



# Contradictions between and within school and university activity systems helping to explain students' difficulty with advanced mathematics

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# Contradictions between and within School and University Activity Systems helping to explain students' difficulty with Advanced Mathematics

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This paper explores how contradictions, as framed by activity theory (Engeström, 1987), can explain first year undergraduate students' experiences of learning advanced mathematics. Analysing qualitative interview and observational data of students and lecturers based in one university mathematics department, we argue that contradictions between the school and university activity systems, as well as those within the latter, help explain some of the difficulties, including the conflict in cognition and identity, students can experience when they encounter advanced mathematics at university. This helps us to adopt a critical stance towards the systems students are expected to learn in, and points to system developments that might better support student learning.

*Keywords:* advanced mathematics; activity theory; contradictions; cognitive conflict

## 1. Introduction

When students enter university they may experience significant challenges in understanding advanced mathematics, especially marked by proof, rigour and precision. This raises serious concerns about students' transition to Advanced Mathematical Thinking (AMT), that is, 'thinking that requires deductive and rigorous reasoning about mathematical notions that are not entirely accessible to us through our five senses' (Edwards *et al.*, 2005, pp. 17-18).

There have been various approaches taken to students' experiences with learning advanced mathematics (only a selection of which are discussed here), each offering a different theoretical lens to conceptualise student difficulty. With the cognitive approach, students are often presented as lacking certain mental abilities, or simply being ill-prepared to conceptually engage with advanced mathematics (e.g. Duval, 2006; Tall, 2008; Thomas & Stewart, 2011). This literature can reduce student difficulty with advanced mathematics to student cognitive processes, and where the sociocultural is discussed, it is often presented as secondary to student cognition. There is research that takes a sociocultural approach, which frequently explains student learning in relation to mathematical communities (e.g. Hemmi,

2008; Perrenet & Taconis, 2009; Biza *et al.*, 2014). These communities are considered to mediate students' learning and understanding of mathematics. In this paper we combine the sociocultural and cognitive dimensions of students' learning of advanced mathematics using activity theory (Engeström, 1987). From this perspective, the 'activity of learning' is essentially a social process in a collective 'activity system', i.e. an ensemble of social relations mediated by cultural artifacts and social norms. We will argue that the contradictions within and between the *school* and *university* activity systems can help to explain some of the difficulties students encounter with advanced mathematics, including conflicts to cognition and identity they may experience. In this way we aim to put the social and cultural in the middle of developing praxis, but also to socioculturally situate the cognitive perspective. In conclusion, we aim to offer a critical sociocultural 're-mediational' perspective that leads to suggestions for developing the learning systems themselves.

## 2. Literature on advanced mathematical thinking at university

The long history of cognitive-based literature suggests that students can experience difficulty with advanced mathematics for a number of reasons, including: challenges with the process of reflective abstraction (Dreyfus, 1991; Duval, 2006); challenges with abduction in proof construction, such as in making inferences (Pedemonte & Reid, 2011); student confusion between natural language and mathematical registers (Mejía-Ramos & Inglis, 2011); a lack of student familiarity with the mathematical notation needed for advanced mathematics (Moore, 1994); student difficulties with definitions (Ouvrier-Buffet, 2011); student issues with reading and identifying proof (Selden & Selden, 2003; Yang, 2011); poor student judgement of what constitutes proof (Weber, 2010); a lack of ability to mentally compress mathematical structures into manageable cognitive units (Barnard, 2002; Harel & Kaput, 1991); and particular subject specific barriers to conceptual understanding (Mahir, 2009). The research that adopts a cognitive perspective generally argues that students' experience challenges with AMT as they face the formidable task of cognitively engaging with mathematics in ways significantly different from that which they experienced previously, e.g. at school, inducing a classically Piagetian form of 'cognitive conflict' (Gray *et al.*, 1999). This cognitive conflict is assumed to occur through a tension between accommodation and assimilation in student cognition (e.g. Edwards & Ward, 2004; Gray *et al.*, 1999; Tall, 1991; Tall, 2008; Thomas & Stewart, 2011).

