

# Essays on Fiscal Policy and Credit Market Frictions

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# Abstract

This thesis aims to study the impact of some of the credit market imperfections on various fiscal decisions. It comprises of two papers, each of which sheds light on how the established results in literature are altered when studied in different environments, with more realistic elements.

In chapter 1, we set up a dynamic stochastic general equilibrium model with financial frictions affecting the cost channel and an endogenously derived probability of default. We then study the effects of an expansionary government spending shock. Our exercise highlights that a positive shock to government spending, a demand side shock, increases the cost of firms' marginal costs and hence, their loan requirements. With higher borrowing, their probability of default goes up. The commercial bank takes this into account and charges a higher finance premium, discouraging the firms from borrowing as much. This results in a lower level of economic activity. The government spending multiplier is thus smaller when risky loans are borrowed to finance working capital, instead of fixed capital. In addition, we derive the multiplier to be less than one. With a lot of start-ups borrowing to meet their day-to-day expenses, this result extends a plausible explanation to why during the Great Recession, the impact of government spending was not as large as it was expected to be.

In chapter 2, we derive the optimal level of capital taxation in the presence of agents with different discount factors. We set up a real business cycle model with patient and impatient households that borrow and lend amongst themselves, as per a borrowing constraint. Our results show that if the Ramsey planner's weights on different households are such that he is indifferent between redistribution towards patient and impatient households, the borrowing constraint is not binding. Moreover, we get the classical result of zero optimal capital taxation in the distant long run. However, if the Ramsey planner chooses the borrowing constraint to be always binding, he will favour redistribution from impatient households to patient households. As time moves forward, this ultimately leads to a negative optimal tax rate on the capital returns of patient households, a contradiction to the seminal Chamley-Judd result.

# Declaration

I declare that no portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning. I also certify that both the chapters presented in the thesis are my own work with sole authorship.

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# Dedication

*To my beloved parents for their love and support*

# Acknowledgements

I started the Ph.D. program at The University of Manchester as a novice in research, eager to learn and critically reflect on various economic questions. Since then, the process has been a long roller-coaster ride with many sleepless nights spent thinking about the Blanchard-Kahn conditions and convergence of Lagrange multipliers, usually followed by sheer exhilaration of deriving the results.

Although not yet there, I have come a step closer to my long-term goals. It would not have been possible to cover this distance without the help of a few people I am grateful to. This thesis is a collective effort of each of them.

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# Introduction

Recent years have seen a rise in studies evaluating policies in the presence of market imperfections. With advancements in thinking and understanding, economists have started accounting for heterogeneity, information asymmetry, uncertainty and constrained economies. These added features have widely affected policy recommendations.

Extensive research has been done on how these imperfections influence monetary policy. However, fiscal policy in the presence of frictions still remains a relatively less chartered territory, with a lot of policy makers gearing their research towards it. This thesis adds to this direction and aims to study the impact of some of such imperfections on various fiscal decisions. It comprises of two papers, each of which sheds light on how the established results in literature are altered when studied in different environments, with more realistic elements.

In the first chapter, we analyze a probable cause behind the impact of government spending following the recent financial crisis. We aim to highlight a channel which has a moderating effect on the government spending multiplier and hence explains why the effect was not as large as it was expected to be. Inspired from the literature, we call this “the risky working capital channel.” The firms borrow to finance their day-to-day operations, or the working capital (in our model, the wages). However, since their production activity is subject to idiosyncratic and aggregate shocks, there exists an uncertainty about their level of production and hence their ability to repay the loans. Accounting for this uncertainty, the financial intermediary asks for a fraction of the firms’ output as collateral and charges a loan rate which is higher than the base rate. If the value of the firms’ collateral is greater than the amount to be repaid, the firms choose to repay the loan. However, if the value of collateral is less than the repayment amount, the firms default. The difference between the loan rate and the base rate is the risk premium. This wedge generates financial frictions

in the economy and has added effects on the key variables.

In such a scenario, a rise in government spending creates additional production demand. To meet the higher production requirements, firms hire more labor and pay them by borrowing more from the financial intermediaries. Higher borrowing increases the repayment amount, increasing the firms' probability of default. Seeing this, the financial intermediary charges a higher finance premium which discourages the firms to borrow as much. This depressed borrowing leads to a lower level of output production, mitigating the effect of initial government spending increase.

Moreover, we use the values generated by the impulse response functions to calculate the cumulative discounted spending multipliers for the first few quarters. We derive the government spending multiplier to be less than one, implying that the increase in output is less than the increase in government spending. Further, compared to the multiplier in the absence of financial frictions, the multiplier is smaller in the presence of financial frictions. Due to the cumulative discounting, the difference between the two set of multipliers increases with every quarter, highlighting the inefficiencies caused by the frictions.

Compared to the literature, our findings are novel because we study how aggregate variables respond to a government spending shock when financial frictions arise due to loans that are used to finance working capital, instead of fixed capital. As discussed in the literature, if the loans are used to finance the fixed capital, financial frictions lead to an accelerator mechanism and an increase in government spending has an expansionary effect on output. The increased demand has an inflationary effect on firms' collateral, without affecting their loan requirements. This reduces their probability of default, lowers the risk premium, boosts borrowing and amplifies the production activity—a result different from ours. With an advent of technology, firms fixed capital requirements have reduced substantially. To add to this, an increase in small and medium sized enterprises has resulted in an increase in demand for working capital loans. It thus becomes imperative to consider the channel we

present and look at transmission mechanisms from a different perspective.

Moving on, we take a theoretical stance in Chapter Two and study a different set of frictions. We study imperfections that evolve from the difference in time preferences of various households and subsequent borrowing limits. This gives rise to two different groups of agents, lenders and borrowers, who discount the future differently. Compared to lenders, borrowers are more impatient and value the present more than the future. Thus, to increase their current consumption, they borrow from the savers. As seen in actuality, this borrowing is subject to a borrowing constraint, resulting in a credit constrained economy. Given the set up, we then derive the optimal level of capital taxation by maximizing the weighted utilities of the two household kinds.

Our results suggest that with heterogeneity in time preferences and a borrowing constraint, it is infeasible for the Ramsey planner to tax the patient households (or the lenders) and have a redistribution preference towards impatient households. This is because a tax on capital will reduce patient households' savings, lowering the level of production and wages in the economy. With low wages in the long run, the only way the impatient households will be able to repay their loans is by making their consumption negative—an improbable result!

Moreover, if the Ramsey planner is indifferent between redistributing amongst the two household types, the borrowing constraint is not binding and both the optimal capital tax rate and the impatient households' consumption approach zero in the long run. Since the borrowing constraint is not binding, the impatient households borrow against the discounted value of their future income, resulting in zero consumption later. The model thus collapses to the representative agent model with zero capital tax rate, as shown in the literature.

Finally, in the presence of a binding borrowing constraint, the Ramsey planner will always favour a redistribution towards patient households, resulting in a negative tax rate, or subsidy, on capital income. With a binding borrowing constraint, the

patient households earn a higher interest on the savings lent to impatient households than on the physical capital savings. Therefore, to equate the returns on the two saving forms and to encourage the patient households to invest in physical capital, the Ramsey planner subsidizes returns on physical capital income. This leads to higher wages for the impatient households, leading to a positive level of impatient households' consumption, even in the future.

Our result is different from the standard Chamley-Judd result of zero optimal level of capital taxation because of two reasons—heterogeneity in the rate of time preference amongst households and the presence of a borrowing constraint. Difference in time preference implies that the impatient households discount their future faster than the patient households, who eventually become the dominant consumers in the economy. Adding to that, the borrowing constraint introduces imperfection in the credit market by not allowing for free flow of funds between lenders and borrowers.

Examined as a whole, what can we infer from this thesis? Chapter One highlights the mitigating effect of the risky working capital channel on the government spending multiplier. Additionally, Chapter Two derives the optimal level of capital income taxation in a heterogeneous and constrained environment. Taken together, the two chapters indicate that credit market imperfections yield altering results for the fiscal policy. It is thus imperative to scrutinize the effects of such various imperfections before making any policy conclusions. Even though a lot of research has been conducted in this direction, a lot remains to be done.

# Chapter 1

## Cost Channel, Financial Frictions and the Government Spending Multiplier



## 1.1 Introduction

The American Recovery and Reinvestment Act (ARRA) of 2009 delivered a fiscal stimulus package of approximately USD 800 billion (5.5% of GDP) with the aim of rescuing the economy from the then ongoing financial crisis. Since then, the impact of the stimulus has been widely studied and debated. Adding to the debate, in this paper, we focus on the transmission of financial frictions through the cost channel and, consequently, their dampening effects on government spending.

The literature recognizes various methods, data sets, modelling approaches, etc. and generates a wide range of fiscal spending multipliers. This range varies from multipliers as high as 2.8 (Carrillo and Poilly (2013)) to more conservative values, closer to zero (for example, Cogan et al. (2010)). Such varied results stem from varied modelling environments. For example, it has been shown that the multipliers are larger in the presence of deep habits (Ravn et al (2012)), binding zero lower bound (Carrillo and Poilly (2013)), financial frictions (Villaverde (2010)), fixed exchange rate regimes (Born et al. (2013)), etc. Similarly, recently, the empirical evidence has shown that the spending multiplier is particularly higher during recessions. Auerbach and Gorodnichenko (2012) conclude that the government spending multiplier is less than one during expansions and greater than one during recessions. Bachmann and Sims (2012), Riera-Crichton, Vegh and Vuletin (2015), Nakamura and Steinsson (2014), Fazzari et al. (2015) and Caggiano et al (2015) also conclude that the fiscal expansions are more expansionary during recessions than during booms. This school of thought, however, is challenged by Ramey and Zubairy (2016) and Bruckner and Tuladhar (2011) who find no state dependence and estimate multipliers that are less than one, irrespective of the amount of slackness in the economy. The debate thus, seems unsettled.

Free flow of funds gets restricted during crises, leading to imperfect financial markets. To study the impact of fiscal policies following the crisis, it would thus be obvious to incorporate certain credit constraints in the theoretical economic models.

For example, Canzoneri et al. (2016) develop a theoretical model with financial frictions to show state dependency of fiscal multipliers which results in higher multipliers during recessions. They argue that during a recession, the financial frictions worsen and thwart borrowing. In such a scenario, when a fiscal stimulus is introduced, output grows and the spread between the deposit and lending rates goes down, encouraging borrowing and spending. Higher borrowing expands the economy and reduces the spread even more, generating the financial accelerator mechanism and resulting in a high multiplier. The same mechanism is present during expansions as well. However, during booms, the initial spread is smaller, yielding a weaker accelerator mechanism. The key here is the countercyclical spread which is stronger in recessions due to exogenously assumed, countercyclical bank intermediation costs.

Going forward, we next discuss more dated models in the literature. Gali, Salido and Valles (2007) discuss the "rule-of-thumb" consumers who consume their wages in entirety and do not participate in asset markets. Hence, when an increase in government spending raises labor demand and wages, consumption of these households rise, ensuing a larger multiplier. Eggertsson and Krugman (2012) use a similar approach to derive a multiplier greater than one. Comparably, Villaverde (2010) studies fiscal multipliers in a DSGE model inspired from the Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto, and Rostango (2010) papers. In the model presented in Villaverde (2010), the entrepreneurs borrow money against their net worth to partly finance their purchase of fixed capital. The entrepreneurs' investment thus gets constrained by the value of their collateral and the amount of loans they obtain. Following an increase in public spending, inflation rises, thereby increasing the value of collateral and availability of cheap credit. This leads to higher investment and an amplification effect resulting in a higher government spending multiplier. Carrillo and Poilly (2013) argue the same.<sup>1</sup>

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<sup>1</sup>Further, literature also explores work on fiscal multipliers in the presence of financial frictions and a zero lower bound (ZLB) on nominal interest rates. During the recent global financial crisis, inadequate demand lead to a fall in prices, lowering expected inflation. This lead to a rise in real rates, which further reduced the demand. In such a scenario, interest rates as a policy tool became

However, in the wake of technological advances and a boom in the online industry, fixed capital requirements of start ups and many other small and medium enterprises (SMEs) have gone down.<sup>2</sup> Firms, instead, rely on working capital loans to ensure their smooth functioning.<sup>3</sup> National Federation of Independent Businesses (NFIB) surveyed 507 small U.S. businesses and questioned them on the allocation of credit. Figure 1.1 summarizes these results. As shown, 63% of the interviewed small businesses responded in the affirmative to using loans for covering daily expenses. In this paper, we use a DSGE model in which borrowers keep a portion of their production as collateral and borrow money to finance their working capital–labor wages, in particular. Thus, the total working capital of the firms include various daily expenses along with interest payments on the borrowed loans. While Christiano, Eichenbaum and Evans (2005) call this the working capital channel, Ravenna and Walsh (2005) address this as the cost channel—a channel which directly relates a firm’s marginal cost to the nominal interest rate.

Given this change in use of credit, we find that compared to a frictionless environment, the government spending multiplier is smaller when financial frictions affect the cost channel. We derive the multiplier to be less than one, implying the increase in output is smaller than the increase in government spending. Christiano, Eichenbaum and Trabandt (2015) argue that when credit is used to cover day-to-day activities, a rise in firms’ working capital increases the credit spread, pushing their marginal cost upward. We use and further extend the argument to show that following a spending shock, the firms demand more labor and loans. This raises the wages and interest rates. Since these payments form a part of the firms’ mar-

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ineffective and policy makers developed a renewed interest in fiscal policy as an alternative tool. In such a scenario, a fiscal stimulus can help to push the prices upwards, increasing the interest rates and expected inflation, thereby rescuing the economy from a deflationary trap. Hall (2009), Erceg and Linde (2010), Christiano, Eichenbaum, and Rebelo (2011), Woodford (2011) and Carrillo and Poilly (2013) use New Keynesian models to show that the government spending multiplier is larger in the presence of the ZLB.

<sup>2</sup>WhatsApp, for example, has a market share equivalent to that of Sony, but a comparatively negligible level of fixed capital investment.

<sup>3</sup>Working capital is the capital required by a firm to carry its day-to-day operations.

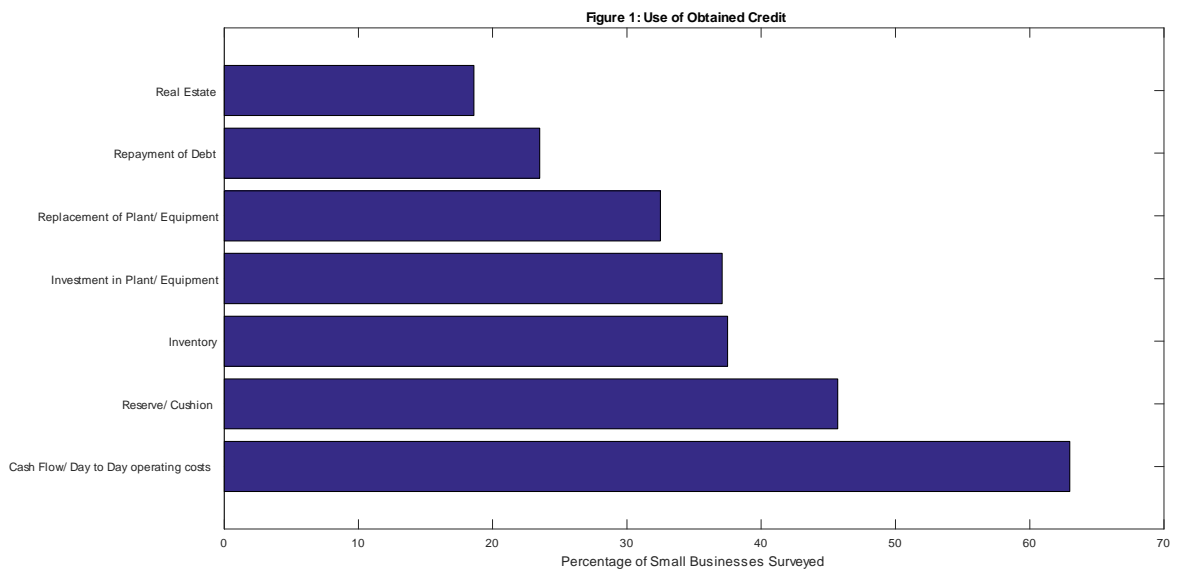


Figure 1.1: Use of Obtained Credit

ginal cost, firms' working capital requirements increase further. With higher loan requirements, the probability of default goes up, encouraging the commercial bank to charge a higher finance premium. This in turn, discourages the firms from borrowing as much and reduces the level of economic activity in the economy. This result hints at one of the reasons why during the Great Recession, the impact of government spending was not as large as it was expected to be. It also bolsters the claims of Krugman and Stiglitz who favoured a larger stimulus.

