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Designing Coalition-Proof Reverse Auctions over Continuous Goods

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***Abstract*—This paper investigates reverse auctions that involve continuous values of different types of goods, general nonconvex constraints, and second stage costs. We seek to design the payment rules and conditions under which coalitions of participants cannot influence the auction outcome in order to obtain higher collective utility. Under the incentive-compatible Vickrey-Clarke-Groves mechanism, we show that coalition-proof outcomes are achieved if the submitted bids are convex and the constraint sets are of a polymatroid-type. These conditions, however, do not capture the complexity of the general class of reverse auctions under consideration. By relaxing the property of incentive-compatibility, we investigate further payment rules that are coalition-proof without any extra conditions on the submitted bids and the constraint sets. Our results are verified with several case studies based on electricity market data.**

I. Introduction

Auctions are effective for allocating resources and determining their values among a set of participants. In an auction, participants submit their valuations of the resources (bids), and then a central operator determines the allocation and the payment for each participant as specified by the mechanism. Recently, there has been a surge of interest from the control community to explore auction mechanisms because of their applications in several problems, such as coordinating electric vehicle charging [2], telecommunication networks [3], demand response management [4], provisioning of a distributed database [5], and selecting a host for a noxious resource (e.g., trash disposal facility) [6]. The central element in any auction is the design of the payment and allocation rules, since the participants have incentives to strategize around these rules. In particular, the operator designs the payment rule to ensure an efficient outcome, that is, an outcome maximizing social welfare. This goal is best achieved by solving for the optimal allocation under the condition that the participants submitted their true valuations.

In this paper, we consider the design of reverse auctions that may involve continuous values of different types of goods, general nonconvex constraints, and second stage costs. This is motivated by the fact that several current energy market problems can be cast within this general class of auctions [7], [8]. Previous work on these markets considers the pay-as-bid mechanism [9] and the locational marginal pricing mechanism [10]. In both mechanisms, participants can bid strategically to influence their payoff since these mechanisms do not incentivize truthful bidding. As an alternative, we analyze the Vickrey-Clarke-Groves (VCG) mechanism, which has several theoretical virtues [11]–[13]. In particular, under the VCG mechanism truthful bidding is the dominant-strategy Nash equilibrium. We refer to this property as incentive-compatibility. Due to this property, several recent works propose the use of the VCG mechanism [14]–[16]. Despite the desirable theoretical properties, in practice, this mechanism is often deemed undesirable since coalitions of participants can strategically bid to increase their collective utility. Consequently, it is susceptible to different kinds of manipulations, such as collusion and shill bidding [17]. These shortcomings are decisive in practicality of the VCG mechanism since in a larger context this mechanism is not truthful.

As is outlined in combinatorial auction literature [17], the shortcomings described above occur when the VCG utilities are not in the *core*. The core is a concept from coalitional game theory where the participants have no incentives to leave the grand coalition, that is, the coalition of all participants [18]. Recently, coalitional game theory has received attention, especially for aggregating power generators [19], and deriving control policies for multi-agent systems [20]. In this paper, we use coalitional game theory to ensure that the VCG mechanism is coalition-proof, in other words, collusion and shill bidding are not profitable. To this end, we derive conditions on the submitted bids and the constraint sets that ensure core VCG utilities by using some recent advances from combinatorial optimization. We show that under convex (or marginally increasing) bids and polymatroid-type constraints the VCG mechanism is coalition-proof.

The restricted market setting for core VCG utilities, however, does not capture the complexity of general reverse auctions arising in electricity markets. Specifically, these markets may involve nonconvex bids (e.g., generator startup costs [14]), and complex constraint sets that are not polymatroids (e.g., DC or AC optimal power flow constraints [10]). Hence, it may not be possible to ensure core VCG utilities. To this end, we focus on payment rules that are coalition-proof without any extra conditions on the bids and the constraints. In particular, we show that selecting the payments from the core yields coalition-proofness. Naturally, these mechanisms relax the incentive-compatibility of the VCG mechanism. In order to alleviate this issue and incentivize truthfulness, we propose a coalition-proof mechanism which minimizes the participants' abilities to benefit from strategic manipulation among all other coalition-proof mechanisms.

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Due to space constraints, a few proofs have been omitted. We have provided all the details of these proofs online, available at [1].

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Let us contrast our work with existing mechanism design research. The authors in [21] design a forward VCG auction for continuous goods by restricting each participant to submitting a single price-quantity pair to the operator. They show that this mechanism, called progressive second price (PSP) mechanism, has a truthful ϵ-Nash equilibrium, which can be attained by best-response dynamics. The work in [22] studies the PSP mechanism subject to quantized pricing assumptions, and proposes a fast algorithm converging to a quantized Nash equilibrium with a high probability. More recently, the work in [2] addresses cross-elasticity in PSP design arising from having a multi-period problem for charging electric vehicles. This work is generalized to decentralized procedures and double-sided auctions in [3]. In regard to these studies, the PSP mechanism cannot be implemented in dominant-strategies, and truthfulness is only in the price dimension for a given quantity. Specifically, there is no single true quantity to declare, and the optimal quantity depends on the bids of other participants [21, §3.2]. Moreover, the convergence analysis of the best response dynamics is limited to strongly concave true valuations and simple market constraints, for example, availability of a fixed amount of a single continuous good [21, §3.1]. On the other hand, the work in [14] applies the VCG mechanism to the wholesale electricity markets and shows that it results in larger payments than the locational marginal pricing mechanism. Nevertheless, none of the aforementioned works consider coalitional manipulations. Finally, the works in [23]–[25] study the design of the participants' utilities such that the selfish behavior of the participants results in a social welfare maximizing outcome. By contrast, in our case, the true valuations of the participants are *a priori* unknown and they are not part of the design. Instead, we are guiding the participants to a social welfare maximizing outcome by designing meaningful incentives through the payment rule.

