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Design optimisation of multi-stream plate fin heat exchangers with multiple fin types

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Abstract

For multi-stream plate fin heat exchanger design, one needs to consider the optimal combination of fin categories and detailed geometries of fin types. Due to the complexity of the design problem for multi-stream plate fin heat exchangers, a major limitation of existing technology is the lack of a general design methodology that can consider mix-and-match fin type selection and imposed constraints simultaneously to take advantage of different fin characteristics. In this work, a new design method is proposed to consider the possibility of mix-and-match fin types. Multi-stream plate fin heat exchangers with mix-and-match fin types is considered as a network of two-stream plate fin heat exchangers. A mixed integer nonlinear programming (MINLP) model is converted into a nonlinear programming (NLP) model, and solved in combination of enumeration for a few remaining binary variables. The objective is to minimise the total capital cost. Case studies are carried out to validate the effectiveness of new proposed design methodology and demonstrate the benefits of mix-and-match fin type combinations in the overall design methodology.

Keywords: Plate fin heat exchanger; Mix and match; Fin types; Optimisation

Nomenclature

A

total heat transfer area of one side, m²

A_c

free flow area of one side, m²

A_{fr}

total heat exchanger front area, m²

b

plate spacing, m

c

fin pitch, m

c_p

heat capacity, J/kg °C

d_h

hydraulic diameter, m

e

wavy fin amplitude, m

ER

relative difference

f

funning friction factor

F_f

fixed cost factor

f_s

ratio of secondary surface area to total surface area for heat transfer

F_v

variable cost factor

h

heat transfer coefficient, W/(m² K)

j

colburn factor

k

thermal conductivity of fins, W/(m K)

l

wavy fin wavelength, m

L

flow length on one side, m

m

mass flow rate, kg/s

MINLP

mixed Integer Non-linear Programming

N

number of layers per stream

NLP

non-linear Programming

ΔP

pressure drop, Pa

Pr

Prandtl number

Q

heat duty of heat exchanger, W

R

fouling resistance factor, $m^2 K/W$

Rw

wall thermal resistance, K/W

Re

Reynolds number

St

Stanton number

TC

capital cost, \$

t_f

fin thickness, m

Δt

stream temperature difference, K

ΔT_{LM}

logarithmic mean temperature difference, K

U

overall heat transfer coefficient, $W/m^2 K$

V

total exchanger volume, m³

W

exchanger width, m

x

fin length, m

Greek letters

α

ratio of total transfer area of one side exchanger to total exchanger volume, m²/m³

β

ratio of total transfer area of one side of exchanger to volume between plates of that side, m²/m³

σ

ratio of free flow area of one side to total frontal area, m²/m²

μ

viscosity, W/m °C

ρ

density, kg/m³

η

fin efficiency

Subscript

i

hot stream

j

cold stream

k

design sequency

y

interval

T

total

h

hot stream

cold stream

1 Introduction

With extremely small minimum approach temperature, large surface area per volume, high heat transfer coefficient and possibility of accommodating several streams (up to 10), multi-stream plate fin heat exchangers, categorized as compact heat exchangers, are widely used in various industrial areas such as chemical, petrochemical, cryogenics and aerospace industries [1]. Plate fin heat exchangers, commonly known as brazed aluminium heat exchangers, consist of a series of fin surfaces sandwiched between the parting sheets and stacked together. The introduced fin, work as a secondary surface, can increase efficient heat transfer area and transfer heat to fluid through parting sheets, enlarge fluid turbulence and enhance the local convective heat transfer coefficient to strength the efficiency of heat transfer [2]. Therefore, compared with conventional shell and tube heat exchangers, plate fin heat exchangers have larger heat transfer load per unit volume. Consequently, the capital cost and the process operating cost will be reduced to a large extent, and the smaller size of plate fin heat exchanger can give more freedom of plant layout [3].

However, why have plate fin heat exchangers not been widely applied in most process industries? In practice, there are some disadvantages of plate fin heat exchangers: they cannot handle high pressure and temperature for material issue, and are normally used for clean and non-corrosive fluids [1]. The lack of general plate fin heat exchanger design methodology that can consider fin type selection and imposed constraints simultaneously also restricts their applications. Even some leading commercial software, such as ASPEN and UniSim, can only simulate plate fin heat exchangers and produce a “first shot” design without optimisation [4,5].

According to Kays and London [6], there are approximately sixty standardised fin parts (plain fin, louvered fin, offset strip fin and wavy fin) with specific heat transfer and pressure drop performance. A major challenge in designing optimal multi-stream plate fin heat exchangers is the large number of combinations of standardised fin geometries for various fin categories and types to choose from, which adds discrete aspects to an already complicated design problem. Therefore, an appropriate fin selection method is important in the plate fin heat exchanger design. In the past, few methodologies considered all possible fin type combinations in the design procedure to guarantee the optimum solution. Shah [7] and Cowell [8] analysed individual fins in a variety of different ways: volume vs. power consumption, frontal area vs. power consumption, area goodness. But when it comes to the actual design, the selection of another surface can have an effect on the overall performance of an exchanger. Therefore, the performance of fins in combination is important if there is any degree of interaction between the two. Lee and Zhu [9] in 1999 proposed two new concepts: identical-fin concept and Z-Y graph to select optimum fin types. The identical fin indicates that the same fin type is employed for all streams in the early design stage. Apart from time-consuming, this screening method ignores mix and match fin types and may not reach the minimum plate fin heat exchanger size. Picon-Nunez et al. [10] rearranged heat balance equations, proposed a new term “Volume Performance Index” and plotted a graph of VPI vs Reynolds number to select appropriate fin types. But the selection is based on the assumed Reynolds number relying on a time-consuming trial and error procedure. Moreover, this method neglects the effect of imposed design constraints on fin selection. Recently, Peng, et al. [11] and Yousefi, et al. [12] took basic fin geometries as variables and integrated into a cross flow plate fin heat exchanger design model to optimise total weight and total heat transfer area respectively. Discrete fin geometry variables result in a very complicated mixed-integer nonlinear programming (MINLP) design problem. To solve these discrete variables, Yousefi, et al. [13] later developed a proposed variant of harmony search algorithm for design optimisation. Guo, et al. [14] set basic fin geometries as continuous variables in plate fin heat exchanger design optimisation, with the final selected fin types to be the closest standardised fin types. Although these design methodologies consider the effect of imposed design constraints on fin selection, there is one common assumption of identical fin category. In other words, only one fin category, such as plain fin or offset strip fin, is employed in the design procedure. However, ignoring the possibility of mix and match fin categories in plate fin heat exchanger design could lead to sub-optimal design solutions.