Barth and Ramey (2001) put forward aggregate and industry-level empirical evidence stressing the importance and existence of cost channel. In the existing literature, the cost channel is theoretically modelled by assuming that firms borrow to fund the factor payments, which are paid in advance, prior to the collection of sales receipts. An increase in interest rates thus raises firms' production cost, reducing the overall output in the economy. Some of the important works in this strand of literature include Farmer (1984, 1988), Blinder (1987), Christiano and Eichebaum (1992), Furest (1992), Christiano, Eichebaum and Evans (1997), Christiano, Eichebaum and Evans (2005), Ravenna and Walsh (2005) and Christiano, Eichebaum and Trabandt (2015).

In our paper, we combine the two strands of literature on frictions and fiscal policy and cost channel into one unifying macroeconomic framework and study the interaction between the two. The structure of the remainder of the paper is as follows: Section 2 describes our model. Section 3 provides the calibrated and steady state values of our key parameters and variables. Section 4 highlights the model dynamics following an expansionary government spending shock. Section 5 conducts robustness analyses. We close with our concluding remarks in section 6. Details of the equilibrium conditions, steady state calculations and log linearizations are given in the appendix.

## 1.2 Model

We follow Agénor, Bratsiotis, and Pfajfar (2014) and use a Dynamic Stochastic General Equilibrium (DSGE) New Keynesian model with price and wage rigidities. By name, DSGE models are general equilibrium models that aid in studying the effects of random shocks (stochastic) over time (dynamic). Drawing a parallel between the DSGE approach and Newtonian Physics, Andrew G Haldane, Chief Economist, Bank of England emphasizes the advantages of such models and the importance of having a unique and stable equilibrium with oscillatory dynamics around it (Haldane, 2016). Our model features an economy with four sectors- households, firms, financial sector and government. A continuum of identical firms,  $j \in (0, 1)$  hire labor from all existing household types,  $i \in (0, 1)$  and produce differentiated consumption goods. Households supply labor to firms, consume goods from all firms, invest in deposits and bonds and, at the end of each period, receive profits from firms. Firms cover labor costs entirely by borrowing against a fraction of output as collateral. Their production of output is subject to aggregate and idiosyncratic productivity shocks. The credit market is represented by a commercial bank that receives deposits from households and extends loans only to firms. The government

determines monetary and fiscal policies.

The sequence of events in the model is as follows. At the beginning of the period, the households receive the earnings on the previous year's deposits and bonds. They also invest in current period's assets. The representative bank receives these deposits and lends them to the firms to aid them with their production activities. For this, the bank charges a loan rate which is derived after taking into consideration the idiosyncratic shocks that a firm's production is subject to. It is hence higher than the risk-free rate by the finance premium—a premium charged by the bank to compensate for the probability of default on loans. The shocks are then realized and production takes place after the optimum level of employment and loans are decided, given the prevalent interest rates in the economy. The firms also set the prices. Following Calvo pricing index, there is price rigidity in the economy and only a fraction of the firms are able to reset their prices. By the end of the period, given their level of production, the firms pay off their loans and transfer the profits to the households. In the case of a default, the firms' collateral is seized and is further used to repay the depositors.

On the fiscal side, similar to Villaverde (2010), the government runs an intertemporal budget constraint. The households pay income taxes, the receipts of which, along with government borrowing, are used towards government spending and repaying old government debt, with interest. Tax rate follows an autoregressive process and changes only if there is an exogenous shock.

We now proceed by setting up the different sectors of the economy.

### **1.2.1 Households**

#### **Utility Maximization**

Household  $i$  derives utility from consumption and disutility from working. Given the utility function, it chooses the optimum levels of consumption, deposits, bonds and number of hours worked by maximizing the following constant relative risk aversion

preference:

$$\max_{C_t, D_t, N_t} E_t \sum_{s=0}^{\infty} \beta^s \left( \frac{C_{t+s}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{h_{i,t+s}^{1+\xi}}{1+\xi} \right),$$

where  $E_t$  is the expectations operator conditional on information available at  $t$ .  $C_t$  is aggregate consumption;  $h_{i,t}$  is the amount of labor supplied by household type  $i$  and  $\beta \in (0, 1)$  is the subjective discount factor. Parameters  $\sigma, \xi > 0$  are such that  $\sigma$  is the intertemporal elasticity of substitution and  $\xi$  is the inverse of the Frisch Elasticity of labor supply. The households' decisions, however, is subject to their budget constraint. The budget constraint, in real terms, is

$$C_t + D_t + B_t = R_{t-1} \frac{D_{t-1}}{\pi_t} + R_{t-1} \frac{B_{t-1}}{\pi_t} + (1 - \tau_{h,t}) \frac{W_{i,t}^n}{P_t} h_{i,t} + PF_t,$$

where  $\pi_t = \frac{P_t}{P_{t-1}}$  is the rate of inflation,  $D_t$  are the deposits at the financial intermediary;  $B_t$  are the government bonds;  $W_{i,t}^n$  is the nominal wage rate paid to household and is taxed at rate  $\tau_{h,t}$ ;  $PF_t$  are the profits of the firms in the economy. Like Gertler, Kiyotaki and Queralto (2012) and Carrillo and Poilly (2013), we assume that the bank deposits and government bonds are perfect substitutes, which pay an uncontingent, risk-free gross interest rate of  $R_t$ . The households are thus indifferent between the two assets. While the left hand side of the budget constraint gives the money outflows in the form of expenditure on consumption, deposits and government bonds, the right hand side represents the inflow of money from the returns on previous time period's deposits and government bonds, tax deductible income on number of hours worked and profit transfers from firms, which are ultimately owned by the households.

On solving the household's problem, we derive the following optimality conditions;

$$\lambda_t = C_t^{-\frac{1}{\sigma}} = \beta E_t \frac{C_{t+1}^{-\frac{1}{\sigma}} R_t}{\pi_{t+1}}, \quad (1.1)$$

$$\frac{W_{i,t}^n}{P_t} = \frac{h_{i,t}^\xi}{(1 - \tau_{h,t}) C_t^{-\frac{1}{\sigma}}}. \quad (1.2)$$

Equation (1.1) represents the Euler equation which gives the optimum level of intertemporal consumption, given the real interest rate. Equation (1.2) gives the marginal rate of substitution between consumption and number of hours worked and is distorted by the income tax rate. A lower tax rate will reduce the margin between consumption and number of hours worked, encouraging households to supply more labor.

### Wage Setting

As in Erceg, Henderson and Levin (2000), each household  $i$  supplies a unique type of labor,  $h_{i,t}$  which is then aggregated by a labor aggregator using the Dixit-Stiglitz (1977) method:

$$N_t = \left( \int_0^1 h_{i,t}^{\frac{\theta_w-1}{\theta_w}} di \right)^{\frac{\theta_w}{\theta_w-1}},$$

where  $\theta_w > 1$  and represents the elasticity of substitution between different type of labor. The household labor demand is thus given by:

$$\Rightarrow h_{i,t} = \left( \frac{W_{i,t}^n}{W_t^n} \right)^{-\theta_w} N_t,$$

This gives the aggregate wage:

$$\Rightarrow W_t^n = \left( \int_0^1 W_{i,t}^{n^{1-\theta_w}} di \right)^{\frac{1}{1-\theta_w}}.$$

To introduce wage stickiness, we assume that in each period a constant fraction of  $1 - \omega_w$  workers are able to re-optimize their wages while a fraction of  $\omega_w$  index their wages according to last period's inflation rate, i.e.

$$W_{i,t}^n = \pi_{t-1} W_{i,t-1}^n.$$



Given the above problem, the wage inflation is derived as:

$$\hat{\pi}_t^w = \frac{(1 - \beta\omega_\omega)(1 - \omega_\omega)}{\omega_\omega(1 + \xi\theta_\omega)} \left( \widehat{MRS}_t - \hat{W}_t \right) + \beta E_t \hat{\pi}_{t+1}^w,$$

where  $\hat{\pi}_t^w = \hat{W}_t - \hat{W}_{t-1}$ , is defined as the wage inflation. The above equation states that wage inflation depends on expected one period ahead wage inflation and the difference between  $\widehat{MRS}_t$ , log-linearized marginal rate of substitution, and the prevalent wage rate. The economy's average marginal rate of substitution is given by

$$MRS_t = \left( \frac{1}{1 - \tau_{h,t}} \right) \frac{N_t^\xi}{C_t^{-\frac{1}{\sigma}}},$$

and is distorted by  $\tau_{h,t}$ , the income tax rate in the economy. Finally, the log-linearized real wage rate,  $\hat{W}_t$ , is given by:

$$\hat{W}_t = \left( \frac{\hat{W}_t^n}{P_t} \right) = \hat{W}_{t-1} + \hat{\pi}_t^w - \hat{\pi}_t.$$

### 1.2.2 Firms

We follow Christiano, Eichebaum and Evans (1997) and Ravenna and Walsh (2005) to model the firms. The firms pay labor wages before they realize their revenue. This introduces a delay between their payments and receipts, creating a need for the firms to borrow from a bank at the given loan rate. The interest payment adds to the marginal cost of labor, generating a cost channel. As defined by Ravenna and Walsh (2005), a cost channel exists when the marginal cost of a firm is directly related to the nominal interest rate.

#### Production

The production of each firm depends on technology and labor services provided by the households. The firm's technology is subject to aggregate and idiosyncratic

shocks and the constant returns to scale production function is thus given by

$$Y_{j,t} = N_{j,t}Z_{j,t}, \quad (1.3)$$

with

$$Z_{j,t} = A_t \varepsilon_{j,t},$$

where  $N_{j,t}$  denotes the amount of labor services hired by firm  $j$ , at the wage  $W_t$ ;  $Z_{j,t}$  is the total level of productivity of firm  $j$ ;  $A_t$  is an aggregate technology shock and  $\varepsilon_{j,t}$  is an idiosyncratic productivity shock, which follows a uniform distribution over the interval  $(\underline{\varepsilon}, \bar{\varepsilon})$ .

The aggregate technology shock evolves according to the following autoregressive process,

$$\log A_t = (1 - \rho^A) \log A + \rho^A \log A_{t-1} + \epsilon_t^A,$$

where,  $\rho^A > 0$  and  $A > 0$  is the steady state aggregate productivity level;  $\epsilon_t^A$  is a normally distributed random shock with zero mean and a constant variance,  $\sigma_A$ .

## Borrowing

Firms borrow from the bank to cover their expected wage costs,  $W_t N_{j,t}$ , at the gross interest rate  $R_t^L$ , and repay their loans at the end of each period. Let  $L_{j,t}$  denote the amount of borrowing by firm  $j$  at time  $t$ , such that

$$L_{j,t} = W_{j,t} N_{j,t}.$$

Similar to Agénor, Bratsiotis, and Pfajfar (2014), in case of default, the bank can seize a fraction,  $\chi$ , where  $\chi \in (0, 1)$ , of the firms' final output. This implies that a fraction  $1 - \chi$  of the output is lost in state verification and contract enforcement

costs. Consequently, a firm will choose to default if

$$\chi Y_{j,t} < R_t^L L_{j,t},$$

where the left-hand side is firm  $j$ 's actual repayment following a default, and the right-hand side is the real contractual repayment. In simple words, a firm will default if the value of the collateral is less than the total amount the firm owes to the bank.

Let  $\varepsilon_{j,t}^M$  be the cut-off point, below which default occurs; that is, the value of  $\varepsilon_{j,t}$  for which the above equation holds as an equality. Thus,

$$\chi (A_t \varepsilon_{j,t}^M) N_{j,t} = R_t^L L_{j,t}, \quad (1.4)$$

$$\Rightarrow \varepsilon_{j,t}^M = (\chi A_t)^{-1} R_t^L W_t. \quad (1.5)$$

From the above equation, since  $\varepsilon_{j,t}^M$  depends on  $A_t$ ,  $R_t^L$  and  $W_t$ , all aggregate variables, we can drop the  $j$  subscript and conclude that the cut-off point is not firm specific.

### Price Setting

Each firm has a Calvo (1983)-type constant probability,  $\omega_p$ , of keeping its price fixed at the previous period's price and a constant probability  $1 - \omega_p$  of adjusting to the new optimal price based on the new real marginal cost and treating the loan rate as given;

$$P_t^{1-\theta_p} = (1 - \omega_p)(P_t^*)^{1-\theta_p} + \omega_p P_{t-1}^{1-\theta_p},$$

where  $\theta_p$  is the elasticity of substitution between goods such that  $\theta_p > 1$ . The total real cost faced by the firms consists of wages and the interest paid on the loan. It is thus given by

$$R_t^L W_t N_{j,t},$$

$$\Rightarrow mc_t^R = \frac{R_t^L W_t}{Z_{j,t}}. \quad (1.6)$$

Given the above information and the assumption of constant returns to scale, the firms' maximization problem can be expressed as,

$$\max_{P_{jt}} E_{t-1} \sum_{\tau=0}^{\infty} \beta^\tau \omega_p^\tau \frac{\lambda_{t+\tau}}{\lambda_t} \left( \frac{P_{jt}}{P_{t+\tau}} - mc_{t+\tau}^R \right) Y_{jt+\tau}^*,$$

where  $\frac{\lambda_{t+\tau}}{\lambda_t}$  gives the households' discount factor. Substituting the CES aggregator  $Y_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\theta_p} Y_t$ , we get

$$\max_{P_{jt}} E_{t-1} \sum_{\tau=0}^{\infty} \beta^\tau \omega_p^\tau \frac{\lambda_{t+\tau}}{\lambda_t} \left[ \left( \frac{P_{jt}}{P_{t+\tau}} \right)^{1-\theta_p} - mc_{t+\tau}^R \left( \frac{P_{jt}}{P_{t+\tau}} \right)^{-\theta_p} \right] Y_{t+\tau},$$

and differentiating with respect to  $P_{jt}$ , we get:

$$E_{t-1} \sum_{\tau=0}^{\infty} \beta^\tau \omega_p^\tau \frac{\lambda_{t+\tau}}{\lambda_t} \left[ \left( \frac{1-\theta_p}{P_{t+\tau}} \right) \left( \frac{P_{jt}}{P_{t+\tau}} \right)^{-\theta_p} + \frac{\theta_p mc_{t+\tau}^R}{P_{t+\tau}} \left( \frac{P_{jt}}{P_{t+\tau}} \right)^{-1-\theta_p} \right] Y_{t+\tau} = 0.$$

On further solving the above equation and log linearizing it around a steady state level of inflation, such that  $\pi = 1$ , we derive the following New Keynesian Phillips Curve (NKPC):

$$\hat{\pi}_t = \frac{(1 - \omega_p \beta)(1 - \omega_p)}{\omega_p} \widehat{mc}_t^R + \beta E_t \hat{\pi}_{t+1}.$$

This gives us the forward looking NKPC that describes the supply side of the economy and states the relationship between current and expected future inflation.

### 1.2.3 Financial Intermediation

The financial sector of the economy is represented by a representative commercial bank. At the beginning of each period, the bank receives deposits from households. These funds are then lent to the firms to finance their working capital. The balance

sheet of the bank is such that its assets equal its liabilities,

$$L_t = D_t.$$

We next move on to derive the lending rate,  $R_t^L$ . Since firms' production is subject to random shocks, firms might default on their repayments to the bank. In order to minimize this asymmetric information, the bank charges a loan rate inclusive of a finance premium. This loan rate is derived by setting up the bank's break-even condition. Specifically, this condition requires that in equilibrium the expected income from lending to firm  $j$ , is equal to the cost of return on deposits to the households. Writing the break-even condition in an equation form,

$$R_t \mathbf{E}_t D_t = \underbrace{\int_{\underline{\varepsilon}^M}^{\bar{\varepsilon}} (R_t^L L_{j,t}) f(\varepsilon_{j,t}) d\varepsilon_{j,t}}_{\text{Revenue if firms pay}} + \underbrace{\int_{\underline{\varepsilon}}^{\varepsilon^M} (\chi Y_{j,t}) f(\varepsilon_{j,t}) d\varepsilon_{j,t}}_{\text{Revenue if firms default}}, \quad (1.7)$$

where the first term on the left hand side shows the expected revenue collected by the bank if the firms' honour their debt obligations. The second term, on the other hand, is the expected value of collateral collected in case of default.  $f(\varepsilon_{j,t})$  is the density function of  $\varepsilon_{j,t}$ .

