunov sense if there exists a continuously differentiable function  $V_T : \mathbb{R}^n \rightarrow \mathbb{R}$  such that there exist class  $\mathcal{K}_\infty$  functions  $\underline{\alpha}, \bar{\alpha}, \alpha$  such that for any compact and invariant set  $\mathcal{O}_x \subset \mathbb{R}^n$  and a sufficiently small number  $\nu > 0$  there exists  $T^* > 0$  such that for all  $T \in (0, T^*)$  and for all  $x \in \mathcal{O}_x$  the following holds:

$$\begin{aligned} \underline{\alpha}(|x|) &\leq V_T(x) \leq \bar{\alpha}(|x|) \\ V_T(F_T(x)) - V_T(x) &\leq -T\alpha(|x|) + T\nu. \end{aligned} \quad (4)$$

The function  $V_T$  is called a SPA stable Lyapunov function. ■

*Definition 2.3. (Semiglobal practical asymptotic (SPA) stabilizability):* A discrete-time system (3) is SPA stabilizable by means of a state feedback if there exists a state feedback control law  $u_k = u_k(x_k)$  such that system (3) with the control  $u_k$  is SPA stable. ■

## 3. REVIEW OF CONTINUOUS-TIME DESIGN

First, we review the result in continuous-time from Karagiannis and Astolfi (2008), which we will base our sampled-data design on later. Consider a general input affine nonlinear system

$$\begin{aligned} \dot{x}_1 &= f(x_1) + g(x_1)x_2, \\ \dot{x}_2 &= u + \phi(x)^\top \theta, \end{aligned} \quad (5)$$

where  $x = [x_1^\top, x_2^\top]^\top \in \mathbb{R}^{n-1} \times \mathbb{R}$  is the state,  $u \in \mathbb{R}$  is the control input,  $f : \mathbb{R}^{n-1} \rightarrow \mathbb{R}^{n-1}$ ,  $g : \mathbb{R}^{n-1} \rightarrow \mathbb{R}^{n-1}$  and  $\phi : \mathbb{R}^n \rightarrow \mathbb{R}^p$  are  $C^1$  mappings,  $\theta \in \mathbb{R}^p$  is an unknown constant vector, and  $\phi(x)^\top \theta$  is a parameterized function whose role is to model the uncertainty (or disturbance) entering the system.

*Assumption 1.* The system  $\dot{x}_1 = f(x_1)$  has a globally asymptotically stable equilibrium at the origin with a positive definite and proper (smooth) Lyapunov function  $V_1(x_1)$  such that  $\frac{\Delta V_1}{T} \leq -\alpha(x_1)$ , where  $\alpha$  is a positive definite and proper function. ■

The control objective is to find a continuous adaptive state feedback control law of the form

$$\dot{\hat{\theta}} = \sigma(x, \hat{\theta}), \quad u = \nu(x, \hat{\theta}) \quad (6)$$

such that all trajectories of the closed-loop system (5) and (6) are bounded and

$$\lim_{t \rightarrow \infty} x(t) = 0. \quad (7)$$

As stated in Karagiannis and Astolfi (2008), the idea of this adaptive control technique follows Astolfi and Ortega (2003); Karagiannis et al. (2005), where the adaptive stabilization problem is formulated as a problem of asymptotically immersing the closed-loop system into the

system that would result from applying the non-adaptive controller.

This continuous-time result is summarized in the following proposition.

*Proposition 2.* (Karagiannis and Astolfi, 2008) Consider system (5) under assumption 1 and the adaptive state feedback control law

$$\dot{\hat{\theta}} = -\frac{\partial\beta}{\partial x_1}(f(x_1) + g(x_1)x_2) - \frac{\partial\beta}{\partial x_2}(u + \phi(x)^T(\hat{\theta} + \beta(x))) \quad (8)$$

$$u = -\kappa x_2 - \epsilon \frac{\partial\beta}{\partial x_1}g(x_1) - \phi(x)^\top(\hat{\theta} + \beta(x)), \quad (9)$$

where

$$\beta(x) = \gamma \int_0^{x_2} \phi(x_1, \chi) d\chi \quad (10)$$

and  $\kappa > 0$ ,  $\epsilon > 0$ ,  $\gamma > 0$  are constants. Then the closed-loop system (5)-(9) has a globally asymptotically stable equilibrium at  $(x, \hat{\theta}) = (0, \theta)$  and (7) holds. ■

#### 4. MAIN RESULT

While dissipativity property, in which stability is included as a special case, is preserved under sampling (Laila et al., 2002), it is well-known that sampling may degrade the quality of a signal. Hence, applying a continuous-time controller/observer in discrete-time through sampling (emulation design) may reduce the performance of the closed-loop system. To avoid this degradation of performance, a direct discrete-time design is then used. In this case, the discrete-time controller/observer is directly designed in discrete-time based on the discrete-time model of the plant to be controlled, therefore the implementation of the controller does not involve further sampling.

