Colour Calibration Theory

Document Version
Accepted author manuscript

Link to publication record in Manchester Research Explorer

Citation for published version (APA):
Oulton, D. (1999). Colour Calibration Theory. In *Proceedings of the Advanced Seminar in Colour and Imaging Science* (Vol. 1, pp. 1 - 7). Industrial Technology Research Institute.

Published in:
Proceedings of the Advanced Seminar in Colour and Imaging Science

Colorimetry Topic 3. The Development of Colour Calibration Transforms

D.P.Oulton UMIST March 1999

An Introduction to Colour Calibration Theory

1. The Background

The CIE Standard Observer Colorimetry System is in essence a system for defining colour identity, based on a set of scales which demonstrate an additive equivalence of effect in n spectral dimensions. In this paper the principles and properties of additive colour mixing are examined, as set out in the first instance by Newton and Grassman[1,2] , and more accessibly by MacAdam [3] . They were implemented as a model of human colour perception in 1931 by the CIE Colorimetry Committee[4,5]. By concentrating on vector-based modelling of the human response, a number of insights have been gained into the analysis and development of transforms used in colour calibration.

Vector notation[6] is widely used to represent complex multi-variable phenomena, and shows the effects of the variables as an 'n' dimensional space with an independent dimension for each variable. The additive effects the contributing variables are predicted as vector-sum displacements, point identities, and relative separations in n dimensions. For the resulting vector space to be a correct model of the phenomenon, it must represent all the variables which contribute to the phenomenon, and also correctly represent the additive result of the combined action of the variables.

A vector space can be described as having a 'Euclidean metric' (i.e. a metric equivalent to the metric of physical space). The class of vector-spaces with a 'Euclidean metric', is however restricted to those in which RMS (root mean square, or Pythagorean) distance between point identities, has a precise significance in relation to the phenomenon being modelled, and the basis-vectors correctly represent the fundamental variables of the phenomenon. In the following account, the vector-spaces used can not in general, be described as having a Euclidean metric.

The vector spaces used are described as vector-additive $\Re^n$ spaces, implying that all n dimensions are equivalent in effect, and displacements along the basis-vectors are equivalent when used to define point-identity. No significance is assumed in relation to RMS separation of point identities.

Before it is possible to gain the advantages and simplicity of a vector-based model, it is first necessary to isolate and characterise the true nature of the contributing vectors. Otherwise, any attempt at calibration must be based on exhaustive (or at least comprehensive) sampling. This results in the need for a large number of calibration points. If however the true basis vectors of the phenomenon are isolated, only those points need to be determined, which define incremental displacement values, relative scaling, and direction of action of the variables.

In most colour calibration problems the presence of the additivity of light stimuli is overlaid by additional`non-linear, processes. Calibration of both colour reproducing, and colour identity capturing devices, are however based on additive mixing processes, and they are thus susceptible to vector-based analysis and modelling.

In the following analysis it is shown that both linear and non-linear device-characteristic phenomena can be successfully mapped into an additive device space, which is referenced to CIE colour-identity space. By doing so, a model is created with true vector-additive predictive power for the characteristics of the device being calibrated.

2. Colour and Colour Mixing

Much detailed work has shown that coloured light stimuli produce an additive sensation in the Human Visual System[4,7,8,9]. It is now often taken as axiomatic that mixing of coloured light is 'additive mixing'. The implications to this simple statement are examined below.

We first review the overall principles of colour mixing, as formulated by Newton, and Grassman :-

1. The colour sensation produced by a light stimulus is a property of the light, and only of the light.
2. The colour sensation produced by a light stimulus is independent of how, and by what mixture of components it was made, and is a strict additive sum of the sensations produced by the component stimuli.
3. The luminous effect of a mixture of coloured lights is the sum of the luminous effects of the component colours, regardless of composition.
4. The colour sensation produced by a mixture of light stimuli is exactly specified by the proportions in which the components are mixed, and depends only on the colour identity of the components, not on their detailed energy distribution.