There is research that focuses more on the social aspects of student learning of mathematics at university. This body of literature has often drawn on the communities of practice model of learning (Lave & Wenger, 1991; Wenger, 1998) to frame student learning as a process of enculturation into the practices of a mathematics community/communities (e.g. Hemmi, 2008; Perrenet & Taconis, 2009; Biza *et al.*, 2014). This process can be problematic as school mathematics often instil a mathematical repertoire that is misaligned with those held by professional or more experienced mathematicians, presumed to be the 'old timers' in the university community of practice (Perrenet & Taconis, 2009). Moreover, studies indicate problems with the teaching of advanced mathematics at university that can hinder the process of student enculturation, such as where there is a lack of transparency in the teaching of proof, i.e. where it is not made explicit in lecturers' presentations, and where teaching lacks meaningful student-lecturer discussions about proof and proof technique (Hemmi, 2008; Biza *et al.*, 2014). This can result in some students believing that proof production is an unattainable skill and something which only lecturers do and not students (Solomon, 2006).

For heuristic purposes (the literature is more nuanced), we can consider the research that takes a cognitive approach to frame learning as 'acquisition' via assimilation and accommodation

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3 whereas research that draws on the communities of practice model typically presents learning  
4 as ‘participation’ in practice. These different approaches can be characterised by ‘acquisition  
5 metaphors’ (AM) and ‘participation metaphors’ (PM) (Sfard, 1998) and have been argued to  
6 separately share much of the same conservative bias. Specifically, with the AM approach  
7 primary importance is often placed on concepts which are framed as empirical and formal  
8 (Engeström & Sannino, 2010). **By contrast, research that draws on the PM approach tends not  
9 to focus on psychology in general and the formation of theoretical concepts in particular, as  
10 researchers from this community of practice also tend to view concepts as formal abstractions  
11 (Engeström & Sannino, 2010).**

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14 Activity theory – or Cultural-historical Activity Theory (ChAT) – invokes both the  
15 psychological and the sociocultural. Activity theory, after Vygotsky’s legacy, **frames** concepts  
16 as arising in social practice (on the intermental plan) and transforming the psyche via the  
17 intramental plain (inner speech and verbal thought). It offers a perspective on how knowledge  
18 can be transferred across learning situations, but suggests that the process of knowledge  
19 transfer is not one that involves the movement of an isolated, intact conceptual entity divorced  
20 from the sociocultural environment (Engeström & Sannino, 2010). Rather, conceptual  
21 knowledge is framed in such a way as to develop and evolve in practice where the process of  
22 transfer involves a change in knowledge and context (Engeström & Sannino, 2010). Indeed,  
23 from the perspective of activity theory, concepts are embedded and distributed in and across  
24 activity systems as well as embodied in individuals. This perspective on how student  
25 cognitive activities, including the cognitive difficulties encountered, are socioculturally  
26 situated helps to offer a more holistic explanation regarding student experiences with AMT,  
27 since it promises to attach research on student’s conceptual difficulties to the sociocultural  
28 practices in which they are situated.  
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### 32 **3. Activity systems and their contradictions**

33 Activity theory is a cross-disciplinary framework, a conceptual and descriptive tool for  
34 analysing and explaining human activity (Sannino *et al.*, 2009). Activity theory is rooted in  
35 dialectical materialism, as developed by Marx and Engels: thus when humans (the subjects)  
36 perform labour activity (using tools) they not only transform nature (the object), but are  
37 transformed themselves in the process (Vygotsky, 1978). According to Vygotsky (1978,  
38 1981), human consciousness is developed and shaped through joint activity with others  
39 mediated by cultural tools including ‘language; various systems of counting; mnemonic  
40 techniques; algebraic symbols systems; works of art; writing; schemes, diagrams, maps and  
41 mechanical drawings; all sorts of conventional signs and so on’ (Vygotsky, 1981, p. 137).  
42 Tools are products of sociocultural evolution, imbued with particular cultures and histories,  
43 and are produced and reproduced in human activity; thus tools can be conceptualised as  
44 carrying accumulated social knowledge that is transmitted with use (Leontiev, 1981). Tools  
45 are reasoned by Vygotsky to mediate activity and so are important to the development of  
46 higher cognitive functioning.  
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49 Engeström (1987) expands on Vygotsky’s **and Leontiev’s** ideas and advances that activity is  
50 situated in a larger community governed by rules and a division of labour which also mediate  
51 activity, regulating individual actions. Importantly, as Barab *et al.*, (2002) state: ‘The  
52 components of activity systems [subject, object, tools, community, rules and division of  
53 labour] are not static components existing in isolation from each other but are dynamic and  
54 continuously interact with the other components through which they define the activity  
55 system as a whole’ (p.79).  
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3 According to Engeström (2001, p. 137), activity systems always harbour contradictions.  
4 Contradictions, not to be confused with the vernacular or standard usage of the term, should  
5 be considered as ‘historically accumulating structural tensions within and between activity  
6 systems’. Contradictions have also been referred to as a general misalignment within and  
7 between the components in an activity system, or between different activities (Kuutti, 1996).  
8