Following the continuous-time result presented in Section 3, in this section we will state the main result of this paper. That is, the sampled-data adaptive control design for a class of nonlinear systems. Consider the continuous-time system (5) and suppose that Assumption 1 holds.

The control objective is to find a discrete-time adaptive state feedback law of the form

$$\hat{\theta}_{k+1} = \sigma_d(x_k, \hat{\theta}_k), \quad u_k = \nu_d(x_k, \hat{\theta}_k) \quad (11)$$

such that the trajectories of (5), (11) are bounded and

$$\lim_{t \rightarrow \infty} x(t) \leq \mu, \quad (12)$$

for  $\mu$  a sufficiently small offset around the origin.

To develop the sampled-data result, we start from the discrete-time model (5). Due to its nonlinearity, it is almost impossible to compute the exact discrete-time model of (5) explicitly. Hence, we use the approximate discrete-time model instead. In this case, the approximate model is required to be one step consistent with respect to the original continuous-time system (Stuart and Humphries, 1996). For simple presentation, we use the Euler approximate model, which is known to satisfy the one-step consistency property:

$$\begin{aligned} x_{1(k+1)} &= x_{1k} + T(f(x_{1k}) + g(x_{1k})x_{2k}) \\ x_{2(k+1)} &= x_{2k} + T(u_k + \phi(x_k)^\top \theta_k) \end{aligned} \quad (13)$$

Now, we state the following proposition, which is the discrete-time counterpart of Proposition 2.

*Proposition 3.* Consider the Euler approximate discrete-time model (13). Suppose Assumption 1 holds for the continuous-time dynamics. The discrete-time adaptive state feedback law

$$\begin{aligned} \hat{\theta}_{k+1} &= \hat{\theta}_k + T\dot{\hat{\theta}}_k \\ &= \hat{\theta}_k + T \left( -\frac{\partial\beta}{\partial x_1}(f(x_1) + g(x_1)x_2) \right. \\ &\quad \left. - \frac{\partial\beta}{\partial x_2}(u + \phi(x)^T(\hat{\theta} + \beta(x))) \right) \end{aligned} \quad (14)$$

$$\begin{aligned} u_T &= -\kappa x_2 - \epsilon \frac{\partial\beta}{\partial x_1}g(x_1) \\ &\quad - \phi(x)^\top(\hat{\theta} + \beta(x)) + T\tilde{u}(x, \hat{\theta}) \end{aligned} \quad (15)$$

where

$$\beta(x) = \gamma \int_0^{x_2} \phi(x_1, \chi) d\chi, \quad (16)$$

and  $\kappa > 0$ ,  $\epsilon > 0$ ,  $\gamma > 0$  are constants. Then the equilibrium point of the closed-loop system (13)-(15) is semiglobally practically asymptotically stable and (12) holds. ■

Define the estimation error  $z = \hat{\theta} - \theta + \beta(x)$ . In discrete-time, the error dynamics can then be written as

$$\begin{aligned} z_{k+1} - z_k &= \hat{\theta}_{k+1} - \hat{\theta}_k + \theta_{k+1} - \theta_k \\ &\quad + \beta(x_{k+1}) - \beta(x_k). \end{aligned} \quad (17)$$

As  $\theta$  is a constant,  $\theta_{k+1} - \theta_k = 0$ . Hence we have

$$\begin{aligned} z_{k+1} - z_k &= \hat{\theta}_{k+1} - \hat{\theta}_k + \beta(x_{k+1}) - \beta(x_k) \\ &= -T \frac{\partial\beta}{\partial x} \phi(x_k)^\top z_k + O(T^2). \end{aligned} \quad (18)$$

Consider the function  $V_2(z) = \gamma^{-1} z^\top z$ . It can be shown that the difference

$$\Delta V_2(z_k) = V_2(z_{k+1}) - V_2(z_k) \leq -2T(\theta_k(x_k)^\top z_k)^2 + T\mu_1, \quad (19)$$

which shows that the error is converging to a small offset  $\mu_1 > 0$ .