A major challenge to consider mix and match fin categories in a single plate fin heat exchanger design is the discrete thermal-hydraulic model associated with various standardised fin types in different fin categories. As a matter of fact, the detailed fin geometry is of great importance to heat transfer and pressure drop performance of plate fin heat exchangers. In other words, each standardised fin type has unique heat transfer and pressure drop performance. When including all possible fin type combinations, the overall design problem is an MINLP problem, which increases the difficulty of finding the optimum design solution. In terms of heat transfer coefficient and pressure drop in the thermal-hydraulic model, Picon-Nunez and Robles [15], and Wang and Sunden [16] regressed Fanning friction factor and Colburn factor of each fin type only as a function of Reynolds number. This regression neglects the effect of detailed fin geometry on heat transfer coefficient and pressure drop performance, and still cannot avoid the discrete design problem. Yousefi et al. [13] employed published expressions of Colburn factor and Fanning friction factor, which are functions of Reynolds number and basic fin geometries, in the design optimisation problem. Sepehr and Hassan [17] and Amin [18] employed genetic algorithm and biogeography-based optimisation algorithm to optimise the heat exchanger configurations. But these designs limit to offset-strip fins and also cannot avoid solving an MINLP problem. Guo et al. [19] integrated published continuous fanning factor and Colburn factor expression of various fin categories into the optimisation design model to select fin types by minimising the heat exchanger volume, which converts the MINLP problem to an NLP problem for easier convergence. But it does not consider mix and match fin types in a single plate fin heat exchanger.

Due to the complexity of the design problem for multi-stream plate fin heat exchangers, a major limitation of existing technology is the lack of a general design methodology that can consider mix-and-match fin type selection

and imposed constraints simultaneously to take advantage of different fin characteristics. Therefore, in this work, a modified design methodology of multi-stream plate fin heat exchangers is developed to consider mix-and-match fin types in designing a single heat exchanger, while taking fin selection optimisation and pressure drop performance into consideration simultaneously (see Fig. 1).

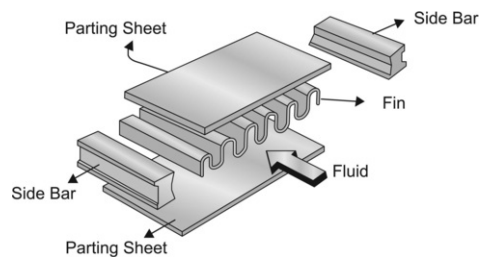


Fig. 1 Basic elements of plate fin heat exchanger.

2 Modelling of mix-and-match fin type selection

From Kays and London [7], four fin categories, plain fin, louvered fin, offset strip fin and wavy fin, shown in Fig. 2, are commonly employed in plate fin heat exchangers. Each fin category has 10-20 fin types with different detailed fin geometries. With four available fin categories and totally 60 fin type options, a simple two-stream plate fin heat exchanger has 3600 possible combinations. In the past few decades, using same fin category for each heat exchange match is employed in the plate fin heat exchangers design to simplify the design, which cannot guarantee a true optimum result for all cases (see Fig. 3).

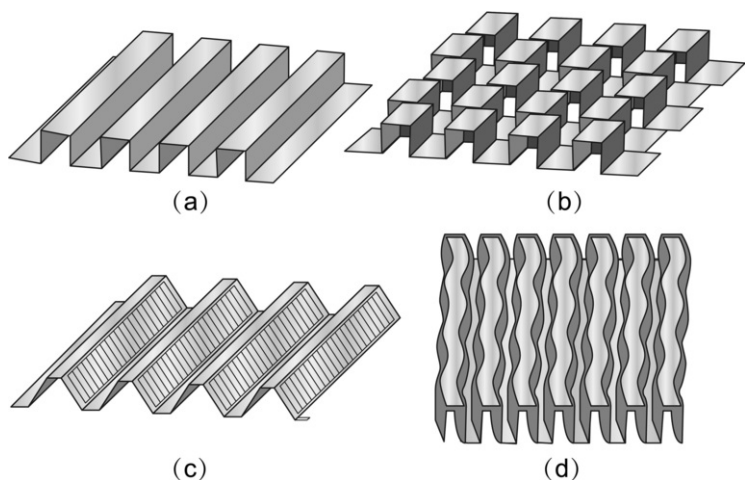


Fig. 2 Common employed fin categories, (a) plain fin (b) offset-strip fin (c) louvered fin (d) wavy fin.

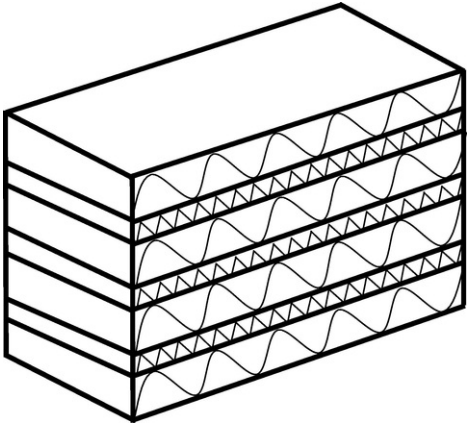


Fig. 3 A mix-and-match fin type model.

In practice, each fin category has unique heat transfer and pressure drop performance. The heat transfer performance of offset strip fins is increased by a factor of about 5 over plain fins of similar geometry, because offset strip fins increase heat transfer coefficient and heat transfer area through repeated growth and destruction of boundary layers and increase the effective surface area, while plain fins simply increase heat transfer area per volume. But the increase of heat transfer coefficient is at the expense of higher pressure drop for offset strip fins [1]. Without appreciably destructing layers and changing heat transfer area, the heat transfer characteristics of louvered fin and wavy fin lie between those of plain and offset strip fins, and their pressure drop performance ranks in between them as well [1].