9  
10 Williams *et al.*, (2001) provide insight into the difficulties students may encounter when  
11 moving from one activity system to another. One student in a lab placement struggled to  
12 understand how graphs are used in an industrial chemistry experiment. They explain the  
13 primary difficulty by the contradiction between workplace and school graphing activity and  
14 its mediating tools: the lab’s graph was read from right to left, plotted more than one variable  
15 on each of the axes, used a logarithmic scale etc. Hence the student was equipped with tools  
16 from school that were misaligned with the tools needed in the lab. In this case the student’s  
17 cognitive conflict – in their conscious actions – embodied the contradiction **between** the two  
18 systems.  
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21 This conflict in cognition from the perspective of activity theory can also be read as one  
22 related to the **multi-voicedness** of activity systems. According to Engeström (2001), activity  
23 systems are always comprised of a community of multiple **perspectives**, traditions and  
24 interests, where the participants carry their own diverse histories. The multi-voicedness of  
25 activity systems increases when they activity systems interact (Engeström, 2001) and we may  
26 imagine that this multi-voicedness can also manifest in different mathematical practices.  
27 Engeström suggests that the multi-voicedness of activity system can be a source of conflict.  
28 Thus, from the perspective of activity theory, cognitive conflict can be conceptualised as an  
29 interpersonal conflict, caused by the differences in mathematical practices, giving rise to  
30 conflict of an intrapersonal kind. Indeed, from the perspective of activity theory we cannot  
31 discuss the cognitive conflict students may experience without turning attention to the conflict  
32 in identity they can also encounter; cognition and identity are unified. As Nardi (1996) argues  
33 ‘consciousness is not a set of discrete disembodied cognitive acts (decision making,  
34 classification, remembering), and certainly it is not in the brain; rather, consciousness is  
35 located in everyday practice: you are what you do’ (p. 8).  
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39 In this paper we frame many of the difficulties students encounter with advanced  
40 mathematics, including the conflict to cognition and identity, as due to, or explained by, the  
41 contradictions between the school and university activity systems. This helps us to situate the  
42 literature that discusses students’ difficulties with advanced mathematics, and with transition  
43 in general in the broader, complex sociocultural landscape of institutional activity systems  
44 (school and university). These activity systems, together with inherent tensions, are embodied  
45 in students and in some cases can challenge students’ identification with mathematics.  
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48 Lawson (2015) describes how most undergraduate mathematics students believe they are  
49 good at mathematics and that **their pre-university examination results** appear to confirm this.  
50 However, when some students cease to find mathematics as straightforward as they found it **at**  
51 **school**, they face the realisation that their positive disposition towards mathematics was  
52 primarily **grounded** on the affirmative emotions generated by success at **school** rather than  
53 due to a **deeper** interest in the subject. In addition to the difficulties students encounter with  
54 mathematics at university, students can experience challenges with aspects associated with  
55 learning at university in general where new demands are made on them. For example,  
56 Pritchard (2015) argues that the familiar didactical contract at school, where teachers provide  
57 regular affirmation of students’ performances and give them enough procedures to pass  
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3 assessments, is threatened when they enter university, because of the greater emphasis placed  
4 on independent learning.  
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6 The challenges that students can experience in transition are noted elsewhere. Williams  
7 (2015) observes that lecturers tend to expect students to learn how to become independent  
8 learners and often expect students to have well developed advanced mathematical  
9 conceptions. However, he stresses that the teaching practices in universities are habitually  
10 transmissionist and they do not foster meaningful student-lecturer dialogue which in turn does  
11 not effectively develop students' conceptual understanding of mathematics or help to nurture  
12 the independent skills required for successful learning at university. As well as causing  
13 conflict, structural tensions within activity systems can also be a source of developmental  
14 change and promote individual learning when they result in 'breakdowns'.  
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17 'Breakdowns' occur when automatic or routine operations are disrupted, which can lead to a  
18 reassessment of these operations involved in activity (Leontiev, 1978). In our paper we report  
19 an occurrence of a breakdown and how this reveals potential for development which may be  
20 of use to universities when reflecting on their own practices in how to support student  
21 transition to AMT.  
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24 Biza *et al.*, (2014) - drawing on the communities of practice framework as well as on the  
25 theory of communities of inquiry, based on the works of Goodchild *et al.*, (2013) and others -  
26 propose that developing communities of inquiry characterised by lecturer-student dialogue  
27 and questioning, or inquiry, around advanced mathematics can help to effectively support  
28 student learning when at university. They also suggest that inquiry can help lecturers to  
29 critically reflect on their teaching and adjust their practices accordingly. They argue that with  
30 the communities of inquiry approach, emphasis is placed on critical alignment where  
31 participants question the practices in which they engage, and that through this process it may  
32 be possible that this 'leads to new forms of practice, new modes of awareness of the problems  
33 and issues in developing effective ways of working, and good outcomes for students'  
34 learning' (p. 164). We argue that by addressing contradictions and thus facilitating  
35 breakdowns one can create such critical alignment.  
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#### 38 4. Methods