Our contributions in the paper are as follows. First, we prove that in the reverse auctions over continuous goods the VCG mechanism is coalition-proof and the VCG utilities lie in the core, if and only if the market objective function is supermodular, see Theorems 2 and 3. These results are direct extensions of the results from the multiple item setting. Second, considering the special setting of continuous goods and complex constraints, we derive novel conditions on the bids and the constraint sets under which the VCG mechanism is coalition-proof, see Theorems 4 and 5. Third, in Theorem 6, we show that in the reverse auctions over continuous goods selecting payments from the core results in a coalition-proof mechanism without any restrictions on the bids and the constraints, extending results from the multiple item setting.

This paper is organized as follows. In Section II, we introduce a constrained optimization problem that models a general class of reverse auctions. Then, we introduce the VCG mechanism and illustrate that coalitions can influence the outcome. For an analysis of this shortcoming, in Section III we bring in tools from coalitional game theory, namely the core. We then investigate conditions under which the VCG mechanism is coalition-proof. Since these conditions do not capture the complexity of the general reverse auctions, alternative payment rules are proposed in Section IV. In Section V, we present case studies based on real-world electricity market data. Due to space limitations, a few proofs have been omitted. We have provided all the details of these proofs online, available at [1].

II. Mechanism Framework

We start with a generic reverse auction model. The set of auction participants consists of the central operator $l = 0$ and the bidders $l \in L$ where $L = \{1, \dots, |L|\}$. Let t be the number of types of goods in the reverse auction. Goods of the same type from different bidders are fungible to the central operator. Each bidder $l \in L$ has a private true cost function $c_l : \mathbb{R}^t_+ \to \mathbb{R}_+$ which is nondecreasing. We further assume that $c_l(0) = 0$. This assumption holds for many existing reverse auctions, for instance, control reserve markets and day-ahead electricity markets that include generators' start-up costs [7], [14]. Each bidder l then submits a bid function to the central operator, denoted by $b_l : \mathbb{R}^t_+ \to \mathbb{R}_+$ and nondecreasing with $b_l(0) = 0$.

Given the bid profile $\mathcal{B} = \{b_l\}_{l \in L}$, *a mechanism* defines an allocation rule $x^*_l(\mathcal{B}) \in \mathbb{R}^t_+$ and a payment rule $p_l(\mathcal{B}) \in \mathbb{R}$ for each bidder l. The central operator's objective is $\bar{J}(x, y; \mathcal{B}) = \sum_{l \in L} b_l(x_l) + d(x, y)$. Here $y \in \mathbb{R}^p$ are additional variables entering the central operator's optimization, in addition to the allocation $x \in \mathbb{R}^{t|L|}$. The function $d : \mathbb{R}^{t|L|}_+ \times \mathbb{R}^p \to \mathbb{R}$ is an additional cost term. For example, in a two-stage auction model, the operator can buy the goods from another market. In this case, y corresponds to the second stage variables and the function d is the second stage cost [7].[1]

In most auctions, the allocation is determined by minimizing the operator's objective subject to some constraints

$$J(\mathcal{B}) = \min_{x,y} \bar{J}(x, y; \mathcal{B}) \text{ s.t. } g(x, y) \leq 0, \tag{1}$$

where $g : \mathbb{R}^{t|L|}_+ \times \mathbb{R}^p \to \mathbb{R}^q$ defines the constraints. (If the optimization is infeasible then $J(\mathcal{B}) = \infty$.) Let the optimal solution of (1) be denoted by $x^*(\mathcal{B}) \in \mathbb{R}^{t|L|}_+$ and $y^*(\mathcal{B}) \in \mathbb{R}^p$. We assume that in case of multiple optima, there is some tie-breaking rule. The solution of the optimization problem (1) is the optimal allocation with respect to the submitted bids, that is, the goods are bought from the bidders with lower bid prices.

The *utility* of bidder l is given by $u_l(\mathcal{B}) = p_l(\mathcal{B}) - c_l(x^*_l(\mathcal{B}))$. A bidder whose bid is not accepted, $x^*_l(\mathcal{B}) = 0$, is not paid and $u_l(\mathcal{B}) = 0$. The total payment made by the central operator is given by $u_0(\mathcal{B}) = -\sum_{l \in L} p_l(\mathcal{B}) - d(x^*(\mathcal{B}), y^*(\mathcal{B}))$, which can be treated as the utility of the central operator. Note that this utility can be an expected utility when the function d is an expected cost. As a remark, if the problem (1) is infeasible, then $u_0(\mathcal{B}) = -\infty$.

Constrained optimization problem (1) defines a general class of reverse auction models. Several current market problems such as stochastic energy market mechanisms [7]–[9], [15], [26] and energy reserve co-optimized markets [14], [27] can be cast within this model. The constraints may correspond to procurement of the required amounts of power supplies, for instance, in the Swiss control reserve markets accepted reserves must have a deficit probability of less than 0.2%. They may also correspond to a transmission network, for instance, in

[1] In Section V, we provide a real-world electricity market case study where the function d incorporates expected daily market prices in a weekly market.

energy markets power injections must satisfy the transmission line limits and the power flow equations.

Three fundamental properties that we desire for a mechanism are individual rationality, efficiency and dominant-strategy incentive-compatibility. A mechanism is *individually rational* if bidders do not face negative utilities, $u_l(\mathcal{B}) \geq 0$, for all $l \in L$. A mechanism is *efficient* if sum of all the utilities $\sum_{l=0}^{|L|} u_l(\mathcal{B})$ is maximized. To define the third property, we first bring in tools from game theory. Let $\mathcal{B}_{-l} = \{b_k\}_{k \in L \setminus l}$. The bid profile $\mathcal{B}$ is a *Nash equilibrium* (NE) if for every bidder l, $u_l(\mathcal{B}_l, \mathcal{B}_{-l}) \geq u_l(\tilde{\mathcal{B}}_l, \mathcal{B}_{-l})$, $\forall \tilde{\mathcal{B}}_l$. The bid profile $\mathcal{B}$ is a *dominant-strategy NE* if for every bidder l, $u_l(\mathcal{B}_l, \hat{\mathcal{B}}_{-l}) \geq u_l(\tilde{\mathcal{B}}_l, \hat{\mathcal{B}}_{-l})$, $\forall \tilde{\mathcal{B}}_l$, $\forall \hat{\mathcal{B}}_{-l}$. Let the truthful bid profile be $\mathcal{C} = \{c_l\}_{l \in L}$. Then, a mechanism is *dominant-strategy incentive-compatible* (DSIC) if the bid profile $\mathcal{C}$ is the dominant strategy NE. In other words, every bidder finds it more profitable to bid truthfully, regardless of the other bids.