Pressure drop is an important reference factor in the fin selection method. Minimising heat exchanger volume in the optimisation design will prefer fin types with high heat transfer coefficient, such as offset strip fin, and give a smaller heat exchanger size. But the pressure drop may not be in the allowed range. If that is the case, the fin type must be switched to other fin types with lower pressure drop.

Therefore, regarding different heat transfer and pressure drop performance, and if there is a big pressure drop difference requirements for two sides, especially in multi-stream heat exchangers, mix-and-match fin types can provide a better design solution. Hence, in this work, mix-and-match fin type selection is introduced to find the optimal plate fin heat exchanger design.

As mentioned before, a simple two-stream plate fin heat exchanger has 3600 possible combinations of fin types with different geometry. For both fin types employed in each combination, they have unique heat transfer and pressure drop performance. Without appropriate algorithms for fin type selection, design of multi-stream plate fin heat exchangers will be a big challenge. To simplify the fin selection problem, Guo, et al. [19] proposed an identical fin category screening method, in which the typical fin geometries of different standardised fin categories and types, such as fin pitch, fin length, fin thickness and fin height, are considered as continuous variables. By employing continuous expressions of Fanning friction factor and Colburn factor in Table 1, which are functions of Reynolds number and basic fin geometry parameters, and regressing some design parameters as a function of basic geometry parameters, the discrete optimisation problem caused by standardised fin parts can be converted into a continuous problem to avoid solving the MINLP problem directly.

Table 1 Continuous colburn factor and funning friction factor expression.

Plain fin [20]	$j = 0.233\text{Re}^{-0.48} \left(\frac{c}{b}\right)^{0.192} \left(\frac{t_f}{b}\right)^{-0.208}$	Re ∈
	$f = 0.029\text{Re}^{-0.09} \left(\frac{c}{b}\right)^{0.034} \left(\frac{t_f}{b}\right)^{-0.169}$	(2700, 10100)
Offset strip fin [21]	$j = 0.6522\text{Re}^{-0.5403} \left(\frac{c}{b}\right)^{-0.1541} \left(\frac{t_f}{x}\right)^{0.1499} \left(\frac{t_f}{c}\right)^{-0.0678} \times \left[1 + 5.269 \times 10^{-5} \text{Re}^{1.34} \left(\frac{c}{b}\right)^{0.504} \left(\frac{t_f}{x}\right)^{0.456} \left(\frac{t_f}{c}\right)^{-1.055}\right]^{0.1}$	Re ∈
	$f = 9.6243\text{Re}^{-0.7422} \left(\frac{c}{b}\right)^{-0.1856} \left(\frac{t_f}{x}\right)^{0.3053} \left(\frac{t_f}{c}\right)^{-0.2659} \times \left[1 + 7.669 \times 10^{-8} \text{Re}^{4.429} \left(\frac{c}{b}\right)^{0.920} \left(\frac{t_f}{x}\right)^{3.767} \left(\frac{t_f}{c}\right)^{0.236}\right]^{0.1}$	(120, 10000)
Louvered fin [22]		Re ∈

	$j = 0.249\text{Re}^{-0.42} b^{0.26} \left(\frac{x}{b}\right)^{1.1} x^{0.33}$	
	$f = 0.494\text{Re}^{-0.39} c^{0.46} \left(\frac{x}{c}\right)^{1.1} \left(\frac{b}{c}\right)^{0.33}$	(300, 4000)
Wavy fin [23]	$j = 0.0836\text{Re}^{-0.2309} \left(\frac{c}{b}\right)^{0.1284} \left(\frac{x}{l}\right)^{-0.326} \left(\frac{c}{2e}\right)^{-0.153}$	Re ∈
	$f = 1.16\text{Re}^{-0.309} \left(\frac{c}{b}\right)^{0.3703} \left(\frac{x}{l}\right)^{-0.1152} \left(\frac{c}{2e}\right)^{-0.25}$	(800, 6500)

Therefore, in this work, the corresponding fin geometrical parameters in each mix-and-match fin type combination is set as continuous variables, which are listed in Table 2. Also, the unique heat transfer and pressure drop performance of different fin types in the same fin category can be expressed generally by continuous Colburn factor and Funning friction factor correlations, which are functions of Reynolds number and typical fin geometrical parameters. With assumptions of continuous basic fin geometry variables and the above continuous Colburn factor and Funning friction factor expressions, one fin category with different detailed fin geometries can be simplified to one fin type option. Therefore, for a two-stream heat exchanger with multiple fin types, 3600 different fin type combinations can be simplified to 16 possible mix-and-match combinations listed in Table 3. In this way, the design problem is simplified to a large extent. To differentiate the remaining sixteen combinations of different fin categories, enumeration can be applied to compare all these possible combination options.

Table 2 Design variables for different fins.

Fin type	Model variables
Plain fin	Fin height (b_1, b_2); Fin pitch (c_1, c_2); Fin thickness (t_{f1}, t_{f2})
Serrated fin	Fin height (b_1, b_2); Fin pitch (c_1, c_2); Fin thickness (t_{f1}, t_{f2}); Fin offset length (x_1, x_2)
Louvered fin	Fin height (b_1, b_2); Fin pitch (c_1, c_2); Fin thickness (t_{f1}, t_{f2}); Louver fin pitch (L_{p1}, L_{p2}); Louver fin cut length (L_{l1}, L_{l2}); Louver fin height (L_{h1}, L_{h2})
Wavy fin	Fin height (b_1, b_2); Fin pitch (c_1, c_2); Fin thickness (t_{f1}, t_{f2}); Wavy fin wave length (L_{d1}, L_{d2}); Heat exchanger length L

Table 3 Possible mix and match combinations.

	Side 1			
Side 2	Plain-Plain	Plain-Serrated	Plain-Louvered	Plain-Wavy
	Serrated-Plain	Serrated-Serrated	Serrated-Louvered	Serrated-Wavy
	Louvered-Plain	Louvered-Serrated	Louvered-Louvered	Louvered-Wavy
	Wavy-Plain	Wavy-Serrated	Wavy-Louvered	Wavy-Wavy

In terms of the objective function, Picon-Nenez, et al. [10] took the plate fin heat exchanger volume rather than heat transfer area as the evaluation criterion of heat exchangers design. In practice, due to material and manufacture feasibility, capital costs of different fin categories are different. Moreover, due to porosity issues, the total weights of different fin categories per unit volume are different as well. Hence, when mix-and-match fin selection are considered in the design methodology, minimising the total capital cost is used as the objective function.