Solving, we get:

$$R_t \mathbf{E}_t D_t = R_t^L \mathbf{E}_t L_{j,t} - \int_{\underline{\varepsilon}}^{\varepsilon^M} (R_t^L L_{j,t} - \chi Y_{j,t}) f(\varepsilon_{j,t}) d\varepsilon_{j,t}.$$

From 1.4,

$$\begin{aligned}
R_t \mathbf{E}_t D_t &= R_t^L \mathbf{E}_t L_{j,t} - \int_{\underline{\varepsilon}}^{e^M} [\chi (A_t \varepsilon_{j,t}^M) N_{j,t} - \chi Y_{j,t}] f(\varepsilon_{j,t}) d\varepsilon_{j,t}, \\
&= R_t^L \mathbf{E}_t L_{j,t} - \int_{\underline{\varepsilon}}^{e^M} \chi N_{j,t} \left[ A_t \varepsilon_{j,t}^M - \frac{Y_{j,t}}{N_{j,t}} \right] f(\varepsilon_{j,t}) d\varepsilon_{j,t}, \\
&= R_t^L \mathbf{E}_t L_{j,t} - \int_{\underline{\varepsilon}}^{e^M} \chi N_{j,t} [A_t \varepsilon_{j,t}^M - A_t \varepsilon_{j,t}] f(\varepsilon_{j,t}) d\varepsilon_{j,t}, \\
&= R_t^L \mathbf{E}_t L_{j,t} - \int_{\underline{\varepsilon}}^{e^M} [\chi A_t N_{j,t} (\varepsilon_{j,t}^M - \varepsilon_{j,t})] f(\varepsilon_{j,t}) d\varepsilon_{j,t}.
\end{aligned}$$

Dividing both sides by  $\mathbf{E}_t L_{j,t}$  ( $= \mathbf{E}_t D_t$ ):

$$\begin{aligned}
R_t &= R_t^L - \frac{\int_{\underline{\varepsilon}}^{e^M} [\chi A_t N_{j,t} (\varepsilon_{j,t}^M - \varepsilon_{j,t})] f(\varepsilon_{j,t}) d\varepsilon_{j,t}}{L_t}, \\
&\Rightarrow R_t^L = R_t + \varrho_t^L,
\end{aligned} \tag{1.8}$$

where  $\varrho_t^L$  is the finance premium with,

$$\begin{aligned}
\varrho_t^L &= \frac{\chi A_t N_t \int_{\underline{\varepsilon}}^{\varepsilon_t^M} (\varepsilon_t^M - \varepsilon_{j,t}) f(\varepsilon_{j,t}) d\varepsilon_{j,t}}{L_t}; \\
&= \frac{\chi A_t}{W_t} \left( \frac{\bar{\varepsilon} - \underline{\varepsilon}}{2} \right) \phi_t^2 = \frac{R_t^L}{\varepsilon_t^M} \left( \frac{\bar{\varepsilon} - \underline{\varepsilon}}{2} \right) \phi_t^2,
\end{aligned} \tag{1.9}$$

and  $\phi_t \in (0, 1)$  is the probability of default,

$$\phi_t = \int_{\underline{\varepsilon}}^{\varepsilon_t^M} f(\varepsilon_t) d\varepsilon_t = \frac{\varepsilon_t^M - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}. \tag{1.10}$$

Analyzing equation 1.10, we notice that  $\phi_t$  is the ratio of the range in which a firm can default to the overall range of the idiosyncratic productivity shock.

## 1.2.4 Government

The government, in the model, determines monetary and fiscal policies.

## Monetary Policy

The government targets nominal interest rate through the Taylor (1993) type rule:

$$\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho^i} \left( \frac{\pi_t}{\pi} \right)^{\phi_\pi (1-\rho^i)},$$

where,  $R = \frac{1}{\beta}$  is the steady state nominal gross interest rate,  $\rho^i \in (0, 1)$  captures the degree of interest rate smoothing and  $\phi_\pi > 0$  is the policy parameter.<sup>4</sup>

## Fiscal Policy

The government intertemporal budget constraint is given by:

$$B_t = G_t + R_{t-1} \frac{B_{t-1}}{\pi_t} - \tau_{h,t} \frac{W_t}{P_t} N_t$$

This makes explicit the reduction in real public debt caused by inflation. As the budget constraint states, the sovereign debt issued by government in a given period equals the government expenditure, plus the payments made on the previous year's public debt, minus the tax receipts from income tax.<sup>5</sup>

We follow Villaverde (2010) closely and set up government expenditure such that it follows an autoregressive process:

$$\hat{G}_t = \alpha_G \hat{G}_{t-1} + B_G \left( \hat{B}_{t-1} \right) + \sigma_G \varepsilon_{G,t}$$

where  $B_G$  determines the sensitivity of expenditures to public debt brought into the period. It is also important to adjust the government expenditure to the level of debt so that the government cannot run a Ponzi scheme; i.e., it is important to ensure that the government does not pay off the old debt by issuing new debt.

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<sup>4</sup>We do not add a response to output in the Taylor rule as it does not alter the results materially. See Schmitt-Grohe and Uribe (2007).

<sup>5</sup>Note that the aim of this paper is to focus on the effects of government spending shock in the presence of a cost channel and not to study the redistributive effects of fiscal policy.

Further, income tax follows an AR(1) process:

$$\hat{\tau}_{h,t} = \alpha_h \hat{\tau}_{h,t-1} + \sigma_{\tau_h} \varepsilon_{\tau_h,t} \text{ where } \hat{\tau}_{h,t} = \log \frac{1 - \tau_{h,t}}{1 - \bar{\tau}_h}$$

where  $\alpha_G$  and  $\alpha_h$  are the persistence parameters,  $\varepsilon_{G,t} \sim N(0, \sigma_G)$  and  $\varepsilon_{\tau_h,t} \sim N(0, \sigma_{\tau_h})$ .

### 1.2.5 Aggregate Equilibrium

Finally, we close the model with the following aggregate resource constraint:

$$Y_t = C_t + G_t. \tag{1.11}$$

## 1.3 Parameterization

We log-linearize the model's equilibrium conditions around the steady state. To solve the base model, we select numerical values for certain key parameters. Table 1.1 shows these values.

We now emphasize on some of the model specific parameters. In case of a default, the fraction of output seized,  $\chi = 0.97$ , implies that three percent of the output value is lost in bankruptcy costs. Following Faia and Monacelli (2007), we aim to derive a probability of default of approximately three percent. We thus take the idiosyncratic shock interval of  $[0.85, 1.15]$  and get a probability of default equal to 3.04% in steady state.

For preference parameters, we fix  $\beta$  such that the average annual real interest rate is 4%. For this we use the data provided by Trabandt and Uhlig (2012) and use the average of U.S. interest rates between 1995 and 2010. We use the result from Kimball and Shapiro (2008) and set the Frisch elasticity of labor supply to be 1.

Moving on to the price parameters, an elasticity of demand of 6 formulates into a price markup of 1.2. Moreover, a price stickiness degree of 0.75 implies a price



Table 1.1: Baseline Parameters

Parameter	Value	Description	Target/Source
$\beta$	0.99	Discount Factor	$R - 1 = 4$ , Data, Trabandt and Uhlig (2012)
$\sigma$	1.00	Intertemporal Substitution in Consumption	Log utility in consumption
$\xi$	1.00	Inverse of the Frisch Elasticity of Labor Supply	Kimball and Shapiro (2008)
$\theta_P$	6.00	Elasticity of Demand- Intermediate Goods	20% price markup, Christiano, Eichenbaum and Evans (2005)
$\omega_P$	0.75	Degree of Price Stickiness	4 quarters price duration, Erceg, Henderson and Levin (2000)
$\theta_W$	6.00	Elasticity of Demand- labor	20% markup
$\omega_W$	0.75	Degree of Wage Stickiness	4 quarters wage duration, Erceg, Henderson and Levin (2000)
$A$	1.00	Productivity Parameter	Normalization
$\underline{\varepsilon}$	0.85	Idiosyncratic Productivity Shock Lower Range	$\phi = 3\%$ , Faia and Monacelli (2007)
$\bar{\varepsilon}$	1.15	Idiosyncratic Productivity Shock Upper Range	$\phi = 3\%$ , Faia and Monacelli (2007)
$\chi$	0.97	Proportion of Output Seized in case of Default	3% bankruptcy cost, Agénor, Bratsiotis, and Pfajfar (2014)
$\rho^A$	0.95	Persistence in Technology Shock	Villaverde, 2010
$\phi_\pi$	1.5	Interest Rate response to Inflation	Taylor Rule
$\rho^i$	0.95	Degree of Interest Rate Smoothing	Villaverde, 2010
$\bar{\tau}_h$	0.22	Steady State value of tax on labor	Data, Trabandt and Uhlig (2012)
$B_G$	-0.01	Sensitivity of expenditure to public debt	Villaverde, 2010
$\alpha_G$	0.95	Persistence in Government Spending Shock	Villaverde, 2010
$\alpha_h$	0.95	Persistence in Income Tax Shock	Villaverde, 2010

Table 1.2: Steady State Values of Key Variables

Variable	Description	Steady State Value
$C/Y$	Consumption to Output	0.8185
$G/Y$	Government Expenditure to Output	0.1815
$R$	Policy Rate	1.0101
$R^L$	Loan Rate	1.0103
$\varrho^L$	Finance Premium	0.0002
$\phi$	Probability of Default	0.0304
$e^M$	Cut-off Point	0.8591

duration of  $\frac{1}{1-0.75} = 4$  quarters and a coefficient of 0.0858 on marginal cost in the NKPC. We hold a neutral stand on the rigidity of wages compared to prices, and thus use the same parameter values. Following the standard Taylor rule parameters, we take the weight on inflation to be 1.5. In simple words, when inflation rises by 1%, the monetary authority would respond by raising interest rates by 1.5%.

Finally, on the fiscal side, we use the data from Trabandt and Uhlig (2012) and take the average value of U.S. labor income tax rates between 1995 and 2010 as the steady state value of income tax rate in our model. This value is 22%. Further, following Trabandt and Uhlig (2012), we target  $G/Y = 0.18$ . With  $\bar{\tau}_h = 0.22$  and  $G/Y = 0.18$ , the steady state value of government bonds is zero. A similar assumption is made by Villaverde (2010). Similar to Villaverde (2010), a negative value of  $B_G (= -0.01)$  ensures that the model has a determined equilibrium. Moreover, a very small value of  $B_G$  is selected to imply that fluctuations in government expenditure triggered by public debt are negligible in the immediate short run. Persistence in all shocks is 0.95.

Table 1.2 states the steady state values of some of the key variables.

## 1.4 Results

After setting the model and calibrating the parameters, we study the effects of a one percent increase in government spending on three economies; 1) a frictionless economy; 2) a low friction economy with  $\chi = 0.97$  and 3) a high friction economy

with  $\chi = 0.75$ . Since  $\chi$  is inversely related to the cut off point, equation (1.5), a lower  $\chi$  implies a higher cut off point. A high friction economy thus features a higher probability of default and a higher finance premium, creating a crisis-like scenario.

Figure 1.2 studies the impact of a one percent rise in government spending on key variables.<sup>6</sup> A positive shock to government spending leads to a demand-driven expansion. To catch up with higher demand, firms increase their labor and loan demands. This results in a rise in labor market wages. Higher wages, along with higher interest rates caused by the demand side shock increases firms' marginal costs. Higher marginal costs reduce firms' markups. This inverse effect of a positive demand shock on firms' markups is in line with Smets and Wouters (2003).

This story stays true for all the three types of economies. However, in the presence of frictions, an extra mechanism comes into play. From equations (1.5) and (1.6), we notice that a higher marginal cost leads to a higher default cut-off point, increasing firms' probabilities of default. This incentivizes the financial intermediary to charge a higher finance premium, leading to a further rise in loan rate. The extra upward push on the loan rate, due to a rise in finance premium, discourages firms to borrow as much as they would have in the absence of financial frictions. A lower level of borrowing results in lower level of production and, hence, a comparatively lower level of output. This effect is more pronounced in the high friction economy than in the low friction economy. The first subplot in figure 1.2 compares the effect of government spending on output in the presence and absence of financial frictions. À la Ravenna and Walsh (2005), the model with no financial frictions features a loan rate which is equivalent to the policy rate, implying no spread between the two rates.

Further, we follow the approach used by Ramey and Zubairy (2017) to quantify our results. We calculate the government spending multiplier of the economy using

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<sup>6</sup>On the y-axis, we plot the percentage deviation of key variables from their respective steady states, following the shock. On the x-axis, we show time elapsed after the shock, in quarters.

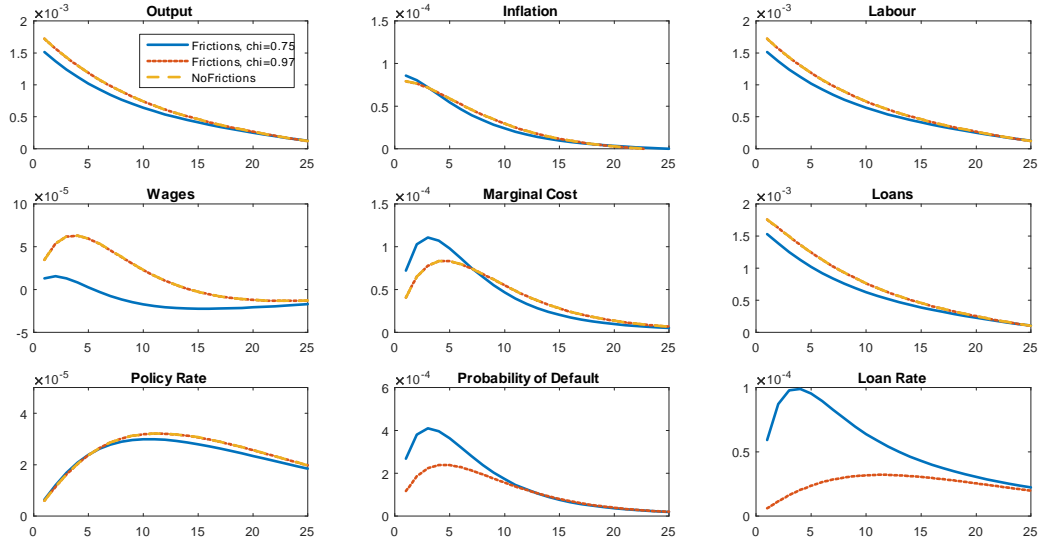


Figure 1.2: IRFs to a Positive Government Spending Shock

the formula:

$$M = \frac{\Delta Y}{\Delta G},$$

where  $M$  is the multiplier and  $\Delta Y$  and  $\Delta G$  are the changes in output and government expenditure respectively. From the impulse response functions, we know,

$$\ln Y_t - \ln Y_{t-1} = \frac{Y_t - Y_{t-1}}{Y_{t-1}}; \quad (1.12)$$

and

$$\ln G_t - \ln G_{t-1} = \frac{G_t - G_{t-1}}{G_{t-1}}. \quad (1.13)$$

Using equations 1.12 and 1.13,  $M$  can be written as,

$$M = \left( \frac{\ln Y_t - \ln Y_{t-1}}{\ln G_t - \ln G_{t-1}} \right) \frac{Y_{t-1}}{G_{t-1}}. \quad (1.14)$$

We use the generated impulse response functions and 1.14 to derive the values for the government spending multipliers in various quarters. For quarters greater than one, we use the cumulative discounted values of changes over time and obtain the

Table 1.3: Government Spending Multiplier

<b>Quarters</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>No Friction</b>	0.950	0.928	0.908	0.889	0.872	0.855
<b>Friction</b> ( $\chi = 0.75$ )	0.929	0.904	0.881	0.860	0.840	0.823
<b>Difference</b>	0.021	0.024	0.027	0.029	0.032	0.032

numbers presented in Table 1.3.