*Remark 4.* Note that as the parameter estimation error (18) does not converge to zero but to a sufficiently small offset of order  $O(T^2)$ , the difference  $\Delta V_2(z)$  has also the offset  $\mu_1$ , which shows the *practical convergence* of the error. In this discrete-time framework, the convergence error depends on the sampling period  $T$ , that we can make the offset very small and get it close to zero by choosing  $T$  sufficiently small. ■

By using  $u_T$  as given by (15), we can write the closed-loop system as:

$$\begin{aligned} x_{1(k+1)} &= x_{1k} + T(f(x_{1k}) + g(x_{1k})x_{2k}), \\ x_{2(k+1)} &= x_{2k} + T \left( -\kappa x_2 - \epsilon \frac{\partial\beta}{\partial x_1}g(x_1) \right. \\ &\quad \left. - \phi(x)^\top z + T\tilde{u} \right). \end{aligned} \quad (20)$$

Suppose that function  $W_1(x)$  is

$$W_1(x) = 2\epsilon V_1(x_1) + x_2^2, \quad (21)$$

then the difference

$$\begin{aligned}\Delta W_1 &= W_{1(k+1)} - W_{1(k)} \\ &= 2\epsilon V_1(x_{1(k+1)}) - 2\epsilon V_1(x_{1k}) + x_{2(k+1)}^2 - x_{2k}^2.\end{aligned}\quad (22)$$

Suppose that  $V_1$  is a quadratic function  $ax_1^2$ , and without loss of generality, we can assume that  $a = 1$ . Hence, we have:

$$\begin{aligned}\Delta W_1 &= 2\epsilon a(x_{1(k+1)}^2 - x_{1k}^2) + x_{2(k+1)}^2 - x_{2k}^2 \\ &= 2\epsilon a((x_{1k} + T(f(x_1) + g(x_1)x_2))^2 - x_{1k}^2) \\ &\quad + (x_{2k} + T(u_T + \phi(x)^T\theta))^2 - x_{2k}^2 \\ &= 2\epsilon a(2Tx_{1k}(f(x_1) + g(x_1)x_2) + \\ &\quad T(f(x_1) + g(x_1)x_2)^2) \\ &\quad + 2Tx_{2k}(u + \phi(x)^T\theta) + T^2(u_T + \phi(x)^T\theta)^2 \\ &\leq 2\epsilon x_{2a}\{2Tx_{1k}g(x_1)x_2 + T^2(2f(x_1) + g^2(x_1)x_2)^2\} \\ &\quad + 2Tx_{2k}(u_T + \phi(x)^T\theta) + 2Tx_{2k}u_T.\end{aligned}$$

By using

$$u_T = u + T\tilde{u}, \quad (23)$$

where  $u$  has the same form as the continuous-time controller, after some calculations, we obtain

$$\Delta W_1 \leq -2Tae\epsilon x_1^2 - T\kappa x_2^2 + 0(T^2). \quad (24)$$

Furthermore, for the augmented system (13), (14) and (15), by using the Lyapunov function

$$W(x, z) = W_1(x) + \kappa^{-1}V_2(z), \quad (25)$$

and following similar steps as in Karagiannis and Astolfi (2008), we can show that with the discrete-time estimator, the closed-loop augmented system is semiglobally practically asymptotically stable. Moreover, by using the following theorem from Nešić and Laila (2002), we conclude the relation between the closed-loop approximate model and the closed-loop exact model. Note that we define the closed-loop approximate discrete-time models  $\mathcal{F}_T^a(\tilde{x}, \cdot)$  as the closed-loop system consisting of the approximate model of the plant controlled by the discrete-time controller, and we define the closed-loop exact discrete-time models  $\mathcal{F}_T^e(\tilde{x}, \cdot)$  as the closed-loop system consisting of the exact model of the plant controlled by the discrete-time controller.