A. Currently used payment rules

We introduce two prominent payment rules widely used for the reverse auctions under consideration. In the *pay-as-bid mechanism*, the payment rule is $p_l(\mathcal{B}) = b_l(x_l^*(\mathcal{B}))$.[2] It follows that each bidder's utility is $u_l(\mathcal{B}) = b_l(x_l^*(\mathcal{B})) - c_l(x_l^*(\mathcal{B}))$. A rational bidder would overbid to ensure positive utility. Consequently, the central operator calculates the optimal allocation for the inflated bids rather than the true costs. There are many Nash equilibria arising from the pay-as-bid mechanism, none of which are incentive-compatible [9]. On the other hand, the *locational marginal pricing (LMP) mechanism* is adopted in the energy markets where transmission networks are present. We refer to [10] for an exposition on the calculation of these payments from the KKT conditions of the optimization problem (1). Under this mechanism, a bidder can manipulate its LMP payment by inflating its bids. As a strategic manipulation, a bidder may also withhold its maximum supply [29]. Similar to the pay-as-bid mechanism, the bidders need to spend resources and learn how to maximize their utilities. Furthermore, a Nash equilibrium may not exist [30].

We see that these payment rules do not satisfy the properties of incentive-compatibility or efficiency.

B. Vickrey-Clarke-Groves mechanism

The *Vickrey-Clarke-Groves (VCG) mechanism* is characterized with the payment rule $p_l(\mathcal{B}) = b_l(x_l^*(\mathcal{B})) + (h(\mathcal{B}_{-l}) - J(\mathcal{B}))$. The function $h(\mathcal{B}_{-l}) \in \mathbb{R}$ must be chosen carefully to ensure individual rationality. A particular choice is the *Clarke pivot rule* $h(\mathcal{B}_{-l}) = J(\mathcal{B}_{-l})$, where $J(\mathcal{B}_{-l})$ is the optimal value of (1) with $x_l = 0$.[3] Note that this is well-defined under the assumption that there exists a feasible solution when a bidder is removed. However, this is not a restrictive assumption in the presence of many bidders and a second-stage market.

Our first result shows that the VCG mechanism derived in [11]–[13] satisfies all three fundamental properties. This result is a straightforward generalization of the works in [11]–[13] which do not consider continuous values of goods, second stage costs, and general constraints.

Theorem 1: Given the market model (1),

(i) The VCG mechanism is DSIC.
(ii) The VCG mechanism is efficient.
(iii) The VCG mechanism ensures nonnegative payments and individual rationality with the Clarke pivot rule.

The proof is provided online in [1]. In summary, all bidders have incentives to reveal their true costs in a VCG mechanism. DSIC makes it easier to enter the auction, without spending resources in computing optimal strategies. Theorem 1-(ii) shows that the optimal allocation with the true costs results in efficiency. In the remainder, we use the Clarke pivot rule for the VCG mechanism since it ensures individual rationality.

Despite many advantages of the VCG mechanism, this mechanism suffers from collusion and shill bidding. Bidders $K \subseteq L$ are *colluders* if they obtain higher collective utility by changing their bids from $\mathcal{C}_K = \{c_l\}_{l \in K}$ to $\mathcal{B}_K = \{b_l\}_{l \in K}$. Bidder l is a *shill bidder* if there exists a set S and bids $\mathcal{B}_S = \{b_k\}_{k \in S}$ such that bidder l finds participating with bids $\mathcal{B}_S$ more profitable than participating with a single truthful bid $\mathcal{C}_l$. To illustrate these issues, we study an energy market example which is a reverse auction of a single type of power supply. Later, we come back to this example in order to discuss conditions to eliminate collusion and shill-bidding.

Example 1: Suppose the central operator has to procure 800 MW of power supply from bidders 1, 2 and 3 who have the true costs \$100 for 400 MW, \$400 for 400 MW and \$600 for 800 MW, respectively. Under the VCG mechanism, bidders 1 and 2 win and receive $p_1^{\text{VCG}} = 100 + (600 - 500) = \$200$ and $p_2^{\text{VCG}} = 400 + (600 - 500) = \$500$. Suppose bidders 1 and 2 collude and change their bids to \$0 for 400 MW. Then, bidders 1 and 2 receive a payment of \$600 each. In fact, bidders 1 and 2 could represent multiple identities of a single losing bidder (that is, a bidder with the true cost greater than \$600 for 800 MW). Entering the market with two shill bids, this bidder receives a payment of $2 \times \$600$ for 800 MW.

It is troubling that the VCG mechanism can result in large payments through coalitional manipulations. Especially, in this example, there exists a bidder who is willing to offer the same amount of good by receiving less payment.

Next, we define the desirable auction outcomes as the *coalition-proof* outcomes. By coalition-proof, we mean that a group of bidders who lose when bidding their true cost, cannot profit by a joint deviation, and a bidder cannot profit from bidding with multiple identities. Consequently, auctions with coalition-proof outcomes are immune to collusion and shill bidding, and they would avoid large payments through coalitional manipulations. We remark that it is not possible to expect being fully immune to collusion from all sets of bidders. For instance, no mechanism can eliminate the case where all bidders inflate their bid prices simultaneously. Hence, we concentrate our efforts on eliminating collusion from the bidders who lose when bidding truthfully.

III. Ensuring Coalition-Proof VCG Outcomes

In coalitional game theory, the *core* defines the set of utility allocations that cannot be improved upon by forming

[2]For instance, several European balancing market are settled under a pay-as-bid mechanism—see [28] and further references therein.