39 The research reported here (doctoral work to be found in full in Jooganah, 2015) was  
40 conducted in association with the Transmaths project and included case study and interviews  
41 in one university mathematics department with the aim of understanding students' learning  
42 experiences of advanced mathematics (for more information on the Transmaths project, see  
43 <http://transmaths.org/>). There is no clear consensus on how activity theory should be applied,  
44 but, as Jonassen & Rohrer-Murphy (1999) argue, because the theory focuses on cultural  
45 practice, a qualitative approach is generally taken (i.e. an approach where emphasis is  
46 generally placed on capturing the multi-dimensionality and complexity of participant  
47 experiences), e.g. in examining how cultural artifacts mediate social practice.  
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50 This paper draws on the semi-structured interviews conducted with eight first-year  
51 undergraduate mathematics students during the course of their first transitional year at  
52 university. They were asked general questions about their learning experiences at school and  
53 university, for example, 'What are the differences between school and university  
54 mathematics, and between teaching styles? What areas are you struggling with and why?'  
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The paper is also informed by 12 unstructured interviews with lecturers. In this form, lecturers were able to discuss issues they felt were significant to students' learning experiences with advanced mathematics, with the additional benefit of yielding rich data that may have been lost within a more structured approach. For example, we used a video-stimulated recall interview with a lecturer who we interviewed about one of his tutorials which we filmed (for which we obtained consent from him and his tutor group as part of our research). This interview involved us showing the lecturer the filmed tutorial and asking him general questions about what happened in the tutorial and what he felt was significant.

In addition, observations of the teaching environment that shape first-year students' learning were useful for constructing an informed assessment of the university activity system of the mathematics department. This included observing interactions in small group tutorials (7) and lectures (10). One of the tutorials observed is presented in this paper as a vignette.

### 5. Findings: contradictions between school and university

In the transition to university, students experience a number of academic changes, for which lecturers reportedly claimed students were 'not prepared'. We were informed that these problems required the mathematics department to adjust their academic programmes and a series of changes had occurred over a number of years in response to such disjunctures. One lecturer, Brian, (all participant names have been replaced with pseudonyms) commented in a manner that typically blames schools and their examination systems for the problems students encounter at university:

Brian: I think they've been poorly prepared by schools and by the examination system. They're really too much spoon-fed. And yet we don't spoon-feed them quite so much, although we do spoon-feed them more than we would have done, say twenty, thirty years ago because of awareness of what's happened previously. I suppose we expect them to study more on their own. I suspect they've had more, closer attention than from when they've been in schools and colleges – been helped more. Whereas here we'd expect them to attend lectures and do enough study. Do lecture notes, do the exercises, go to the tutorials, read books by themselves and be more responsible for their own learning. I wonder whether schools and colleges have done enough of that.

