Energy Efficient Resource Allocation in Downlink Non-Orthogonal Multiple Access (NOMA) System

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Abstract—This paper investigates the resource allocation scheme to maximize the energy efficiency for a downlink non-orthogonal multiple access (NOMA) system. An optimization problem is formulated taking into account the total power and the minimum user rate requirements to balance the system energy efficiency and the total system throughput. Due to the complexity of the objective function, we used the Dinkelbach approach to convert the non-linear fractional programming problem into a simpler subtractive form. Then, the equivalent subtractive form-objective function problem is solved by using iterative programming. A subgradient based resource allocation algorithm is proposed to allocate the power for each user. Simulation results justify the effectiveness of the proposed method and show how it approaches the optimal solution. It also shows that the proposed schemes for NOMA provide better performance than the orthogonal frequency division multiple access (OFDMA) in terms of the energy efficiency and sum rate.

Index Terms—Energy efficiency, NOMA systems, subgradient method, power allocation.

I. INTRODUCTION

Future generations of mobile networks (5G and beyond) are expected to have a high traffic demand and provide a variety of services. Non-orthogonal multiple access (NOMA) is one of the promising air interface schemes for next generation mobile networks to satisfy such needs. In NOMA, users are multiplexed in the power domain on the transmitter side and all are allowed to use the whole radio spectrum as shown in Fig. 1. On the receiver side, users are separated using the successive interference cancellation (SIC) techniques. Another goal for 5G networks is to be energy efficient transmitting at very high rates while maintaining high quality of service (QoS). The conventional definition of network energy efficiency (EE) is the ratio of the total network throughput to the total power consumption [1]. However, recent articles [2], [3] considered EE with per-user QoS requirements. This implies that the transmission techniques could be exploited to serve the users in an energy-efficient way while maintaining a certain level of QoS [4], [5].

Recently, NOMA attracted more attention in the literature, e.g., combination of NOMA with MIMO [6], [7], power allocation techniques [8]–[12], NOMA for multiple antenna relay network [13], capacity analysis [14], inter-cell interference mitigation [15], fairness for NOMA [16], superposition coding for NOMA [17], design of NOMA receiver [18], pairing based NOMA for multiuser scenario [19], etc. However, the investigation in energy efficient NOMA is very limited as previous works focused mainly on sum rate maximization and transmission power minimization.

In this paper, we propose an energy-efficient resource allocation approach in multiuser downlink NOMA system that guarantees a QoS level for each user. This paper presents an iterative based sub-optimal power allocation technique that achieves energy efficient power allocation in downlink NOMA system. Within each iteration, the power allocation is derived according to the iterative subgradient method along with Lagrange multipliers. An optimization problem is formulated to maximize the EE of NOMA system under the constraints of the total transmission power and minimum data rate requirements. In the simulations, the proposed power allocation approach is compared to the numerically obtained optimal NOMA system, optimal OFDMA, and the hierarchical pairing based power allocation (HPPA) approach that was proposed in [11], [19]. The rest of the paper is organized as follows. Section II addresses the system model of the downlink NOMA system. In addition, Section III presents the energy efficiency optimization problem and the subgradient based solution. The simulation scenarios and results are discussed in Section IV. Finally, Section V concludes the paper.

II. SYSTEM MODEL

The considered downlink NOMA system consists of a cell of diameter $D$ with a single base station surrounded by $U$ uniformly distributed users. The total bandwidth $W_T$ is divided into $S$ resource blocks (RBs), and all users are allowed to transmit in all resource blocks. Without loss of generality, for each RB, the users are arranged in ascending order based on their channel power, i.e., the $U$-th user has the best channel power. An illustration for a two user case is shown in Fig. 1. In this figure, the horizontal axis represents the bandwidth in terms of RBs, and the vertical axis is the assigned power for each RB. In case of OFDMA, the RBs are exclusively allocated to one of the users and the power assigned to user 1 and 2 at the $s$-th RB is denoted to be $P_s^{(1)}$ and $P_s^{(2)}$ respectively. For the case of NOMA, both users occupy the whole bandwidth (all the RBs) and the user with a better channel at the $s$-th RB will be assigned the power $P_s^{(2)}$, and the weaker one with $P_s^{(1)}$. Using all the available $S$ resource blocks, the achievable rate of the system is given by

$$R = B_s \sum_{s=1}^{S} \sum_{u=1}^{U} \log_2 (1 + \gamma_{u,s})$$

(1)
and the path loss exponent on the propagation environment. On the other hand, the total power including both the dynamic and the static power consumed by other circuit components is:

\[ P_{tc} = (1 + \tau) \sum_{u=1}^{U} \sum_{s=1}^{S} P_{u,s} + P_c \]

where \( \tau \) represents the drain efficiency of the power amplifier and \( P_c \) stands for the circuit power.