[3]In the case of an auction of a single item, the VCG mechanism with the Clarke pivot rule is equivalent to the second-price (Vickrey) auction.

coalitions.[4] Here, we show that if the VCG outcome lies in the core, then the VCG mechanism eliminates any incentives for collusion and shill bidding. Keeping this in mind, our goal is to derive sufficient conditions to ensure that the VCG outcome lies in the core.

For any $S \subseteq L$ and $\mathcal{B}_S = \{b_l\}_{l\in S}$, let $J(\mathcal{B}_S)$ be defined as

$$J(\mathcal{B}_S) = \min_{\substack{x,y:\\ x_{-S}=0}} \sum_{l\in S} b_l(x_l) + d(x,y) \text{ s.t. } g(x,y) \leq 0, \quad (2)$$

where the stacked vector $x_{-S} \in \mathbb{R}_+^{t(|L|-|S|)}$ is defined by omitting the subvectors from the set S. This function is nonincreasing, $J(\mathcal{B}_R) \geq J(\mathcal{B}_S)$ for $R \subseteq S$. This holds since $b_l(0) = 0$ for all $l \in L$. Next, we define the core.

Definition 1: For every set of bidders $R \subseteq L$, the core $Core(\mathcal{C}_R) \in \mathbb{R}\times\mathbb{R}_+^{|R|}$ is defined as follows

$$\Big\{u \in \mathbb{R}\times\mathbb{R}_+^{|R|} \,|\, u_0 + \sum_{l\in R} u_l = -J(\mathcal{C}_R), \quad u_0 + \sum_{l\in S} u_l \geq -J(\mathcal{C}_S),\ \forall S \subset R\Big\}. \quad (3)$$

Note that there are $2^{|R|}$ linear constraints that define a utility allocation in the core for the set of bidders R. The core is nonempty in an auction because the utility allocation $u_0 = -J(\mathcal{C}_R)$ and $u_l = 0$ for all $l \in R$ lies in the core. This corresponds to the utility allocation of the pay-as-bid mechanism under truthful bidding $\mathcal{C}_R$.

We highlight the implications of the constraints in (3). Restricting the utility allocation to the nonnegative orthant yields individual rationality. The equality constraint implies that the mechanism is efficient, since the term on the right is maximized by the optimal allocation. We say that a utility allocation is unblocked if there is no set of bidders that could make a deal with the operator from which every member can benefit. This condition is satisfied by the inequality constraints.

The VCG outcome attains the maximal utility in the core for every bidder. Note that under the VCG mechanism each bidder's utility is given by $u_l^{\text{VCG}} = J(\mathcal{C}_{-l}) - J(\mathcal{C})$. Then, for every bidder l, $u_l^{\text{VCG}} = \max\{u_l \,|\, u \in Core(\mathcal{C})\}$, see [9]. In general, this maximal point may not lie in the core. The following example gives visual insight about the core and illustrates the VCG mechanism corresponding to Example 1. In this example, shill bidding and collusion are profitable.

Example 2: We revisit Example 1. Without loss of generality, assume that in case of a tie the central operator prefers bidders 1 and 2 over bidder 3. We can visualize the core outcomes for the bidders 1 and 2 by removing the losing bidder 3, $p_3^{\text{VCG}} = 0$, and the operator. Core outcomes and the VCG payments (p_i^{VCG}) are given in Figure 1.

We are ready to investigate the conditions under which the VCG outcome lies in the core. To this end, we provide three conditions that ensure core VCG outcomes for the model (1). Notice that there are $2^{|L|}$ linear constraints in $Core(\mathcal{C})$ in (3). First, we derive the following equivalent characterization with significantly lower number of constraints.

Lemma 1: Let $W \subseteq L$ be the winners of the reverse auction (1) for the set of bidders L, that is, they are all allocated a positive quantity. Let $u \in \mathbb{R}\times\mathbb{R}_+^{|L|}$ be the corresponding utility allocation. Then, $u \in Core(\mathcal{C})$ if and only if $u_0 = -J(\mathcal{C}) - \sum_{l\in L} u_l$ and

$$\sum_{l\in K} u_l \leq J(\mathcal{C}_{-K}) - J(\mathcal{C}),\ \forall K \subseteq W. \quad (4)$$

[4]Throughout the paper, we use the term utility allocation and the auction outcome interchangeably.

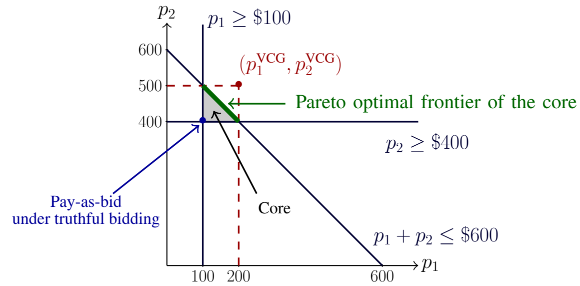

Fig. 1. Core outcomes and the VCG payments under truthful bidding

The proof relies on removing the redundant constraints in the core, and it is provided in [1]. The following proposition is our first sufficient condition for core VCG outcomes.

Proposition 1: The VCG outcome is in the core, $u^{\text{VCG}} \in Core(\mathcal{C})$, if the market in (1) is infeasible whenever any two winners $l_1, l_2 \in W$ are removed from the set of bidders L.

Proof: Notice that if the optimization problem (1) is infeasible then the objective value is $J(\mathcal{B}) = \infty$. Given the property in Proposition 1, inequality constraints (4) in Lemma 1 simplify to constraints on the utilities of single bidders, that is, $u_l^{\text{VCG}} \leq J(\mathcal{C}_{-l}) - J(\mathcal{C})$, for all $l \in W$. This inequality follows directly from the definition of the VCG utility, $u_l^{\text{VCG}} = J(\mathcal{C}_{-l}) - J(\mathcal{C})$, for all $l \in W$. Equality constraint in Lemma 1 is satisfied by definition. ■

The above condition can only be present in some specialized instances. It is not possible to guarantee that this condition will hold in a market. We soon show that supermodularity provides an equivalent condition for core VCG outcomes.