As we notice from Table 1.3, the model generates a government spending multiplier of less than one, implying that the increase in output is less than the increase in government spending. Cogan et al. (2009) study the magnitude of the spending multiplier in the Smets and Wouters (2007) model and argue that the multipliers in "New-Keynesian" models are smaller compared to the "Old-Keynesian" models. This is mainly due to the to a contraction in private sector investment/ consumption, which follows an increase in government spending. Moreover, in our model, we show that the multipliers in the presence of financial frictions are lower than the ones in the absence of frictions. This result overturns the established results, according to which the government spending multiplier is larger (for example, Villaverde (2010) calculates a government spending multiplier which is greater than one) in the presence of financial frictions. The reason for contrast between the two results is that in the established literature, people have studied financial frictions which arise when borrowing is used to finance fixed capital like the physical capital. A rise in inflation generates the "Fisher effect," increasing the value of borrowers' collateral and hence reducing the probability of default. This encourages firms to borrow and hence produce more. In our model however, loans are used to finance the working capital, giving rise to the "risky working capital channel" as discussed in Christiano, Eichebaum and Trabandt (2015). Since the financial frictions affect the economy through the cost channel, an increase in working capital exerts an upwards inflationary pressure on the loan rate, reducing the demand for borrowing in the economy. Intuitively, following a demand shock, firms swell up their balance sheets by undertaking more risk, thereby increasing their probability of default.

## 1.5 Robustness Analyses

Further, we conduct robustness analyses to support our result that compared to the frictionless environment, the government spending multiplier is smaller when financial frictions affect the cost channel. For this, we study the government spending multiplier when 1) the wages are flexible and 2) the government runs a balanced budget constraint.

### 1.5.1 Flexible Wages

In the presence of flexible wages, household's utility is given by

$$\max_{C_t, D_t, N_t} E_t \sum_{s=0}^{\infty} \beta^s \left( \frac{C_{t+s}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \eta_N \frac{N_{t+s}^{1+\xi}}{1+\xi} \right),$$

where  $N_t$  is the amount of labour supplied. The household's budget constraint is

$$C_t + D_t = R_{t-1} \frac{D_{t-1}}{\pi_t} + (1 - \tau_{h,t}) W_t N_t + R_{t-1} \frac{B_{t-1}}{\pi_t} + PF_t.$$

Moreover, on solving household's problem with respect to labor, we derive that the household's marginal rate of substitution is equivalent to the wage rate in the economy, i.e.

$$W_t = \frac{\eta_N N_t^\xi}{(1 - \tau_{h,t}) C_t^{-\frac{1}{\sigma}}}.$$

The rest of the model remains similar to the baseline model.

Next, we conduct the same experiment. We shock the government spending positively by one percent of its steady state value and study its effects on the dynamics of the model.

Figure 1.3 shows these results. As evident, the dynamics of our model remain the same. Moreover, our result that the government spending multiplier is smaller when financial frictions affect the cost channel still holds. We can see this from

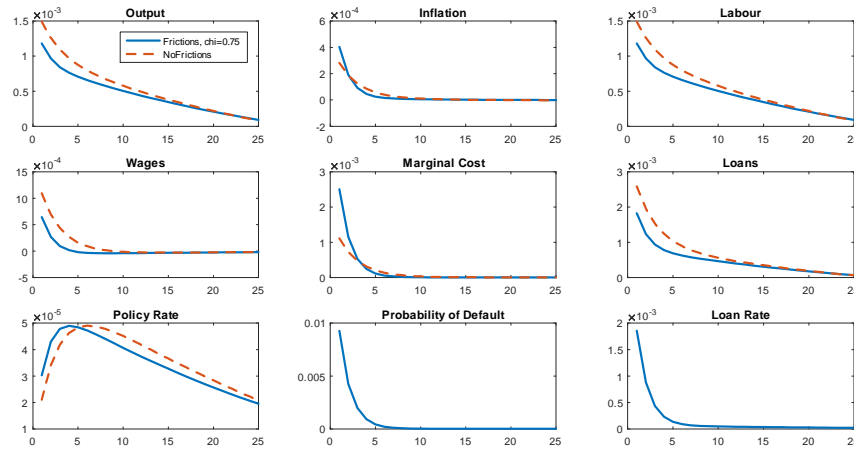


Figure 1.3: IRFs to a Positive Government Spending Shock (when wages are flexible)

Table 1.4: Government Spending Multiplier, when wages are flexible

Quarters	1	2	3	4	5	6
<b>No Friction</b>	0.822	0.776	0.739	0.711	0.688	0.669
<b>Friction (<math>\chi = 0.75</math>)</b>	0.724	0.672	0.638	0.614	0.598	0.585
<b>Difference</b>	0.098	0.104	0.101	0.097	0.090	0.084

the "Output" panel of figure 1.3– the change in output in a frictionless environment (depicted by the red, dashed line) is higher than the change in output in the presence of frictions (blue, solid line).

Quantitatively, however, the multipliers change. Table 1.4 lists the government spending multipliers in the friction and no friction cases, when wages are flexible.

Compared to the wage rigidity case, the multipliers are smaller in the presence of flexible wages. This is because, with flexible wages, an increase in government demand increases the marginal cost even more, leading to higher inflation and interest rates and thus, lower equilibrium borrowing and production.

Also, compared to the wage rigidity case, the difference between the two set of multipliers (no friction and friction) is larger. With flexible wages, the rise in marginal cost is higher. This results in higher loan demand, higher cut-off point and hence, higher probability of default. Flexible wages thus have a larger impact on the finance premium, leading to a larger difference between the multipliers.

## 1.5.2 Balanced Government Budget Constraint

Moving on, we study our model in the presence of a balanced government budget constraint. The government runs a balanced budget constraint when there is no intertemporal public borrowing, that is, there are no government bonds in household and government budget constraints. In such a scenario, the government finances its expenditure from its tax revenues. The government budget constraint is given by:

$$G_t = (G_{t-1})^{\alpha_G} (\tau_{h,t} W_t N_t)^{1-\alpha_G} (\varepsilon_{G,t})^{\sigma_G},$$

where the government expenditure is equivalent to the income tax receipts and is subject to an exogenous shock.

The remainder of the model is akin to the above developed model, with flexible wages.

Similar to the above case, our qualitative dynamics of an increase in government spending, remain the same— the government spending multiplier is smaller when financial frictions enter the economy via the cost channel. Figure 1.4 shows the same.

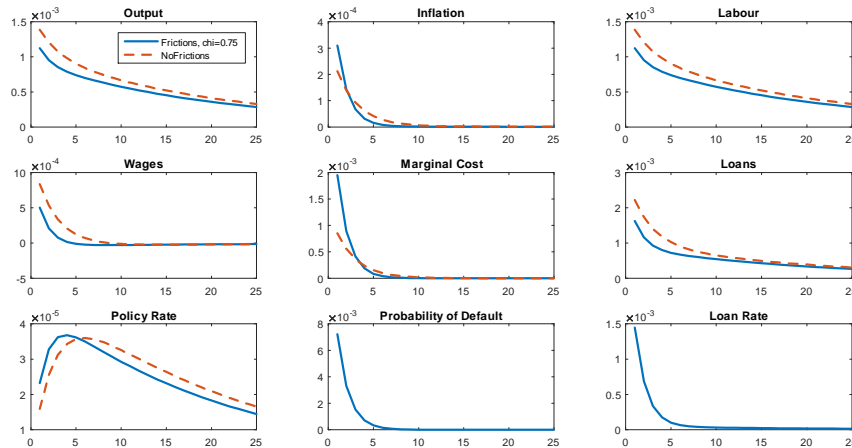


Figure 1.4: IRFs to a Positive Government Spending Shock (when the government runs a balanced budget)

Quantitatively, in the balanced budget case, one less lag variable (government



Table 1.5: Government Spending Multiplier, when the government runs a balance budget

Quarters	1	2	3	4	5	6
<b>No Friction</b>	0.756	0.719	0.691	0.668	0.65	0.635
<b>Friction (<math>\chi = 0.75</math>)</b>	0.683	0.642	0.615	0.596	0.583	0.572
<b>Difference</b>	0.073	0.077	0.076	0.072	0.067	0.063

bonds) in the model, attenuates the propagation mechanism of the shock. Compared to the baseline model, the multipliers are thus smaller and are as shown in Table 1.5.

Finally, to summarize, we compare the different cases in Figure 1.5. As shown, the government spending multiplier is always lower in the presence of the risky working capital channel. This bolsters our main result.

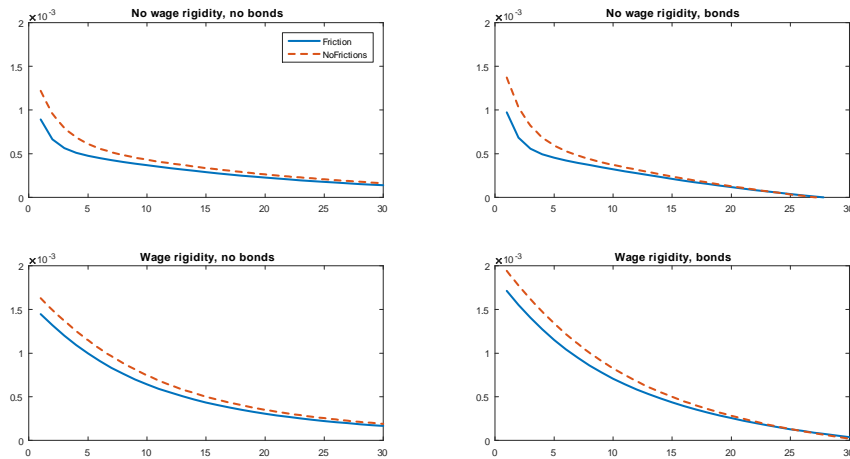


Figure 1.5: IRFs of Output to Government Spending Shock in Different Scenarios

## 1.6 Conclusion

This paper studies the transmission of fiscal policy through the "risky working capital channel." In a DSGE New-Keynesian model with price and wage rigidities, we show that when financial frictions strongly affect the cost channel, the government spending multiplier shrinks. An increase in government spending leads to a rise in

prices, which increases the marginal cost, finance premium and hence the loan rate. The additional increase in loan rate discourages production activity in the economy, reducing the government spending multiplier. Furthermore, the government spending multiplier is less than one. Given the increasing reliance of start-ups on working capital funding, this gives a plausible explanation about why the effect of government spending was not as large as it was expected to be.

Even though the paper highlights a new transmission channel of fiscal policy, the strength of the channel still remains a question mark. As an empirical exercise, using real life data, it will thus be interesting to study the intensity of the channel. Moreover, studying the channel in the presence of the zero lower bound and conducting welfare analysis to derive the optimal policy will also make intriguing research exercises. Further, the paper assumes that the commercial bank lends all its deposits in entirety. However, it will be compelling to introduce equity and capital requirements and study the interaction between macroprudential regulation and fiscal policy in such a set-up.

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## 1.A Equilibrium Conditions

The following 20 equations could be solved for the 20 variables:

$$Y_t, C_t, G_t, D_t, B_t, R_t, R_t^L, \hat{\pi}_t, \hat{\pi}_t^w, mc_t^R, W_t, Z_t, L_t, N_t, MRS_t, A_t, \phi_t, \varepsilon_t^M, \varrho_t^L, \hat{\tau}_{h,t}$$

$$Y_t = C_t + G_t$$

$$C_t^{-\frac{1}{\sigma}} = \beta E_t \frac{C_{t+1}^{-\frac{1}{\sigma}} R_t}{\pi_{t+1}}$$

$$mc_t^R = \frac{R_t^L W_t}{Z_{j,t}}$$

$$L_t = W_t N_t$$

$$L_t = D_t$$

$$\hat{\pi}_t^w = \frac{(1 - \beta\omega_\omega)(1 - \omega_\omega)}{\omega_\omega(1 + \xi\theta_\omega)} \left( \widehat{MRS}_t - \hat{W}_t \right) + \beta E_t \hat{\pi}_{t+1}^w$$

$$W_t = \left( \frac{\theta_w}{\theta_w - 1} \right) MRS_t$$

$$MRS_t = \left( \frac{1}{1 - \tau_{h,t+s}} \right) \frac{\eta_N N_t^\xi}{C_t^{-\frac{1}{\sigma}}}$$

$$Y_t = N_t Z_t$$

$$Z_{j,t} = A_{j,t} \varepsilon_{j,t}$$

$$\log A_t = (1 - \rho^A) \log A + \rho^A \log A_{t-1} + \varepsilon_t^A$$

$$R_t^L = R_t + \varrho_t^L$$

$$\varrho_t^L = \frac{\chi A_t}{W_t} \left( \frac{\bar{\varepsilon} - \underline{\varepsilon}}{2} \right) \phi_t^2$$

$$\phi_t = \frac{\varepsilon_t^M - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}$$

$$\varepsilon_t^M = (\chi_t A_t)^{-1} R_t^L W_t^R$$



$$\hat{\pi}_t = \frac{(1 - \omega_p \beta)(1 - \omega_p)}{\omega_p} \widehat{m} c_{t+\tau}^R + \beta E_t \hat{\pi}_{t+1}$$

$$\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho^i} \left( \frac{\pi_t}{\pi} \right)^{\phi_\pi (1 - \rho^i)} \epsilon_t^i$$

$$B_t = G_t + R_{t-1}^B \frac{B_{t-1}}{\pi_t} - \tau_{h,t} \frac{W_t}{P_t} N_t$$

$$\hat{G}_t = \alpha_G \hat{G}_{t-1} + B_G \left( \hat{B}_{t-1} \right) + \sigma_G \varepsilon_{G,t}$$

$$\hat{\tau}_{h,t} = \alpha_h \hat{\tau}_{h,t-1} + \sigma_{\tau_h} \varepsilon_{\tau_h,t}$$

## 1.B Steady State

Resource Constraint:

$$Y = C + G$$

Deposit/ Bond Rate:

$$R = \frac{1}{\beta}$$

Production Function:

$$Y = NZ$$

Productivity:

$$Z = A \left( \frac{\underline{\varepsilon} + \bar{\varepsilon}}{2} \right)$$

Loan Rate:

$$R^L = R + \frac{\chi A}{W} \left( \frac{\bar{\varepsilon} - \underline{\varepsilon}}{2} \right) \phi^2$$

Real Wage:

$$W = \left( \frac{\theta_w}{\theta_w - 1} \right) MRS$$

Marginal Rate of Substitution:

$$MRS = \eta_N N^\xi C^{\frac{1}{\sigma}} (1 - \bar{\tau}_h)^{-1}$$

Total Loans:

$$L = WN$$

Marginal Cost

$$mc = \frac{R^L W}{Z}$$

At zero inflation steady state,

$$mc = \frac{\theta_p - 1}{\theta_p}$$

Cut-off point:

$$\varepsilon^M = \frac{R^L W}{\chi A}$$

Firm's probability of default:

$$\phi = \frac{\varepsilon^M - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}$$

Government Expenditure:

$$G = \bar{\tau}_h WN$$

Solving the steady state for  $W$ :

$$\begin{aligned} \frac{mc \times Z}{W} &= R + \frac{\chi A}{W} \left( \frac{\bar{\varepsilon} - \underline{\varepsilon}}{2} \right) \phi^2 \\ \Rightarrow \frac{1}{W} \left[ mc \times A \left( \frac{\underline{\varepsilon} + \bar{\varepsilon}}{2} \right) - \chi A \left( \frac{\bar{\varepsilon} - \underline{\varepsilon}}{2} \right) \phi^2 \right] &= R \\ \Rightarrow \frac{A}{2W} &= \frac{R}{mc(\underline{\varepsilon} + \bar{\varepsilon}) - \chi \phi^2 (\bar{\varepsilon} - \underline{\varepsilon})} \\ \Rightarrow W &= \frac{A [mc(\underline{\varepsilon} + \bar{\varepsilon}) - \chi \phi^2 (\bar{\varepsilon} - \underline{\varepsilon})]}{2R} \end{aligned}$$

Solving the steady state for  $C$ :

