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Order picking with heterogeneous technologies: An integrated article-to-device assignment and manpower allocation problem

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Abstract: Current order picking technologies are characterized by different degrees of automation. Irrespective of the automation level, order picking remains a labor-intensive process. Hence, the decisions on the deployment of installed technologies and on labor utilization are interdependent. We develop two planning approaches for an integrative decision support. In comparison to the simultaneous approach the sequential one induces coordination deficits, but less computational effort. In order to inquire into the behavior of both approaches, we conduct a numerical study using sampled data of a pharmaceutical wholesaler.

1 Problem description

In a warehouse multiple devices (non-automated or automated equipment) are used to pick articles according to customer orders. At each *device* the activities of order picking and storage slot replenishment are performed. Devices vary in the number of workplaces and storage slots, slot dimension, time per pick and per slot replenishment. Their capacity can only be utilized, if manpower is allocated to it. There are a number of specialized *operators* qualified to work at a certain device only and a pool of generalists able to work at all devices, yet with a lower efficiency. *Article* data includes information on demand, dimension and eligibility for being picked at specific devices. Regular fluctuations of *workload* induce a sequence of slack and peak periods per day. In slack periods all slots of devices are replenished so as to reduce the number of replenishments during the peak periods.

Two basic *decisions* are relevant for the article-to-device assignment and manpower allocation (ADAMA) problem: Which articles have to be picked at which devices? How much manpower of which kind has to be allocated to each device? Further two decisions have to be made for each device: How many slots have to be occupied by each article? How many replenishments have to be performed for each article? All decisions have to be made subject to the operational *objective* of minimizing the makespan.

Despite of its relevance, the ADAMA problem has not been discussed yet in the literature, but there is research on structurally similar problems: *Forward-reserve assignment and allocation problems* (FRAAP) occur in warehouses with two types of storage areas: *Reserve areas* hold the bulk storage and replenish forward areas. Order picking at these areas is possible, but time-consuming. *Forward areas* allow for fast order picking, but have very limited storage space. This induces replenishments, which are the more frequent, the more articles share the forward area [3]. Accordingly, there is a *trade-off* between picking and replenishment time when the article assignment changes [8]. The question is, how much of the limited forward area space has to be allocated to each article in order to minimize the total demand fulfillment time [5]. To solve this ADA problem a heuristic based on a ranking index is developed in [5]. A branch-and-bound procedure to find optimal solutions is developed in [3]. Further FRAAP approaches consider objects with limited divisibility [4, 8, 9]. Such discrete problems are found to be generalized knapsack problems that require heuristics for solving real-world instances in reasonable time. In comparison to the ADAMA problem, existing FRAAP approaches do not consider the following aspects: manpower needed for picking and replenishment activities; more than two picking devices (except for [4]); articles can be picked simultaneously at several devices.

More general analyses are performed under the topic *dual resource constrained systems* (DRC). DRC are production systems with capacity restricted by both, machine and labor [10]. From this point of view the interdependent sub-problems of machine loading (ML), job dispatching (JD) and manpower allocation (MA) are to be solved. Two approaches are similar to the ADAMA problem. Integrative ML-MA decisions in a cellular manufacturing system (CMS) with multiple work zones and a pool of differently skilled workers are analyzed in [1]. A simultaneous and a sequential ML-MA optimization approach as well as a heuristic approach are developed and compared. In the context of CMS a sequential approach for the MA-ML problem is developed in [7] and remarkably generalized in [2]. The ADAMA problem substantially differs from the situations analyzed in [1] and [2, 7] in two regards: Instead of loading the whole system by releasing orders, articles are assigned to multiple types of devices and occupy one or multiple storage slots there; replenishment activities have to be considered in addition to picking activities.