Definition 2: A function $f : 2^L \to \mathbb{R}$ is *supermodular* if $f(S) - f(S_{-l}) \leq f(R) - f(R_{-l})$ for all $S \subseteq R \subseteq L$ and for all $l \in S$. Or, equivalently, for all $S, R \subseteq L$, $f(S\cup R) + f(S \cap R) \geq f(S) + f(R)$ must hold. A function $f : 2^L \to \mathbb{R}$ is *submodular* if $-f$ is supermodular. Furthermore, a function is nondecreasing if $f(S') \leq f(S)$, for all $S' \subseteq S$.

For the remainder, the objective function J in (2) is said to be supermodular if supermodularity holds under any bid profile. Our main result of this section proves that supermodularity is necessary and sufficient for ensuring core VCG outcomes.

Theorem 2: For any bid profile $\mathcal{C}$ and for any set of participating auction bidders $R \subseteq L$, the outcome of the VCG mechanism is in the core, if and only if the objective function J in (2) is supermodular.

The proof is provided online in [1]. The proof significantly simplifies the arguments in a similar result in [17] for submodularity of forward combinatorial auctions. Our proof generalizes this result to reverse auctions with continuous goods and arbitrary objectives and constraints as modeled in (1).

As anticipated, we now prove that core outcomes, hence supermodularity, makes the VCG mechanism coalition-proof.

Theorem 3: For the bidders L, consider a VCG auction modeled by (1). If the objective function J is supermodular, then,

(i) A group of bidders who lose when bidding their true values cannot profit by a joint deviation.
(ii) Shill bidding is unprofitable for any bidder.

The proof is provided online in [1]. Theorem 3 shows that if the operator has a supermodular objective (or equivalently, the VCG outcomes are in the core), then the VCG mechanism is coalition-proof. Given this result, we next investigate sufficient conditions on the bids and the constraint sets in order to ensure supermodularity and thus coalition-proof outcomes. In the following we derive conditions for two classes of markets.

A. Markets for a single type of good

We start by considering simpler auctions where the operator has to procure a fixed amount M of a single type of good. Each bidder l has a true cost $c_l : \mathbb{R}_+ \to \mathbb{R}_+$ that is nondecreasing with $c_l(0) = 0$. These auctions are mainly characterized by single-stage decisions with mutually exclusive bids. This means that a bidder can offer a set of bids, of which only one can be accepted. We first show that such bids fit into our model. Here, bidder l submits truthful bids for n_l discrete amounts as $\{(c_{l,i}, x_{l,i})\}_{i=1}^{n_l}$ where $c_{l,i} \in \mathbb{R}_+$ and the amounts offered by each bidder $x_{l,i} \in \mathbb{R}_+$ must be equally spaced by some increment m which is a divisor of M, that is,

$$x_{l,i} = im, \text{ for some } i \in \mathbb{Z}_+. \tag{5}$$

There is an equivalent representation of the form $c_l(x) \in \mathbb{R}_+$ for $x > 0$ as $c_l(x) = \min_{i=1,\dots,n_l} \{c_{l,i} \,|\, x_{l,i} \geq x\}$, and $c_l(0) = 0$. This form equivalently represents that all amounts up to the size of the winning bid are available to the operator. Moreover, prices of this form are piecewise constant and continuous from the left. Consequently, we consider auctions cleared by

$$J(\mathcal{C}_S) = \min_{x \in \mathbb{R}_+^{|S|}} \sum_{l \in S} c_l(x_l) \text{ s.t. } \sum_{l \in S} x_l \geq M, \tag{6}$$

for $S \subseteq L$. Note, we can equivalently assume that x_l, above, takes values in $\{x_{l,i} \,|\, i \in \mathbb{Z}_+\} \subseteq \mathbb{R}_+$ and doing so, we let $x^* = \{x_l^*\}_{l \in S}$ be the optimal values in these sets.

The model (6) is within the auction model (1). We can now derive conditions on bidders' costs to ensure supermodularity of J. Thus, we derive conditions under which the VCG outcome would lie in the core.

Theorem 4: Given (5), if the true costs are marginally increasing, namely, $x_{l,b} - x_{l,a} = x_{l,d} - x_{l,c}$ implies that $c_{l,b} - c_{l,a} < c_{l,d} - c_{l,c}$ for each $l \in L$ and for each $0 \leq x_{l,a} < x_{l,c} < x_{l,d}$, then the function J is supermodular.

The proof relies on an important lemma we prove showing that the allocations of every bidder is nondecreasing when a bidder is removed from the auction (6). The proof is provided online in [1]. As a corollary, marginally increasing costs imply coalition-proof VCG outcomes for (6) and thus eliminate collusion and shill bidding. The analogue of Theorem 4 also holds for continuous bids and for strictly convex bid curves. This could also be seen as the limiting case, by taking the limit where the incerement m goes to 0.

We illustrate Theorem 4 by revisiting Example 1.

Example 3: Revisiting Example 1 and Example 2, we observe that the bid from bidder 3 does not satisfy the conditions in Theorem 4, because the bid price for 400 MW is not submitted. Assume instead bidder 3 provides the mutually exclusive bids of \$300 for 400 MW and \$600 for 800 MW. Suppose bidders 1 and 2 change their bids to \$0 for 400 MW. Then, bidders 1 and 2 are the winners and each receives a payment of \$300. If they were shill bids of a single bidder, sum of their payments would decrease from \$700 to \$600.

B. Markets for different types of goods

We now consider auctions where the operator is procuring different types of goods. Each bidder has a true cost function $c : \mathbb{R}_+^t \to \mathbb{R}_+$ that is nondecreasing with $c(0) = 0$. We assume that this cost has an additive from, $c(x) = \sum_{\tau=1}^{t} c_\tau(x_\tau)$. Typically, in these markets, bids are submitted separately for each type with an upper-bound on the amount, $\bar{X}_\tau \in \mathbb{R}_+$ [7]. The operator treats these bids as bids from different identities, and then distributes the payments. In this case, the set L is the extended set of bidders such that the bid profile $\mathcal{C}$ is given by the bids of the form $c_l : \mathbb{R}_+ \to \mathbb{R}_+$, $\bar{X}_l \in \mathbb{R}_+$, for all $l \in L$.