The unit capital cost can be estimated on the basis of various involved fin types capital costs, which are related to heat exchanger weight, heat transfer area and heat exchanger volume.

$$A_1 = \alpha_1 VA_2 = \alpha_2 V \quad (1)$$

where α is relates the total heat transfer area of one side of heat exchanger to the total heat exchanger volume. 1 and 2 represent one side and the other side of the heat exchanger.

Then the total weight can be determined by:

$$We_1 = \rho\alpha_1 Vt_{f,1} \quad We_2 = \rho\alpha_2 Vt_{f,2} \quad (2)$$

where ρ is fin material density, t_f is fin thickness.

The total capital cost TC is calculated as:

$$TC_1 = F_{f,1} + F_{v,1}\rho\alpha_1 V_{t_{f,1}} \quad TC_2 = F_{f,2} + F_{v,2}\rho\alpha_2 V_{t_{f,2}} \quad (3)$$

$$TC = F_{f,1} + F_{v,1}\rho\alpha_1 V_{t_{f,1}} + F_{f,2} + F_{v,2}\rho\alpha_2 V_{t_{f,2}} \quad (4)$$

where F_f and F_v are fixed cost factor and variable cost factor.

The converted non-linear problems can be solved by commercial solver in GAMS. The most common solvers for NLP problem in gams are MINOS and CONPT. CONPT, equipped a fast method for finding a first feasible solution, is well suited for models with very non-linear constraints. MINOS is more suited for the cases with less non-linear constraints, many more variables and less equations. CONPT has a pre-processing step in which recursive equations and variables are solved and removed from the model. Therefore, the CONPT solver is selected in this paper.

3 Design methodology of two-stream plate fin heat exchangers with multiple fin types

The overall design methodology is based on the assumptions of constant fluid physical properties, constant heat transfer coefficients, common wall temperature and steady state operation. Counter-current flow is set as default flow arrangement in this work, and only single phase heat transfer is considered in this research. It is assumed that the design of distributor and its corresponding pressure drop will not affect the selection of fin types.

The thermal-hydraulic model employed in this work, adopted from Picon-Nunez [15] and Guo, et al. [19], is summarised as below. The basic heat balance equation for a two-stream heat exchanger is shown as:

$$Q = UA\Delta T_{LM} \quad (5)$$

where Q is the heat load, U is the overall heat transfer coefficient, A is the total heat transfer area and ΔT_{LM} is the logarithmic mean temperature difference.

Including surface fouling and fin effects, the overall heat transfer coefficient is defined as [24]:

$$\frac{1}{UA} = \frac{1}{U_1 A_1} = \frac{1}{U_2 A_2} = \frac{1}{\eta_1 A_1} \left(\frac{1}{h_1} + R_1 \right) + \frac{1}{\eta_2 A_2} \left(\frac{1}{h_2} + R_2 \right) + R_w \quad (6)$$

where U_1 and U_2 are overall heat transfer coefficients in terms of the surface area of each side; A_1 and A_2 represent the total heat transfer area of each side; h_1 and h_2 are the film heat transfer coefficients of each side; R_1 and R_2 are the fouling resistance factor of each side. R_w is the wall thermal resistance, which can be neglected compared to fouling in this study.

Neglecting the wall thermal resistance and combining Eqs. (5) and (6) gives

$$A_1 = \frac{Q}{\Delta T_{LM}} \left[\frac{1}{\eta_1} \left(\frac{1}{h_1} + R_1 \right) + \frac{1}{\eta_2} \left(\frac{A_1}{h_2 A_2} + R_2 \right) \right] \quad (7)$$

By introducing a parameter α that relates the total heat transfer area of one side of heat exchanger to the total heat exchanger volume, the heat exchanger volume can be expressed as [8]:

$$V_T = \frac{Q}{\Delta T_{LM}} \left[\frac{1}{\eta_1 \alpha_1} \left(\frac{1}{h_1} + R_1 \right) + \frac{1}{\eta_2 \alpha_2} \left(\frac{1}{h_2} + R_2 \right) \right] \quad (8)$$

The overall surface efficiency η_1 and η_2 of fin types employed in the design can be generally calculated by following equation [6]:

$$\eta = 1 + f_s \left\{ \frac{\tanh \left[\left(\frac{2h}{k_f t_f} \right)^{1/2} \frac{b}{2} \right]}{\left[\left(\frac{2h}{k_f t_f} \right)^{1/2} \frac{b}{2} \right]} - 1 \right\} \quad (9)$$

where k_f is the thermal conductivity of fin metal; t_f is the fin thickness.

Based on the definitions of Colburn factor j, Prandtl number Pr and Stanton number St, the heat transfer coefficient h can be calculated as below [24]:

$$j = St Pr^{2/3} \quad (10)$$

$$St = \frac{hA_c}{\dot{m}C_p} \quad (11)$$

$$Pr = \frac{\mu C_p}{k} \quad (12)$$

$$h = \frac{\dot{m}C_p j}{A_c Pr^{2/3}} \quad (13)$$

where m is stream mass flow rate.

The whole pressure drop of plate fin heat exchanger should include the pressure drop within the core, the pressure drop at the core entrance due to sudden contraction and the pressure drop at the core exit. Generally, the core pressure drop is a dominating term. The entrance effect represents the pressure loss and the exit effect in many cases represents a pressure rise. Thus, the net effect of entrance and exit pressure losses is usually compensating.