From the steady state government budget and production function:

$$\begin{aligned}
G &= NZ - C = \bar{\tau}_h WN \\
\Rightarrow NZ - C &= \bar{\tau}_h WN \\
\Rightarrow -C &= -N [Z - \bar{\tau}_h W] \\
\Rightarrow C &= N [Z - \bar{\tau}_h W]
\end{aligned} \tag{1.15}$$

Solving the steady state for  $N$ :

From labor supply:

$$C = \left[ \frac{MRS(1 - \bar{\tau}_h)}{\eta_N N^\xi} \right]^\sigma \tag{1.16}$$

Equating 1.15 and 1.16:

$$\begin{aligned}
N^{-\xi\sigma} \left[ \frac{MRS(1 - \bar{\tau}_h)}{\eta_N} \right]^\sigma &= N [Z - \bar{\tau}_h W] \\
\Rightarrow N^{\sigma\xi+1} &= \left[ \frac{MRS(1 - \bar{\tau}_h)}{\eta_N} \right]^\sigma \left[ \frac{1}{Z - \bar{\tau}_h W} \right] \\
\Rightarrow N &= \left[ \frac{MRS(1 - \bar{\tau}_h)}{\eta_N} \right]^{\frac{\sigma}{\sigma\xi+1}} \left[ \frac{1}{Z - \bar{\tau}_h W} \right]^{\frac{1}{\sigma\xi+1}}
\end{aligned}$$

## 1.C Log-Linearization

Log-linearized variables are denoted by hat and represent log-deviations around their steady state values, or percentage point deviations in the case of interest rate and the inflation rate. The log-linearized equations are as follows,

Resource Constraint:

$$\hat{Y}_t = \frac{C}{Y} \hat{C}_t + \frac{G}{Y} \hat{G}_t$$

Intertemporal condition, deposits/ bonds:

$$\hat{C}_t - E_t \hat{C}_{t+1} = -\sigma \left( \hat{R}_t - \hat{\pi}_{t+1} \right)$$

Marginal Cost:

$$\widehat{mc}_t = \hat{R}_t^L + \hat{W}_t - \hat{Z}_t$$

Total Loans (in real terms):

$$\hat{L}_t = \hat{W}_t + \hat{N}_t$$

Wages:

$$\hat{W}_t = \left( \frac{\hat{W}_t^n}{P_t} \right) = \hat{W}_{t-1} + \hat{\pi}_t^w - \hat{\pi}_t$$

Wage inflation:

$$\hat{\pi}_t^w = \frac{(1 - \beta\omega_\omega)(1 - \omega_\omega)}{\omega_\omega(1 + \xi\theta_\omega)} \left( \widehat{mrs}_{t+s} - \hat{W}_t \right) + \beta E_t \hat{\pi}_{t+1}^w$$

MRS:

$$\widehat{MRS}_t = \frac{1}{\sigma} \hat{C}_t + \xi \hat{N}_t - \hat{\tau}_{h,t}$$

Production:

$$\hat{Y}_t = \hat{N}_t + \hat{Z}_t$$

Idiosyncratic Shock:

$$\hat{Z}_t = \hat{\varepsilon}_t + \hat{A}_t$$

Technology Shock:

$$\hat{A}_t = \rho^A \hat{A}_{t-1} + \epsilon_t^A$$

Probability of Default

$$\hat{\phi}_t = \phi_t \left( \frac{\varepsilon^M}{\varepsilon^M - \underline{\varepsilon}} \right) \left( \hat{R}_t^L + \hat{W}_t - \hat{A}_t \right)$$

Cut off point:

$$\hat{\varepsilon}_t^M = \hat{R}_t^L + \hat{W}_t - \hat{A}_t$$

Finance Premium

$$\hat{\varrho}_t^L = 2\hat{\phi}_t - \hat{W}_t + \hat{A}_t$$

Total Loans:

$$\hat{L}_t = \hat{D}_t$$

Lending Rate:

$$\hat{R}_t^L = \frac{1}{R^L} \left( R\hat{R}_t + \varrho^L \hat{\varrho}_t^L \right)$$

NKPC:

$$\hat{\pi}_t = \frac{(1 - \omega_p \beta)(1 - \omega_p)}{\omega_p} \widehat{m}c_{t+\tau} + \beta E_t \hat{\pi}_{t+1}$$

Interest Rate Policy Rule:

$$\hat{R}_t = \rho^i \hat{R}_{t-1} + (1 - \rho^i) \phi_\pi \hat{\pi}_t + \epsilon_t^i$$

Government Budget Constraint:

$$\hat{B}_t = (\bar{\tau}_h W N) \hat{G}_t + R \hat{B}_{t-1} - \bar{\tau}_h W N \left( \hat{W}_t + \hat{N}_t + \hat{\tau}_{h,t} \right) + W N \hat{\tau}_{h,t}$$

Government Expenditure:

$$\hat{G}_t = \alpha_G \hat{G}_{t-1} + B_G \left( \hat{B}_{t-1} \right) + \varepsilon_{G,t}$$

Tax on labor income:

$$\hat{\tau}_{h,t} = \alpha_h \hat{\tau}_{h,t-1} + \sigma_{\tau_h} \varepsilon_{\tau h,t}$$

## **Chapter 2**

# **Optimal Capital Income Taxation in the Borrower-Saver Model**

## 2.1 Introduction

In 1928, Ramsey wrote about heterogeneous degrees of patience amongst individuals and conjectured that in the long run, "the thrifty enjoy bliss and the improvident live at the subsistence level." Does Ramsey's conjecture hold in the Ramsey problem? We find the answer to be affirmative. In simple words, we derive the optimal level of capital taxation that maximizes the social welfare of a society inhabited by people of varying degrees of patience. We model two types of households with one being more impatient than the other. With the aim of increasing their current utility, the impatient households borrow from the patient households, subject to a borrowing constraint. Taking a social discount factor, which is a linear weighted average of the two different discount factors, we show that if the Ramsey planner is indifferent between redistribution between patient and impatient households, the borrowing constraint is not binding. With a non-binding borrowing constraint, the impatient agents bring forward their consumption by borrowing against the discounted value of their future income. This reduces their future disposable income and hence consumption to zero, shrinking the heterogeneous agent model to a representative agent model. Thus, similar to the representative agent model, the optimal taxation approaches zero in the distant long run. On the other hand, if the borrowing constraint is to be binding, the planner favours a redistribution towards patient households, resulting in positive consumption for both the households. To achieve this, the Ramsey planner subsidizes returns on capital income. Since the impatient households value the future less than the patient households, with passing time, the Ramsey planner will have the opportunity to tax them and transfer the tax receipts to patient households, who value future more.

Empirical estimates of capital income taxation show high tax rates, especially in the large industrial countries. Mendoza et al. (1994), for example, use national accounts and revenue statistics to calculate tax rate estimates, from 1965 to 1988.



During this period, as per their computations, the capital income tax rate in the US varied from 37.2% in 1965 to 49.2% in 1970. From then, the capital income tax rate fluctuated between 43% and 47%, until a drop in 1983 to 39%. It however, started rising again towards the late 80s. Trabandt and Uhlig (2012) revise the exercise conducted by Mendoza et al (1994) and extend the data up to 2010. Their calculations suggest a similar range for the capital income tax rate in the following years, with an average capital income tax rate of 41.2% between 1995 and 2010.

Compared to the US, UK is shown to have a higher capital income tax rate. Even though, year 1965 experienced a tax rate of around 39%, the capital income tax rate peaked as high as 74% in 1981. For most of the 80s, it was in the range of 61%-63%. Going further, according to the calculations conducted by Trabandt and Uhlig (2012), the average income tax rate in the UK was around 52.3% between 1995 and 2010. Moreover, both Mendoza et al (1994) and Trabandt and Uhlig (2012) find the capital income tax rate estimates for the European region to be lower than those in the US and the UK. Trabandt and Uhlig (2012) find the average capital income tax rate for the EU-14 region to be around 37% for the period 1995-2010.

Having presented the empirical statistics, we now focus on the theoretical literature which concludes otherwise. An important result in the literature on optimal taxation is the one by Chamley (1986) and Judd (1985) who show that the optimal capital taxation should be zero in the long run. They argue that taxing capital would reduce the incentive of people to invest in capital, lowering the production activity and wages in the economy—a cost which exceeds the benefits of redistribution. However, both the papers assume perfect capital markets and agents with same time preferences. While Chamley (1986) uses an infinitely lived representative agent model to show the result, Judd (1985) includes heterogeneity in the form of capitalists and workers. The framework used by Judd (1985) resembles the Savers-Spenders model in which one section of the society optimizes intertemporally by saving for the future while the other section survives on its wages.

However, the Savers-Spenders model, lacks the borrowing side of the market. In reality, agents vary in their levels of patience and preferences towards present and future levels of consumption and savings. Models should thus have at least two different groups of agents—lenders and borrowers (Quadrini, 2011). Empirically, compared to the benchmark representative agent model, Krussel and Smith (1998) show that when heterogeneity in discount factors is taken into account, the model better captures the very skewed wealth distribution observed in the data. In this paper, accounting for this heterogeneity, we study the optimal level of capital taxation in the Borrower-Saver model, which, as stated by Bilbiie, Monacelli and Perotti (2012), differs from the Savers-Spenders model in four aspects: first, there are savers and borrowers who have different time preferences and hence, different discount factors; second, both kind of agents are intertemporal optimizers with the borrowers borrowing from the savers; third, the borrowers face a borrowing constraint which introduces financial frictions in the economy; and finally, the decision on the equilibrium level of lending and borrowing is endogenized, subject to the borrowing constraint.

In the presence of borrowing constraints, Aiyagari (1995) shows that the result of zero long-run capital taxation is not welfare optimizing. Accounting for future uncertainty and the possibility of being borrowing constrained in the future, agents indulge in an over accumulation of savings in the short run. Compared to the marginal product of capital, the return on capital is lower hence making it optimal to tax capital income even in the long run. Later, Chamley (2001) generalized this result by showing that in the presence of a borrowing constraint, whenever there is positive relationship between consumption and savings, savings should be taxed. However, if there is a negative relationship between consumption and savings, savings should be subsidized. As argued in subsequent sections, our result is a manifestation of the latter.

Recently, the Borrower-Saver model has started gaining momentum in the Macro-

economic literature with Kiyotaki and Moore (1997) using an extended version of the model to study credit cycles. Some other important papers which work on or extend this model are Iacoviello and Neri (2010), Monacelli and Perotti (2011), Eggertsson and Krugman (2012), McKay and Reis (2016) and Alpanda and Zubairy (2016). With a lot of macroeconomists, policy makers and central bankers using the model to study optimal policies and various policy implications, it becomes even more important to know the very basic—the optimal level of capital taxation in a model with different degrees of patience and a borrowing constraint.

The structure of the remainder of the paper is as follows: Section 2 highlights the modelled setup of our economy. Sections 3 and 4 respectively discuss the equilibrium and steady state conditions of the decentralized economy. Section 5 covers the benevolent social planner’s first best allocation. Section 6 is the main focus of the paper and details the Ramsey optimal taxation. Finally, section 7 concludes. All the detailed mathematical calculations are presented in the appendix.

## **2.2 The Model**

We use a deterministic, real business cycle, closed economy model that comprises of heterogeneous households that differ in their discount factors. The setup also comprises of firms which hire labor and rent capital to carry out their production activities. The government levies distortionary capital income taxes to finance its expenditure and runs a balanced budget constraint.

### **2.2.1 Households**

Households maximize their utilities subject to their respective budget constraints and differ in two respects—the rate of time preference and labor supply. The patient households do not work and earn interest income on their savings. The impatient

households, on the other hand, supply one unit of inelastic labor and earn wages.<sup>1</sup> Their preferences are given by the following log utility function:

$$\max E_t \sum_{t=0}^{\infty} \beta_s^t \ln c_{s,t},$$

where  $\beta \in (0, 1)$  is the discount factor and  $s = P$  and  $I$ , denotes patient and impatient households respectively. Following a few important papers using the Borrower-Saver model, for example Monacelli and Perotti (2011), Eggertsson and Krugman (2012) and McKay and Reis (2016) to name a few, we choose a logarithmic utility function which implies a constant savings rate of  $\beta$ , an important characteristic which helps us derive an elegant tax function. In the literature of optimal capital taxation, Lansing (1999) showed that the logarithmic utility function is a knife-edged case which generates a non-zero optimal level of capital taxation. Later, Straub and Werning (2014) show that the result holds even when the preferences are not logarithmic. The focus of our paper is to study optimal level of capital taxation in the Borrower-Saver model, which primarily uses log utility function. Study of other utility functions in such a set up has been left for future work.

Patient households are more patient and value future more. They thus indulge in consumption smoothing by saving today. Impatient households, on the other hand, derive a higher marginal utility from consuming today and hence borrow to increase their level of current consumption. This heterogeneity is highlighted through the difference in their discount factors,  $\beta_P$  and  $\beta_I$  respectively, with  $\beta_P > \beta_I$ .

Further, following Campbell and Hercowitz (2005) and keeping our analysis close to Judd (1985) and Section 2 of Straub and Werning (2014), we assume that patient

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<sup>1</sup>Our results hold true even when both the household types supply labor. However, to keep our analysis simple, we assume that the patient households receive capital income while the impatient households receive labor income. The assumption of impatient households not earning capital income follows from the fact that they have a higher marginal utility of consumption and would thus like to consume more. This is also shown in literature. For example, Becker and Foias (1987) show that, in the long run, only the most patient households in the economy, save. Iacoviello and Neri (2010) and Alpanda and Zubairy (2016) also assume different budget constraints for patient and impatient households.

households do not work and accumulate enough wealth to fulfill their consumption needs. Impatient households, on the other hand, supply inelastic labor that we normalize to one.

### Patient Households

Patient households choose the optimal level of consumption  $c_{P,t}$  to maximize

$$\max E_t \sum_{t=0}^{\infty} \beta_P^t \ln c_{P,t},$$

subject to

$$c_{P,t} + d_{t+1} + k_{t+1} - (1 - \delta) k_t = (1 + r_t^d) d_t + r_{k,t} k_t - \tau_{k,t} (r_{k,t} - \delta) k_t, \quad (2.1)$$

where  $k_t$  is the level of capital stock at the beginning of the period,  $d_t$  is the amount lent to impatient households,  $\delta$  represents the rate of depreciation and  $\tau_{k,t}$  is the capital income tax rate.<sup>2</sup> Further  $r_t^d$  and  $r_{k,t}$  represent the rate of return on private lending to impatient households and physical capital, respectively.

We derive the following first order conditions for the above problem:

$$\frac{1}{\beta_P} \left( \frac{c_{P,t+1}}{c_{P,t}} \right) = 1 + r_{t+1}^d, \quad (2.2)$$

$$r_{t+1}^d = (1 - \tau_{k,t+1}) (r_{t+1}^k - \delta). \quad (2.3)$$

Equation 2.2 is the standard Euler equation which highlights the relationship between intertemporal consumption choices of the patient households and the interest rate. Equation 2.3 represents the no arbitrage condition between the lending rate  $r_t^d$  and the net return on capital.

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<sup>2</sup>We do not tax returns on private lending, as that imposes certain restrictions on the tax rate to have a stable decentralized equilibrium. Refer to section A of the appendix for more details.

## Impatient Households

Similar to the patient households, the impatient households maximize their utility

$$\max E_t \sum_{t=0}^{\infty} \beta_I^t \ln c_{I,t},$$

subject to

$$c_{I,t} + (1 + r_t^d) d_t = w_t + d_{t+1} + T_t, \quad (2.4)$$

where  $c_{I,t}$  is consumption,  $d_t$  is the level of borrowing from the patient households,  $w_t$  is the wage,  $r_t^d$  is the interest rate on borrowings and  $T_t$  represents lump sum transfers (when positive) or taxes (when negative) from/ to the government. Further, impatient households' level of borrowing is subject to a borrowing constraint. In the spirit of simplicity, following Monacelli and Perotti (2011), we choose the constraint to be an exogenous, fixed borrowing limit with  $\bar{D} > 0$ :<sup>3</sup>

$$d_{t+1} \leq \bar{D}. \quad (2.5)$$

Solving the impatient households' maximization problem subject to their budget and borrowing constraints, we derive the following first order condition:

$$\left( \frac{1 - \mu_{I,t}}{\beta_I} \right) \left( \frac{c_{I,t+1}}{c_{I,t}} \right) = 1 + r_{t+1}^d. \quad (2.6)$$

Equation 2.6 gives the impatient households' Euler equation.  $\frac{\mu_{I,t}}{c_{I,t}}$  is the Lagrange multiplier associated with the borrowing constraint and  $\mu_{I,t}$  measures the value of marginally relaxing the borrowing constraint, in units of  $c_{I,t}$ . The borrowing constraint alters the equilibrium by introducing credit market imperfections. It puts a limit on the amount impatient households can borrow which results in a stable steady state and a positive level of consumption for both the households.