Brian indicates how students' subjective experiences of education at school stand in contrast with the learning culture at university. The lecturers generally contrasted the procedural approaches taken by students at school with the greater level of rigour and conceptual generality demanded by university approaches to mathematics. Martin put it thus: 'They get taught methods at school and then they come here and of course, shall we say, we do it properly, and the students will say 'why are you doing it this way, we could answer the question using the method we used at school', but the problem with the method at school is that it's just a method which works in the cases to answer the questions at A-Level. It probably doesn't generalise properly and that's not the correct way of looking at situations.'

Another lecturer, who taught first-year students, expressed a similar opinion.

Mel: A-Level is very methods based. So they learn a method and then they apply it to lots of different examples. And they don't worry about where these methods come from, how they developed and why they are there. They just believe the teacher. If the teacher says, if

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<sup>1</sup> In the UK system, after compulsory education (ending at the age of 16) students may go on to study various subjects, typically 3 or 4, at Advanced Level, or 'A-Levels'. An A-Level in mathematics is often a prerequisite to study for a mathematics degree at university.

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3 you differentiate  $x$  cubed and (you get) three  $x$ -squared, and they'll take that on board and  
4 they'll go and differentiate functions which are made up of these standard functions and  
5 they have rules for doing that. They never worry about how these rules work, they just  
6 unquestionably take them on board and they do expect something similar at university.  
7

8  
9 These sentiments are echoed in the literature on AMT. For example, Tall & Vinner (1981)  
10 argue that students may be so secure in their own interpretations of mathematical notions that  
11 they regard formal theory as inoperative and superfluous. Moreover, there may be instances  
12 when cognitive conflicts arise from student intuitions that have stabilised and become  
13 resistant to change (Tirosh *et al.*, 1998). According to activity theory the 'cognitive intuitions'  
14 that cause conflicts are embedded in unexamined 'operational conditions' that fall below the  
15 level of consciousness and are all the more powerful for being unexamined and uncritically  
16 accepted (Leontiev, 1978). It is these conditions that have to be examined at breakdown  
17 moments, for instance when university practices assert they are invalid, and not 'proper'  
18 mathematics. We now turn to such a moment.  
19

### 20 21 **5.1 Vignette: tutorials, contradictions and breakdown**

22 Robert, a first-year lecturer, and the students of his tutorial discussed a question from the  
23 problem sheet: 'To find the inverse function of  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = 2x+3$ '. Several students had  
24 answered this in a way that Robert indicated '[W]as not *wrong*, but likely to cause problems  
25 due to its imprecision'. He explained that the students had rearranged the formula  $y = 2x+3$  to  
26 get  $x = \frac{1}{2}(y - 3)$ , then rewritten this by exchanging the  $x$  and the  $y$  to arrive at the 'inverse'  $y$   
27  $= f(x) = \frac{1}{2}(x-3)$ .  
28

29  
30 This is a correct but arguably an incomplete answer, as the domain and co-domain are left  
31 implicit. In fact both are the set of real numbers ' $\mathbb{R}$ ', and so for the inverse function too, as  $f$  is  
32 in this case a bijection from  $\mathbb{R}$  to  $\mathbb{R}$ . It is hardly surprising that the students might leave this as  
33 implicit, even if they had perhaps thought it was important. Right or wrong, why is this  
34 method such a powerful attractor for the students? At school, as one can see in various texts, a  
35 function is described as a rule mapping  $x$ -values to  $y$ -values, usually represented horizontally  
36 and vertically as real number lines in a graph. Thus, typical tasks/questions might ask students  
37 to sketch a graph and its image after a transformation, e.g. noting the relationship between  
38 graphs of  $f(x+3)$  and  $f(x-3)$ .  
39