With regard to the problem discussed in this paper FRAAP and DRC are complementary approaches. This paper aims at combining both in order to allow for a more efficient manpower and device utilization. The extent of efficiency improvement is dependent from the ability to coordinate interdependent deployment decisions. Planning approaches that make both decisions sequentially cannot bring about a better coordination than a simultaneous one, but will reduce computational effort. Hence, the question is, how a sequential approach balances the trade-off between coordination deficit and solution time.

The remainder of the paper is organized as follows: Section 2 presents the simultaneous and the sequential decision models for the ADAMA problem. Both approaches are compared with respect to solution quality and solution time under different operating conditions (section 3). Finally conclusions on the applicability of the approaches are drawn in section 4.

2 Decision models

Based on the problem description a simultaneous ADAMA model can be formulated as follows (cf. table 1 for notations and co-domains of variables):

ADAMA

(1) $\min m$

s.t.

(2) $m \geq d_i \quad \forall i$

(3) $d_i \leq \bar{d} \quad \forall i$

(4) $\sum_i e_{ij} \cdot a_{ij} = 1 \quad \forall j$

(5) $\sum_i s_i^f \leq 1$

(6) $w_i \cdot s_i^s + w^f \cdot s_i^f \leq p_i \quad \forall i$

(7) $y_j \cdot a_{ij} \leq \rho_{ij} \cdot \lfloor h_i / g_j \rfloor \quad \forall i, j$

(8) $\sum_j o_{ij} \leq l_i \quad \forall i$

(9) $o_{ij} \leq \rho_{ij} \quad \forall i, j$

(10) $o_{ij} \geq a_{ij} \quad \forall i, j$

(11) $\sum_j \left(y_j \cdot a_{ij} \cdot t_{ij}^p + \left(\rho_{ij} - o_{ij} \right) \cdot t_{ij}^r \right) \leq \left(w_i \cdot s_i^s + \lambda \cdot w^f \cdot s_i^f \right) \cdot d_i \quad \forall i$

Table 1. Notations

Indices		*Variables*	
i	device $i = 1,...,I$	a_{ij}	share of j^{th} demand assigned to i, $a_{ij} \in [0,1]$
j	article $j = 1,...,J$	d_i	total time to fulfill article demand assigned to i, $d_i \in \mathcal{R}_0^+$
Parameters		m	makespan $m \in \mathcal{R}_0^+$
$\bar{d}$	duration of peak period	o_{ij}	number of storage slots occupied by j at i, $o_{ij} \in \mathcal{N}_0$
e_{ij}	eligibility of j to be picked at i	r_{ij}	number of storage slot replenishments for j at i, $r_{ij} \in \mathcal{R}_0^+$
g_j	size of j	ρ_{ij}	total storage slot usage for j at i, with $\rho_{ij} = r_{ij} \cdot o_{ij}$, $\rho_{ij} \in \mathcal{R}_0^+$
h_i	length of one storage slot at i	s_i^f	share of generalists allocated to i, $s_i^f \in [0,1]$
λ	output ratio between generalists and specialists $0 < \lambda < 1$	s_i^s	share of specialists allocated to i, $s_i^s \in [0,1]$
l_i	number of storage slots available at i	*Indicators*	
t_{ij}^p	time per piece to pick j at i	$\tilde{s}_i^f$	estimated share of generalists allocated to i
t_{ij}^r	time to replenish one slot at i with j	$\tilde{s}_i^s$	estimated share of generalists allocated to i
p_i	number of workplaces at i		
w_i	number of specialists available for i, with $w_i < p_i$		
w^f	number of available generalists		
y_j	demand of j		