3. Vector-Based Representation of Additive mixing

The visual sensation of colour identity produced by light stimuli appears to have true vector properties. This means (by inference from the above principles) that light fluxes have both equivalence of effect in magnitude, and a colour identity which defines direction of action in an n-dimensional colour-space. In consequence the stated principles predict that additive mixing of coloured light stimuli can be modelled exactly by some vector space. The success of CIE Standard Observer Colorimetry in predicting the human sensation of colour identity is one of the strongest possible arguments for the generality and correctness of the stated principles of additive mixing.

The trichromatic theory of colour vision implies that the human colour sensation of colour identity, can be defined by a set of just three independent vectors. Berns, Robertson and Brill give a more detailed review of the mathematics of CIE colorimetry in the proceedings of the CIE Symposium [7] "Advanced Colorimetry '93".

CIE Standard Observer Colorimetry implies that colour stimuli display linear additivity in both n spectral dimensions, and three psycho-physical response dimensions. Independence of detailed energy distribution however, requires that the $\Re^n \rightarrow \Re^3$ mapping from spectral data to CIE space is not injective, allowing the existence of metamerically or conditionally matching colour identities.

It should be noted that while the central core of recognition of colour identity is strictly three dimensional, additional contributing variables such as the intensity and quality of adapting illumination modify the sensation. In consequence these can be modelled by additional dimensions, as described for example by Fairchild[10]. The resulting 'n' space is then transformed back to three dimensions.

4. 0 Additivity in CIE XYZ colour-space

The following features of the CIE Colorimetric Transform express additive equivalence of light energy, implement principles 2, 3, and 4 stated above, and require principle 1 to be true:-

1. The CIE Standard Observer Tristimulus colour identity definition is based on the additivity of spectral stimuli. Colour identity is defined as a linear sum of mono-chromatic (or in practice narrow-band) CIE Tristimulus Values. These are themselves defined by reference to the equal energy illuminant S_E. The key additive property used for a light stimulus, is thus light energy.
2. The unit of measurement for establishing the vector-sum location in CIE XYZ colour-space is the Trichromatic unit (or T-Unit) whose properties are explained in some detail by Wright [9]. It scales light energy addition by reference to its relative luminous efficiency. The use of T-Unit scaling ensures additive equivalence in n spectral dimensions, and also between the three basis-vectors [X], [Y], and [Z].
3. The CIE primaries [X] [Y] and [Z] thus have independent action, equivalence of value, and linear additivity. They were not however selected to represent the fundamental response vectors of the human colour sensation.
4. The use of the CIE primaries [X] [Y] and [Z] as basis vectors modifies the relative location of colour identities in the vector-space, and the meaning of RMS distance in CIE XYZ space is directly determined by the properties of the basis-vectors, not the properties of the phenomenon modelled.
5. The CIE system models the luminosity of all colour sensations, by projecting this property exclusively onto the [Y] dimension of XYZ space via the photometric definition of Luminance. The Y response is thus a central reference characteristic in colour calibration.
6. The CIE Colorimetric Transform uses the additive equivalence of spectral vectors, to model the $\Re^n \rightarrow \Re^3$ resolution of an n-dimensional (spectrally) defined colour identity into three-dimensional psycho-physical response dimensions.
7. Any device or system defining colour identities can in principle be referenced to CIE colour-space, using a (potentially non-linear) $\Re^3 \rightarrow \Re^3$ colour identity mapping from device-primary space to CIE XYZ space.

4.1 Vector based modelling by injection into CIE XYZ colour-identity space

Each device that is a candidate for calibration by reference to CIE XYZ colour identity space, can be described by injective mapping of colour identities from device colour identity-space into CIE XYZ space. In general, an n-space is required at the first stage of the transform, with a vector for each device characteristic variable, giving the overall transform an $\Re^n \rightarrow \Re^3 \rightarrow \Re^3$ structure. The first $\Re^n \rightarrow \Re^3$ stage, maps from device drive or response values to colour identity. The overall mapping is almost always a complex combination of non-linear cross-dependency. The separation of the overall transform into a linear, vector-additive, $\Re^3 \rightarrow \Re^3$ cross-dependency and a set of non-linearities is an important conceptual simplification.