40  
41 Robert had selected this particular example to discuss with his students in the tutorial as he  
42 had marked their work beforehand and, in his own words, was 'gob-smacked' at the students'  
43 lack of understanding and by the procedural approach they took to resolve the problem. He,  
44 therefore, used the tutorial as an opportunity to make explicit to his students the need for  
45 complete writing of the mathematical proof (with explicit definition of the sets  $X$  and  $Y$   
46 involved). Robert wrote on the board how he would prove  $f$  is a bijection, starting with the  
47 definition of an inverse function in which the domain and co-domain are included explicitly,  
48 thus:  
49

50  
51 Definition: the inverse function of  $f: X \rightarrow Y$  [that is, a function that maps any  $x$  in  $X$  to a  
52 single value  $y$  in  $Y$  given by a rule  $y = f(x)$ ] is a function  $g$  that maps a value  $y$  in  $Y$  to a  
53 single value  $x$  in  $X$ , (i.e.  $g(y) = x$ ) such that  $g f(x) = x$ , for all  $x$  in  $X$ .  
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55  
56 He began to work through the solution but was interrupted by a student, Ben, who said he had  
57 obtained the right answer in his own way, and explained what he had done. Robert responded  
58 'It is not wrong but why exchange the values  $x$  and  $y$ ?', which he claimed was likely to lead  
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3 to confusion of the domain and co-domain. Robert encouraged Ben to illustrate the reasoning  
4 behind his answer on the board. Indeed, most of the class seemed to agree: 'That's the way  
5 we were taught at A-Level!'

6  
7 Ben then chose to exemplify his approach using exponential functions from a later worksheet  
8 question: he argued that the expression  $y = \exp(x)$  was the same as  $x = \log(y)$ , and that  
9 switching  $x$  and  $y$  was required to obtain the inverse function  $y = \log(x)$ . Implicitly, one  
10 assumes the graph of  $y = \exp(x)$  is the same as that of  $x = \log(y)$ , and therefore does not  
11 represent the inverse. However, Robert noted, 'Oh yes it might be, if you consider the graph  
12 as a representation of the function mapping  $y$  to  $x$ '. The notion that the same graph  
13 represented two inverse functions appeared to counter how the students were taught at school.  
14  
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16 In Robert's tutorial, the students believed  $\exp(x)$  and  $\log(x)$  to be the inverse functions,  
17 whereas Robert pointed out that these expressions are not functions, but values. Students  
18 would have been introduced to the idea that the graphs of  $f$  and its inverse  $g$  would be  
19 reflections/images of each other in the line  $y=x$ . The suggestion that one graph can represent  
20 two functions ( $y$  as a function of  $x$  and its inverse  $x$  as a function of  $y$ ) clearly contradicts the  
21 practice taught in school and explains the cognitive conflict here revealed in the tutorial  
22 dialogue.  
23  
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25 Activity systems are multi-voiced and this multiplicity of voices can increase when activity  
26 systems interact (Engeström, 2001), which can give rise to interpersonal as well as  
27 intrapersonal conflict of a cognitive nature. Although Robert was aware of students' informal  
28 approach to learning as shaped by the school activity system, he informed us he had been  
29 unaware of the depth of the clash of values and practices between school and university  
30 involved in this example. By questioning their mathematical practices, he may have caused  
31 the students to reflect on the conflict they experienced. This created an unexpected learning  
32 opportunity. Although the students' approach was 'not wrong', Robert encouraged them to  
33 recognise the value of precision and rigour in mathematics, not fully anticipated prior to the  
34 tutorial.  
35  
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37 Robert: Well it was very, very clear wasn't it what I learnt in that particular class. One of  
38 the things I learnt was that something has changed about the way students are taught to  
39 write functions at A-Level and their inverses and I damn well better understand what that  
40 is, and why it is... When I'm doing this class next year I will be very much more prepared  
41 and alerted.  
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44 Robert's session stood out from the other tutorials observed. Even Robert said that it deviated  
45 from the typical tutorial, which often resembled 'feedback' sessions. By contrast, Robert's  
46 tutorial seemed to us to achieve its intended purpose, i.e. to have an open dialogue about the  
47 learning material based on his prior assessment of some of the students' work.  
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## 50 6. Findings: Contradictions within the university activity system