*Theorem 5.* (Nešić and Laila, 2002) Suppose that: (i) The family of the closed-loop approximate discrete-time models  $\mathcal{F}_T^a(\cdot)$  is Lyapunov SPA stable; (ii) The Lyapunov function  $W(x, z)$  is locally Lipschitz in  $x$ . (iii)  $F_T^a$  is one-step weakly consistent with  $F_T^e$ ; (iv)  $u_T$  is uniformly locally bounded. Then, the family of the closed-loop exact discrete-time models  $\mathcal{F}_T^e(\cdot)$  is SPA stable. ■

Since the exact discrete-time model of a system matches fully the continuous-time model of the system, by using Nešić et al. (1999), we can relate the property of the closed-loop exact discrete-time model with the closed-loop sampled-data system, in which we can conclude the same property for the closed-loop sampled-data system, in which the continuous-time plant is controlled by the discrete-time controller. Hence, as all three conditions in Theorem 5 are satisfied, we can conclude SPA stability of the closed-loop exact discrete-time model, and furthermore, the SPA stability of the sampled-data system where we apply the adaptation law (14) and controller (15) to control the continuous-time plant (5).

## 5. ILLUSTRATIVE EXAMPLE

Consider a two-dimensional nonlinear system:

$$\begin{aligned}\dot{y} &= x, \\ \dot{x} &= u + x^2\theta,\end{aligned}\quad (26)$$

where  $\theta$  is an unknown constant parameter. We define the state vector  $\mathbb{X} = [y, x]^T$ . The objective of this control problem is to find the parameter estimate  $\hat{\theta}$  and the control law  $u$  such that both states  $y$  and  $x$  converge to zero.

First, a continuous-time adaptive controller is designed, to achieve the design objective. Second, a discrete-time controller is synthesized based on the constructed continuous-time controller, to obtain improved performance when applying the controller in a sampled-data setting.

### 5.1 Continuous-time controller design

First, taking  $V_1(y) = \frac{1}{2}y^2$ , we see that Assumption 1 is satisfied as

$$\frac{\partial V_1}{\partial y}f(y) = 0 \leq -\kappa(y). \quad (27)$$

We define the error variable

$$z = \hat{\theta} + \beta(\mathbb{X}) - \theta \quad (28)$$

By choosing  $\beta(\mathbb{X}) = \gamma\frac{x^3}{3} := \beta(x)$ , and using (8), we obtain the parameter estimator

$$\begin{aligned}\dot{\hat{\theta}} &= -\frac{\partial\beta}{\partial x}\left(u + x^2\left(\hat{\theta} + \beta(x)\right)\right) \\ &= \gamma x^2\left(u + x^2\left(\hat{\theta} + \gamma\frac{x^3}{3}\right)\right),\end{aligned}\quad (29)$$

and the error dynamics

$$\dot{z} = -\gamma x^4 z. \quad (30)$$

Finally, the control law is obtained as

$$u = -\epsilon y - \kappa x - x^2(\hat{\theta} + \beta(x)), \quad (31)$$

for any  $\epsilon > 0$ ,  $\kappa > 0$ .

### 5.2 Discrete-time controller design

Based on the continuous-time controller (31), we design the discrete-time controller, satisfying (15). Hence, the objective of this discrete-time design is to find the extra term  $\tilde{u}$ . In this case, we replace the continuous-time estimator (29) with the discrete-time estimator satisfying (14).

The Euler discrete-time model of the system is

$$\begin{aligned}y_{k+1} &= y_k + T x_k \\ x_{k+1} &= x_k + T(u_k + x_k^2\theta).\end{aligned}\quad (32)$$

Suppose the discrete-time parameter estimator is

$$\hat{\theta}_{k+1} = h_T(y_k, x_k, \hat{\theta}_k), \quad (33)$$

With the error definition as (28), we can write

$$\begin{aligned}
z_{k+1} - z_k &= \hat{\theta}_{k+1} - \hat{\theta}_k + \beta(x_{k+1}) - \beta(x_k) \\
&= T \left( -\frac{\partial \beta}{\partial x} (u + x^2(\hat{\theta} + \beta(x))) + \frac{\gamma}{3} (x_{k+1}^3 - x_k^3) \right) \\
&= T \left( -\frac{\partial \beta}{\partial x} (u + x^2(\hat{\theta} + \beta(x))) + \frac{\gamma}{3} ((x + T(u + x^2\theta))^3 - x^3) \right) \\
&= T \left( -\frac{\partial \beta}{\partial x} (u + x^2(\hat{\theta} + \beta(x))) \right) \\
&\quad + \frac{\gamma}{3} (3Tx^2(x^2\theta + u) + 3T^2x(x^2\theta + u)^2 + T^3(x^2\theta + u)^3) \\
&\leq -T \frac{\partial \beta}{\partial x} (x^2z) + O(T^2) \leq -Tx^4z + O(T^2)
\end{aligned}$$

which shows that the error dynamic is SPA stable, independently from  $u$ .