The model aims at minimizing the makespan (1), which is the longest time one device needs for fulfilling demand of assigned articles (2). Constraints (3, 4) prevent tardy *demand fulfillment*. The time needed must not exceed the peak period's duration (3). All suitable devices can be used to completely fulfill article's demand (4). Constraints (5, 6) avoid infeasible *MA*. Manpower of flexible operators can be allocated to each device up to its maximum extent (5). The number of specialists and generalists deployed at one device must not exceed its number of workplaces (6). Constraints (7-10) prohibit unrealizable *ADA*. Storage slot requirements of an ADA have to be fulfilled by occupying and replenishing storage slots (7). At a device the number of occupied storage slots cannot be greater than the number of available storage slots (8). Constraint (9) requires using all occupied slots, while constraint (10) requires an article to occupy at least one slot at a device if any fraction of the article's demand is assigned to that device. Constraint (11) reflects the *ADA-MA interdependency*: The workload induced by an ADA has to be met by MA within device's utilization time. ADAMA represents a mixed-integer quadratically constrained program (MIQCP).

In order to avoid non-linearity the described planning problem can be decomposed to a sequential approach. At its *top level* the ADA problem is solved assuming that at each device an estimated number of workplaces is manned. Therefore, in the *ADA model* constraints (5) and (6) are not relevant and (11) becomes linear:

(11t) $$\sum_j \left(y_j \cdot a_{ij} \cdot t_{ij}^p + \left(\rho_{ij} - o_{ij} \right) \cdot t_{ij}^r \right) \le \left(w_i \cdot \tilde{s}_i^s + \lambda \cdot w^f \cdot \tilde{s}_i^f \right) \cdot d_i \quad \forall i$$

The $\tilde{s}$- values are estimated based on *anticipated decision behavior of the base level* [6] which is assumed to be in line with preferring (a) manpower of more efficient operators and (b) manpower allocation to more productive devices. From top level's objective and preference (a) follows $\tilde{s}_i^s = 1$. For setting $\tilde{s}_i^f$ the allocation rule AR represents preference (b).

AR

1. Initialize: $\tilde{s}_i^f := 0$, $s^e := 1$.
2. Determine: $U = \{ i \mid i = 1, \ldots, I \wedge p_i - \tilde{s}_i^s \cdot w_i - \tilde{s}_i^f \cdot w^f > 0 \}$.
3. If $U = \varnothing$, go to 7, else go to 4.
4. Calculate:
$$q_i = \frac{q_i^u}{\sum_{i \in U} q_i^u} \quad with \quad q_i^u = \frac{\max_i \left(\sum_j y_j \cdot e_{ij} \cdot t_{ij}^p \Big/ \sum_j y_j \cdot e_{ij} \right)}{\sum_j y_j \cdot e_{ij} \cdot t_{ij}^p \Big/ \sum_j y_j \cdot e_{ij}} \quad \forall i \in U$$
$$\Delta \tilde{s}_i^f = \min(q_i \cdot s^e ; (p_i - \tilde{s}_i^s \cdot w_i - \tilde{s}_i^f \cdot w^f) / w^f) \quad \forall i \in U$$
5. Update:
$$s^e := \sum_{i \in U} \max(0; q_i \cdot s^e - (p_i - \tilde{s}_i^s \cdot w_i - \tilde{s}_i^f \cdot w^f) / w^f)$$
$$\tilde{s}_i^f := \tilde{s}_i^f + \Delta \tilde{s}_i^f \quad \forall i \in U$$
6. If $s^e > 0$, go to 2, else go to 7.
7. Stop.

The solution to ADA provides the fixed values $\overline{m}$, $\overline{a}_{ij}$, $\overline{\rho}_{ij}$, $\overline{o}_{ij}$ and the instruction for the *base level* to fulfill workload (induced by $\overline{a}_{ij}$, $\overline{\rho}_{ij}$, $\overline{o}_{ij}$) within $\overline{m}$ with minimum manpower. Hence, the objective of the *MA problem* is

(12) $$\min \sum_i (w_i \cdot s_i^s + w^f \cdot s_i^f)$$

Furthermore constraints (5) and (6) are relevant and (11) becomes linear:

(11b) $$\sum_j \left(y_j \cdot \bar{a}_{ij} \cdot t_{ij}^p + \left(\bar{\rho}_{ij} - \bar{o}_{ij} \right) \cdot t_{ij}^r \right) \le \left(w_i \cdot s_i^s + \lambda \cdot w^f \cdot s_i^f \right) \cdot \bar{m} \quad \forall i$$

Thus, the MA problem is a linear program and its solution provides information on s_i^s and s_i^f .