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<sup>3</sup>With  $\bar{D} = 0$ , the model will approach the natural debt limit where the impatient households will borrow against the discounted value of their future income.

In the case of a perfect capital market, with no borrowing constraint and no  $\mu_{I,t}$ , 2.6 would become

$$\frac{1}{\beta_I} \left( \frac{c_{I,t+1}}{c_{I,t}} \right) = 1 + r_{t+1}^d, \quad (2.7)$$

implying (from equating 2.7 and 2.2)

$$\frac{1}{\beta_I} = \frac{1}{\beta_P},$$

in steady state. With  $\beta_P > \beta_I$ , this is not true. Thus, in a model with heterogeneous patience and no borrowing constraint, a stable steady state does not exist. This is because, without a borrowing constraint, the impatient households trade future labor income with current consumption. They borrow against the future discounted value of their wages, leading their consumption to approach zero asymptotically.

### 2.2.2 Firms

A perfectly competitive firm hires labor from impatient households to produce goods and pays wages  $w_t$  to its employees. The production of the firms is given by a Cobb-Douglas production function

$$y_t = F(k_t, n_{I,t}) = k_t^\alpha n_{I,t}^{(1-\alpha)}, \quad (2.8)$$

where  $\alpha \in (0, 1)$  is the output elasticity of capital.

Since  $n_{I,t} = 1$  in equilibrium,  $f(k_t) = F(k_t, 1)$ .

The firm maximizes its profits and thus obtain the following conditions:

$$f'(k_t) = r_t^k, \quad (2.9)$$

$$f(k_t) - f'(k_t) k_t = w_t. \quad (2.10)$$

Equation 2.9 states that in equilibrium, marginal product of capital is equal

to the return on capital. Similarly, equation 2.10 states that in equilibrium, the marginal product of labor is equal to the wage paid.

### 2.2.3 Government

The government runs a balanced budget constraint with no debt. The inflows from current period's tax receipts on net returns on capital are used on an exogenous level of government expenditure,  $g$ , and lump sum transfers,  $T_t$ , to impatient households

$$g + T_t = \tau_{k,t} (r_t^k - \delta) k_t. \quad (2.11)$$

The government thus uses capital income tax as a tool of income redistribution between households.  $\tau_{k,t} > 0$  implies a tax incidence on patient households, the receipts of which are distributed to impatient households in the form of lump sum transfers. However, when  $\tau_{k,t} < 0$ , taxes are imposed on impatient households in lump sum and redistributed to patient households as subsidies on capital income.

### 2.2.4 Aggregate Resource Constraint

Finally, the aggregate resource constraint is as follows:

$$y_t = c_{P,t} + c_{I,t} + g + k_{t+1} - (1 - \delta) k_t. \quad (2.12)$$

## 2.3 Equilibrium (Decentralized Economy)

**Definition 2.1** (*Competitive Equilibrium*): A competitive equilibrium consists of government policies  $\{ \tau_{k,t}, T_t \}_{t=0}^{\infty}$ , prices  $\{ w_t, r_t^k, r_t^d, \mu_{I,t} \}_{t=0}^{\infty}$  and private sector allocations  $\{ c_{P,t}, k_{t+1}, d_{t+1}, c_{I,t}, y_t \}_{t=0}^{\infty}$ , satisfying:

- (i) private sector optimization taking government policies and prices as given, that is,



- the households' budget constraints 2.1 and 2.4, borrowing constraint 2.5, and optimality conditions 2.2, 2.3 and 2.6;
  - the production function 2.8, and firm's optimality conditions 2.9 and 2.10;
- (ii) market clearing condition 2.12 and;
- (iii) the government's budget constraint 2.11.

## 2.4 Steady State (Decentralized Economy)

**Proposition 2.1** *The borrowing constraint is always binding in steady state.*

**Proof.** Analyzing the two Euler equations, 2.2 and 2.6, in steady state

$$\begin{aligned} \left[ \frac{(1 - \mu_I) - \beta_I}{\beta_I} \right] &= \frac{1 - \beta_P}{\beta_P}, \\ \Rightarrow \mu_I &= 1 - \frac{\beta_I}{\beta_P}. \end{aligned}$$

Given our choice of  $\beta_I$  and  $\beta_P$  such that  $\beta_P > \beta_I$ ,  $\mu_I$  will be greater than zero in steady state for all  $\tau_k$ , ensuring that the borrowing constraint will always be binding.<sup>4</sup> ■

## 2.5 First Best Allocation

The social planner maximizes the following weighted utility function:

$$\omega \beta_P^t u(c_{P,t}) + (1 - \omega) \beta_I^t u(c_{I,t}), \quad (2.13)$$

where  $\omega$  is the weight assigned by the social planner to the patient households.

**Definition 2.2** (*First Best*): *The first-best equilibrium consists of allocations  $\{c_{P,t}, c_{I,t}, k_t\}_{t=0}^{\infty}$*

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<sup>4</sup>Refer to section B of the appendix for other steady state calculations.

that maximize 2.13, subject to the production function 2.8, and the market clearing condition 2.12.

The social planner's problem thus becomes,

$$\max_{\{c_{I,t}, c_{P,t}, k_{t+1}\}} \sum_{t=0}^{\infty} \beta_P^t \left[ \omega \ln c_{P,t} + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \ln c_{I,t} \right],$$

subject to

$$f(k_t) - c_{P,t} - c_{I,t} - g - k_{t+1} + (1 - \delta) k_t = 0.$$

The next crucial step is to determine the social planner's discount factor. We could either use the linear average weight of the two agents,  $\omega \beta_P^t + (1 - \omega) \beta_I^t$ , or the weight of an average agent,  $[\omega \beta_P + (1 - \omega) \beta_I]^t$ . Lengwiler (2005) shows that

$$\omega \beta_P^t + (1 - \omega) \beta_I^t > [\omega \beta_P + (1 - \omega) \beta_I]^t,$$

for  $t > 1$ . As  $t$  grows larger, the inequality gets stronger. This is because, in  $\omega \beta_P^t + (1 - \omega) \beta_I^t$ , the impatient households' weights keep falling with time and they assign almost negligible weights on distant future. In such a scenario, the average weight is thus dominated by the patient households' preferences. This implies that the discount factor is time varying and does not generate a geometric progression, as in the case of a representative agent model where the discount factor is  $\beta_P^t$ .<sup>5</sup>

Analyzing the two discount factors, we notice that, the discount rate (inverse of the discount factor) obtained from the average weight will be less than the discount rate obtained from the weight of an average agent. Weitzman (1998) and Gollier (2002) argue that while evaluating problems for very long horizons, the social planner should be conservative and should consider minimum growth by using the average discount factor which gives a lower discount rate. Moreover, a higher discount rate leads to a lower present value of the social welfare, implying that the derived outcome

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<sup>5</sup>Mitra (1979) discusses about a time varying discounted growth model.

is not the first best. We thus choose to use  $\omega\beta_P^t + (1 - \omega)\beta_I^t$  as the social planner's discount factor.

**Proposition 2.2** *As  $t \rightarrow \infty$ ,  $c_{I,t} \rightarrow 0$  will maximize the social welfare.*

**Proof.** Let  $\lambda_{F,t}$  be the Lagrange multiplier associated with the feasibility/ aggregate resource constraint. Maximizing, we get the following first order conditions:

$$\frac{\partial \mathcal{L}}{\partial c_{P,t}} : \omega c_{P,t}^{-1} = \lambda_{F,t} \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right], \quad (2.14)$$

$$\frac{\partial \mathcal{L}}{\partial c_{I,t}} : (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t c_{I,t}^{-1} = \lambda_{F,t} \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right]. \quad (2.15)$$

Equating 2.14 and 2.15 we get:

$$\left( \frac{1 - \omega}{\omega} \right) \left( \frac{\beta_I}{\beta_P} \right)^t c_{P,t} = c_{I,t},$$

where as  $t \rightarrow \infty$ ,  $\left( \frac{\beta_I}{\beta_P} \right)^t \rightarrow 0$ , implying  $c_{I,t} \rightarrow 0$ . ■

This result is consistent with the early conjecture of Ramsey (1928) that the most patient consumer will be the dominant consumer, while the impatient consumers will survive at the subsistence level. Eventually, in the long run, impatient households' consumption will reach zero. This result has also been shown in the existing literature (see for example, Becker (2012), Le Van et al (2007) and Goenka et al (2012)).

Further, on differentiating the social planner's problem with respect to capital, we derive:

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} : (f'(k_{t+1}) + 1 - \delta) = \frac{\lambda_{F,t} \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right]}{\lambda_{F,t+1} \beta_P \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^{t+1} \right]},$$

$$(f'(k_{t+1}) + 1 - \delta) = \frac{c_{I,t}^{-1}}{\beta_I c_{I,t+1}^{-1}}.$$

The literature analyzes the above equation (see for example, Le Van and Vailakis (2003) and Becker (2012)) and shows the following two main properties of optimal capital path in a model with many agents and varying degrees of patience. Since our first best allocation problem is subject to the aggregate resource constraint only, it is similar to the heterogeneous agent problem studied in the literature and these results apply here as well.

First, Le Van and Vailakis (2003) prove that with heterogeneous discount factors, the optimal capital path eventually approaches  $k^s$ -steady state level of capital from the representative agent model with only patient households. Here  $k^s$  is such that  $f'(k^s) + 1 - \delta = \frac{1}{\beta_P}$ . Mathematically, the heterogeneous agent model collapses to the representative agent model with  $c_{I,t} \rightarrow 0$  and  $(f'(k_{t+1}) + 1 - \delta) = \frac{c_{I,t}^{-1}}{\beta_I c_{I,t+1}^{-1}} \rightarrow \frac{1}{\beta_P}$ .

Second, even though the optimal sequence approaches  $k^s$ ,  $k^s$  is not the steady state of the model. As discussed in Le Van and Vailakis (2003), this property implies that if the optimal capital sequence starts from  $k^s$ , it will be optimal to initially deviate from  $k^s$  and asymptotically converge to  $k^s$  in the long run. This yields a non-monotonic capital path. Going a step further, Becker (2012) shows that the generated optimal capital accumulation path exhibits the turnpike property. According to this property, irrespective of initial capital levels and non-monotonicity of optimal capital path, as time approaches infinity, all optimal capital sequences become monotonic and asymptotically converge to  $k^s$ .

The aim of our paper, however, is to solve the optimal level of capital taxation and we thus focus on the second best problem or the Ramsey planner's problem.

## 2.6 Ramsey Optimal Taxation

Unlike the social planner, the Ramsey planner aims to maximize the social welfare, given the decentralized economy's equilibrium conditions.

Substituting 2.2 and 2.3 in 2.1, we derive the patient household's implementabil-

ity constraint:

$$\beta_P + \beta_P c_{P,t}^{-1} (d_{t+1} + k_{t+1}) - c_{P,t-1}^{-1} (d_t + k_t) = 0. \quad (2.16)$$

Further, equating 2.2 and 2.6, we get:

$$(1 - \mu_{I,t-1}) \beta_P c_{I,t-1}^{-1} c_{P,t}^{-1} - \beta_I c_{I,t}^{-1} c_{P,t-1}^{-1} = 0. \quad (2.17)$$

The Ramsey planner's problem could thus be defined as:

**Definition 2.3** (*Ramsey Problem*): *Ramsey policy maker's problem is to maximize 2.13, choosing  $\{\tau_{k,t}\}_{t=0}^{\infty}$  and allocations  $\{c_{P,t}, c_{I,t}, k_t, \mu_{I,t}\}_{t=0}^{\infty}$  subject to 2.16, 2.17, 2.12 and 2.5 with  $\lambda_{I,t}$ ,  $\lambda_{E,t}$ ,  $\lambda_{F,t}$  and  $u_{cp,t} \lambda_{B,t}$  being the Lagrange multipliers on the implementability constraint, equation equating the two Euler equations, feasibility constraint and borrowing constraint respectively.*<sup>6</sup>

$$\max_{\{c_{I,t}, c_{P,t}, k_{t+1}, d_{t+1}, \mu_{I,t}\}} \sum_{t=0}^{\infty} \beta_P^t \left[ \omega \ln c_{P,t} + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \ln c_{I,t} \right],$$

subject to

$$\beta_P + \beta_P c_{P,t}^{-1} (d_{t+1} + k_{t+1}) - c_{P,t-1}^{-1} (d_t + k_t) = 0,$$

$$(1 - \mu_{I,t-1}) \beta_P c_{I,t-1}^{-1} c_{P,t}^{-1} - \beta_I c_{I,t}^{-1} c_{P,t-1}^{-1} = 0,$$

$$f(k_t) - c_{P,t} - c_{I,t} - g - k_{t+1} + (1 - \delta) k_t = 0,$$

$$d_{t+1} \leq \bar{D}.$$

As argued in the case of a social planner, we use the linear weighted average of the two discount factors,  $\omega \beta_P^t + (1 - \omega) \beta_I^t$ , as the Ramsey planner's discount factor.

The Lagrange multiplier  $\frac{\lambda_{B,t}}{c_{P,t}}$  helps the shadow price  $\lambda_{B,t}$ , measure the value, in

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<sup>6</sup>Note that from Walras' Law, if all but one markets clear in the economy, the last one will clear too. Following the law, and keeping the analysis similar to Straub and Werning (2014), we drop impatient household's budget constraint while setting the Ramsey problem.

units of  $c_{P,t}$ , of marginally relaxing the borrowing constraint. Further, the borrowing constraint is binding, that is, it holds with equality whenever  $\lambda_{B,t} > 0$ , and is not binding when  $\lambda_{B,t} = 0$ .