Furthermore, looking at the closed-loop system, we define the Lyapunov function

$$W_1 = 2\epsilon V_1(y) + x^2. \quad (34)$$

The Lyapunov difference is

$$\begin{aligned}
\Delta W_1 &= W_{1(k+1)} - W_{1(k)} \\
&= 2\epsilon V_1(y_{k+1}) - 2\epsilon V_1(y_k) + x_{k+1}^2 - x_k^2 \\
&= 2\epsilon y_{k+1}^2 - 2\epsilon y_k^2 + x_{k+1}^2 - x_k^2
\end{aligned} \quad (35)$$

Splitting the terms that do not contain  $u$  and the terms containing  $u$ , and applying Proposition 3, we obtain the controller

$$\begin{aligned}
u_{dt} &= -\epsilon y - \kappa x - x^2\theta + T\tilde{u} \\
&= u_{ct} + T\tilde{u},
\end{aligned} \quad (36)$$

with

$$\tilde{u} = \frac{1}{2} \left( \epsilon x + x^3(\hat{\theta} + \beta(x))^2 \right). \quad (37)$$

### 5.3 Simulation results and discussion

We apply both controllers in a sampled-data setup, in the sense that we apply the controller in discrete-time, to control the original system in continuous-time (26). In this case, the continuous-time controller (31) is discretized using a zero-order hold to obtain the discrete-time emulation of the controller, while the controller (36) is directly applied as it is already in discrete-time form. We tested the controllers by applying the same sampling period of  $T=0.3$  seconds for both controllers. The simulation results are presented in Figures 1 and 2. In this simulation, we have applied the initial conditions  $x_1 = 4$ ,  $x_2 = 4$ ,  $\hat{\theta}_0 = 0$ , and we have  $\theta = 2$ .

Figure 1 shows that the discrete-time controller yields a closer closed-loop response to the continuous-time controller. However, from Figure 2 we can observe that although it is bounded, the parameter estimate  $\hat{\theta}$  from the discrete-time estimator does not converge to the correct value of  $\theta$ , compared to the parameter estimate from the continuous-time estimator. This is expected due to the practical stability, rather than global asymptotic stability property of the error dynamics  $z$  for the discrete-time design. Nevertheless, the smoother response with the direct discrete-time controller is possible since the extra term  $\tilde{u}_k$  in the controller helps to add the damping to the closed-loop system.

By reducing the sampling period to  $T=0.15$ , the parameter estimation result improves, as seen in Figures 3 and 4. This consistent estimation can be achieved by constructing