The $\Re^n \rightarrow \Re^3 \rightarrow \Re^3$ mapping, models the phenomenon by injection of colour identities from a device space into CIE XYZ space. If the device either creates or responds to light energy, any correct non-linear transform representing the device characteristics, must be *constrained to represent and maintain*, both the additivity of the phenomenon, and of the subjective human response.

Proceeding by analogy, the constraints which deliver correct vector additivity in CIE XYZ Tristimulus space are :-

1. Use of light fluxes in n spectral dimensions defined in T-Unit quantities, modelling the additive equivalence of light energy [9].
2. Use of linear weighting (i.e. n-dimensional scaling) to define the spectral response of the receptor channels.
3. Use of the neutral response (i.e. the response to equal energy at all wavelengths) to define numeric equivalence of displacement along the basis vectors. The equal energy Illuminant S_E has by definition, vector co-ordinates $X = Y = Z$ (i.e. a precise T-Unit balance).
4. Use of basis vectors (CIE [X], [Y] and [Z]), which are defined in direction of action by reference to the spectral response, using a weighted vector sum of that response.

The $\Re^n \rightarrow \Re^3$ transform from n spectral sensations to a three-channel response model (the CIE Standard Observer Colorimetric Transform) thus maintains linear additivity of response to spectral energy.

Processes known as 'White Point Balancing', 'Grey Scale Tracking' and 'Gamma Correction' are often used empirically in colour calibration transform development. They approximate the non-linear relationship of device drive or response values with light energy output, and all three processes can be related directly to constraints 1 - 3 above. A set of calibration points is used in this context, that have at least approximate T-Unit properties. This serves to control if not to completely separate the presence of non-linearity and cross-dependency.

We suggest in the following analysis, that the input and the output of the transform should be related to T-Units. The use of T-Units enables a precise theoretical separation of the non-linearities from the cross dependency, and the quality of the transform appears to improve in direct relation to the accuracy of T-Unit colour definition.

A device needing calibration usually defines light output (or senses light input) by a combination of RGB primary values. If the RGB channels are behaving strictly as independent contributors to the light energy mixture, changes in value for a single primary demonstrate constant direction of action in CIE XYZ space. The presence of constancy validates the use of a three-dimensional vector model. In such a model, a linear cross dependency exists between vector-additive device primary definitions, and the CIE primaries [X], [Y], and [Z].

Any departure from constancy of chromaticity indicates the presence of additional variables, and in principle additional vectors are then needed in the model. CRT primary chromaticity for example, does not always display full channel independence, due for example to back-scattered light, or variable cross excitation of phosphors.

The method given by Sproson [11] for characterising CRT displays, is a clear and concise account of the process of developing such a linear cross dependency. In Sproson's transform development method for CRT colour the RGB primaries of the device space, and its neutral or white-point, are precisely defined in terms CIE x,y,z chromaticity, and are thus related to the T-Unit-scaled primaries of CIE XYZ space. A linear summation matrix based on CIE chromaticity co-ordinates delivers the required cross-dependency.

In effect Sproson's method describes unit quantity of RGB device output precisely, using T-Unit definitions as both the input and output of the transform development. The resulting cross-dependency maps T-Unit quantities of device output into T-Unit quantities of CIE Standard Observer response. In principle this is a definitive unique and precise mapping. The mapping is however only unique and precise, if the units of both the reference and device spaces are correctly defined using T-Units, and the device primaries concerned display constant chromaticity.

It is common practice, used also by Sproson, to invoke single dimension Gamma functions to map the non-linearity of drive values into CIE co-ordinate specified colour output values. Based on the analysis given below, we regard Gamma functions as inadequate for this purpose because attention is not given to balanced equivalence of effect, and response often differs markedly from an exponential curve. Post and Calhoun give a comprehensive review of alternatives to single dimension Gamma curve fitting[12].