Note that there is a time change—  $t = 0$  and  $t > 0$ . At  $t = 0$ ,  $k_0$  is given and there is no need to impose the Euler equation. The planner thus maximizes with respect to  $c_{P,-1}$ , leading to  $\lambda_{I,0} = 0$ . This derivation is detailed in section C of the appendix. Further, on simplification, maximizing with respect to  $\mu_{I,t}$ , we find that varying  $\mu_{I,t}$  imposes no opportunity costs on the government. Given the Ramsey planner's optimal choice for  $c_{P,t}$ ,  $c_{I,t}$  and  $k_t$ , the value of  $\mu_{I,t}$  will adjust accordingly to achieve the desired levels. The problem thus reduces to

$$\begin{aligned} \mathcal{L} = & E_0 \sum_{t=0}^{\infty} \beta_P^t \left[ \omega \ln c_{P,t} + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \ln c_{I,t} \right] \\ & + \lambda_{I,t} \beta_P^t \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right] [\beta_P + \beta_P c_{P,t}^{-1} (d_{t+1} + k_{t+1}) - c_{P,t-1}^{-1} (d_t + k_t)] \\ & + \lambda_{F,t} \beta_P^t \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right] [f(k_t) - c_{P,t} - c_{I,t} - g - k_{t+1} + (1 - \delta) k_t] \\ & + \lambda_{B,t} c_{P,t}^{-1} \beta_P^t \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right] [\bar{D} - d_{t+1}]. \end{aligned}$$

The necessary first order conditions are:

$$\lambda_{I,0} = 0,$$

$$\lambda_{B,t} \geq 0,$$

$$c_{P,t}^{-1} \beta_P \left\{ \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right] \lambda_{I,t} - \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^{t+1} \right] \lambda_{I,t+1} \right\} = \lambda_{B,t} c_{P,t}^{-1} \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right], \quad (2.18)$$

$$(1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t c_{I,t}^{-1} = \lambda_{F,t} \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right], \quad (2.19)$$

$$\lambda_{I,t+1} \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^{t+1} \right] = \lambda_{I,t} \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right] + \frac{(1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t c_{I,t}^{-1} - \omega c_{P,t}^{-1}}{\beta_P c_{P,t}^{-2} (d_{t+1} + k_{t+1})}, \quad (2.20)$$

$$(f'(k_{t+1}) + 1 - \delta) = \frac{c_{I,t+1}}{\beta_P \beta_I c_{I,t}} \left\{ 1 - (1 - \beta_P) \left( \frac{\beta_P}{\beta_I} \right)^t \left( \frac{\omega}{1 - \omega} \right) \frac{c_{I,t}}{c_{P,t}} \right\}. \quad (2.21)$$

Analyzing the necessary optimal conditions, we observe that given a value of  $\omega$ , equation 2.20 can generate one of the three possible scenarios:

- **Case 1:** Redistribution preference towards impatient households;  $(1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t c_{I,t}^{-1} > \omega c_{P,t}^{-1}$
- **Case 2:** Indifference between redistribution;  $(1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t c_{I,t}^{-1} = \omega c_{P,t}^{-1}$
- **Case 3:** Redistribution preference towards patient households;  $(1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t c_{I,t}^{-1} < \omega c_{P,t}^{-1}$

and discuss each of the three possibilities in detail.<sup>7</sup>

### 2.6.1 Redistribution Towards Impatient Households

**Proposition 2.3** *A welfare maximizing Ramsey allocation does not exist when*

$$(1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t c_{I,t}^{-\sigma} > \omega c_{P,t}^{-\sigma}.$$

**Proof.** Ramsey planner has a redistribution preference towards impatient households when the weighted, discounted marginal utility of impatient households is greater than the weighted, discounted marginal utility of patient households; that is,  $(1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t c_{I,t}^{-1} > \omega c_{P,t}^{-1}$ . As  $t \rightarrow \infty$ ,  $\left( \frac{\beta_I}{\beta_P} \right)^t \rightarrow 0$ . For  $(1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t c_{I,t}^{-1} > \omega c_{P,t}^{-1}$  to hold as  $t \rightarrow \infty$ ,  $c_{I,t}$  will become negative—a contradiction!

Further, assume  $(1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t c_{I,t}^{-1} > \omega c_{P,t}^{-1}$ . With  $\lambda_{I,0} = 0$ , from 2.20,  $\lambda_{I,1} > 0$ . Substituting  $\lambda_{I,1} > 0$  in 2.18, we get  $\lambda_{B,0} < 0$ . This defies the Kuhn-Tucker condition according to which  $\lambda_{B,t} \geq 0$ . Hence, for a solution to exist in the presence of the borrowing constraint,  $(1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t c_{I,t}^{-1} \leq \omega c_{P,t}^{-1}$ .<sup>8</sup> ■

<sup>7</sup>Refer to section D of the appendix for the derivation of equation 21.

<sup>8</sup>Refer to section E of the appendix for a formal proof of the violation of the Kuhn-Tucker condition.

Next, we go on to the case when the Ramsey planner is indifferent between redistribution between households, that is, he values the weighted discounted marginal utilities of both the households equally.

## 2.6.2 Indifference Between Redistribution

**Proposition 2.4** *When  $(1 - \omega) \left(\frac{\beta_I}{\beta_P}\right)^t c_{I,t}^{-1} = \omega c_{P,t}^{-1}$ , the borrowing constraint is not binding and the optimal capital taxation approaches zero in the distant long run.*

**Proof.** *Let  $(1 - \omega) \left(\frac{\beta_I}{\beta_P}\right)^t c_{I,t}^{-1} = \omega c_{P,t}^{-1}$ , implying that the Ramsey planner is indifferent to redistribution between the two household types. With  $\lambda_{I,0} = 0$ , from 2.20,  $\lambda_{I,1} = 0$ . Further,  $\lambda_{I,t} = 0$  for all  $t$ . Substituting  $\lambda_{I,t} = 0$  in 2.18, we get  $\lambda_{B,t} = 0$  for all  $t$  and the borrowing constraint is never binding. In such a scenario, 2.21 becomes*

$$(f'(k_{t+1}) + 1 - \delta) = \frac{c_{I,t+1}}{\beta_I c_{I,t}},$$

*and the Ramsey planner's problem reduces to the social planner's problem. As discussed in Section 5, as  $t \rightarrow \infty$ ,  $f'(k_{t+1}) + 1 - \delta \rightarrow \frac{1}{\beta_P}$  implying that in the long run, marginal product of capital equals the return on capital and the optimal level of capital tax rate approaches zero. ■*

Intuitively, in the long run, with  $(1 - \omega) \left(\frac{\beta_I}{\beta_P}\right)^t c_{I,t}^{-1} = \omega c_{P,t}^{-1}$ , as  $t \rightarrow \infty$ ,  $\left(\frac{\beta_I}{\beta_P}\right)^t \rightarrow 0$ , implying  $c_{I,t} \rightarrow 0$ . The heterogeneous agent model thus collapses to the representative agent model. The socially optimal level of capital and patient household's capital savings eventually, become equal and the optimal capital tax rate approaches zero. With a higher weight on patient households, that is, with a higher  $\omega$ , the tax rate will approach zero faster. On the contrary, with a lower  $\omega$ , that tax rate will take longer to approach zero.

Finally, we discuss the case of redistribution preference towards patient households.



### 2.6.3 Redistribution Towards Patient Households

**Proposition 2.5** *When  $(1 - \omega) \left(\frac{\beta_I}{\beta_P}\right)^t c_{I,t}^{-1} < \omega c_{P,t}^{-1}$ , the borrowing constraint is always binding and the optimal level of capital taxation is negative.*

**Proof.** *Let  $(1 - \omega) \left(\frac{\beta_I}{\beta_P}\right)^t c_{I,t}^{-1} < \omega c_{P,t}^{-1}$ . With  $\lambda_{I,0} = 0$ , from 2.20,  $\lambda_{I,1} < 0$ . Further,  $\lambda_{1,t+1} < \lambda_{1,t}$  for all  $t$ . Substituting  $\lambda_{1,t+1} < \lambda_{1,t}$  in 2.18, we get  $\lambda_{B,t} > 0$  for all  $t$ . Hence the borrowing constraint will always be binding. Using 2.3, the optimal tax rate on net return to wealth is given by*

$$\tau_{k,t+1} = 1 - \frac{r_{t+1}^d}{(r_{t+1}^k - \delta)} = 1 - \frac{r_{t+1}^d}{[f'(k_{t+1}) - \delta]}.$$

Using 2.2 and 2.21, the above equation can be rewritten as

$$\tau_{k,t+1} = 1 - \frac{\frac{1}{\beta_P} \left(\frac{c_{P,t+1}}{c_{P,t}}\right) - 1}{\frac{c_{I,t+1}}{\beta_P \beta_I c_{I,t}} \left\{ 1 - (1 - \beta_P) \left(\frac{\beta_P}{\beta_I}\right)^t \left(\frac{\omega}{1-\omega}\right) \frac{c_{I,t}}{c_{P,t}} \right\} - 1}. \quad (2.22)$$

Alternatively, the optimal tax rate on the gross return to wealth is

$$1 - \frac{\frac{1}{\beta_P} \left(\frac{c_{P,t+1}}{c_{P,t}}\right)}{\frac{c_{I,t+1}}{\beta_P \beta_I c_{I,t}} \left\{ 1 - (1 - \beta_P) \left(\frac{\beta_P}{\beta_I}\right)^t \left(\frac{\omega}{1-\omega}\right) \frac{c_{I,t}}{c_{P,t}} \right\}}.$$

From the representative agent's model, we know that as  $t \rightarrow \infty$ ,  $\frac{1}{\beta_P} \left(\frac{c_{P,t+1}}{c_{P,t}}\right) \rightarrow \frac{1}{\beta_P}$ . Also, from the social planner's problem, we know,  $\frac{c_{I,t+1}}{\beta_I c_{I,t}} \rightarrow \frac{1}{\beta_P}$  with time. Thus in the long run, wealth tax approaches

$$1 - \frac{\beta_P}{\left\{ 1 - (1 - \beta_P) \left(\frac{\beta_P}{\beta_I}\right)^t \left(\frac{\omega}{1-\omega}\right) \frac{c_{I,t}}{c_{P,t}} \right\}}.$$

Further,  $(1 - \omega) \left(\frac{\beta_I}{\beta_P}\right)^t c_{I,t}^{-1} < \omega c_{P,t}^{-1}$  implies that:

$$\begin{aligned} &\Rightarrow (1 - \beta_P) \left(\frac{\beta_P}{\beta_I}\right)^t \left(\frac{\omega}{1 - \omega}\right) \frac{c_{I,t}}{c_{P,t}} > (1 - \beta_P), \\ &\Rightarrow 1 - \left[ (1 - \beta_P) \left(\frac{\beta_P}{\beta_I}\right)^t \left(\frac{\omega}{1 - \omega}\right) \frac{c_{I,t}}{c_{P,t}} \right] < \beta_P \text{ for all } t. \end{aligned}$$

In such a scenario,  $\frac{\beta_P}{\left\{1 - (1 - \beta_P) \left(\frac{\beta_P}{\beta_I}\right)^t \left(\frac{\omega}{1 - \omega}\right) \frac{c_{I,t}}{c_{P,t}}\right\}} > 1$ , implying a negative wealth tax. ■

Unlike the previous two cases,  $(1 - \omega) \left(\frac{\beta_I}{\beta_P}\right)^t c_{I,t}^{-1} < \omega \beta_P^t c_{P,t}^{-1}$  implies  $c_{I,t} > 0$  for all  $t$ . Since impatient households consume their wages, a positive function of capital, the socially optimal level of capital in such a circumstance will be higher than the one derived from the representative agent problem.

Moreover, when the borrowing constraint is binding, the impatient households borrow  $\bar{D}$  amount of loans from the patient households. With the maximum amount of borrowing from the impatient households, the return on private lending is higher than the return on physical capital, creating an arbitrage opportunity for the patient households. Thus, in order to encourage patient households to save more, the Ramsey planner will subsidize returns on capital—a negative tax rate!

The results derived from the second best allocation exercise can be seen as an application of Chamley (2001) who shows that if consumption and savings are negatively related, in the presence of a binding borrowing constraint, it would be optimal to have the returns on capital subsidized in the long run. He does not, however, make any claim on the rate of subsidization. Similar to our impatient households who are poor, consume less in the long run and have a negative relationship between consumption and savings (or positive relationship between consumption and dissavings, here borrowing), in Chamley (2001) the "very old" are poor and consume less in distant time periods and, hence, generate the negative correlation between consumption and savings. Moreover, the results obtained, could also be concluded to have preserved Ramsey's conjecture, according to which, the patient households

are the dominant consumers and enjoy bliss in the long run.

## 2.7 Conclusion

In our paper, we consider the case of heterogeneous agents who differ in their rates of time preference—impatient households value today’s consumption more than the patient households. To increase their current level of consumption, they even borrow from the patient households as per a borrowing constraint. We study the optimal level of capital income taxation in such a scenario. The results show that if the Ramsey planner is indifferent between redistribution towards patient and impatient households, the optimal level of capital taxation will approach zero in the long run. Further, when the borrowing constraint is binding, the capital income tax rate is not zero in the long run. On the contrary, the Ramsey planner chooses to give subsidies to patient households. These subsidies are funded by transfers from impatient households who anyway do not favour distant future much.

Although the derived results are welfare maximizing, they are not pareto improving. The increase in the utility of patient households comes at the cost of impatient households’ utility. It will thus be interesting to derive the optimal level of pareto improving capital taxation. As mentioned earlier, it will also be interesting to check the validity and stability of our results with different utility functions. For example, Kemp et al (1993) questions the stability of Judd (1985) results and concludes that in certain cases, cycles around steady state could establish. Straub and Werning (2014) shows that such a case exists when the intertemporal elasticity of substitution is greater than one and there is redistribution preference towards capitalists (here, patient households). It is thus intriguing to check if different discount factors and/or the borrowing constraint can stop the formation of such cycles.

For future research purposes, it will be enthralling to conduct a similar study in an open economy setup and analyze the welfare optimizing allocations. With the widely debated concept of the "Global Savings Glut," it will be riveting to theoret-

ically capture how in the presence of patient countries (like China) and impatient ones (like US) global interest rates are affected and how international organizations could intervene to achieve an overall welfare optimization. As a starting point, Leff and Sato (1993) document the existence of international differences in savings behavior across countries.

Analyzing the same problem in the presence of other market imperfections, say for example price rigidity, and checking if our results still hold would make another interesting study. Monacelli and Perotti (2011) for example, show that in the presence of sticky prices, government spending multiplier is larger when taxes are levied on patient households instead of favouring a transfer from impatient to patient households. But does the result hold with respect to capital income taxation as well? These and many other such issues remain a few potential subjects for related future work.

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## 2.A Tax on Private Lending Returns

If we tax the returns on private lending,  $\tau_{d,t}$ , patient household's budget constraint becomes

$$c_{P,t} + d_{t+1} + k_{t+1} - (1 - \delta) k_t = (1 + r_t^d) d_t - \tau_{d,t} r_t^d d_t + r_{k,t} k_t - \tau_{k,t} (r_{k,t} - \delta) k_t. \quad (2.23)$$

Solving the patient household's problem, in such a scenario, we derive the first order conditions to be

$$\frac{1}{\beta_P} \left( \frac{c_{P,t+1}}{c_{P,t}} \right) = 1 + (1 - \tau_{d,t+1}) r_{t+1}^d, \quad (2.24)$$

and

$$(1 - \tau_{d,t+1}) r_{t+1}^d = (1 - \tau_{k,t+1}) (r_{t+1}^k - \delta). \quad (2.25)$$

Equation 2.24 gives the Euler equation, which is now distorted by the tax on private lending. Equation 2.25 states the no arbitrage condition between the two assets— physical capital and private lending.

Impatient household's problem remains the same. We now analyze the competitive equilibrium of such an economy. If we tax the returns on private lending, we need to impose restrictions on the tax rate to have a stable decentralized equilibrium. That is, in steady state for the Euler equations of the two households to be equal (refer to equations 2.24 and 2.6),

$$\begin{aligned} \left[ \frac{(1 - \mu_I) - \beta_I}{\beta_I} \right] &= \frac{1 - \beta_P}{\beta_P (1 - \tau_d)}; \\ \Rightarrow (1 - \mu_I) &= \beta_I \left[ \frac{1 - \beta_P}{\beta_P (1 - \tau_d)} \right] + \beta_I; \\ \Rightarrow \mu_I &= 1 - \frac{\beta_I (1 - \beta_P \tau_d)}{\beta_P (1 - \tau_d)}. \end{aligned}$$

Given our choice of  $\beta_I$  and  $\beta_P$  such that  $\beta_P > \beta_I$  and to ensure that in steady

state,  $\mu_I \geq 0$ ,  $\tau_d \leq \frac{\beta_P - \beta_I}{\beta_P(1 - \beta_I)}$ .

Further, we show that even if we consider this limit on private lending taxation, our result of Ramsey optimal negative capital income taxation in the long run, in the presence of a binding borrowing constraint, doesn't change. To prove this, we substitute equation 2.24 and equation 2.25 in equation 2.23 and derive the implementability constraint as follows

$$\beta_P + \beta_P c_{P,t}^{-1} (d_{t+1} + k_{t+1}) - c_{P,t-1}^{-1} (d_t + k_t) = 0.$$

This is similar to equation 2.16. The Ramsey Planner's problem thus remain the same, resulting same set of first order condition. The two cases of redistribution preference towards impatient households and indifference between the two households will remain the same. So will the case of a redistribution preference towards patient households. To show that, we consider this case in detail. From equation 2.25

$$\tau_{k,t+1} = 1 - \frac{r_{t+1}^d}{(r_{t+1}^k - \delta)} = 1 - \frac{(1 - \tau_{d,t+1}) r_{t+1}^d}{[f'(k_{t+1}) - \delta]}.$$

Using 2.24 and 2.21, the above equation can be rewritten as

$$\tau_{k,t+1} = 1 - \frac{\frac{1}{\beta_P} \left( \frac{c_{P,t+1}}{c_{P,t}} \right) - 1}{\frac{c_{I,t+1}}{\beta_P \beta_I c_{I,t}} \left\{ 1 - (1 - \beta_P) \left( \frac{\beta_P}{\beta_I} \right)^t \left( \frac{\omega}{1 - \omega} \right) \frac{c_{I,t}}{c_{P,t}} \right\} - 1},$$

which is similar to equation 2.22. The rest of the derivation hence follows. Thus, our result of negative capital income taxation in the distant long run, with a binding borrowing constraint, holds even when  $\tau_{d,t} > 0$ .