3 Numerical study

Real data of a pharmaceutical wholesaler is used. Per peak period order picking is performed by a workforce of 12 operators ($\lambda = 0.9$), working at 4 automated and 2 manual devices[1]. A representative sample of demand data reveals that orders are fulfilled from an assortment of over 73,000 articles. We restrict attention in this study to the 4% of articles eligible for both, automated and manual order picking. In each problem instance, y_j is sampled from a Poisson distribution with the parameter equal to the average observed demand, g_j is sampled from the empirical distribution of standardized article size and t_{ij}^r is dependent on g_j and h_i . We conduct a 3^k full-factorial study with $k = 3$ factors that characterize the specific *problem instances*: (I) number of articles (500, 1000, 1500), (II) number of storage slots at automated devices (25%, 50%, 75% of total slot requirements), and (III) the fraction of flexible workforce (25%, 50%, 75%). For each combination of factor levels, 3 problem instances are randomly generated, which altogether yields $3^3 \cdot 3 = 81$ instances. In contrast to the sequential approach, the simultaneous one failed to solve 5 of the instances[2].

A *comparison of approaches* reveals that the sequential approach exceeds the minimum makespan (m) on average by 5.3% (CV 2.1%), but reduces solution time (st) on average to 0.3% (CV 62.7%). Correlations between (I), (II), (III) and m, st are quantified by *multiple regression analyses* based on absolute values observed with each approach and their ratios (table 2).

Table 2. Results of regression analyses

Subject	m	α	β_I	β_{II}	β_{III}	r^2	st	α	β_I	β_{II}	β_{III}	r^2
Sim	linear	821	13	-307	-1542	0.990	expon.	36.12	1.00	0.53	3.94	0.893
Seq	linear	228	14	-175	-404	0.991	expon.	0.25	1.00	0.05	1.31	0.870
seq/sim	linear	1.00	0.00	0.00	0.11	0.999	expon.	0.01	1.00	0.18	0.20	0.499

In both approaches the same *tendencies* can be noticed: (I) is by far the strongest factor and positively correlated with m and st . (II) is negatively correlated with both indicators and concerning m weaker than (III). The correlation direc-

[1] Data for instance generation and generated instances can be provided by the authors.

[2] We used a MINLP solver (BARON 15.9) for solving ADAMA, and a MIP solver (Gurobi 7.0.2) for solving ADA-MA on a MacBook Pro computer (2 GHz Intel Core i5 with two cores).

tions of (III) are negative/positive for m, resp. st. *Ratios* of observed values are not correlated with (I), but positively/negatively correlated with (II) and (III) in case of m, resp. st. Correlations of (III) are much stronger than those of (II) in case of m. That is, the coordination deficit is noticeable positively correlated with (III). Since (III) is considered in the sequential approach at the top level by anticipating the base level, a deficit reduction could be achieved by improving AR.

4 Conclusions

For warehouses with heterogeneous order picking technologies we propose two approaches that assign articles and allocate manpower to devices in an integrative way. The simultaneous approach is a MIQCP. To avoid non-linearity, a hierarchical decomposition leads to a sequential approach composed of a MIP (top level) and a LP (base level). A numerical study reveals that the simultaneous approach cannot handle real-world problems in acceptable time and fails sometimes. In contrast, the sequential approach was able to solve all instances, allows for a strong reduction of solution time, but slightly reduces solution quality. A regression analysis indicates the fraction of flexible workforce as a driver of this coordination deficit. Therefore, continuing research will be directed to the sequential approach, in particular towards a better anticipation of the base level's behavior.

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