To be a correct model, the final transform must reflect the fundamental additive equivalence of the light energy generated by the device, expressed by its CIE co-ordinate colour identity. This fundamental additivity of the device, is modelled in the intermediate $\Re^3$ device-characteristic space, in which the device primaries display linear additive displacement along each device basis vector, ***and in addition*** have additive equivalence ***between the basis vectors***.

RGB primary drive or response values do not demonstrate either additive property, and the required properties must be developed in any non-linear model, using appropriate functions. Methods based on curve fitting in a single dimension address the first constraint but not the second constraint.

Full additive equivalence must be developed simultaneously across all three of the non-linear functions. Additive equivalence across a non-linear transform is achieved by relating both the input and the output of the transform to T-Unit colour identity definitions. Two distinct sets of T-Unit specified (i.e. CIE co-ordinate defined) colour-calibration identities are required. The first, a set of neutrals, is used to map drive-value units into T-Unit increments of device primary change. The second set is used to characterise individual single-channel output.

In the case of CRT characteristics, a set of gun-drive triplets is first established for each level of T-Unit balance. This delivers additive equivalence between the basis vectors, but does not address the non-linearity of increment size, relative to Y along each vector. In general, each triplet (or set of n values) defines an equivalence point across n device-drive scales. The most commonly used of these equivalence points is the 'White Point'.

The intervals between device drive triplets, define the size of drive-value increments between incrementally linear equivalence points. This allows a function to be derived delivering linearity relative to increments in Y. The combined and simultaneous operation of both T-Unit balance and linearization with Y develops the required additivity across three (not necessarily identical) simultaneous non-linearities.

Oulton and Porat[13] use precise CIE co-ordinate defined test colours. Input to the transform is derived from a grey scale, and individual primary characterisation. Carefully measured CIE XYZ D_{65} neutrals define drive-value balanced equivalents. The white point provides the appropriate T-Unit balance, and device-primary chromaticities provide the correct cross-dependency. The non-linear interpolation method recommended by Post and Calhoun, can then be used to derive intermediate drive values for each of the three basis vectors of the device colour space.

Non-linearity is expressed for equal light energy increments (linearity with CIE Y). An interpolated function for drive values is used with respect to Y. The calibration transform is overall a non-linear cross-dependency. This includes a unique linear cross-dependency combined with three simultaneous single dimension non-linearities, constrained to neutral balance at all levels.

To summarise :-

1. Using T-Unit based. (i.e. CIE co-ordinate specified) colour definitions of the device primaries, allows the establishment of a device-primary space, with a unique linear cross-dependency defining the direction of action of the device-primary basis vectors with respect to CIE XYZ reference space.
2. Device drive-value units are mapped non-linearly onto T-Unit, device-primary, light output vectors, using T-unit defined test colours in order to meet the constraint of additive equivalence of effect.
3. The process of 'Drive value to T-Unit mapping', has two components. One (the non-linearity) delivers additive incremental displacement along the basis-vectors. The other (grey-scale tracking) delivers additive equivalence between the basis vectors. The two characteristics must be developed simultaneously from a single data-set.
4. The processes of 'calibrating' additive colour mixture effects, appears to function by implementing vector additive constraints.

4.2 The Effects and Advantages of a true Vector-Additive Transform model

A non-linear three-dimensional relationship (for example between screen-drive RGB space and CIE XYZ colour identity space), can be represented by a cube of equivalent colour identities i.e. an injective mapping. If the cube is densely populated (typically 1-2000 points), reasonably accurate calibration can be achieved using three-dimensional non-linear interpolation methods. This is however a time consuming, and computationally intensive method.

By contrast with the 'intensive evaluation' method just described, a correctly constructed vector-based model requires only the properties of the basis-vectors to be evaluated. The remaining points in the device space are then accurately predicted by vector-sum principles. In practice this means that in CRT monitor calibration [12,13] a set of less than 30 calibration identities serves to predict the colour identity of all the (approximately) 16.77 million combinations of RGB drive values. If the model is constrained to have true vector additive properties, prediction to within a ΔE of 0.5 CMC (2:1) units, across the full gamut of colour reproduction is shown (in the following tests) to be possible.