Let $[t] = \{1, \dots, t\}$. Define the set $\{A^\tau\}_{\tau=1}^t$ to be a partition of the set L where each set $A^\tau \subseteq L$ is the set of bidders submitting a bid for goods of type τ. Specifically, we consider auctions cleared by the optimization problem:

$$\begin{aligned} J(\mathcal{C}_S) = &\min_{\substack{x \in \mathbb{R}_+^{|L|}, \\ x_{-S}=0}} \sum_{l \in S} c_l(x_l) \\ &\text{s.t.} \sum_{l \in A^T} x_l \geq M(T),\, \forall T \subseteq [t],\; x_l \leq \bar{X}_l,\, \forall l \in L, \end{aligned} \tag{7}$$

where $A^T = \bigcup_{\tau \in T} A^\tau$. Here, the function $M : 2^{[t]} \to \mathbb{R}_+$ defines the amount the operator wants to procure from possible combinations of different types of goods. We assume that $M(\emptyset) = 0$ (normalized).

In the following we see that if c_l and M satisfy convexity and supermodularity respectively, then J is supermodular.

Theorem 5: The objective function J given by (7) is supermodular when c_l is increasing and convex for all $l \in L$, M is supermodular and nondecreasing.

The proof is provided online in [1], and it builds upon recent advances in polymatroid optimization [31], [32].[5] As a corollary, Theorem 5 imply core VCG outcomes. The conclusions of Theorem 3 are further corollaries of this result. In [1], we provide an example illustrating that the VCG outcome may not lie in the core for convex bids and a polyhedral constraint set that is not of the form of a polymatroid.

IV. Core-Selecting Mechanisms

In this section, we investigate further payment rules that are coalition-proof without any restrictions on the bidders and the constraints. In particular, we show that any mechanism that selects its utilities from the core is coalition-proof. Under such mechanisms, losing bidders cannot profit from collusion, and a shill bidder cannot profit more than its truthful VCG utility.

Notice that the core in (1) can be defined with respect to the submitted bids. We refer to it as the revealed core $Core(\mathcal{B})$. We then define utilities with respect to the submitted bids. The revealed utility of bidder l is defined by $\bar{u}_l(\mathcal{B}) = p_l(\mathcal{B}) -$

[5] Polymatroid is a polytope associated with a submodular function. The first set of constraints in (7) is a contra-polymatroid.

$b_l(x_l^*(\mathcal{B}))$, and the revealed utility of the operator is defined by $\bar{u}_0(\mathcal{B}) = u_0(\mathcal{B}) = -\sum_{l\in L} p_l(\mathcal{B}) - d(x^*(\mathcal{B}), y^*(\mathcal{B}))$.

A mechanism is said to be *core-selecting* if it is selecting its payments such that the revealed utilities lie in the revealed core. Then, the payment rule is given by $p_l(\mathcal{B}) = b_l(x_l^*(\mathcal{B})) + \bar{u}_l(\mathcal{B})$, where $\bar{u} \in Core(\mathcal{B})$. We highlight that the revealed core can be defined without the true costs. As a remark, the pay-as-bid mechanism is a core-selecting mechanism where $\bar{u}_l = 0$ for all $l \in L$. Our main result of this section shows that these mechanisms give rise to coalition-proof outcomes.

Theorem 6: Consider a core-selecting auction for model (1).

(i) A group of bidders who lose when bidding their true values cannot profit by a joint deviation.

(ii) Shill bidding is unprofitable for all bidders with respect to the VCG utilities.

First, we need the following lemma.

Lemma 2: If $\bar{u} \in Core(\mathcal{B})$, then, for all $K \subseteq L$ we have $\sum_{l\in K} \bar{u}_l(\mathcal{B}) \le J(\mathcal{B}_{-K}) - J(\mathcal{B})$.

Proof: Since $\bar{u}_0 = -J(\mathcal{B}) - \sum_{l\in L} \bar{u}_l$, we reorganize the inequality constraint in (1) as $-J(\mathcal{B}) - \sum_{l\in L\setminus S} \bar{u}_l \ge -J(\mathcal{B}_S), \forall S \subseteq L$. Setting $K = L \setminus S$ yields the statement. ■

Proof of Theorem 6: (i) Let K be a set of colluders who would lose the auction when bidding their true values $\mathcal{C}_l = c_l$, when bidding $\mathcal{B}_l = b_l$ they become winners, that is, they are all allocated a positive quantity. We define $\hat{\mathcal{C}} = (C_K, \mathcal{B}_{-K})$ and $\mathcal{B} = (\mathcal{B}_K, \mathcal{B}_{-K})$ where $\mathcal{B}_{-K} = \{b_l\}_{l\in L\setminus K}$ denotes the bidding profile of the remaining bidders. As a remark, the profile $\mathcal{B}_{-K}$ is not necessarily a truthful or a strategic profile. The total utility that colluders receive, $\sum_{l\in K} u_l(\mathcal{B})$, is

$$\begin{aligned}
&= \sum_{l\in K} \bar{u}_l(\mathcal{B}) + b_l(x_l^*(\mathcal{B})) - c_l(x_l^*(\mathcal{B})) \\
&\le J(\mathcal{B}_{-K}) - J(\mathcal{B}) + \sum_{l\in K} b_l(x_l^*(\mathcal{B})) - c_l(x_l^*(\mathcal{B})) \\
&= J(\hat{\mathcal{C}}) - \Big[\sum_{l\in L} b_l(x_l^*(\mathcal{B})) + d(x^*(\mathcal{B}), y^*(\mathcal{B})) \\
&\qquad - \sum_{l\in K} b_l(x_l^*(\mathcal{B})) + \sum_{l\in K} c_l(x_l^*(\mathcal{B}))\Big] \\
&= J(\hat{\mathcal{C}}) - \Big[\sum_{l\in K} c_l(x_l^*(\mathcal{B})) + \sum_{l\in L\setminus K} b_l(x_l^*(\mathcal{B})) + d(x^*(\mathcal{B}), y^*(\mathcal{B}))\Big] \\
&\le 0 = \sum_{l\in K} u_l(\hat{\mathcal{C}}).
\end{aligned}$$