### III. Energy Efficiency Optimization

#### A. Problem Formulation

By defining the EE as the ratio between the total throughput and the total power consumption [5], the objective function is expressed as

\[ EE = \frac{R}{P_{tc}}. \] (3)

Mathematically, the EE maximization problem subject to a proportional fairness constraint is formulated as:

\[
\begin{align*}
\text{maximize} & \quad EE \\
\text{Subject to} & \quad \sum_{u=1}^{U} \sum_{s=1}^{S} P_{u,s} \leq P_t \\
& \quad P_{u,s} \geq 0, \forall s, u \\
& \quad R_i : R_j = \Phi_i : \Phi_j \\
& \quad \text{where } j \in \left\{ 1, 2, ..., \frac{U}{2} \right\}, \quad i \in \left\{ \frac{U}{2} + 1, \frac{U}{2} + 2, ..., U \right\}.
\end{align*}
\] (4-8)

where the power constraints (5) and (6) are set to guarantee that the total power is within its limit and the allocated power is non-negative. In addition, the proportional fairness constraint (7) is to control the achievable throughput by all users and it guarantees the minimum rate requirement in the system, where \( \Phi_i \) and \( \Phi_j \) are the minimum rate requirements for the user with the best channel conditions and the worst channel conditions, respectively. The minimum rate requirement is assigned to each user based on the large scale fading factor (the distance based path loss and the Log-normal shadowing factor) experienced by that user. It is clear that the objective function is nonlinear and of fractional nature. The fractional characteristic is due to the fact that the function is a ratio of two functions. Solving such problem is very complex mathematically due to its lack of convexity. To simplify the objective function and make it more tractable, we transform the objective function into a subtraction form. This is possible thanks to the theorem that for any optimization problem with a fractional objective function, there is an equivalent optimization problem with a subtraction objective function and both of them have the same optimal solution [20]–[24]. Dinkelbach approach is used to transform the objective function from its fractional form into an equivalent subtraction form [20]–[24]. The transformed version of the objective function is a concave function with the parameter \( \alpha \), and it will be in the form of

\[ F(P_{u,s}, \alpha) = A(P_{u,s}) - \alpha O(P_{u,s}) \] (9)

where \( A \) and \( O \) are the numerator and the denominator of the original fractional objective function and both are functions of \( P_{u,s} \), and \( \alpha \) is a weighting factor. Based on the Dinkelbach approach, \( \alpha \) will reach its optimal value when \( A(P_{u,s}) - \alpha O(P_{u,s}) = 0 \). Then the problem formulation will be

\[
\begin{align*}
\text{maximize} & \quad F(P_{u,s}, \alpha) \\
\text{Subject to} & \quad (5), (6), (7).
\end{align*}
\] (10-11)

The solution of this problem could be reached through an iterative method where the solution of the power allocation in each iteration is derived by the subgradient method based on Lagrange multipliers [20], [23]–[25].

#### B. Subgradient-based Solution

Assume that the optimal solution of this problem is \( P_{u,s}^* \) and that \( \alpha = \frac{A(P_{u,s}^*)}{O(P_{u,s}^*)} \). Next, solving the problem in (10)-(11) is achieved by determining the roots of the equality \( F(P_{u,s}^*, \alpha) = \)
0. Then, if the value of $F(P_{u,s}, \alpha)$ is positive, it means that $\alpha$ is less than its optimal value. On the contrary, negative $\alpha$ means that it is higher than its optimal value. Finally, if the solution equals to zero it means that $\alpha$ has reached its optimal value. This is due to the fact the maximum EE is maintained only if $A(P_{u,s}, \alpha) - \alpha \cdot O(P_{u,s}) = 0$. It is worth mentioning that $\alpha$ could be used as a constant.