A similar procedure can be applied to express the core pressure drop. The core pressure drop across the core of a heat exchanger is [24]:

$$\Delta P = \frac{2fL\dot{m}^2}{\rho d_h A_c^2} \quad (14)$$

As mentioned in Section 2, for optimisation process, basic fin geometry parameters, such as fin pitch c , plate spacing b and fin thickness t_p , are set as continuous variables to minimise the total capital cost. The optimisation (fin type, layer number etc.) for each mix-and-match combination option is optimised in GAMS. To help with the convergence, one random feasible design solution is set as the initial point. Because of standardised fin parts, the fin type with optimum basic fin geometry parameters may not exist. The closest standardised fin type is selected, and the heat exchanger dimensions and cost are recalculated. The selected fin type combination is considered as the local optimum design of current mix-and-match combination option. Afterwards, switch the design calculation to next possible mix and match combination until all possible combinations are examined. By comparing total capital costs of all combinations, the general optimum fin types for both sides with the minimum capital cost and heat exchanger dimensions can be determined. Therefore, the optimisation problem can be converted to multiple continuous NLP problem, which can save calculation workload and easily obtain the optimum design results.

The overall design optimisation methodology for a two stream plate-fin heat exchanger with multiple fin types is summarised in Fig. 4.

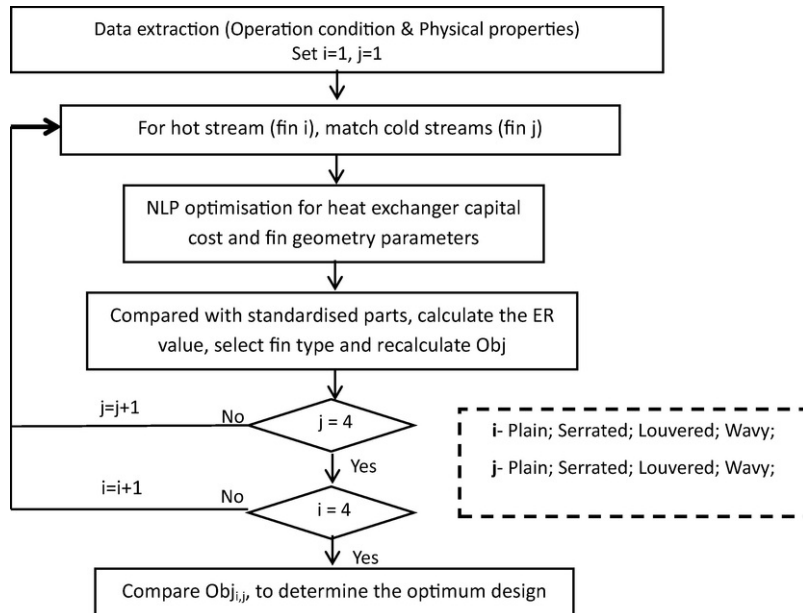


Fig. 4 Overall design optimisation methodology for a two stream plate-fin heat exchanger with multiple fin types.

4 Design methodology of multi-stream plate fin heat exchangers with multiple fin types

Multi-stream plate fin heat exchangers can be considered as a network of two-stream plate fin heat exchangers. With the help of pinch technology [25], the network can be graphically represented by composite curves, which include hot and cold composite curves. Based on assumed constant fluid physical properties, composite curves are formed by several straight lines with several kink points. The kink points indicate certain process stream inlet or outlet points. Therefore, the composite curves can be divided into several enthalpy intervals by kink points shown in Fig. 5 [25]. Each enthalpy interval has fixed inlet and outlet temperature, flow rate, permissible pressure drop and heat load.

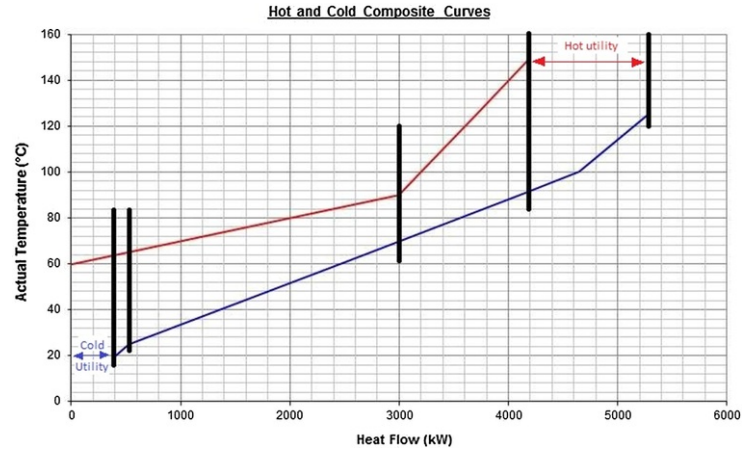


Fig. 5 Composite curves and interval decomposition.

The allowable pressure drop per stream in each interval is assumed to be distributed linearly on the basis of heat load fraction [10].

$$\Delta P_{i,k} = \Delta P_{i,total} \left(\frac{\Delta H_{i,k}}{\Delta H_{i,total}} \right) \quad (15)$$

where i is the stream number, k is the interval number.

As a result of fixed entry and exit temperature in each enthalpy interval, each hot stream can match any cold stream in the same enthalpy interval. Therefore, in this work, a superstructure based heat exchanger network is employed in the design stage. Each hot stream is split into several streams to match every cold stream in the same enthalpy interval. Correspondingly, each cold stream is split to allow a match with all involved hot streams. Fig. 6 illustrates a simple heat exchanger superstructure example. In an enthalpy interval, the splitting heat load ratio distribution of a hot stream is based on the heat load fractions of cold streams.

$$Q_{i,j,k} = Q_{i,k} \times \frac{Q_{j,k}}{\sum_j Q_{j,k}} \quad (16)$$

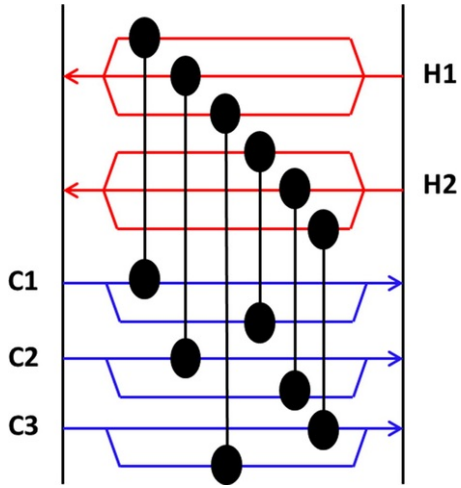


Fig. 6 Enthalpy Interval superstructure.