The first equality follows from the core-selecting payment rule, where $\bar{u}(\mathcal{B})$ is the revealed utility allocation. The first inequality follows from Lemma 2. The second equality comes from the fact that K was a group of losers, so $J(\mathcal{B}_{-K}) = J(\hat{\mathcal{C}})$. After substituting these terms, we see that the term in brackets is the cost $\bar{J}$ of $\hat{\mathcal{C}}$ but evaluated at a feasible suboptimal allocation $(x^*(\mathcal{B}), y^*(\mathcal{B}))$. Then, $\sum_{l\in K} u_l(\mathcal{B})$ is upper bounded by 0. As a result, there is at least one colluder not facing any benefit in collusion. Moreover, they cannot increase their collective utility by a joint deviation. Hence, collusion is not profitable for the losing bidders.

(ii) Define $\mathcal{C} = (\mathcal{C}_{-l}, \mathcal{C}_l)$, where $\mathcal{C}_{-l}$ denotes the bidding profile of the remaining bidders. The profile $\mathcal{C}_{-l}$ is not necessarily truthful. Shill bids of bidder l are given by $\mathcal{B}_S = \{b_k\}_{k\in S}$. We define a merged bid $\tilde{\mathcal{B}}_l$ as $\tilde{b}_l(x_l) = \min_{x_k\in\mathbb{R}^t_+, \forall k} \sum_{k\in S} b_k(x_k)$ s.t. $\sum_{k\in S} x_k = x_l$. We then define $\tilde{\mathcal{B}} = (\mathcal{C}_{-l}, \tilde{\mathcal{B}}_l)$. The total utility obtained from shill bidding under $\mathcal{B} = (\mathcal{C}_{-l}, \mathcal{B}_S)$, $\sum_{k\in S} u_k(\mathcal{B})$, is given by

$$\begin{aligned}
&= \sum_{k\in S} [\bar{u}_k(\mathcal{B}) + b_k(x_k^*(\mathcal{B}))] - c_l(\sum_{k\in S} x_k^*(\mathcal{B})) \\
&\le [J(\mathcal{B}_{-S}) - J(\mathcal{B})] + \sum_{k\in S} b_k(x_k^*(\mathcal{B})) - c_l(\sum_{k\in S} x_k^*(\mathcal{B})) \\
&= [J(\mathcal{C}_{-l}) - J(\tilde{\mathcal{B}})] + \tilde{b}_l(\sum_{k\in S} x_k^*(\mathcal{B})) - c_l(\sum_{k\in S} x_k^*(\mathcal{B})) \\
&= u_l^{\text{VCG}}(\tilde{\mathcal{B}}) \le u_l^{\text{VCG}}(\mathcal{C})
\end{aligned}$$

The first inequality follows from the core-selecting payment rule and Lemma 2. The second equality holds since we have $J(\tilde{\mathcal{B}}) = J(\mathcal{B})$. This follows from the definition of the merged bid and the following implication. Since the goods of the same type are fungible for the central operator, the functions g and d in fact depend on $\sum_{l\in L} x_l$. The third equality follows from the definition of the VCG utility. The second inequality is the DSIC of the VCG mechanism. Therefore, the total utility that l receives from shill bidding is upper bounded by the utility that l would receive by bidding truthfully as a single bidder in a VCG auction. Making use of shills, hence, is not profitable with respect to the VCG utilities. ■

This proof can be used as an alternative approach to prove Theorem 3. We remark that the proof method differs from Theorem 3 since this proof does not require supermodularity.

The VCG mechanism is known to be the only DSIC efficient mechanism [33]; however, this mechanism can be subject to collusion and shill bidding [34]. We showed that this occurs since the VCG outcome may not lie in the core and thus DSIC is relaxed under the core-selecting mechanisms. To alleviate this issue, we investigate core-selecting mechanisms that can approximate the DSIC property.

A revealed utility allocation $\bar{u} \in Core(\mathcal{B})$ is *bidder-Pareto-optimal* if there is no $\tilde{u} \in Core(\mathcal{B})$ such that $\tilde{u}_l \ge \bar{u}_l$ for each $l \in L$, and $\tilde{u}_l > \bar{u}_l$ for some bidder $l \in L$. In Figure 1, the set of bidder-Pareto-optimal points correspond to the line segment of the core with the maximum total payment. Nash equilibria of the pay-as-bid mechanism for the model (1) are also given by these points in the core, see [9, Theorem 1].

It is shown that a core-selecting mechanism minimizes the tendencies to deviate from truthful bids, among all other core-selecting mechanisms, if and only if the mechanism chooses a bidder-Pareto-optimal revealed utility allocation [35, Theorem 4]. This follows from the fact that the maximum gain by a deviation from truthful bidding is given by the difference between the VCG payment and the core-selecting one [36]. We call such mechanisms *bidder optimal core-selecting (BOCS) mechanisms*. From Figure 1, we observe that there are many utility allocations satisfying this property.[6] To obtain a unique outcome, we minimize the Euclidean distance to the VCG utilities [37]. In the numerics, we discuss a computationally efficient method for computing the BOCS mechanism.

[6]We remark that if the VCG outcome lies in the core, then it is the unique bidder-Pareto-optimal point, see [9]. As a result, the VCG and BOCS mechanisms are equivalent under the supermodularity condition.