A ΔE of 0.5 CMC (2:1) units is very close to the average RGB quantization step size. It indicates that the reverse process of predicting the RGB drive values required for a given colour identity is being correctly achieved on average, to the nearest digit.

The CRT calibration model tested, was carefully constrained to be vector-additive of drive-value inputs. The model used the principles described above, and consists of a reversible two stage transform mapping, of device-drive identities into CIE XYZ colour identity space. Nine re-calibration and test cycles were used to test the model. The test-set colours cover CIEL*a*b* space at 5 degree hue angle intervals, with 1000 - 2300 samples per hue page, covering RGB combinations out to the gamut limit (R and G and B ranging from 0 to 255). All measurements were made with a Minolta CA100 screen-colour analyser.

Table 1 below, summarises the results from 124,968 individual test measurements at all possible L, C and H^{0} combinations with an L, C step size of 5 CIEL*a*b* units. The larger deviations are concentrated near the gamut limit where RGB quantization step size is greatest.

Examples of Test Pages	Min Error (ΔE CMC(2:1))	Max Error (ΔE CMC(2:1))	Mean Error (ΔE CMC(2:1))
H = 30° (1908 samples)	0.025	1.35	0.49
H = 60° (1682 samples)	0.015	1.97	0.58
H = 90° (1729 samples)	0.042	1.87	0.51
H = 120° (2073 samples)	0.027	1.75	0.46
H = 150° (2135 samples)	0.037	1.75	0.43
H = 180° (1427 samples)	0.047	1.65	0.43
H = 210° (1159 samples)	0.058	1.61	0.40
H = 240° (1095 samples)	0.042	1.37	0.42
H = 270° (1330 samples)	0.050	1.53	0.45
H = 300° (2304 samples)	0.015	1.77	0.57
H = 330° (2301 samples)	0.033	1.44	0.52
H = 360° (1777 samples)	0.032	1.47	0.47
Full Monitor Gamut	0.100	1.969	0.484

Table 1. The results of tests for CRT calibration accuracy.

5. Conclusions

Vector representation of contributing variables, and vector analysis based methods have been used to investigate the calibration of device characteristics based on additive colour mixing. An immediate and pleasing outcome is a healthy respect for the CIE Colorimetry Committee of 1931 who developed the vector-additive logic of Standard Observer Colorimetry.

5.1 The Principles of Transform Construction

It is in particular suggested that, *if the principles of additive mixing have been correctly defined, and if they represent a distinct core component of the human colour sensation* then : -

1. The principles of additive colour mixing predict that a vector space can be constructed, which is a complete and accurate model of the human colour-identity receptor response.
2. CIE Standard Observer Colorimetry is confirmed from first principles as an $\Re^3$ additive model, which successfully predicts colour identity using CIE Tristimulus Values. This includes prediction of all possible (both unconditionally and metamerically matching) additive mixtures of spectral component sensations.
3. Description by three independent additive vectors, models the presence of three separate contributing information channels, each of which form indistinguishable sub-components of the overall human colour-identity response to mixtures of light energy.

5.2 The methods of Transform Construction

Colour calibration transforms modelling apparently additive colour-mixing phenomena, can be constructed by a variety of methods :-

1. At the simplest level use is made of an injective 1:1 mapping of defined colour-identity, from a device-specific space into CIE XYZ space. No vector additive constraints are applied. This allows an arbitrary choice of transform type, but does not deliver vector-additive prediction of the colour identity for combinations of the variables present in the device colour-space. Calibration methods employing this strategy must in principle use comprehensive sampling, and n-dimensional interpolation to give a reasonable level of predictive power.