51 These relatively small group tutorials had been designed to support students in first-year  
52 transition by offering a more interactive dialogue, as one lecturer Mel told us: 'You can get  
53 these ideas over and can find out if they've understood it. You can ask them questions. You  
54 can find out where their misunderstandings are and you can sort of fill in the gaps... you will  
55 never be able to do that in a lecture.'  
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3 However, getting students to participate was not always easy. Robert noted how ‘in a class of  
4 ten or eleven they feel threatened that they may get picked on – I’ve got their written work to  
5 see that they are not doing very well, and they’re apprehensive, [they are] cagey about  
6 admitting that they don’t understand anything. They’ll just sit there and be totally lost’. The  
7 challenges concerning the division of labour in the form of low levels of student participation  
8 often made tutorials appear stilted; students were positioned ‘purely as recipients of the  
9 information’, as Robert indicated. Of Ben, the student who spoke up in his tutorial, Robert  
10 said: ‘he, unlike some of them, was willing to speak up and to listen, and I bet he’s got a  
11 better understanding now of what was happening. Many of the others in the class I’ve no  
12 idea’.

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15 The tutorial situation was difficult for some lecturers who were mainly used to lecturing,  
16 telling us that they were torn between getting students through the course and engaging them  
17 with the material at a relational level. Students also reported having difficulties: ‘That was  
18 really annoying when we did turn up and when we had a question that when we did get to the  
19 problem [the tutor] didn’t really answer the question as to why. He’d just like do it himself  
20 and then expect us just to know what he’s done and why he’s done it.’

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23 Lecturers were aware of issues such as lack of sufficient time to present all the worksheet  
24 solutions as well as develop general understandings. Lecturer discourse indicated a lack of  
25 agency in the face of the rules governing the undergraduate curriculum and assessment  
26 structure generally: as one lecturer said: ‘We are not free in teaching. Many syllabuses are  
27 dominated by, well students have to survive the exams. There’s this gun that I have in my  
28 back which says, ‘you cannot explain what your topic is actually about. You have to prepare it  
29 in a way that is examinable’, to bring them in the situation so that at least they have some  
30 tools to pass exams, so that most of the students pass as required’. Thus labour time is at the  
31 root of the contradiction: primary contradictions relate to the essential conflict between  
32 exchange (as determined by labour time) and use value (Engeström, 1987).

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35 It appears that the very same characteristic of school learning that is often blamed for  
36 students’ cognitive deficits applies equally to university: procedural practices continue to  
37 flourish at the expense of conceptual development. One lecturer, Gert, commented: ‘When I  
38 came here two years ago at the beginning I was very outraged. If I would teach what I would  
39 like to, they wouldn’t understand it, or they wouldn’t like to understand it, and I don’t blame  
40 the students, it’s just the university does not allow the students to understand.’

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43 Gert’s discourse suggests how the contradictions within the university activity system can  
44 mediate student motivation and understanding, or lack thereof, regarding advanced  
45 mathematics. His discourse also illustrates how opposing forces within activity systems  
46 manifest in tensions in individual subjectivity. These tensions in subjectivity caused by  
47 contradictions are also observed in the discourse of students. As described earlier, when  
48 students transition to the university activity system, they experience changes in the social and  
49 material conditions that mediate their experiences with mathematics. These changes not only  
50 give rise to a conflict to student cognition but can also cause students to question their own  
51 mathematical identities. One first year student, Mark, claimed he chose to study mathematics  
52 at university as he thought ‘just stick with what you know’. However, the mathematics he  
53 experienced at university came as a ‘shock’.

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56 Mark: It was a shock to the system that people, like, can’t understand why they only got  
57 50-60% in the exams, ‘cos they’re used to getting 90-100’s at A-Level. It was a sort of,

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3 'hold on, we understood everything at A-Level, why are we not understanding this straight  
4 away' ... I probably don't like it for that reason, because it didn't click... They see  
5 something like a proof and it scares them, they don't try and just think about it. I was one  
6 of those. I guess I still am.  
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8 We asked Mark if he considered himself to be a mathematician and he replied, 'I thought I did  
9 before university, I'm not so sure now'. We thus observe from Mark, and overall in this  
10 paper, how activity systems, together with their inherent tensions between and within them,  
11 are embodied and can mediate students' learning of advanced mathematics at university and  
12 even challenge their identification with the subject.  
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## 15 7. Conclusion