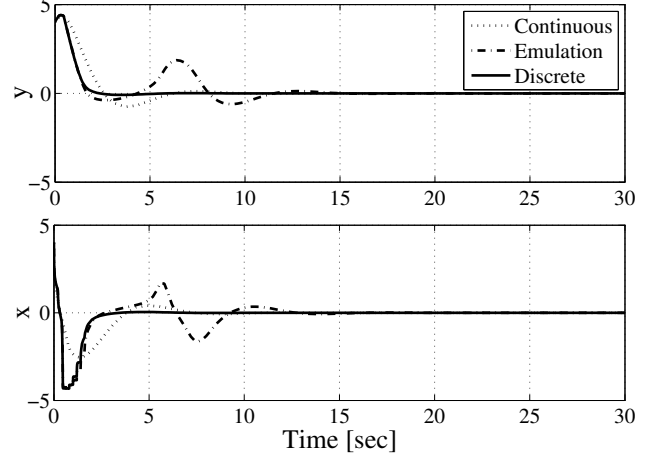


Fig. 1. The response of the states with  $T=0.3$  seconds

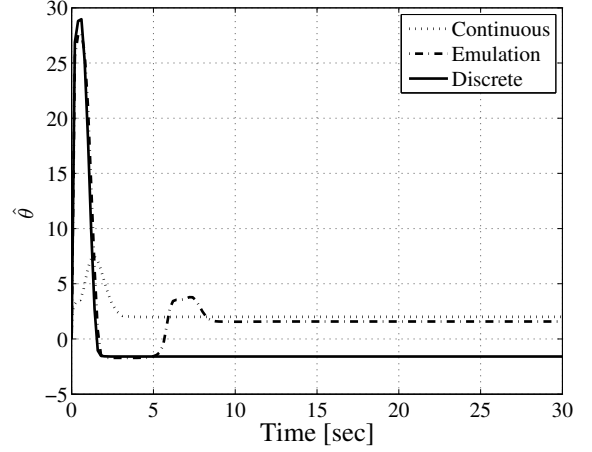


Fig. 2. The parameter estimate  $\hat{\theta}$  with  $T=0.3$  seconds.

a discrete-time estimator that yields a global asymptotic stability property of the error dynamics  $z$ .

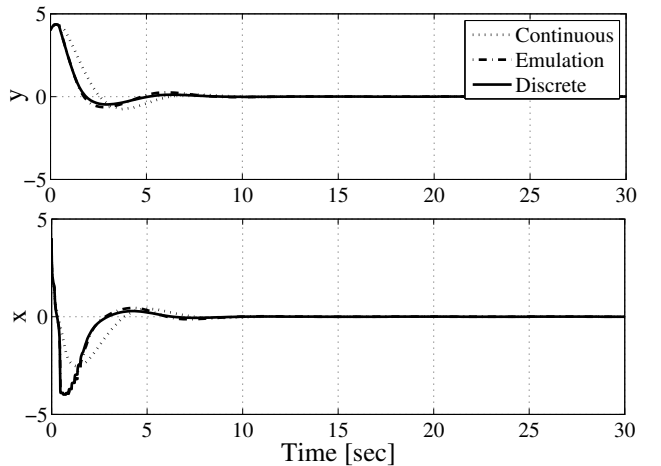


Fig. 3. The response of the states with  $T=0.15$  seconds

*Remark 6.* We have observed in the example that due to the practical convergence of the parameter estimator, we can not make the estimation error zero. However, we can make it very small by reducing  $T$ . The convergence

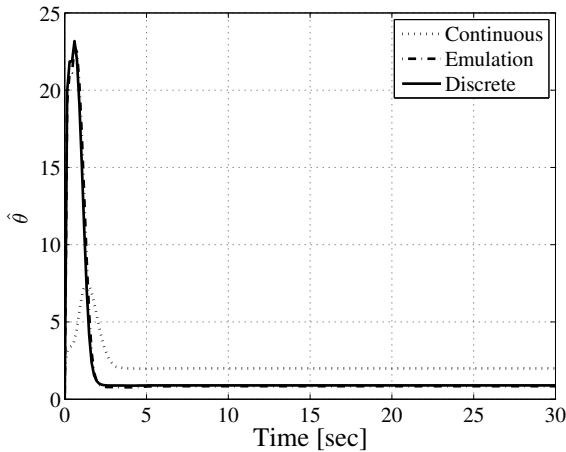


Fig. 4. The parameter estimate  $\hat{\theta}$  with  $T=0.15$  seconds.

of the parameter estimator to a constant rather than to the real value of the parameter is a common phenomenon in classical adaptive control. It is possible to obtain a stronger result to achieve zero parameter estimation error by assigning  $\hat{\theta}$  which is globally stable rather than only practically stable. ■

## 6. SUMMARY

In this paper, a sampled-data control algorithm for a class of nonlinear systems with parametric uncertainty has been presented. It has been shown that the proposed design can be seen as a redesign of the control law proposed in Karagiannis and Astolfi (2008). The controller has been tested to a simple nonlinear system, and the performance has been compared to the controller obtained from sample and hold (emulation) of controller (Karagiannis and Astolfi, 2008). It has been shown that the proposed controller improves performance of the emulation design, which shows the potential of this control redesign approach. This result can be extended to more general class of systems as those considered in Karagiannis and Astolfi (2008). We could also construct a discrete-time parameter estimator that attains convergence to zero of the parameter estimation error. This means that we may achieve global stabilization of the sampled-data system.

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