Using one of the standard optimization techniques in [20], the Lagrangian function of optimization problem in (9), (11) can be expressed as

$$F(P_{u,s}, \alpha) = B_u \sum_{u=1}^{U} \sum_{s=1}^{S} \log_2 (1 + \gamma_{u,s}) - \alpha P_{te} - \lambda \left( \sum_{u=1}^{U} \sum_{s=1}^{S} P_{u,s} \right) - \mu_u \left( B_u \sum_{s=1}^{S} \log_2 (1 + \gamma_{u,s}) \right),$$

where $\alpha$ is a weighting factor, $\lambda$ and $\mu$ represent the Lagrange multipliers. By differentiating (12) against $P_{u,s}$ for all the users except the one with the best channel conditions (i.e., the $U$-th user), we have

$$\frac{dF}{dP_{u,s}} = -\lambda + \frac{B_u |h_{u,s}|^2}{\left( \sum_{m=1}^{U} P_{m,s} |h_{u,s}|^2 + B_u N_0 \right) (1 + \gamma_{u,s}) \ln(2)} + \alpha (1 + \tau) + \frac{\Phi_u B_u |h_{u,s}|^2 \mu_u}{\sum_{m=1}^{U} P_{m,s} |h_{u,s}|^2 + B_u N_0} (1 + \gamma_{u,s}) \ln(2).$$

For the user with the best channel conditions, the derivative of (12) is

$$\frac{dF}{dP_{U,s}} = \frac{|h_{U,s}|^2}{N_0 (1 + \tau_{U,s}) \ln(2)} - \frac{|h_{U,s}|^2 \mu_u}{N_0 (1 + \tau_{U,s}) \ln(2)} - \alpha (1 + \tau) - \lambda.$$  

Finally, the optimal solution can be obtained by solving (13) and (14) with respect to $P_{u,s}$ and $P_{U,s}$, respectively, as follows

$$P_{u,s} = \left[ \frac{B_u \left( 1 + \mu_u \Phi_u \right)}{\left( \alpha (1 + \tau) + \lambda \right) \ln(2)} - \frac{\sum_{m=1}^{U} P_{m,s} |h_{u,s}|^2 + B_u N_0}{|h_{u,s}|^2} \right]^+$$  

$$P_{U,s} = \left[ \frac{B_u \left( 1 - \sum_{u=1}^{U} \mu_u \right)}{\left( \alpha (1 + \tau) + \lambda \right) \ln(2)} - \frac{B_u N_0}{|h_{U,s}|^2} \right]^+.$$

Using the subgradient steps depicted in Algorithm 1, the optimal power allocation can be obtained. Within this algorithm, the Lagrangian multipliers will be updated using

$$\lambda_u^{(i+1)} = \lambda_u^{(i)} - \phi^{(i)} \left( P_t - \sum_{u=1}^{U} \sum_{s=1}^{S} P_{u,s} \right)^+$$

$$\mu_u^{(i+1)} = \left[ \mu_u^{(i)} - \tau^{(i)} \left( \sum_{u=1}^{U} \sum_{s=1}^{S} B_u \log_2 (1 + \gamma_{u,s}) \right) \right]^+$$

where $\phi^{(i)}$ and $\tau^{(i)}$ are small step sizes to be updated at each iteration and chosen to be $0.1/\sqrt{t}$ [25].

### Algorithm 1 Subgradient based power allocation method

- **Initialization** $P_{u,s} = 0$, $P_{U,s} = 0$, $\lambda_u^{(0)} = 0.01$, $\mu_u^{(0)} = 1$.
- **while** $\lambda_u$ and $\mu_u$ are not convergent, **do**
  - **Calculate** $P_{u,s}$ and $P_{U,s}$ from (15) and (16), respectively.
  - **Update** $\lambda_u$ and $\mu_u$ using (17) and (18), respectively.
- **end while**
- **Return** $P_{u,s}$ and $P_{U,s}$

The solution to the problem in (4)-(8) is summarized in Algorithm 2.