2. The existence of a distinct class of colour calibration transforms is suggested, which has a central $\Re^3$ vector additive 'device colour-space' representing colour-identities, defined in terms of device primary vectors. These vectors are constrained to deliver additive equivalence of effect in n dimensions, both along and between the basis-vectors of the model, reflecting the apparent additive equivalence of light energy.
3. Demonstration of this equivalence is regarded as a pre-requisite for correctness, in a transform modelling additive colour mixing phenomena.
4. A vector additive device colour-space modelling the characteristics of scanners, or cameras, or CRT screens can in principle be constructed as the central $\Re^3$ component of an overall $\Re^n \rightarrow \Re^3 \rightarrow \Re^3$ transform. This central $\Re^3$ component has properties that are defined by a linear $\Re^3 \rightarrow \Re^3$ transform of device primaries into CIE vectors on one hand, and an $\Re^n \rightarrow \Re^3$ input transform of non-linear device characteristics on the other.
5. The non-linear $\Re^n \rightarrow \Re^3$ stage is a theoretically and practically distinct sub-component of the transform. Its function is to map units of drive value in n distinct channels into T-Unit displacements along the device-primary vectors describing colour identity.
6. Resolution of n simultaneous non-linearities is achieved by mapping T-Unit defined inputs, into T-Unit defined outputs of the non-linear relationship.
7. Calibration colours are selected, having two distinct sets of T-Unit properties. The first is a comprehensive set of neutrals. It defines inter-vector equivalence, and intra-vector linearity with Y, and serves to relate T-Unit quantities of device output across the non-linearities to drive values. This allows distinct functions for each of the n non-linearities to be derived. The second set of calibration colours are single channel chromaticities, which define the linear cross-dependency of RGB and XYZ basis vectors thereby defining the linear $\Re^3 \rightarrow \Re^3$ transform component.
8. If the vector additive device colour-space, and its related non-linear input transform are correctly defined, the overall transform will have full predictive properties for the colour mixture phenomena, characteristic of the device concerned.
9. A calibration model based on the above principles, is shown to be a precise predictor of the CIE colour identity, to the available quantization and measurement accuracy limits, for the full range of CRT gun-drive-value combinations in a tested CRT monitor. A maximum of 30 colour identities, defined by their CIE co-ordinates, are used to characterise the basis vectors of device-drive colour space, as functions of incremental displacement and direction of action in CIE XYZ space.
10. T-Unit colour definitions are shown to model the apparent additive properties of light energy. Constraining colour calibration transforms to represent this apparent additivity, is achieved by use of T-Unit defined colour definitions for the input and output of each stage of the transform.

Bibliography

1. Isaac Newton 'New Theory about Light and Colours' Phil. Trans. Roy. Soc. 80 pp 3075-3087 (1671/2).
2. Herman G. Grassman 'Theory of Compound Colours' Annalen der Physik und Chemie 89 pp69-84 (1853)
3. David L.MacAdam 'Sources of Color Science' MIT Press 1970 ISBN 0262 13061 0.
4. W.D.Wright 'Historical and Experimental Background to the 1931 CIE system of Colorimetry' in Golden Jubilee of Colour Publ. Soc. Dyers and Colorists 1981 ISBN 0901956-34-1
5. CIE Publication No. 15.2 (1986) Colorimetry 2nd Edition.
6. H. F. Davis & A .D. Snider 'Introduction to Vector Analysis' Publ. Wm C.Brown 1991 ISBN 0-697-06814-5.
7. Roy Berns Proc of the CIE Symposium '93 on Advanced Colorimetry CIE Publication x007 - 1993
8. W.Thornton Proc of the CIE Symposium '93 on Advanced Colorimetry CIE Publication x007 - 1993
9. W.D.Wright 'The Measurement of Colour' Publ. 1969 by Adam Hilger. ISBN 85274-134-0.
10. Mark D.Fairchild & Roy S.Berns, 'Image Color-Appearance Specification Through Extension of CIELAB", Color Res. and App. V18. No. 3, June 1993, pp 178-189.
11. W.N.Sproson 'Colour Science in Television and Display Systems' Publ. Adam Hilger 1983 ISBN 0-85274-413-7.
12. D.L.Post and C.S.Calhoun, Col.Res. and App 1989, vol 14, p 172.
13. D.P.Oulton & I.Porat 'Control of Colour By Using Measurement and Feedback', J.Text.Inst Vol.83, No.3 p 453.