TABLE I
TOTAL PAYMENTS OF THE IEEE 14-BUS TEST SYSTEM

Mechanism	14-bus	14-bus with line limits
Pay-as-bid	\$7642.6	\$9715.2
LMP	\$10105.1	\$10361.0
BOCS	\$10513.4	\$11220.1
VCG	\$10513.4	\$11432.1

TABLE II
TOTAL PAYMENTS OF THE TWO STAGE AUCTION

Type	SR	PTR	NTR
Procured MWs	409 MW	100 MW	114 MW
Total Pay-as-bid payment		2.293 million CHF	
Total BOCS payment		2.437 million CHF	
Total VCG payment		2.529 million CHF	

V. NUMERICAL RESULTS

Our goal is to compare the effectiveness of the proposed mechanisms and methods based on electricity market data. First, we consider the IEEE 14-bus test system. We compare the total payment under the LMP, the BOCS, and the VCG mechanisms. We verify that the VCG outcome is in the core. Then, with small modifications to the line limits, we show that the VCG outcomes do not necessarily lie in the core for DC power flow models. Finally, we study the two-stage Swiss reserve procurement auction [7]. In order to illustrate the applicability of the BOCS mechanism, we provide the wall-clock time for the computations performed. We solve all optimization problems with GUROBI [38], called through MATLAB via YALMIP [39], on a computer equipped with a 32 GB RAM and a 4.0 GHz quad-core Intel i7 processor.

Before discussing the numerical results, we explain the computation method for the BOCS mechanism. Finding a Pareto-optimal core outcome is computationally difficult for auctions involving many bidders because one needs to solve the auction problem (1) for $2^{|L|}$ different subsets to define the core constraints. Invoking Lemma 1, we can reduce the number of constraints to $2^{|W|}$, which grows exponentially only in the number of winners. Unfortunately, this approach may also not be computationally feasible since there can be many winners. The state of the art approach for computing a core outcome is to use the constraint generation algorithm [36]. This algorithm was initially used in the 1950s in order to solve linear programs that have too many constraints [40]. At every iteration, the method generates the constraint with the largest violation for a provisional solution. In practice, it requires the generation of only several constraints to converge to the optimal solution. We call this iterative approach *the core constraint generation (CCG) algorithm*. Formulation of the CCG algorithm for the BOCS outcome of the reverse auction (1), and analysis of the BOCS mechanism is relegated to the extended online version [1, §4].

A. IEEE 14-bus test system

We consider the IEEE 14-bus test system [41] with polytopic DC power flow constraints. We assume all bidders are truthful and cost curves are convex quadratics, provided in [41]. In practice, truthfulness can only hold in the VCG mechanism since it is DSIC. The corresponding total payments of the mechanisms are shown in the first column of Table I.

We observe that the VCG outcomes is in the core. Since 14-bus system do not have any line limits, it has the form of (6). Invoking Theorem 4, we conclude that supermodularity holds. This test system is a specialized instance and it does not necessarily conclude that the VCG mechanism is coalition-proof for DC power flow models. We examine this shortcoming of the VCG mechanism in our next simulation.

Next, we consider the IEEE 14-bus test system, with a line limit on lines exiting node 1, connecting node 1 to nodes 2 and 5. We set this line limit to be 10 MW. We again assume all bidders are truthful. The corresponding total payments of the mechanisms are shown in the second column of Table I.

We observe that the VCG outcome does not lie in the core. Since there are line limits, the constraints of the problem does not have the form of (6). Hence, shill bidding and collusion can be profitable. Moreover, we observe that the BOCS mechanism yields a larger total payment than the LMP mechanism. We underline that this does not necessarily hold for the payment of a single bidder, because the BOCS payment depends on the Pareto-optimal point chosen. For some bidders, the BOCS payment can be equal to the pay-as-bid payment (which is smaller than the LMP payment), whereas for some others it can be equal to the VCG payment (which is larger than the LMP payment [14]). For the 14-bus example with the line limits, computation times for the VCG mechanism and the CCG algorithm are 3.6 and 8.2 seconds respectively. After the VCG mechanism, the CCG algorithm converges in 4 iterations.

B. Swiss reserve procurement auctions

The following simulations are based on the bids placed in the 46th weekly Swiss reserve procurement auction of 2014 [7]. The reverse auction involves 21 power plants bidding for secondary reserves, 25 for positive tertiary and 21 for negative tertiary reserves. The objective includes a second stage cost corresponding to the uncertain daily auctions. There are complex constraints arising from nonlinear cumulative distribution functions. The constraints imply that the deficit of reserves cannot occur with a probability higher than 0.2%. Moreover, the constraints include coupling between first and second stage decision variables corresponding to the weekly and daily reserve auctions. The corresponding total payments and procured MWs of the pay-as-bid mechanism, the BOCS mechanism, and the VCG mechanism are shown in Table II.

While the VCG payment rule yields the highest total payment, the BOCS yields the second highest. Note that the bids from the power plants are not marginally nondecreasing for all quantities and the constraints are nonstandard. Hence, the supermodularity condition does not hold and we observe that the VCG outcome does not lie in the core. Consequently, shill bidding and collusion are profitable for bidders under the VCG mechanism. For this electricity market, directly computing the BOCS outcome is not computationally feasible, because there are 28 winners and the optimal cost calculation takes 8 seconds. This approach would require 68 years. Computation times for the VCG mechanism and the CCG algorithm are 580.6 and 659.2 seconds respectively. After the VCG mechanism, the CCG algorithm converges in 4 iterations.

VI. Conclusion

We introduced a constrained optimization problem to model reverse auctions that may involve continuous values of different types of goods, general nonconvex constraints, and second stage costs. We discussed the game theoretic analysis of these auctions under different payment rules. We first showed that the VCG mechanism results in a DSIC Nash equilibrium. Through examples, we then showed that this mechanism suffers from collusion and shill bidding. Motivated by this problem, we derived three different conditions under which collusion and shill bidding are not profitable, and hence the VCG mechanism is coalition-proof. Since these conditions are restrictive and they may not capture the problems under consideration, we investigated the closest we can achieve to the property of DSIC under a coalition-proof mechanism. By removing incentives for manipulations, we expect the bidding process to be simplified, and this can help promote participation in the market. Finally, we verified our results in several case studies based on electricity market data.

As a future work, we will explore learning Nash equilibria in such markets to model the behavior of the bidders in a repeated setting. As an extension, we will consider budget balance in double-sided auctions. Also, we will address the issue of pricing intermittent generation.

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