### Algorithm 2 Energy efficient power allocation

- **Initialization** the maximum tolerance $\Delta$ and the maximum number of iterations $I_{max}$
- **Set** $i = 0$ (the iteration index) and $\alpha_0 = 0$
- **while** $|F(P_{u,s}, \alpha)| \geq \Delta$ or $i \leq I_{max}$ **do**
  - **For** $u = 1$ to $U$
    - **For** $s = 1$ to $S$
      - **Apply** Algorithm 1 to find the optimal $P_{u,s}$ and $P_{U,s}$
      - **Solve** (9) using known $\alpha_u$ and the obtained power $P_{u,s}$
    - **end**
  - **end**
  - **Calculate** $|F(P_{u,s}, \alpha)|$ and $\alpha_{i+1} = \frac{\lambda^*}{\mu^*}$
  - **i = i + 1**
- **end while**
- **Return** $(P_{u,s}, \alpha)$

### IV. SIMULATION RESULTS

A downlink NOMA system is simulated with a number of users uniformly distributed in a circular area of diameter 300m. To model the wireless channel, a frequency selective fading channel is used based on the ITU pedestrian - B model with a six-path profile, where the average power of the multi-path are [0 dB, -0.9 dB, -4.9 dB, -8 dB, -7.8 dB, -23.9 dB]. Other simulation parameters are listed in Table I. It must be noted that channel estimation is assumed to be perfectly applied and the CSI is assumed to be perfectly known at the BS. The proposed power allocation approach is simulated and compared to the optimal NOMA which was obtained numerically and it will also be compared to the sum-rate maximized HPPA approach that was proposed in [11], [19]. In addition, the optimal OFDMA system will be included in the comparison to show the advantage of NOMA. This optimal OFDMA is obtained using an optimization problem set up that is similar to that of the optimal NOMA scheme and subject to the same power and minimum rate constraints (i.e., as in (4) to (7)) with subcarrier allocation as in [26].

While the optimization problem is to maximize EE, it is crucial to evaluate the sum rate performance as that is an
imported measure for future networks. Fig. 2 shows the sum rate comparison among the four schemes where the sum rate increases with the transmission power level as expected. In general, all the NOMA schemes outperform the optimal OFDMA system, once again demonstrating the promise of NOMA. The proposed subgradient based scheme shows a very close performance to the optimal one. In Fig. 3, the EE of the systems with 15 users is plotted for different transmission and circuit power levels. It is obvious that NOMA can achieve a higher EE than OFDMA. This figure also shows the close behavior of the compared schemes in terms of the cumulative distribution function (CDF) of the achievable user rates. It confirms the superiority of NOMA over OFDMA and also proves that the subgradient based NOMA achieves comparable rates to the optimal one.

Fig. 4 demonstrates the behavior of the compared schemes with two circuit power levels of $P_c = 5\text{ W}$ and $P_c = 10\text{ W}$. In general, EE decreases with larger cell size due to higher path loss, which requires more power to provide sufficient coverage to all users. Moreover, as expected, the higher the circuit power consumption, the poorer the EE.

Finally, the EE performance for different cell sizes is presented in Fig. 6 with two circuit power levels of $P_c = 5\text{ W}$, and $P_c = 10\text{ W}$. In general, EE decreases with larger cell size due to higher path loss, which requires more power to provide sufficient coverage to all users. Moreover, as expected, the higher the circuit power consumption, the poorer the EE.
Fig. 5. CDF trends against the achievable rates at $P_t = 33$dBm.

Fig. 6. EE against different cell diameters, with $U = 30$, $P_t = 30$dBm.

performance for all systems.

V. CONCLUSION

This paper presents a low-complexity suboptimal power allocation scheme for downlink NOMA system to improve the EE. An EE optimization problem is formulated under the total power and minimum rate constraints. Since the formulated problem is non-convex and has a fractional form, obtaining the optimal solution is very difficult. Using the properties of fractional programming, we transform the problem into a simpler subtraction form, then the transferred problem is solved using the subgradient approach. All NOMA schemes have better EE and sum rate performance compared to the optimal OFDMA approach, demonstrating its potential for 5G networks. More importantly, the proposed subgradient method achieves very close performance to the optimal numerical solution, but with the advantage of computational complexity.

REFERENCES