In the enthalpy interval superstructure, every match can be regarded as a two-stream plate-fin heat exchanger with a volume $V_{i,j,k}$ and capital cost $TC_{i,j,k}$. The optimisation methodology of one match can refer to the optimisation design of a two-stream plate fin heat exchanger, explained in Section 3. The whole enthalpy interval is a two-stream plate-fin heat exchanger network with several involved streams. Consequently, the capital cost of each enthalpy interval is the sum of every match included. The capital cost of whole plate-fin heat exchanger is the sum of capital cost of all enthalpy intervals involved.

$$TC_k = \sum_i \sum_j TC_{i,j,k} \quad (17)$$

$$TC = \sum_k TC_k = \sum_k \sum_i \sum_j TC_{i,j,k} \quad (18)$$

In the interval design process, the pressure drop performance in each split should be no larger than the allowed pressure drop in this enthalpy interval. Due to structural features, the maximum allowed pressure drop of both streams cannot be utilised simultaneously. Therefore, the reference stream, the pressure drop of which will be maximised in the optimisation, should be set in the early design stage. In this design, to avoid local optimum design solutions caused by the definition of the reference stream, the design sequence γ is introduced. In other words, each stream in one interval can be set as the reference stream to search the general optimum design. Once the reference stream is set, the match included reference stream should be optimised firstly to determine the fin type of the reference stream. In order to ensure the unified pressure drop for the reference stream in the same interval, the fin type for the reference stream will be fixed. The fin types for remaining streams can be determined through optimisation. By comparing the total capital cost of that interval, the optimum heat exchanger network configuration can be determined. The optimum heat exchanger network design for other enthalpy intervals can be completed in a similar way.

Based on selected fin types, the heat exchanger volume should be recalculated and used to determine the interval exchanger dimensions (width W , length L and height H) and the number of layers per stream.

The frontal area A_{fr} can be obtained by free flow area A_c [6]:

$$\sigma = \frac{A_c}{A_{fr}} = \frac{\alpha d_h}{4} \quad (19)$$

The length L can be determined by [6]:

$$L = \frac{V_T}{A_{fr}} \quad (20)$$

With the assumed width W , the height of exchanger H and the number of layers per stream can be deduced by [6]:

$$H = \frac{A_{fr}}{W} \quad (21)$$

$$N = \frac{A_{fr}}{W(b_1 + b_2)}$$

The overall design optimisation methodology of a multi-stream plate fin heat exchanger is summarised in Fig. 7.

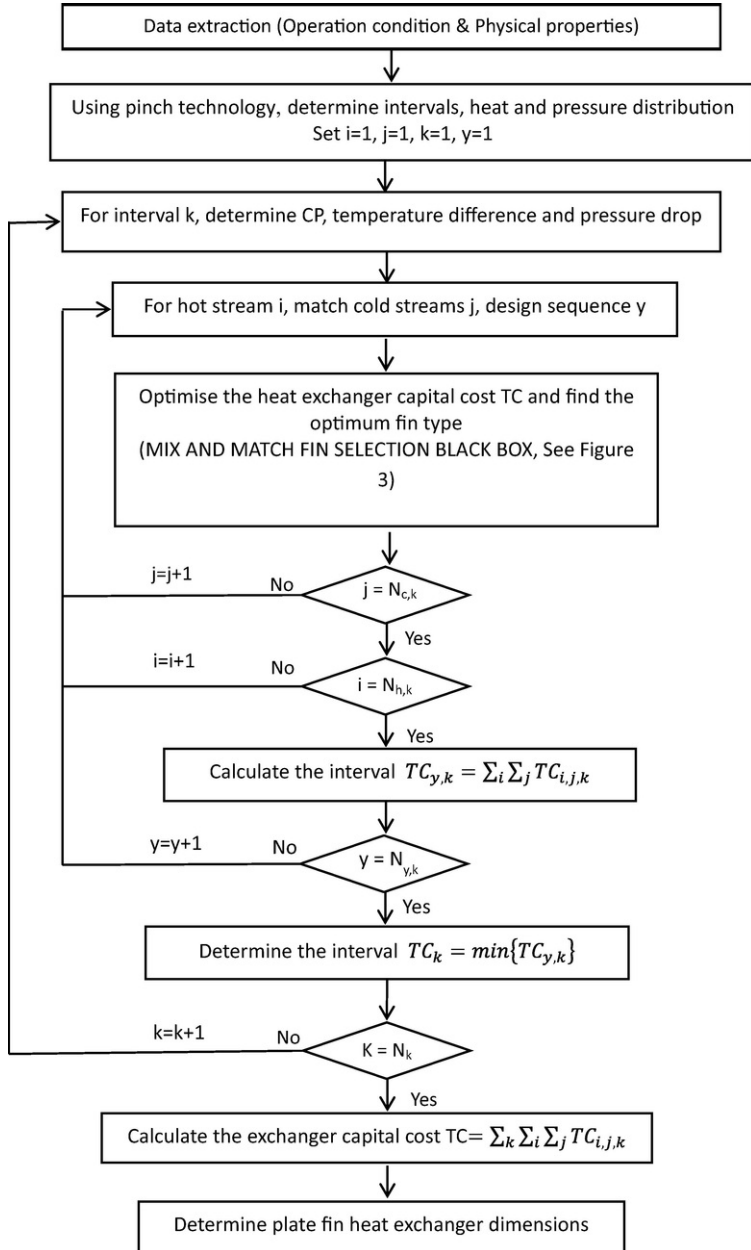


Fig. 7 Overall optimisation design algorithm of multi-stream plate fin heat exchanger with multiple fin types.

5 Case studies

5.1 Case study 1: model validation

A two-stream plate-fin heat exchanger from Picon-Nunez [26] is studied in this work to validate the effectiveness of the new design model. The basic process information and physical information and physical properties are listed in Table 4. The thermal conductivity of fin metal is set as $90 \text{ W m}^{-1} \text{ K}^{-1}$. In order to predict the heat transfer coefficient, the outlet temperature and pressure drop, the fin type Strip-fin 1/10-19.35 and Strip-fin 1/9-24.12 are selected for streams 1 and 2 respectively in this case. The accuracy of the new proposed design methodology is validated by comparing these results of the new proposed model and the published design model as shown in Table 5 [26].