16 The main point of this paper has been to argue that student difficulty with advanced  
17 mathematics can be understood as formed by a series of contradictions between the school  
18 and university mathematical activity systems, as well as those located within the latter.  
19 This helps to direct our attention from the individual student in understanding their difficulty  
20 with advanced mathematics to the activity systems that mediate their activity. The implication  
21 of this is important. Analysing the components that make up activity systems can facilitate  
22 critical reflection on the broader sociocultural environment, which forms a complex web of  
23 forces, some of which opposing, shaping individual disposition, motives, and engagement  
24 with advanced mathematics. This can help us shift 'blame' from the student (paraphrasing  
25 Gert) and apply a critical stance to the activity systems themselves, and how it is these  
26 activity systems which shape student (dis)engagement with mathematics.  
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29 Activity theory offers several other advantages including that it helps to socioculturally  
30 situate the cognitive conflict students can experience with AMT at university. Activity theory  
31 frames consciousness as being stretched beyond the head of the individual and formed in  
32 historically developed and tool-mediated practice with others. Thus, in order to understand the  
33 development of student mathematical thinking from the perspective of activity theory, we  
34 need to look at all components of activity systems such as the object, tools, rules, community,  
35 and division of labour, and the contradictions within and between them. These contradictions,  
36 as well as those between different activity systems, can cause conflict to student cognition and  
37 even identity.  
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40 The transition to university mathematics has been observed to cause student difficulties where  
41 different learning demands are placed on them (Pritchard, 2015; Williams, 2015) and where  
42 their relationship to mathematics as shaped by experiences at A-Level are challenged when  
43 they encounter mathematics at university (Lawson, 2015). This difficulty that students can  
44 encounter with the general transition to university and with university level mathematics can  
45 be conceptualised as one associated with identity.  
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48 When individuals transition from one activity system to another, this can result in a tension in  
49 subjectivity and thus identity. We observe this in Mark's discourse, where the advanced  
50 mathematics encountered in the university activity system resembled a different way of  
51 knowing mathematics and being a mathematician, which made him question his identity of  
52 being a mathematician. The contradictions found in activity systems and how these can be  
53 manifest in a conflict of identity is also witnessed in relation to Gert; he felt torn between  
54 various motives and alluded to a lack of agency in relation to the contradictions that shaped  
55 the learning and teaching environment within the university activity system. This conflict to  
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3 identity as a result of contradictions between and within activity systems and how this is tied  
4 to student cognition is explored further elsewhere (Jooganah, 2015).

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6 Another key advantage with activity theory is the focus it offers on the systems that might be  
7 developed to support students' learning of advanced mathematics. On entering the university  
8 system students are likely to be equipped with tools that may be no longer fit for purpose in  
9 advanced mathematics. Additionally, the kind of activity that might help students manage this  
10 is not always supported within the university system due to internal contradictions there, e.g.  
11 the resource constraint makes small group tutorials institutionally unattractive compared to  
12 large lectures.  
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15 Barab *et al.*, (2002) argue that contradictions in activity systems cannot be controlled from the  
16 outside or through some form of written guidance, but need to be managed in situ, as each  
17 context is different. They suggest that reading the rich descriptions and struggles those in  
18 education encounter can provide insight others can use in addressing the contradictions  
19 affecting their own pedagogic practice. In the case illustrated in this paper, we found some  
20 merit in the creation of a breakdown moment in tutorial dialogue that exposed a core  
21 contradiction in the practice of mathematical functions between school and advanced  
22 mathematics at university, and from which it seemed the students and lecturer had an  
23 opportunity to confront the associated cognitive conflict and develop conceptually (if not  
24 procedurally). Such breakdowns can be important as student understanding of concepts like  
25 functions can be considered as boundary objects (Star & Griesemer, 1989). They have  
26 contested meanings within the school and university activity systems and such differences  
27 may be implicit such that lecturers and even students may not necessarily be aware of them. It  
28 could be argued that creating breakdown moments can aid in the creation of communities of  
29 inquiry where there is critical alignment with participants questioning the practices in which  
30 they engage. This, as Biza *et al.*, (2014) argue, can lead to new forms of practice that can  
31 effectively support students' learning.  
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