Intuitively, with taxation on private lending, in equilibrium,  $r_t^d$ , the private lending interest rate, goes up. Thus, given the marginal product of physical capital (given by equation 2.21), for there to be no arbitrage opportunity between the two asset types, the Ramsey planner will choose to subsidize (or impose negative taxation) on capital returns. This result is driven by equation 2.21, which uses the

Ramsey planner's discount factor as the linear weighted average of the two discount factors. In the long run, this implies a transfer from impatient households, who do not value future consumption as much, to patient households.

## 2.B Decentralized Equilibrium Steady State

From Section 4, we know that the borrowing constraint is binding in steady state. This implies that  $d = \bar{D}$  in steady state. Solving further, we derive the following as the decentralized steady state equilibrium of the model:

$$\begin{aligned} \frac{1 - \beta_P}{\beta_P} &= r^d, \\ \frac{1 - \beta_P}{\beta_P(1 - \tau_k)} + \delta &= r^k, \\ \left[ \frac{\alpha\beta_P(1 - \tau_k)}{1 - \beta_P + \delta\beta_P(1 - \tau_k)} \right]^{\frac{1}{1-\alpha}} &= k, \\ \left[ \frac{\alpha\beta_P(1 - \tau_k)}{1 - \beta_P + \delta\beta_P(1 - \tau_k)} \right]^{\frac{\alpha}{1-\alpha}} &= y, \\ (1 - \alpha) \left[ \frac{\alpha\beta_P(1 - \tau_k)}{1 - \beta_P + \delta\beta_P(1 - \tau_k)} \right]^{\frac{\alpha}{1-\alpha}} &= w, \\ \left( \frac{1 - \beta_P}{\beta_P} \right) \left[ \bar{D} + \left\{ \frac{\alpha\beta_P(1 - \tau_k)}{1 - \beta_P + \delta\beta_P(1 - \tau_k)} \right\}^{\frac{1}{1-\alpha}} \right] &= c_P, \\ \left[ \frac{\alpha\beta_P(1 - \tau_k)}{1 - \beta_P + \delta\beta_P(1 - \tau_k)} \right]^{\frac{\alpha}{1-\alpha}} \left[ 1 - \frac{\alpha(1 - \tau_k)(1 - \beta_P + \delta\beta_P)}{1 - \beta_P + \delta\beta_P(1 - \tau_k)} \right] - g - \left( \frac{1 - \beta_P}{\beta_P} \right) \bar{D} &= c_I. \end{aligned}$$

## 2.C Ramsey Planner's Problem

For any  $(k_0, d_0, c_{P,-1}, c_{I,-1}, \mu_{I,-1})$  we define the value function  $V(k_0, d_0, c_{P,-1}, c_{I,-1}, \mu_{I,-1})$  as:

$$V(k_0, d_0, c_{P,-1}, c_{I,-1}, \mu_{I,-1}) = \max \sum_{t=0}^{\infty} \beta_P^t \left[ \omega \ln c_{P,t} + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \ln c_{I,t} \right] \text{ s.t.}$$

$$\begin{aligned}\beta_P + \beta_P c_{P,t}^{-1} (d_{t+1} + k_{t+1}) &= c_{P,t-1}^{-1} (d_t + k_t), \\ (1 - \mu_{I,t-1}) \beta_P c_{I,t-1}^{-1} c_{P,t}^{-1} &= \beta_I c_{I,t}^{-1} c_{P,t-1}^{-1}, \\ F(k_t) + (1 - \delta) k_t &= c_{P,t} + c_{I,t} + g + k_{t+1}.\end{aligned}$$

Using Envelope condition:

$$\begin{aligned}V_{c_{P,-1}}(k_0, d_0, c_{P,-1}, c_{I,-1}, \mu_{I,-1}) &= 0 = \frac{\partial \mathcal{L}}{\partial c_{P,-1}}; \\ \Rightarrow \lambda_{I,0} c_{P,-1}^{-2} (d_0 + k_0) + \lambda_{E,0} c_{P,-1}^{-2} \beta_I c_{I,0}^{-1} &= 0.\end{aligned}\tag{2.26}$$

Similarly,

$$\begin{aligned}V_{\mu_{I,-1}}(k_0, d_0, c_{P,-1}, c_{I,-1}, \mu_{I,-1}) &= 0 = \frac{\partial \mathcal{L}}{\partial \mu_{I,-1}}; \\ \Rightarrow -\lambda_{E,0} \beta_P c_{I,-1}^{-1} c_{P,0}^{-1} &= 0; \\ \Rightarrow \lambda_{E,0} &= 0.\end{aligned}\tag{2.27}$$

Substituting 2.27 in 2.26, we get  $\lambda_{I,0} = \lambda_{E,0} = 0$ . Further, we solve the Ramsey planner's welfare problem.

$$\frac{\partial \mathcal{L}}{\partial \mu_{I,t}} : \lambda_{E,t+1} \beta_P^{t+1} \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^{t+1} \right] c_{I,t}^{-1} c_{P,t+1}^{-1} = 0.$$

With  $\lambda_{E,0} = 0$  and the above FOC,  $0 = \lambda_{E,1} = \lambda_{E,2} = \lambda_{E,3} = \lambda_{E,4} = \dots$ . This reflects the fact that varying  $\mu_{I,t}$  imposes no opportunity costs on the government. Given the Ramsey planner's optimal choice for  $c_{P,t}$ ,  $c_{I,t}$  and  $k_t$ , the value of  $\mu_{I,t}$  will

adjust accordingly to achieve the desired levels. The problem thus reduces to

$$\begin{aligned}
\mathcal{L} = & E_0 \sum_{t=0}^{\infty} \beta_P^t \left[ \omega \ln c_{P,t} + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \ln c_{I,t} \right] \\
& + \lambda_{I,t} \beta_P^t \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right] [\beta_P + \beta_P c_{P,t}^{-1} (d_{t+1} + k_{t+1}) - c_{P,t-1}^{-1} (d_t + k_t)] \\
& + \lambda_{F,t} \beta_P^t \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right] [f(k_t) - c_{P,t} - c_{I,t} - g - k_{t+1} + (1 - \delta) k_t] \\
& + \lambda_{B,t} c_{P,t}^{-1} \beta_P^t \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right] [\bar{D} - d_{t+1}].
\end{aligned}$$

## 2.D Derivation of 2.21

Maximizing with respect to capital:

$$\begin{aligned}
& \lambda_{F,t+1} \beta_P \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^{t+1} \right] (f'(k_{t+1}) + 1 - \delta) \\
= & \lambda_{F,t} \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right] + c_{P,t}^{-1} \left\{ \begin{array}{l} \beta_P \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^{t+1} \right] \lambda_{I,t+1} \\ - \beta_P \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right] \lambda_{I,t} \end{array} \right\}.
\end{aligned}$$

From 2.20, this can be written as:

$$\begin{aligned}
& \lambda_{F,t+1} \beta_P \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^{t+1} \right] (f'(k_{t+1}) + 1 - \delta) \\
= & \lambda_{F,t} \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right] + \frac{(1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t c_{P,t} - \omega c_{I,t}}{c_{I,t} (d_{t+1} + k_{t+1})}.
\end{aligned}$$

Substituting 2.19,

$$\begin{aligned}
& \frac{\lambda_{F,t+1}}{\lambda_{F,t}} (f'(k_{t+1}) + 1 - \delta) \\
= & \frac{\left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right]}{\beta_P \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^{t+1} \right]} \\
& + \left( \frac{\beta_P}{\beta_I} \right)^t \frac{c_{I,t}}{(1 - \omega)} \left[ \frac{(1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t c_{P,t} - \omega c_{I,t}}{\beta_P c_{I,t} (d_{t+1} + k_{t+1})} \right] \frac{\left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right]}{\left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^{t+1} \right]}.
\end{aligned}$$

Since the utility function is logarithmic, there is a constant savings rate such that  $c_{P,t} = (1 - \beta_P) (1 + r_t^d) (d_{t+1} + k_{t+1})$  and  $(d_{t+1} + k_{t+1}) = \beta_P (1 + r_t^d) (d_t + k_t)$ .

$$\Rightarrow (d_{t+1} + k_{t+1}) = \frac{\beta_P}{(1 - \beta_P)} c_{P,t}.$$

Substituting,

$$\begin{aligned}
& \frac{\lambda_{F,t+1}}{\lambda_{F,t}} (f'(k_{t+1}) + 1 - \delta) \\
= & \frac{\left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right]}{\beta_P \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^{t+1} \right]} \\
& + \left( \frac{\beta_P}{\beta_I} \right)^t \left( \frac{1 - \beta_P}{1 - \omega} \right) \left[ \frac{(1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t c_{P,t} - \omega c_{I,t}}{\beta_P^2 c_{P,t}} \right] \frac{\left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right]}{\left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^{t+1} \right]}.
\end{aligned}$$

$$\begin{aligned}
& (f'(k_{t+1}) + 1 - \delta) \\
= & \frac{\lambda_{F,t}}{\lambda_{F,t+1}} \frac{\left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right]}{\beta_P \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^{t+1} \right]} \left\{ 1 + \left( \frac{\beta_P}{\beta_I} \right)^t \left( \frac{1 - \beta_P}{\beta_P} \right) \left[ \left( \frac{\beta_I}{\beta_P} \right)^t - \left( \frac{\omega}{1 - \omega} \right) \frac{c_{I,t}}{c_{P,t}} \right] \right\}, \\
= & \frac{\lambda_{F,t}}{\lambda_{F,t+1}} \frac{\left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right]}{\beta_P \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^{t+1} \right]} \left\{ 1 + \left( \frac{1 - \beta_P}{\beta_P} \right) - \left( \frac{\beta_P}{\beta_I} \right)^t \left( \frac{1 - \beta_P}{\beta_P} \right) \left( \frac{\omega}{1 - \omega} \right) \frac{c_{I,t}}{c_{P,t}} \right\}, \\
= & \frac{\lambda_{F,t}}{\lambda_{F,t+1}} \frac{\left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right]}{\beta_P \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^{t+1} \right]} \left\{ \frac{1}{\beta_P} - \left( \frac{\beta_P}{\beta_I} \right)^t \left( \frac{1 - \beta_P}{\beta_P} \right) \left( \frac{\omega}{1 - \omega} \right) \frac{c_{I,t}}{c_{P,t}} \right\}, \\
= & \frac{\lambda_{F,t}}{\lambda_{F,t+1}} \frac{\left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^t \right]}{\beta_P^2 \left[ \omega + (1 - \omega) \left( \frac{\beta_I}{\beta_P} \right)^{t+1} \right]} \left\{ 1 - (1 - \beta_P) \left( \frac{\beta_P}{\beta_I} \right)^t \left( \frac{\omega}{1 - \omega} \right) \frac{c_{I,t}}{c_{P,t}} \right\}.
\end{aligned}$$

From 2.19,

$$\begin{aligned}
(f'(k_{t+1}) + 1 - \delta) &= \frac{\left( \frac{\beta_I}{\beta_P} \right)^t c_{I,t}^{-1}}{\beta_P^2 \left( \frac{\beta_I}{\beta_P} \right)^{t+1} c_{I,t+1}^{-1}} \left\{ 1 - (1 - \beta_P) \left( \frac{\beta_P}{\beta_I} \right)^t \left( \frac{\omega}{1 - \omega} \right) \frac{c_{I,t}}{c_{P,t}} \right\}, \\
&= \frac{c_{I,t+1}}{\beta_P \beta_I c_{I,t}} \left\{ 1 - (1 - \beta_P) \left( \frac{\beta_P}{\beta_I} \right)^t \left( \frac{\omega}{1 - \omega} \right) \frac{c_{I,t}}{c_{P,t}} \right\},
\end{aligned}$$

which gives us 2.21.

## 2.E Kuhn-Tucker Conditions

As per the Kuhn-Tucker theorem, if the objective function and inequality constraint are at least once continuously differentiable and are concave on an open convex domain, there exists a solution to the maximization problem iff there exists  $\lambda_{B,t}$  such that the following Kuhn-Tucker conditions are satisfied:

1.  $\frac{\partial \mathcal{L}}{\partial x} = 0$

$$2. \lambda_{B,t} \geq 0$$

$$3. \text{ Complementary Slackness Condition: } \lambda_{B,t} (\bar{D} - d_{t+1}) = 0$$

$$4. \bar{D} - d_{t+1} \geq 0$$

In our model, the Hessian matrix of the objective function,  $\omega\beta_P^t \ln c_{P,t} + (1 - \omega)\beta_I^t \ln c_{I,t}$ , is given as follows:

$$H = \begin{bmatrix} -\beta_P^t \omega c_{P,t}^{-2} & 0 \\ 0 & -\beta_I^t (1 - \omega) c_{P,t}^{-2} \end{bmatrix}.$$

Since  $\det(H) > 0$ , the Hessian matrix is negative definite and is thus a strictly concave function. Moreover, the inequality constraint is linear and is concave as well. A solution to the problem will thus exist if the above mentioned Kuhn-Tucker conditions are satisfied.

In Case 1, condition number two is violated and, hence, a welfare maximizing Ramsey allocation does not exist.

Further, for the complementary slackness condition to hold, either  $\lambda_{B,t} = 0$  or  $\bar{D} - d_{t+1} = 0$ . When  $\lambda_{B,t} = 0$ ,  $\bar{D} - d_{t+1}$  might or might not be equal to zero and we say that the constraint is not binding (similar to our result from Case 2). However, if  $\lambda_{B,t} > 0$ ,  $\bar{D} - d_{t+1} = 0$  implying that the constraint is always binding (similar to our result from Case 3).



# Summary and Conclusion

Following the recent financial crisis, much of macroeconomics research has been geared towards studying the credit market imperfections and their impact on various policy decisions. Contributing to the literature, this thesis aims to conduct a few fiscal experiments in the light of some of these frictions.

In chapter 1, we examine the role of “risky working capital” channel on the government spending multiplier. We find that in the presence of a risk premium, when the loans are used to finance the working capital, an increase in government spending increases the aggregate financial risk in the economy, and, hence has a moderating effect on the aggregate output. As detailed above, a rise in government spending creates additional demand and raises the price level. Along with the prices, the marginal cost of production goes up, increasing firms’ loan requirements and, hence, their chances of default. This results in a higher lending rate, discouraging borrowing and production.

This moderating effect is evident from the government spending multipliers we calculate. We find the spending multiplier to be less than one. Further, as discussed, the multiplier is even smaller in the presence of frictions, highlighting the inefficiencies caused by financial frictions.

Compared to the existing results which study the multipliers in a setup where loans are used to finance fixed capital, for example physical capital, we study the effects of financing working capital. With an increasing number of start-ups with low fixed capital and high working capital requirements, it is important to scrutinize this channel more extensively and study the relevant policy implications. Even though our thesis presents a simplified theoretical framework to study the channel, its strength and significance still needs to be examined, making way for an interesting empirical research question.

Going further, in the second chapter, we explore the optimal level of capital

income taxation in the borrower-saver model. The impatient borrowers increase their current consumption by borrowing from the patient savers as per a borrowing constraint. The borrowing constraint restricts the free flow of capital in the economy, generating imperfections in the credit market. Given the borrowing constraint, the Ramsey planner optimizes social welfare by favoring a redistribution towards patient agents and subsidizing the returns on physical capital income.

This contrasts with the well-established Chamley-Judd result which is studied in a perfect market setup. Our paper uses the logarithmic utility function and provides the first step in attempting to derive the optimal level of capital taxation in a model with time preference heterogeneity and borrowing constraints. From a theoretical perspective, we still need to check if our results hold true with other utility functions. As an extension, it will also be intriguing to parameterize the model and derive the optimal level of subsidy.

In the end, we can conclude by restating that different credit market frictions, via different channels, have different consequences for the fiscal policy. The topic requires a much-detailed study and opens avenues for many more enriching future research questions and policy implications.