Table 4 Process information and physical properties for case study 1.

	Stream 1	Stream 2
Mass flow rate (kg s^{-1})	49.0	49.0
Allowed pressure drop (Pa)	8800	8800
Inlet temperature ($^{\circ}\text{C}$)	524	290
Outlet temperature ($^{\circ}\text{C}$)	313	501
Density (kg m^{-3})	0.55	0.55
Heat capacity ($\text{J kg}^{-1} \text{ K}^{-1}$)	1059	1059
Thermal conductivity ($\text{W m}^{-2} \text{ K}^{-1}$)	0.0780	0.0789
Viscosity (cP)	0.0509	0.0509

Table 5 Results comparison of exchanger performance.

Stream	Published model			New model			Difference		
	T_{out} ($^{\circ}\text{C}$)	h ($\text{W/m}^2 \text{ K}$)	ΔP (Pa)	T_{out} ($^{\circ}\text{C}$)	h ($\text{W/m}^2 \text{ K}$)	ΔP (Pa)	T_{out} ($^{\circ}\text{C}$)	h (%)	ΔP (%)
Stream 1	313	340.40	8800	313	343.26	8800	0	0	0
Stream 2	501	317.72	737.3	504	330.05	747.6	0	3.8	1.4

It is obvious from Table 5 that heat transfer coefficients in the new model are slightly higher than those in the published model and the outlet temperature of stream 2 is 3°C higher, because the Colburn factor expression employed in the new model is related to Reynolds number and basic fin geometry parameters as well. Similarly, the pressure drop of stream 2 is higher than the original value, because the Fanning friction factor is considered as a function of Reynolds number and basic fin geometry parameters in the new model, other than only the function of Reynolds number. Based on these results, it can be seen that the results obtained by applying the new proposed methodology are in good agreement with the published thermal-hydraulic model. Therefore, the new proposed thermal-hydraulic model can be employed to find the optimum multi-stream plate fin heat exchanger design.

5.2 Case study 2: the need of considering mix-and-match fin types

The same case is further studied to examine the benefits of mix-and-match fin selection and verify how pressure drop performance affects fin type selection. In this case study, the allowed pressure drop of stream 2 varies from 502 Pa to 40,000 Pa, while other process information and physical properties remain the same. Basic fin geometry parameters are set as variables, and the capital cost or volume is optimised to select the optimum fin type. Design results are listed in Table 6. Due to unavailable cost data, the capital cost ratio of offset-strip fin, louvered fin, wavy fin and plain fin is assumed as 10: 5: 2: 1 in this work to examine the important of mix-and-match fin types (see Table 7).

Table 6 Design results comparison for different allowed pressure drop.

Case	Allowed ΔP for Stream 2 (Pa)	Fin type (stream 1)	Fin type (stream 2)	Total volume (m ³)	Total capital cost (\$)
1	502.6	ST1/10-19.74	ST1/10-19.74	1.86	20689.7
2	521.3	PF19.86	PF19.86	2.54	12510.1
3	7469.7	PF19.86	LF1/4-11.1	1.69	10197.8
4	39109.1	PF19.86	ST1/10-19.74	1.93	11962.9

*PF-plain fin, ST-offset strip fin, LF-louvered fin.

Table 7 Process data for case study 3.

Stream	T_s (°C)	T_t (°C)	Flow rate (kg/s)	ΔP (kPa)	ρ (kg/m ³)	C_p (J/kg °C)	μ (cP)	k (W m ⁻² K ⁻¹)
H1	150	60	25.0	46	700	800	0.3	0.12
H2	90	60	106.7	60	700	750	0.4	0.12
C1	20	125	27.7	30	750	900	0.5	0.12
C2	25	100	37.5	86	750	800	0.5	0.12

Comparing Case 1 with Case 2, the pressure drop performances of both cases are around 500 Pa. In Case 1, strip fin 1/10-19.74 is employed for both streams with a volume of 1.86 m³, while plain fin PF19.86 is selected in Case 2 with a volume of 2.54 m³. Although the volume of Case 2 is larger than Case 1, the capital cost is much cheaper. Therefore, different objective functions will lead to different design solutions. If the space is priority such as in the aerospace industry, the total heat exchanger volume should be set as the objective function. But when cost is prioritised such as in oil refineries, the objective function should be the total capital cost instead.

In Case 3, the allowed pressure drop of stream 2 is similar to stream 1, Louvered fin LF1/4-11.1 is selected with a smaller volume of 1.69 m³ with a cheaper capital cost. If the pressure drop allowance is increased further to around 40 kPa in Case 4, strip fin ST1/10-19.74 is the best choice and the volume and capital cost are even smaller than those of Case 2. It is clear that mix-and-match may give a better design solution when there is very different pressure drop allowance between streams.

Therefore, mix-and-match fin types can provide more flexibility for exploiting the optimum design solutions, especially in the occasion of big difference in pressure drop allowance. Also, it is worth to note that the pressure drop is a key factor for fin selection, and should be taken into consideration simultaneously in the plate fin heat exchanger design.

5.3 Case study 3: a multi-stream plate fin heat exchanger design

A multi-stream plate fin heat exchanger [10,17] is designed to examine the benefits of mix and match and check the robustness of modified design methodology. The process stream and physical properties are shown in Table 6. The minimum approach temperature remains as the published literature [10,17] at 20 °C. The width of heat exchanger is set as 0.75 m to calculate each interval dimension.

According to pinch analysis, process stream is represented graphically by hot and cold composite curves. The whole process is divided into three enthalpy intervals, shown in Fig. 4. Table 8 presents the interval decomposition details (inlet and outlet temperature, heat load). Based on linear distribution, the pressure drop allowance of each interval is divided by the fraction of heat load and listed in Table 9.

Table 8 Interval decomposition results for Case Study 3.

Interval	$T_{H,in}$ (°C)	$T_{H,out}$ (°C)	$T_{C,in}$ (°C)	$T_{C,out}$ (°C)	ΔT_{LM} (°C)	ΔH (kW)
1	65.3	64	20	25	42.13	124.65
2	90	65.3	25	70	29.00	2417.8

3	150	90	70	91.8	35.74	1200.0
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Table 9 Pressure drop distribution for case study 3.

Stream	ΔP distribution (kPa)		
	Interval 1	Interval 2	Interval 3
H1	0.72	12.62	30.66
H2	2.60	49.40	–
C1	1.43	12.86	6.23
C2	–	51.60	25.00

In this design, basic fin geometry parameters are considered as continuous variables, and the total capital cost is taken as the objective function. Overall, there are 144 variables and 128 nonlinear equations. The CONOPT solver in GAMS version 23.4 is used to optimise the plate fin heat exchanger design. The CPU time of whole design is 68 s on a 2.6 GHz 4th Intel Core i5 PC with 8 GB memory.

The design results for three intervals are shown in Tables 10–12. The basic case is from the published literature. The design results shown in the second line is based on identical fin category and minimising the total heat exchanger volume. The new design results is obtained from minimising the total capital cost, including all possible mix and match combinations. In this case, if offset strip fin and louvered fin are employed, the length of the heat exchanger may not exist because of higher heat transfer coefficient and pressure drop performance. So only wavy fin and plain fin are mixed and matched in this design.

Table 10 Interval 1 design details.

	Interval Dimensions				Fin types			Number of layers			TC (\$)
	Vol. (m ³)	L (m)	W (m)	H (m)	H1	H2	C1	H1	H2	C1	
Base Case	0.012	0.05	0.75	0.313	PF 15.08	PF 11.94T	PF 15.08	4.9	19.7	24.6	265.1
Guo [18]	0.010	0.17	0.75	0.715	PF 19.86	PF 19.86	PF 19.86	1.2	4.7	5.9	247.8
New Design	0.010	0.17	0.75	0.715	PF 19.86	PF 19.86	PF 19.86	1.2	4.7	5.9	247.8

*PF–plate fin.

Table 11 Interval 2 design details.

	Interval Dimensions				Fin types				Number of layers				TC (\$)
	V (m ³)	L (m)	W (m)	H (m)	H1	H2	C1	C2	H1	H2	C1	C2	
Base Case	0.204	0.868	0.75	0.313	PF 15.08	PF 15.08	PF 15.08	PF 15.08	9.0	36.2	20.5	24.7	1405.1
Guo [18]	0.204	1.008	0.75	0.272	PF 19.86	PF 19.86	PF 19.86	PF 19.86	4.5	16.8	9.7	11.6	1399.6
New Design	0.187	0.819	0.75	0.290	PF 19.86	W 17.8	PF 19.86	PF 19.86	5.4	29.8	16.5	18.7	1385.3

*W-wavy fin.

Table 12 Interval 3 design details.

	Interval Dimensions				Fin types			Number of layers			TC (\$)
	V (m ³)	L (m)	W (m)	H (m)	H1	C1	C2	H1	C1	C2	
Base Case	0.178	0.78	0.75	0.313	PF 15.08	PF 15.08	PF 15.08	20.4	9.2	11.1	855.6
Guo [18]	0.111	0.53	0.75	0.282	PF 19.86	PF 19.86	PF 19.86	22.2	10	12.2	789.9
New Design	0.102	0.51	0.75	0.267	PF 19.86	W 17.8	PF 19.86	21.8	9.6	12.2	744.5

From Tables 10-12, it is clear that the new design results keep consistence with that in Guo's research work [19] in the first interval, but in the following two intervals, wavy fin W17.8 appears in our optimum design result. Take interval 3 as an example, in the base case, plain fin 15.08 is employed with a biggest heat exchanger volume and expensive capital cost. In the second case, plain fin PF19.86 is used and have a smaller volume and cheaper capital cost, while in the new design, plain fin PF19.86 and wavy fin W17.8 are selected, and both the heat exchanger volume and capital cost are reduced by 42% and 13% respectively.

As shown in Tables 10-12, the final number of layers are fractional. For a real application, however, their number must be integer. But this change affects the heat transfer coefficient and pressure drop of the streams involved. A decision can be still be made whether go for the nearest upper or lower integer for a given stream.

In the base case, the Reynolds number is assumed in the early design stage, and the fin selection method is based on trial and error. In the second case, the identical fin category limits some possible fin type combinations, resulting in sub-optimal design results. In the new design, mix and match is introduced to consider all possible fin type combinations to find the optimum fin type combination for minimum total capital cost. The number of layers in each enthalpy interval changed correspondingly. The capital cost for the whole heat exchanger is reduced by 8%.

6 Conclusions

In this study, a new design algorithm is proposed to optimise the mix-and-match of different fin types in multi-stream plate fin heat exchanger design. Under a heat exchanger network superstructure, fin selection is optimised within pressure drop constraints. The problem is MINLP in nature, with two categories of integer variables, one relating to the selection and combination of different fin types, and the other relating to the choice of discrete fin geometry parameters. The first category of integer variables is overcome using enumeration to compare all the possible options, while the second category is dealt with by treating basic fin geometries of each fin in one interval as continuous variables, and considering thermal hydraulic performance of different fin types as a function of basic fin geometries. Therefore, the design model is converted into several continuous NLP problems. The optimal fin types for the whole enthalpy interval and their corresponding design parameters can be obtained by minimising the total interval volume or capital cost.

However, this design methodology is developed on the assumption of constant fluid physical properties and single phase. Phase change and various physical properties are other big challenges of plate fin heat exchangers design. Varied heat transfer coefficient and physical properties caused by phase change force designers to cut the whole heat exchanger into countless small pieces, which can guarantee the accuracy of the heat transfer coefficient and pressure drop. Also, the lack of widely acknowledged experienced formula for heat transfer coefficient and pressure drop during phase change will limit the design procedure.

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Highlights

- Mix-and-match fin types concept is proposed and applied in the design methodology.
- Continuous heat transfer and pressure drop expressions simplify MINLP to NLP.
- Case study validate and demonstrate the effectiveness of new design methodology.

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Answer: It can be modified as: Note: ST refers to offset strip fin type, PF refers to plain fin type, and LF refers to louvered fin type.

Query: Please provide a definition for the significance of [asterisk] in Tables 10 and 11.

Answer: For Table 10, it can be modified as: Note: PF refers to plate fin type. And for Table 11, it can be modified as: Note: W refers to wavy fin type.